EQUATION SOLVING IN SOLAR ENERGY ENGINEERING COURSES

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Abstract

With the equation solving program EES (Engineering Equation Solver) students can now have the capability to program important equations, store the resulting functions, and use those functions just like sines, cosines or other mathematical functions. If the arguments are supplied, the value of the function is computed. Functions have been written for most of the important equations in *Solar Engineering of Thermal Processes* [5]. These functions greatly reduce the time and effort in solving equations, and offer the opportunity for the students to use the functions to do new kinds of problems (parametric studies, "what if" problems, and design problems). The need remains to assure that the student has an adequate understanding of the equations that are being used.

Introduction

A typical modern course in solar energy starts with consideration of the nature of solar radiation, its measurement, the data that are available (usually radiation on a horizontal surface), and how those data can be processed to get the information wanted (usually the radiation on a sloped surface). Absorption, emission, and transmission of radiation are then treated. Heat transfer topics important in collectors and other components are studied at some time early in the course. At this point the student is ready to tackle the theory and calculation of collector performance. Storage may be treated next; the solution of time-dependent store energy balances is an essential part of solar process calculations. At this point the elements for solution of systems problems are in hand, and simulations to determine long-term performance are possible. Economics of solar processes may follow, and some attention may be given to correlation, utilizability, and "shortcut simulation" design methods.

All of these topics require the solution of sets of equations, mostly algebraics. With the exception of the energy balance equations, most of them can be solved quite readily with a hand-held calculator, although the process may be tedious.

A solar energy engineering course is, by nature, quantitatively oriented. The objective is to develop in the student the ability to understand and predict how solar energy processes perform, and based on performance calculations, design the processes using economic and other criteria. This ability to predict process performance is based on an understanding of how components (collectors, stores, loads, heat exchangers, PV generators, etc) will work, given time-dependent external forcing functions (radiation, temperature, etc.). Pedagogical questions then are: How do we assist the student to understand and calculate equipment performance? How do we provide him/her with some "feeling" or judgment about what constitutes a reasonable answer to a question? How do we use the capability that modern computing developments provide to expand the student's learning?

There are now available several programs which have the capability of numerically solving large systems of equations. Examples are TK-Solver Plus [1], MathCad [2], and EES [3,4]. These programs provide microcomputers with equation solving capability that reduces many problems that were previously tedious to solve to near trivial levels. It provides the opportunity for students to solve new kinds of problems.

The EES USERLIB Functions

We have chosen EES to implement the functions needed for solar energy applications. EES is different from others in that there is built into the program thermodynamic and property data for fluids commonly found in engineering systems. Also, experience at UW has shown EES to be somewhat easier for students to learn. An important capability of EES versions 3.36 (and later) is the ability to incorporate user written functions that are frequently used. For example, when the function for a collector top loss coefficient is entered into a USERLIB file, it is available for use just as is a SIN or LOG function; when the arguments are supplied, the result is the top loss coefficient. Once these functions are available, the calculations for such quantities as loss coefficients are greatly simplified.

We have written EES USERLIB functions for most of the important equations in part I of Duffie and Beckman [5]. In a trial in the fall semester of 1993, we made these functions freely available to students, and assigned some new types of problems to be worked with these functions.

EES and the USERLIB functions were provided through the college computing services, where students can go to use any of a variety of machines. They were provided with a list of the functions and information about their use; the parts of this handout that are used in the examples in this paper are shown in Table 1. In addition, the student could view or print listing of individual functions if they so desired. Problem assignments in the course (which were due daily) included some from Appendix A in [5] and new problems designed to take advantage of the capability of EES. This paper is a summary of our experience with these new tools.

Three examples of the use of USERLIB functions are shown, to illustrate what is involved in writing the functions, storing them in the USERLIB file, and making use of them. They vary from simple to complex. The first involves calculation of the effect of slope ontop loss coefficient for a flat-plate collector. The second is on effects of cover material characteristics on absorbed radiation. The third is an example that requires the application of several equations to calculate collector performance from design parameters and operating conditions. In each example, a problem using the function is posed, the EES equations needed to solve the problem are shown, and the solution is presented. These problems could be done without the functions but the time and effort involved would prohibit its use as a daily student problem, and the solutions offer insights on some aspects of solar processes.

Example 1: Calculation of Effects of Slope on Convective and Overall Loss Coefficients

Top loss coefficients of flat-plate collectors can be estimated with the Klein approximation, Equation 6.4.9 in [5]:

$$U_t = \left(\frac{N}{\frac{C}{T_{pm}}\left[\frac{(T_{pm} - T_a)}{(N+f)}\right]^e} + \frac{1}{h_w} \right)^{-1} + \frac{\sigma (T_{pm} + T_a)(T_{pm}^2 + T_a^2)}{(\varepsilon_p + 0.00591\ N h_w)^{-1} + \frac{2N + f - 1 + 0.133\varepsilon_p}{\varepsilon_g} - N} \qquad (1)$$

where
N = number of glass covers
f = $(1 + 0.089 h_w - 0.1166 h_w \varepsilon_p)(1 + 0.07866 N)$
C = $520(1 - 0.000051\beta^2)$ for $0^o < \beta < 70^o$. For $70^o < \beta < 90^o$, u s e$\beta = 70^o$
e = $0.430(1 - 100/T_{pm})$
β = collector tilt (degrees)
ε_g = emittance of glass (0.88)
ε_p = emittance of plate
T_a = ambient temperature (K)
T_{pm} = mean plate temperature (K)
h_w = wind heat transfer coefficient (W/m^2C)

The use of this equation is not difficult, but it is easy to make errors and it takes time. Example 6.4.2 in [5] shows such a calculation.

An EES USERLIB function[1] for this equation can be written:

```
FUNCTION U_TOP_(T_pl, T_amb, Slope, WindCoeff, Emitt, Ncov)          {Eq. 6.4.9}
f = (1+0.089*WindCoeff-0.1166*WindCoeff*Emitt)*(1+0.07866*Ncov)
e = 0.430*(1-100/T_pl)
If(Slope>70) Then Slope=70
c = 520*(1-0.000051*Slope^2)
x = (((T_pl-T_amb)/(Ncov+f))^e)*c/T_pl
y = 5.67e-8*(T_pl+T_amb)*(T_pl^2+T_amb^2)
z = (Emitt+0.00591*Ncov*WindCoeff)^(-1)+(2*Ncov+f-1+0.133*Emitt)/0.88-Ncov
U_TOP_ = (Ncov/x+1/WindCoeff)^(-1)+y/z
```

---

[1] [1] Note that the nomenclature used in the program is typical of that used in programming; the translations are obvious. Comments (in this case equation numbers from [5]) are in brackets{} and do not affect the program in any way.

END

With this function in the USERLIB file, the calculation of the coefficients is simple. The function is called, and the numerical values of the arguments are provided either in the call statement or in separate equations. Thus the EES equations to do the calculation using the function for top loss coefficients are:

```
U_t = U_top_(T_pl,T_amb,Slope,WindCoeff,Emitt,Ncov)
T_pl = 373;  T_amb = 283;  Slope = 45;  WindCoeff = 10;  Emitt = 0.95;  Ncov = 1
```

Or:

```
U_t = U_top_(373, 283, 45, 10, 0.95, 1)
```

The solution is 6.64 W/m^2C. With EES it takes less than a minute to program and do this calculation. Note that the user of the function does not need to know how the function is written, but to apply it with judgment and discretion, the user should know the basis of the equation and its limitations.

With the capabilities of EES, including the functions, students can be asked to do other kinds of problems. Parametric studies become easy. For example, the effects of collector slope on top loss coefficient can readily be shown.

For the circumstances of Example 6.4.2, show the dependence of collector top loss coefficient on the slope of the collector.

For this problem, the EES equation is nearly the same as shown above:

```
U_t = U_top_(373, 283, Slope, 10, 0.95, 1)
```

Various slopes will be entered in a parametric table and the coefficient computed for each slope. The result, for slopes of 0^o, 15^o, 30^o, 45^o, 60^o, 75^o, and 90^o (with input data shown in bold type) is:

Slope, deg	U_t, W/m2C
0	6.87
30	6.77
60	6.45
75	6.28
90	6.28

In this computation, the effect of slope is not great. If other methods of calculating the effects of slope were preferred, such as the "exact" method of Sections 3.11 and 6.4, a somewhat more complicated set of equations (but still quite simple) would be needed.

<u>Example 2: Calculation of Effects of Cover Properties on Absorbed Radiation</u>

There is a class of problems that are sometimes referred to as "what if" problems. They are useful in illustrating and evaluating the potential effects of new developments on solar process operations. EES functions make it easy to get answers to many of these. The following is one such problem:

The Hypothetical Glass Company has come up with a new method of coating glass that in effect gives it refractive index in the range of 1.300 to 1.525, with other properties similar to ordinary glass. Would this be of interest in solar energy applications? Can you speculate about what factors might be important in deciding whether it would have such applications?

The following EES function computes cover transmittance as a function of the number of covers, the angle of incidence of the radiation, and the cover properties *KL* (the extinction coefficient-thickness product) and refractive index. It is based on the approximate method of Equation 5.3.4, and Equations 5.1.4, 5.1.1, 5.1.2, 5.1.9, 5.2.2, and 5.3.4 in [5].

```
FUNCTION TRANSINCANG_(Ncov,IncAng,KL,RefrInd)
RefrAng=arcsin(sin(IncAng)/RefrInd)                          {Eq. 5.1.4}
REFperp=(sin(RefrAng-IncAng)/sin(RefrAng+IncAng))^2          {Eq. 5.1.1}
REFpara=(tan(RefrAng-IncAng)/tan(RefrAng+IncAng))^2          {Eq. 5.1.2}
TRANSperp=(1-REFperp)/(1+(2*Ncov-1)*REFperp)
TRANSpara=(1-REFpara)/(1+(2*Ncov-1)*REFpara)
TRANSref=(TRANSperp+TRANSpara)/2                             {Eq. 5.1.9}
TRANSabs=exp(-Ncov*KL/cos(RefrAng))                          {Eq. 5.2.2}
TRANSINCANG_=TRANSabs*TRANSref                               {Eq. 5.3.4}
END
```

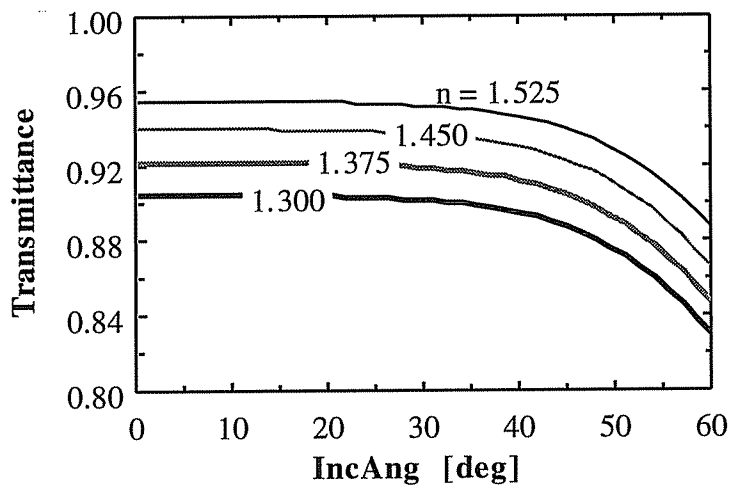
The use of this function in studying the problem is illustrated below. The problem is to assess the potential importance of the availability of a glass with refractive index less than 1.525 for collector covers. This is done by calculating the effects of refractive index on transmittance of cover systems. Sets of parameters to consider must be selected. Here we use those of Figure 5.3.1 of [5], for the best value of *KL*, for angles of incidence of 0, 20, 40, and 60, and for 1 cover.

These transmittances of the cover with the lowest refractive index is compared to those of ordinary glass with an index of 1.525 by the percentage improvement in transmittance for the low refractive index glass. To show how the range of refractive indices affects transmittance, the function is applied four times for four refractive indices.

```
Trans1.300 = TRANSINCANG_(1, IncAng, 0.0125, 1.300)
Trans1.375 = TRANSINCANG_(1, IncAng, 0.0125, 1.375)
Trans1.450 = TRANSINCANG_(1, IncAng, 0.0125, 1.450)
Trans1.525 = TRANSINCANG_(1, IncAng, 0.0125, 1.525)
Improvement = (Trans1.300 - Trans1.525)*100/Trans1.525
```

There are two parts to the answer. The first has to do with the improvement in transmittance and how this affects collector performance (and ultimately solar process system performance). The results of the EES computations are in the table. An EES plot is also shown.

IncAng	τ1.300	τ1.375	τ1.450	τ1.525	Imp.
0	0.955	0.940	0.923	0.906	5.4
20	0.954	0.939	0.922	0.905	5.4
40	0.946	0.930	0.913	0.896	5.6
60	0.888	0.866	0.847	0.830	7.0



It is clear from this result that there are significant gains in transmittance as refractive index decreases, and that the gains are highest at large angles of incidence. For a collector operating under typical conditions with an efficiency of about 50%, an improvement in transmitance of over 5% means an increase in collector output (or a decrease in required collector area) of around 10% from that of ordinary glass.

(Transmittance increases are almost the same for *KL* of 0.0370. A similar calculation for two covers, done by insereting Ncov = 2, shows the expected larger improvements.)

The final factor to be considered is cost. If the cost increase of the low index glass is less than system performance increase, it will pay (assuming the same *KL*, durability, etc.).

<u>Example 3: Calculation of Collector Output</u>

The availability of the EES USERLIB functions makes problems involving many different equations relatively simple. For example, calculation of collector output for a sloped collector, starting with an hour's meteorological data (including radiation on a horizontal surface), the location and time, the design parameters of the collector, and the fluid properties and flow rate, can be calculated from a simple set of equations.

Calculate the output of a flat-plate collector with design parameters and operating conditions as indicated in the equations below.

This problem requires the solution of a set of 34 equations and 34 unknowns. They can be solved sequentially by hand, but the solution is easy with the EES functions. The set of equations is as follows. The first five are the USERLIB functions, the sixth adds back losses to U_t to get U_L, and the remainder give the values of design parameters and operating conditions.

```
S=S_T'LJ_(I,Lat,n,Slope,SurfAzAng,GrRef,HrAng,Ncov,KL,RefrInd,ABS_n)
U_t=U_TOP_(T_pl,T_amb',Slope,WindCoeff,Emitt,Ncov)
CollEffFact=COLLEFFFACT_(U_L,Cond_pl,Thick_pl,TubeSp,TubeDiam,BondCond,h_fi)
F_R=F_R_(MassFR,C_P,A_c,U_L,CollEffFact)
Q_u=Q_U_(A_c,F_R,S,U_L,T_in,T_amb)
U_L=U_t+k/l
k=0.045; l=0.050; I=1.50; Lat=45; n=50; Slope=60; SurfAzAng=0; GrRef=.3
HrAng=7.5; Ncov=1; KL=0.0125; RefrInd=1.526; ABS_n=.92
T_pl=70+273; T_amb'=10+273; WindCoeff=10; Emitt=.10
Cond_pl=385, Thick_pl=0.0005; TubeSp=0.15
TubeDiam=0.010; BondCond=10^6; h_fi=300
MassFR=0.015; C_P=4180; A_c=5.00; T_amb=10; T_in=60
```

The results are:

CollEffFact = 0.911	Q_u = 3.41 MJ	U_L = 4.088 W/m^{2}C
F_R = 0.789	S = 1.60 MJ/m2	U_t = 3.188 W/m^{2}C

If data on h_{fi} and h_w had not been available, functions for these coefficients could have been included. The HDKR model [5] for S_T could have been used in place of the Liu & Jordan model. With this program, effects of changes in design parameters are easily established; the results of changing a parameter like plate conductivity can be assessed by removing the Cond_pl = 385 equation and creating a parametric table for the desired range of conductivity. The effects of alternative ways of estimating variables like S_T are also easy to assess.

Experience with EES USERLIB

We have made EES with the USERLIB functions available in the September-December semester of 1993. On the basis of this limited experience, it is clear that we can accomplish more in the course by the reduction of the time it takes students to do problems. Our first experience with the functions has been encouraging; the next step is to make additional modifications in lectures to put more emphasis on concepts and less on computational methods and change examinations to take maximum advantage of the program.

(The program and functions were also used in a one week short-course given to a group of six engineers. A computer was set up in the conference room where the course was held, and at the beginning of each day EES and the USERLIB were brought up. Thus at any point during the lectures and discussions, problems could be solved by anyone in the room, and questions of the effects of design parameters on component or system performance could be quickly and quantitatively answered. In this setting, the program was a great asset to the learning process.)

Student Reaction

The students in the semester course had ready access to EES and the USERLIB functions, and the decisions on how to do problems were entirely up to them. A survey after the course was over indicated the following:

All students made extensive use of EES. Comments included appreciation of the time saved and the reduction in mathematical errors.

Essentially all did many problems by hand, as a better way to get a feeling of what is involved in the equations.

The examination structure in the course was such that students felt the need for preparing to do problems by hand.

Many found the parametric problems to be interesting.

Conclusions

The availability of EES and the functions for important equations in solar energy engineering makes it possible for students to undertake new and different kinds of problems that provide a better understanding of solar processes and how they work. More complex problems can be done, parametric studies are easier, and new kinds of "open-ended" problems can be assigned. These can be done simply because the time involved in the mechanics of solving the problems is greatly reduced.

From an instructor's viewpoint, the challenge is twofold: to be sure that the students understand what they are doing when they use the functions and appreciate the ideas behind the equations, and to take advantage of EES to increase what the students can effectively learn. Our students found that they first needed to do problems by hand before using the functions (in part because they knew they would be faced with examinations in which EES would not be available). We see the day coming when all students will have EES (or its equivalent) available to them at all times on their own personal computers that they will carry in their backpacks; then full advantage of these methods can be taken in the educational process.

References

1. TK-Solver Plus, Universal Technical Systems, Inc., 1220 Rock Street, Rockford, IL 61101.

2. MathCad, MathSoft, Inc., 1 Kendall Sq., Cambridge, MA 02139.

3. Klein, S. A. and Alvarado, *EES, Engineering Equation Solver*, Users Manual, F_Chart Software, 4406 Fox Bluff Road, Middleton, WI. 53562 (1993).

4. Klein, S. A., Computer Applications in Engineering Education, **1**(3), 265 (1993). *Development and Integration of an Equation-Solving Program for Engineering Thermodynamics Courses.*

5. Duffie, J. A. and W. A. Beckman, *Solar Engineering of Thermal Processes,* 2nd ed., John Wiley & Sons, New York (1991).

Table 1

EES FUNCTIONS, PROCEDURES, and Nomenclature

The nomenclature in these functions and procedures is close to that in [5]. Where differences have been required or are convenient, obvious variable names have been used, such as Dec for declination. There are two exceptions in the matter of style. First: EES does not allow subscripts in variable names, so they are indicated as in the following examples: I_T is substituted for I_T, R_b for R_b, I_zero for I_o, and A_c for A_c, etc.. Second: Variables that are ratios can not be shown with a slash "/" as it denotes a division operation; the reverse slash is used in variable names that are ratios, such as: F_R'\F_R for F_R'/F_R, and I_d\I for the diffuse fraction I_d/I.

Units: Units of all angles are degrees. Units of all linear dimensions are meters unless otherwise specified. Other units are specified in the last column of the table. Be sure that Unit System in the Options menu shows SI, Mass Basis, Degrees, kPa, and Celsius.

Function names: You may see part or all of function names in mixed upper and lower case or in upper case only. It makes no difference; EES does not distinguish between upper and lower case.

Eq'n	FUNCTION	Comments	Units
1.6.1	**Dec_**(n)	Declination	
1.10.4	**I_zero_**(n,Lat,HrAng1,HrAng2)	Extraterrestrial radiation on horizontal surface between times 1 and 2	I_zero MJ/m^2

5.3.4	**TransIncAng_**(Ncov,IncAng,KL,RefrInd)	Transmittance of Ncov covers for radiation incident at IncAng using the approximate method	
5.9.1	**S_T'LJ_**(I,Lat,n,Slope,SurfAzAng, GrRef,HrAng,Ncov,KL,RefrInd,Abs_n)	Hour's absorbed radiation on sloped surface using Liu & Jordan model for south facing surface	I MJ/m^2
6.4.9	**U_top_**(T_pl,T_amb,Slope, WindCoeff,Emitt,Ncov)	Approximate solution for collector top loss coefficient	T_pl, T_amb K; WindCoeff W/m^2K
6.5.18	**CollEffFact_**(U_L,Cond_pl,Thick_pl, TubeSp,TubeDiam,BondCond,h_fi)	Calculates collector plate efficiency factor *F'*	U_L, h_fi W/m^2C Cond_pl W/mC BondCond W/mC
6.7.5 W/m^2C;	**F_R_**(MassFR,C_p,A_c,U_L,CollEffFact) removal factor F_R	Calculates collector heat C_p J/kgC;	MassFR kg/s; U_L A_c m^2
6.7.6 using *T_in* and F_R	**Q_u_**(A_c,F_R,S,U_L,T_in,T_amb)	Calculates collector useful U_L W/m^2C; T_in,T_amb C	A_c m^2; gain S MJ/m^2;