A New Correlation for the Prediction of the Frequency Distribution of Daily Solar Radiation

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ABSTRACT

The frequency of occurrence of daily radiation levels, that is, the relative numbers of good and bad days, is of interest in two contexts. First, information on the frequency distribution is important in weather generation when only monthly average solar radiation is known. If the distribution of radiation is known, it is possible to synthesize statistically meaningful values for use in energy system simulations. Second, the distribution of radiation affects the utilizability of solar radiation (i.e. the fraction above a specified level). Utilizability is an important radiation statistic useful for predicting the long-term average performance of solar energy systems, including active, passive and photovoltaic applications. Refinements and generalizations of utilizability methods have raised the need for improvement of the underlying meteorological correlations. In order to calculate the utilizability, the frequency distribution of daily solar radiation needs to be determined. This paper proposes an improved method to determine the frequency distribution of daily solar radiation by using the daily maximum clearness index, $K_{t,max}$.

1. INTRODUCTION

The daily clearness index, K_t, is defined as the ratio of a particular day's solar radiation on a horizontal surface to the extraterrestrial radiation for that day [1,2]. For any day of the year, analytical expressions are available in the literature [e.g.2] for the extraterrestrial solar radiation incident on a horizontal surface. However, the solar radiation reaching the ground can not be evaluated in such a simple manner.

Liu and Jordan [3] were the first to generalize the distribution of daily solar radiation. They developed the set of curves shown in Figure 1 of K_t versus f which are functions of $\overline{K}_t$, the monthly clearness index (the ratio of the monthly average daily solar radiation on a

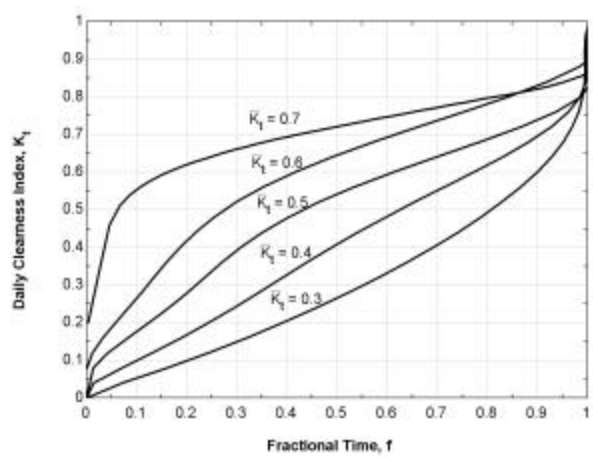

Fig. 1: The generalized monthly average K_t curves [3]

horizontal surface to the monthly average daily extraterrestrial radiation). Thus, if a location has a $\overline{K}_t = 0.7$, 60 % of the days will have a value of K_t that is less than 0.75 or in other words 60 % of the days will have a solar radiation that is less than 75 % of the extraterrestrial radiation.

Bendt et al. [4] proposed a model that can analytically represent the Liu and Jordan curves. Their model was formulated in terms of the maximum daily

clearness index, $K_{t,max}$, the minimum clearness index, $K_{t,min}$, and the monthly average clearness index, $\overline{K}_t$. They have used the probability density function as well as the statistical arguments to produce their model. Hollands and Huget [5] used classical probability theory to produce their frequency distribution model. The core of their theory is the probability density function that is a function that describes the distribution of probability for a continuous random variable. This function has the major property that the total area under the function is unity [6]. Hollands and Huget designed their model to fit the Liu and Jordan curves. To do so they found that the value of $K_{t,max}$, should be 0.864. Examining the measured hourly data for different locations in the United States [7] revealed that a recommended value of 0.864 overpredicts the actual data as can be seen from Table1. Table 1 lists the measured maximum value of the daily clearness index for 5 cities in 5 states of different latitudes, weather conditions and elevation at different times in the year.

Table 1: Values of measured $K_{t,max}$ for different US locations.

City	***January***	***March***	***May***
Albuquerque, NM	0.75	0.78	0.79
Madison, WI	0.7	0.76	0.74
Seattle, WA	0.55	0.71	0.7
Miami, FL	0.72	0.73	0.73
Honolulu, HI	0.68	0.7	0.71
City	***July***	***Sept.***	***Nov.***
Albuquerque, NM	0.76	0.76	0.75
Madison, WI	0.71	0.71	0.66
Seattle, WA	0.7	0.68	0.6
Miami, FL	0.68	0.69	0.68
Honolulu, HI	0.69	0.69	0.71

This motivates the authors to generate a correlation for $K_{t,max}$. The proposed correlation is to be used with Hollands and Huget model to improve its capability of generating the frequency distribution. It will be shown that this suggested model [Hollands and Huget with the present correlation for $K_{t,max}$] will generate a frequency distribution closer to the actual measured data than the Bendt et al. and Herzog model does.

2. EXISTING MODELS

2.1. Bendt et al. Model

Bendt et al. presented their model for the prediction of the frequency distribution based on measured values of daily total hemispherical horizontal irradiation of 90 stations in the contiguous US with approximately 20 years of observations. They used the probability density function assuming random radiation sequences, Taylor expansion and the statistical arguments to determine the different coefficients needed in their model. The complete model is given by Equations 1 and 2.

$$\overline{K_t} = \frac{\left(K_{t,\min} - \frac{1}{g}\right)\exp\left(g\ K_{t,\min}\right)}{\exp\left(g\ K_{t,\min}\right) - \exp\left(g\ K_{t,\max}\right)} - \frac{\left(K_{t,\max} - \frac{1}{g}\right)\exp\left(g\ K_{t,\max}\right)}{\exp\left(g\ K_{t,\min}\right) - \exp\left(g\ K_{t,\max}\right)} \tag{1}$$

$$f = \frac{\exp\left(g\ K_{t,\min}\right) - \exp\left(g\ K_t\right)}{\exp\left(g\ K_{t,\min}\right) - \exp\left(g\ K_{t,\max}\right)} \tag{2}$$

Bendt et al. proposed a value for $K_{t,min}$ of 0.05 and $K_{t,max}$ has been chosen to agree with the observational values of $\overline{K}_t$. In other words, the value of $K_{t,max}$ will be one of the inputs to the model but its value is not fixed but it depends on the value of $\overline{K}_t$.

Therefore, in order to use Bendt et al. model $\overline{K}_t$ should be known as well as K_{tmax}. In order to use the model Equation 1 solved for the value of γ and used in Equation 2 to generate the required f-K_t distribution. Different values of $\overline{K}_t$ and $K_{t,max}$ can be used and the corresponding distribution can be generated. Herzog [8] suggests an explicit equation for the calculation of the value of γ as:

$$g = -1.498 + \frac{1.184\,x - 27.182\exp\left(-1.5\,x\right)}{K_{t,\max} - K_{t,\min}} \tag{3}$$

where

$$x = \frac{K_{t,\max} - K_{t,\min}}{K_{t,\max} - \overline{K_t}} \tag{4}$$

Therefore, if Herzog model for γ is used with Bendt et al. to determine the frequency distribution the only input to the combined model will be $\overline{K}_t$ only while $K_{t,max}$ will be of the results of solving the set of equations, from equation 1 to equation 4, simultaneously. The proposed correlation in the present work for $K_{t,max}$ will not be used with Bendt et al. model because, as will be shown later,

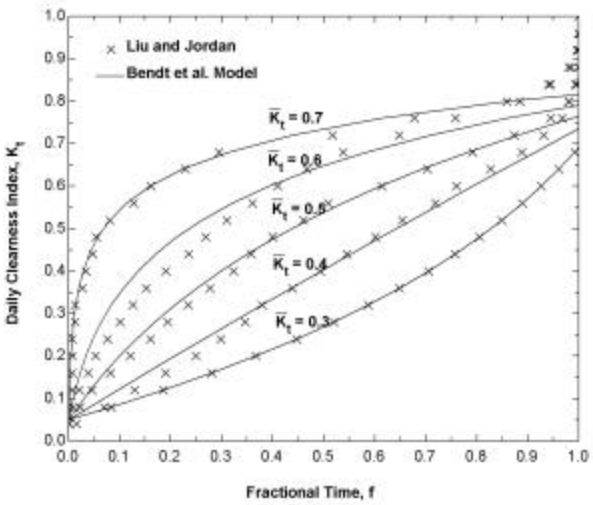

Figure 2: Comparison between combined the Bendt et al. and Herzog model and the Liu and Jordan curves.

the proposed correlation requires $\overline{K}_t$ as one of its inputs. Figure 2 shows the frequency distribution generated using Bendt et al. and Herzog model compared with the initial Liu and Jordan curves. From the figure it can be seen that this model represents the Liu and Jordan curves except at the high values of K_t

2.2. Hollands and Huget model

Hollands and Huget tested the standard forms of the probability density function, discussed earlier, against that obtained by numerically differentiating the Liu and Jordan curves. They found that the probability density function that gives the best fit for the Liu and Jordan curves is in the form of:

$$P(K_t, \overline{K}_t) = C \frac{K_{t,\max} - K_t}{K_{t,\max}} e^{l K_t} \qquad (5)$$

Then they used the properties of the probability density function to develop their model. They assumed that the domain for K_t extends from zero to $K_{t,max}$, therefore their model does not involve $K_{t,min}$.

$$\overline{K}_t = \frac{\left(\frac{2}{l} + K_{t,\max}\right)\left(1 - e^{l K_{t,\max}}\right) + 2 K_{t,\max} e^{l K_{t,\max}}}{e^{l K_{t,\max}} - 1 - l K_{t,\max}} \qquad (6)$$

and

$$l = \frac{2\ \Gamma\ -17.519\ \exp(-1.3118\ \Gamma)}{K_{t,\max}} - \frac{1062\ \exp(-5.0426\ \Gamma)}{K_{t,\max}} \qquad (7)$$

where

$$\Gamma = \frac{K_{t,\max}}{K_{t,\max} - \overline{K}_t} \qquad (8)$$

and

$$f = \frac{C}{K_{t,\max}\, l\, g_1}\left[e^{l K_t}\left(1 - g_1 K_t\right) - 1\right] \qquad (9)$$

where

$$g_1 = \frac{l}{1 + l K_{t,\max}} \qquad (10)$$

and

$$C = \frac{l^2 K_{t,\max}}{e^{l K_{t,\max}} - 1 - l K_{t,\max}} \qquad (11)$$

Figure 3 compares the Liu and Jordan curves with the Hollands and Huget model. It can be seen form the figure that Holland and Huget model presents the Liu and Jordan curves adequately. However, in order to do so, they found that the value of K_{tmax} should be fixed at a constant value of 0.864. This value, as discussed before and as can be seen from Table 1, over-predicts the observed values of $K_{t,max}$, in different locations in US at different times in the year.

It should be mentioned that if $\overline{K}_t$ is known then Bendt et al. and Herzog model could be used directly with no need for the suggested correlation. However, the present correlation can be used as a replacement for the Herzog model. Replacement of Herzog model with the present correlation will help improving the ability of the Bendt et al. model prediction of the f-K_t distribution for the actual data. But using the present correlation with Hollands and Huget model results in f-K_t distributions closer to the actual data than using it with Bendt et al. as will be shown later.

PROPOSED MODEL FOR $K_{t,max}$

A correlation for the maximum daily clearness index is proposed in the present paper to be used with either Hollands and Huget model or Bendt et al. model. The suggested correlation involves the different factors that might affect the value of the maximum clearness

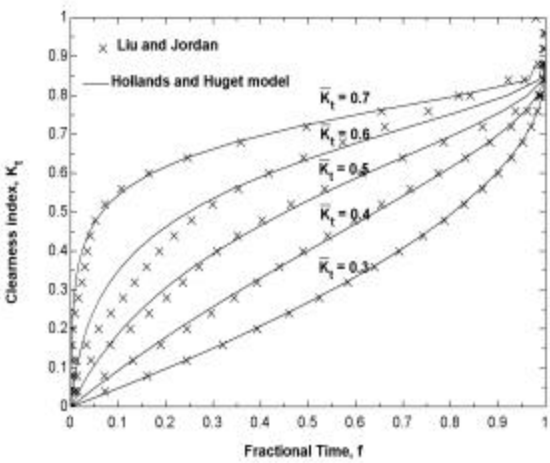

Figure 3: Comparison between the Hollands and Huget model and the curves of Liu and Jordan.

index. First, the monthly average clearness index will affect the value of $K_{t,max}$. That can be seen from Liu and Jordan curves, for different $\overline{K}_t$ values, the value of $K_{t,max}$ changes. Second, the time of the year will affect the value of $K_{t,max}$. For example, during the winter time the maximum daily clearness index is expected to be less in the summer time. This effect is characterized by the declination angle, δ, in degrees. The average value of the declination angle for the different months of the year is given in Table 2. Third, the location of the place may affect the value of the maximum clearness index. It is expected that the tropical areas have higher $K_{t,max}$ than polar areas. The effect of the location is characterized by the latitude angle, $\boldsymbol{f}$ in degrees.

Finally the effect of the elevation above the sea level, Z in meters, may affect the value of $K_{t,max}$ because elevated locations tend to avoid the effect of the atmospheric dust and moisture.

These different factors are used to fit the data from 20 years of hourly measurements for Madison, WI, 20 years for Miami, FL, 10 years for Albuquerque, NM, and one year for Seattle, WA. The resulting correlation has the following form:

$$K_{t\max} = 0.51585 + 0.34847\,\overline{K}_t + 2.3028\times10^{-4}\boldsymbol{d} + 3.4108\times10^{-4}\,\boldsymbol{f} + 9.5709\times10^{-6}\,Z \quad (12)$$

Table 2: Recommended average declination (degrees) for months [2]

Month	***Jan***	***Feb***	***Mar***	***Apr***	***May***	***June***
d	-20.9	-13.0	-2.4	9.4	18.8	23.1
Month	***July***	***Aug***	***Sept***	***Oct***	***Nov***	***Dec***
d	21.2	13.5	2.2	-9.6	-18.9	-23.0

4. RESULTS

The proposed correlation was used with Hollands and Huget model as well as with Bendt et al. model to predict the frequency distribution for different locations in the United States.

Figure 4 shows the fraction, *f*, of the days that

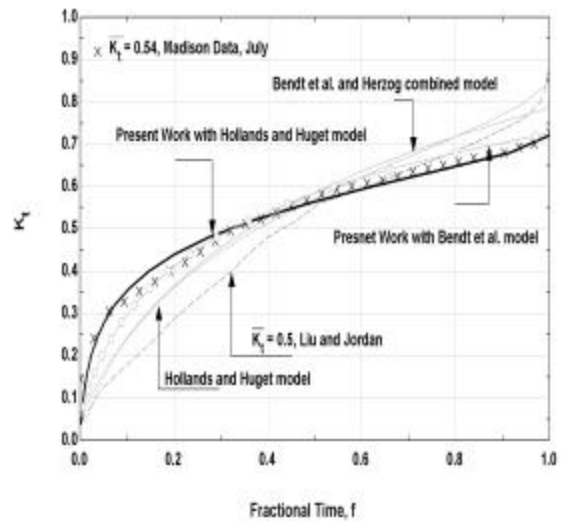

Fig. 4: Comparison of present work to other models and 10 years of measured data in July for Madison, WI.

have daily clearness indices less than K_t for Madison, WI in July. The data used to generate this frequency distribution curve consists of 310 points (10 years) although only 31 points are plotted. The figure shows the predicted frequency distribution using the combined Bendt et al. and Herzog model, the Hollands and Huget model, the Liu and Jordan data (for $\overline{K}_t = 0.5$). Also sown are both the Hollands and Huget model and the Bendt et al. model but with $K_{t,max}$ obtained from Equation 12. It can be seen from the figure that Hollands and Huget model is better when used with the suggested correlation for $K_{t,max}$. It can also be seen that the Bendt et al. model, when used with the suggested correlation, predicts a value for $K_{t,max}$ at frequency greater than one which is impossible. That is why the curve is not complete in Figure 4. Therefore, in presenting the rest of

the results, only the Hollands and Huget model will be used.

Figure 5 shows the frequency distribution for Albuquerque, NM in July. Albuquerque has a much sunnier climate than Madison as shown by the higher value of monthly average clearness index. Moreover, Albuquerque has a higher elevation than Madison ($Z_{Albq} = 1619\ m$, $Z_{Mad} = 262\ m$).

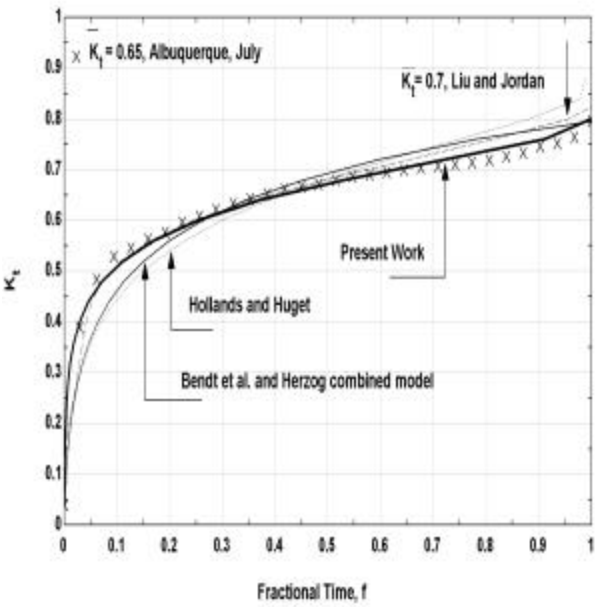

Fig. 5: Comparison of the present work to other models and 10 years of measured data in July for Albuquerque, NM

Again, it can be seen from figure 5 that Hollands and Huget model, when used with the proposed correlation for the $K_{t,max}$, represents the frequency distribution for the measured data more accurately than the other models. Figure 6 is a repeat of Figures 4 but for different time of the year to check the model against the variation in the declination angle.

Figure 7 shows a comparison of the Hollands and Huget model when used alone to predict the frequency distribution and when used with the proposed model for $K_{t,max}$ to predict the frequency distribution in Honolulu, HI in April.

From that, it can be seen that the use of Equation 12 significantly improves the ability of the Hollands and Huget model to represent the frequency distribution of daily radiation.

5. CONCLUSIONS

It can be concluded from the previous results that Liu and Jordan curves do not represent the actual

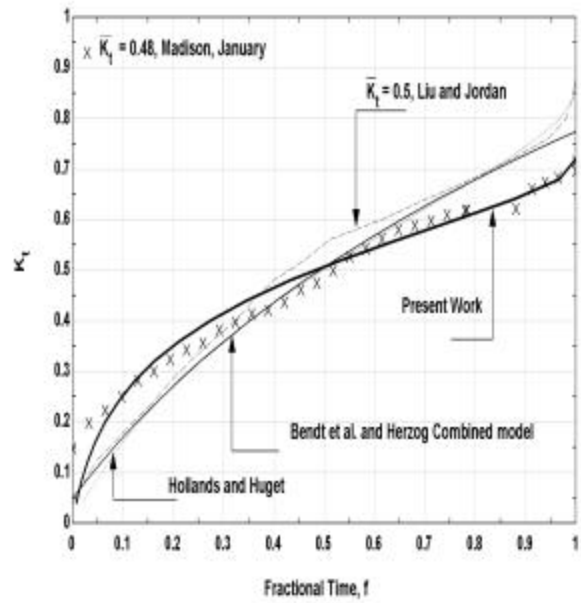

Fig. 6: Comparison of present work to other models and 10 years of measured data in January for Madison, WI.

data. Moreover, they usually overpredict the maximum value of the daily clearness index. Since Bendt et al. and Hollands and Huget presented their analytical models to fit the Liu and Jordan curves, then these models need to be modified to represent the actual data. Modifying their

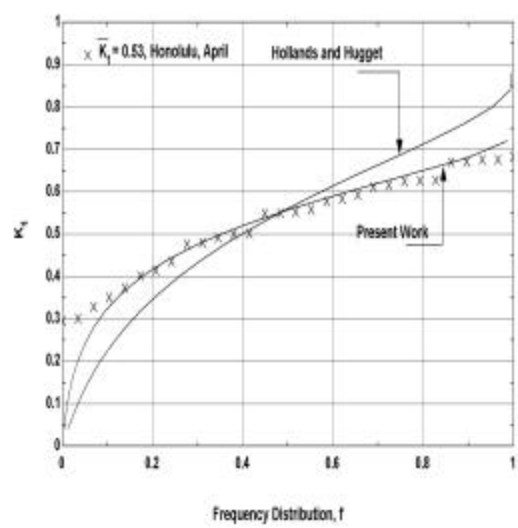

Fig. 7: Comparison of present work to Hollands and Huget model and 1 years of measured data in April for Honolulu, HI.

maximum and minimum daily clearness index estimation technique can modify these models. The

present work suggests a model to estimate the value of $K_{t,max}$. It can be concluded from the shown results that the proposed correlation for $K_{t,max}$ when used with the Hollands and Huget presents an improved model for the prediction of the frequency distribution for different locations in the United States.

6. REFRENCES

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