Methodologies for Optimal Control of Chilled Water Systems Without Storage

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ABSTRACT

In this paper, two methodologies are presented for determining the optimal control settings for chilled water systems that do not have significant thermal storage. A component-based nonlinear optimization algorithm was developed as a simulation tool for investigating optimal system performance. Results of this algorithm, implemented in a computer program, led to the development of a simpler system-based methodology for near-optimal control that is simple enough for on-line implementation.

INTRODUCTION

A central cooling plant consists of one or more chillers, cooling towers, pumps, and air handlers controlled so as to satisfy the cooling requirements of one or more buildings. At any given time, it is possible to meet the cooling needs with any number of different modes of operation and setpoints. Optimal supervisory control of the equipment involves determination of the control that minimizes the total operating cost. The optimal control depends upon time, through changing cooling requirements and ambient conditions. Currently, the operators of central cooling plants determine control practices that yield "reasonable" operating costs by experience gained through trial-and-error operation over a long period of time. Little research has been performed in developing general methodologies that would be suitable for optimal control of large, centralized cooling systems.

Figure 1 shows a simplified schematic of a typical centralized chilled water system. Such a plant has many operating variables that can be controlled in a manner to minimize operational costs. The optimization is complicated by the fact that there are both "discrete" and "continuous" control variables. Discrete control variables are not continuously adjustable, but have discrete settings. Discrete control variables include the number of chillers, tower cells, and pumps operating at a given time. For multiple-speed fans or pumps, the speed settings are also considered as discrete controls. Continuous control variables include chilled water and supply air setpoints, the relative condenser and evaporator flow rates for multiple chillers, and the relative operating speeds for variable-speed fans or pumps. The control variables may interact strongly and, with proper control, it is possible to significantly reduce operating costs.

Most of the control studies related to cooling systems have dealt with the local control of an individual component or subsystem needed to maintain a prescribed setpoint rather than the global determination of optimum setpoints that minimize operating costs. Global optimum plant control has been studied by Marcev (1980), Arnold (1984), Sud (1984), Lau (1985), Hackner (1984, 1985), Johnson (1985), and Nugent (1988). These studies primarily demonstrated the potential savings associated with the use of optimal control. They did not, however, produce general algorithms suitable for on-line optimal plant control. Optimal control setpoints were identified in Hackner's study for a specific plant through the use of performance maps. These maps were generated by many simulations of the plant over the range of expected operating conditions. The use of established performance maps for on-line plant control is advocated by Johnson. However, this procedure lacks generality and is not easily implemented.

In this paper, two methodologies are presented for determining optimal values of the independent control variables that minimize the instantaneous cost of operating chilled water systems in response to the uncontrolled (e.g., weather) variables. First, a modular component-based optimization algorithm is presented. Each hardware component is represented with a mathematical model in the simulation of a system. Information concerning the cost of operation of individual components and the manner in which the components are interconnected are used to perform the optimization in an efficient manner. Each component may also have constraints associated with its operation.

In addition, a "simple" system-based methodology is presented for near-optimal control. An overall empirical

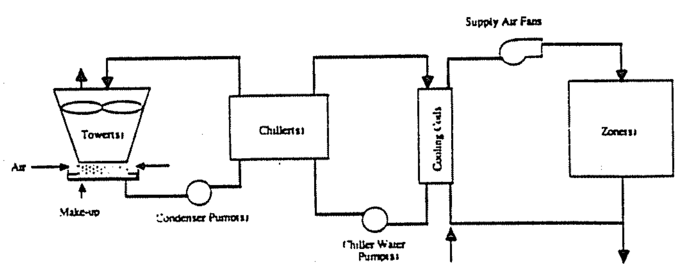

Figure 1 *Schematic of a typical chilled water system*

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cost function for the total power consumption of the cooling plant is inferred from the cost functions associated with the components utilized in chilled water systems. This cost function lends itself to rapid determination of optimal control variables and may be fit to measurements using linear regression techniques. Results of the system-based and component-based algorithms are compared.

There are three intended uses for the component-based optimization algorithm developed in this study:

1. *Analysis of control and design options:* The methodology may be used as a simulation tool for comparing conventional and optimal control strategies. Conventional control strategies, such as fixed temperature setpoints, are implemented through the imposition of constraint equations. Design comparisons, such as variable vs. fixed-speed equipment, may be performed for systems that are optimally controlled.
2. *On-line control optimization:* The algorithm may be used for on-line optimization of the simulation of an operating system using "simple" component models. The simulation could proceed in parallel with the actual system operation with the possibility of updating parameters of the component models using on-line measurements.
3. *Near-optimal control algorithms:* Results of the detailed optimization procedure can be useful for developing "simple" near-optimal control algorithms.

Braun (1988) applied the component-based algorithm to typical systems to study both design and control issues. Existing optimization packages proved to be extremely inefficient for these systems and had difficulties handling the nonlinear equality constraints that arose. The mathematical description of the component-based algorithm is intended as documentation for a method that works well for chilled water systems and may be skipped without loss of clarity. On the other hand, the system-based methodology for near-optimal control presented in this paper is a practical result that is useful to plant engineers.

OPTIMAL SUPERVISORY CONTROL

The optimal control problem associated with a central chilled water system may be thought of as having a two-level hierarchical structure. The first level involves local loop control in response to prescribed setpoints. An example of a first-level control variable would be the compressor speed for a variable-speed chiller that varies in order to maintain a fixed chilled water supply temperature. The second-level controls are independent variables that may be adjusted to minimize the operating costs while still satisfying the load requirements. In the previous example, the chilled water supply temperature is a second-level variable.

The dynamics of the first-level (local loop) controls must be considered in order to maintain prescribed setpoints in an efficient manner. However, for systems without significant thermal storage, the system dynamics may be neglected (Lau 1985) in the determination of the optimal second-level control setpoints. In this study, local loop control is not considered and the first-level (local loop) control is considered to be entirely dependent upon the second-level setpoints.

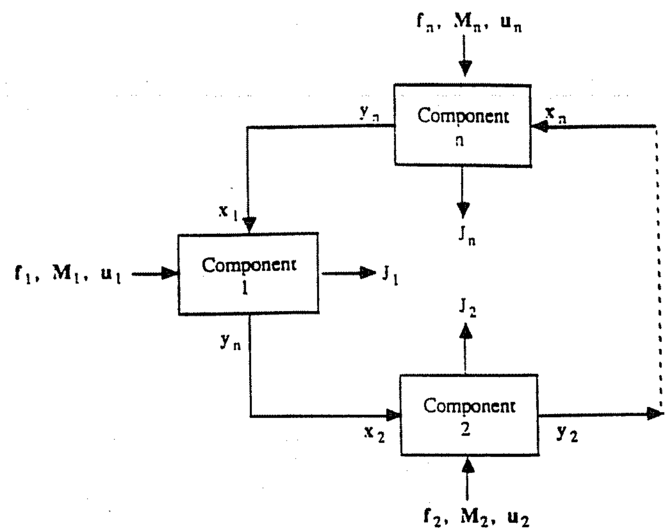

Figure 2 *Schematic of the modular optimization problem*

Optimal control of a system involves minimizing the total power consumption of the chillers, cooling tower fans, condenser water pumps, chilled water pumps, and the air-handling fans at each instant of time with respect to the independent continuous and discrete control variables. The optimization problem, at any time, is to minimize

$$J = \text{function}\ (\boldsymbol{f}, \boldsymbol{u}, \boldsymbol{M}) \tag{1}$$

with respect to $\boldsymbol{u}$ and $\boldsymbol{M}$, where J is the cost of operation, $\boldsymbol{f}$ is a vector of uncontrolled variables, $\boldsymbol{u}$ is a vector of continuous controls, and $\boldsymbol{M}$ is a vector of discrete controls. The uncontrolled variables are measurable quantities that may not be controlled but that affect the component outputs and/or costs, such as load and ambient dry-bulb and wet-bulb temperature. The optimization of chilled water systems is also subject to equality and inequality constraints. One example of an equality constraint that arises when two or more chillers are in operation is that the sum of their relative loadings must be unity. The simplest inequality constraints to handle are bounds on control variables. For example, lower and upper limits are necessary for the chilled water set temperature, in order to avoid freezing in the evaporator and to provide adequate dehumidification for the zones.

A COMPONENT-BASED OPTIMIZATION ALGORITHM

Figure 2 depicts the general nature of the modular optimization problem. The performance of each component in a system is represented with a separate set of mathematical relationships organized into a model. Its output variables and operating cost are functions of parameter, input, output, uncontrolled, and controlled variables. The structure of the complete set of equations to be solved for the entire system is dictated by the manner in which the components are interconnected. The optimization problem is formally stated as the minimization of the sum of the operating costs of each component, J_i, with respect to all discrete and continuous controls or minimize

$$J(\boldsymbol{f}, \boldsymbol{M}, \boldsymbol{u}) = \sum_{i=1}^{n} J_i\,(\boldsymbol{x}_i, \boldsymbol{y}_i, \boldsymbol{f}_i, \boldsymbol{M}_i, \boldsymbol{u}_i) \tag{2}$$

subject to equality constraints of the form

$$g(f, M, u) = \begin{bmatrix} g_1(f_1, M_1, u_1, x_1, y_1) \\ g_2(f_2, M_2, u_2, x_2, y_2) \\ \bullet \\ g_n(f_n, M_n, u_n, x_n, y_n) \end{bmatrix} = 0 \tag{3}$$

and inequality constraints of the form

$$h(f, M, u) = \begin{bmatrix} h_1(f_1, M_1, u_1, x_1, y_1) \\ h_2(f_2, M_2, u_2, x_2, y_2) \\ \bullet \\ h_n(f_n, M_n, u_n, x_n, y_n) \end{bmatrix} \geqslant 0 \tag{4}$$

where, for any component i,

x_i = vector of input stream variables
y_i = vector of output stream variables
f_i = vector of uncontrolled variables
M_i = vector of discrete control variables
u_i = vector of continuous control variables
J = cost of operation
g_i = vector of equality constraints
h_i = vector of inequality constraints

Typical input and output stream variables for chilled water systems are temperature and mass flow rate. Any equality constraint may be rewritten in the form of Equation 3 such that when it is satisfied, the constraint equation is equal to zero. Similarly, inequality constraints may be expressed as Equation 4, so that the constraint equation is greater than or equal to zero to avoid violation.

The mathematics associated with the optimization algorithm utilized in this study are well known. However, special advantage is taken of the characteristic that the operating costs associated with each component in a chilled water system may be modeled with quadratic functions. A background for the development that follows is presented by Gill (1981). In the next section, an algorithm is presented for determining optimal values of the continuous control variables for the special case where all component costs are quadratic functions and outputs are linear functions of these variables. This algorithm is the basis for the more general nonlinear method that follows. The procedure for handling constraints is also given and the complete algorithm including the determination of the optimal discrete controls and implementation in a computer program is summarized.

Quadratic Costs and Linear Outputs

A simple function for which an optimum exists and may be determined analytically is a quadratic function. Braun (1987) has shown that the power consumption of a chiller may be adequately represented as a quadratic function of the load and the temperature difference between leaving condenser and evaporator water temperatures. The other energy-consuming components in a chilled water system are pumps and fans. The power requirement of a continuously adjustable pump or fan (either variable-speed or variable-pitch) may be accurately represented with a quadratic function of its control variable through a second-order Taylor series approximation or a single quadratic correlating function (Braun 1988). As a result, the cost of operating any of these components (chillers or auxiliary equipment) may be expressed in a general form as a quadratic function of its continuous control and/or output stream variables or

$$J_i = u_i^T A_i u_i + y_i^T B_i y_i + y_i^T C_i u_i + p_i^T u_i + q_i^T y_i + r_i \tag{5}$$

where

A_i, B_i, and C_i are coefficient matrices; p_i and q_i are coefficient vectors; and r_i is a scalar constant. The coefficients of this cost function may depend upon the component operating modes (discrete controls) and uncontrolled variables.

The optimization problem is simplified if the output variables for each component are linear functions of the input and continuous control variables or

$$y_i = \theta_i u_i + \phi_i x_i + \xi_i \tag{6}$$

where θ_i and ϕ_i are coefficient matrices and ξ_i is a coefficient vector. The inputs to component i are outputs from other components, so that the solution for all output variables is of the form

$$y = \tilde{\theta} u + \tilde{\xi} \tag{7}$$

where the coefficient matrix $\tilde{\theta}$ and vector $\tilde{\xi}$ depend upon the coefficients for the individual components and the interconnections between components.

The total cost of operation at any time is the sum of the individual component operating costs. With individual component costs and outputs represented by Equations 5 and 7, the total cost may be reduced to

$$J = u^T \tilde{A} u + \tilde{b}^T u + \tilde{c} \tag{8}$$

where

$$\tilde{A} = A + \tilde{\theta}^T [B\tilde{\theta} + C]$$

$$\tilde{b} = p + \tilde{\theta}^T [2B^T \tilde{\xi} + q] + C^T \tilde{\xi}$$

$$\tilde{c} = r + [\tilde{\xi}^T B + q^T]\, \tilde{\xi}$$

$$A = \begin{bmatrix} A_1 & & & \\ & A_2 & & \\ & & \bullet & \\ & & & A_n \end{bmatrix} \quad B = \begin{bmatrix} B_1 & & & \\ & B_2 & & \\ & & \bullet\bullet & \\ & & & B_n \end{bmatrix} \quad C = \begin{bmatrix} C_1 & & & \\ & C_1 & & \\ & & \bullet & \\ & & & C_n \end{bmatrix}$$

$$p = \begin{bmatrix} p_1 \\ p_2 \\ \bullet \\ p_n \end{bmatrix} \qquad q = \begin{bmatrix} q_1 \\ q_2 \\ \bullet \\ q_n \end{bmatrix} \qquad r = \sum_{i=1}^{n} r_i$$

The first-order condition for a minimizing or maximizing point requires that the Jacobian of the cost function be equal to zero. The Jacobian is a vector containing the partial derivatives of the cost function with respect to each of the control variables. For the cost function of Equation 8

$$[\partial J / \partial u]_{u^*} = u^* (\tilde{A} + \tilde{A}^T) + \tilde{b}^T = 0 \tag{9}$$

where u^* represents the optimal control vector. Solving for u^* gives

$$u^* = -[\tilde{A} + \tilde{A}^T]^{-1} \tilde{b} \tag{10}$$

In general, the cost functions that arise with chilled water systems are globally convex (i.e., $\tilde{A} + \tilde{A}^T$ is a positive-definite matrix), so that a single global minimum exists.

Nonlinear Optimization

Some component outputs depend nonlinearly upon controls or input variables and some component costs are only quadratic locally, so that an iterative technique is required to determine the optimal control values. At each iteration, an overall quadratic for the system cost expressed as Equation 8 is formed from individual quadratic relationships for each component cost (Equation 5) and a linearization of the output variables (Equation 7). All output variables are linearized with respect to the continuous controls using a first-order Taylor series expansion about the last iteration point. The Jacobian of the outputs with respect to control variables is determined numerically using forward differences.

An estimate of the minimum point may be determined from Equation 10. However, for points that are "far away" from the minimum, this may give a point that has a greater cost than the last iteration. A common procedure is to perform a one-dimensional search between the previous iteration and the point defined by Equation 10. At the ith iteration, a new estimate of the optimal control point is

$$\boldsymbol{u}^i = \boldsymbol{u}^{i-1} + s\,(\boldsymbol{u}' - \boldsymbol{u}^{i-1}) \tag{11}$$

where $\boldsymbol{u}'$ is from Equation 10 and the step length, s, is determined by minimizing $J(\boldsymbol{u}^{i-1} + s(\boldsymbol{u}' - \boldsymbol{u}^{i-1}))$ with respect to s.

The optimal step length is approximated with polynomial interpolation. The costs at step lengths of zero, one-half, and one are used to construct a quadratic function for the cost as a function of the step length. The optimal value of s is estimated as the minimum of this quadratic function, constrained between zero and one. In some cases, the polynomial may be a poor approximation of the real function and the estimated optimal step length may result in a cost greater than that associated with a step length of zero. Under this circumstance, the interpolation is repeated with the last computed optimal step length becoming the new step length of unity.

It is necessary to iteratively solve for the outputs of each component at each iteration of the optimization procedure with the most recent controls. A simple method that is employed in the TRNSYS (1984) program is successive substitution. Outputs are successively fed as inputs to connecting components until the values do not change significantly. However, this method can be extremely inefficient for solving systems of algebraic equations, even if they are linear. Solution efficiency is important because the equations must be solved at each iteration of the optimization procedure and a high degree of accuracy is required for determining numerical derivatives.

A better approach is to utilize the Newton-Raphson method applied to a set of equations that measure the residual error for the independent variables. These residual equations could be defined as differences between input values and output values that feed those inputs for one component in each recyclic loop of components. At each iteration of the Newton-Raphson procedure, new estimates of the independent variables are determined by assuming that the residual equations are globally linear using coefficients determined with a local linearization. As a result, this method converges in one iteration for linear equations. However, it is necessary at each iteration to solve a linear system of equations involving a Jacobian matrix. The Jacobian contains the partial derivatives of each of the residual equations with respect to the independent variables and must be determined numerically.

In the program developed in this study, an alternative method is employed that is a compromise between the methods of successive substitution and Newton-Raphson. As with Newton-Raphson, a set of residual equations is defined such that, at the solution, they are identically zero. However, these equations are solved using a series of one-dimensional applications of the secant method. The advantage of this method is that it is not necessary to compute a Jacobian, so that the computation associated with updating the independent variables is much less than that for Newton-Raphson. However, since the residual equations are not coupled, convergence is slower than for Newton-Raphson. For the chilled water systems considered in this study, the coupling between recyclic loops is relatively small and the solution algorithm works well.

Constraints

Linear Equality Constraints. For constraints that are linear with respect to the control variables, the constraint equations may be written in the form

$$\boldsymbol{g}(\boldsymbol{u}) = \alpha + \beta\boldsymbol{u} = 0 \tag{12}$$

where $\boldsymbol{g}$ is a vector of constraint equations, β is a coefficient matrix, and α is a vector.

A common method for solving optimization problems with linear constraints is the method of Lagrange multipliers. This method involves redefining the cost function so that at the minimum, the constraint function is automatically satisfied. The modified cost function, termed the Lagrangian, is given as

$$\tilde{J}(\boldsymbol{u},\lambda) = J(\boldsymbol{u}) + \lambda^T\boldsymbol{g}(\boldsymbol{u}) \tag{13}$$

where λ is a vector of Lagrange multipliers. The modified optimization problem involves minimizing the Lagrangian with respect to both $\boldsymbol{u}$ and λ. The first-order conditions for a minimum applied to the quadratic cost function and linear constraints of Equations 8 and 12 yield

$$\boldsymbol{u}^* = [\tilde{\boldsymbol{A}} + \tilde{\boldsymbol{A}}^T]^{-1}\,[\beta^T\lambda - \tilde{\boldsymbol{b}}] \tag{14}$$

$$\lambda = [\beta(\tilde{\boldsymbol{A}} + \tilde{\boldsymbol{A}}^T)^{-1}\beta^T]^{-1}\,[\beta(\tilde{\boldsymbol{A}} + \tilde{\boldsymbol{A}}^T)^{-1}\tilde{\boldsymbol{b}} - \alpha] \tag{15}$$

Nonlinear Equality Constraints. Nonlinear constraints are handled through linearization and the use of Lagrange multipliers. The Jacobian of the constraints with respect to control variables is determined numerically using forward differences coincidently with the computation of the Jacobian of the output stream variables with respect to controls.

With linearization applied to nonlinear constraints, the first-order condition applied to the Lagrangian cost function does not guarantee that the constraints will be satisfied at any point except the solution. More importantly, during the determination of the optimal step length, the Lagrangian does not provide a good measure of the degree to which the constraints are violated. To alleviate this problem, the optimal step length is computed using a cost function that is the sum of the original cost function and a quadratic penalty function.

$$\hat{J}(\boldsymbol{u}) = J(\boldsymbol{u}) + \boldsymbol{g}(\boldsymbol{u})^T\boldsymbol{g}(\boldsymbol{u}) \tag{16}$$

At the i^{th} iteration, a new estimate of the optimal control point is found with Equation 11, with $\boldsymbol{u}'$ determined from Equations 14 and 15 and the step length, s, determined by minimizing $\hat{J}(\boldsymbol{u}^{i-1} + s(\boldsymbol{u}' - \boldsymbol{u}^{i-1})$ with respect to s.

Inequality Constraints. The only inequality constraints considered in this study are simple bounds on the control variables. These become linear equality constraints when violated and are handled with Lagrange multipliers. In order to determine the optimal control points subject to inequality constraints, the optimal control values are first determined at each iteration assuming that no inequality constraints are violated. If some control bounds are exceeded, then linear equality constraints representing these limits are added and the optimal controls are recomputed. Additional constraints are added if violated and the process is repeated until the number of constraints is not changing or equals the number of control variables. The constrained control values represent the next iterates in the overall nonlinear optimization. It is not possible to solve a problem in which the number of constraints exceeds the number of control variables. In this situation, some of the constraints are not satisfied.

Algorithm Summary and Program Implementation

The steps associated with the constrained optimization of the continuous controls are summarized below.

1. Solve for component outputs with current controls.
2. Linearize outputs and equality constraints with respect to controls to get Equations 7 and 12.
3. Determine the coefficients of the quadratic Equation 8 from the component quadratic Equation 5 and the linearized output Equation 7.
4. Determine the control point associated with a step length of unity with Equations 14 and 15.
5. Estimate optimal step length with polynomial interpolation applied to minimizing $\hat{J}(\boldsymbol{u}^{i-1} + s(\boldsymbol{u}' - \boldsymbol{u}^{i-1})$ with respect to s, where the augmented cost function is defined by Equation 16.
6. Determine next estimate of control point, $\boldsymbol{u}^i$, with Equation 11.
7. If $\hat{J}(\boldsymbol{u}^i) > \hat{J}(\boldsymbol{u}^{i-1})$, then set $\boldsymbol{u}' = \boldsymbol{u}^i$ and go to step 5.
8. If no controls exceed their bounds or the number of constraints equals the number of controls, then go to step 11.
9. Add equality constraints for any controls that exceed bounds unless the number of constraints equals the number of controls.
10. Determine a constrained optimum with Equations 14 and 15 and go to step 8.
11. If the change in cost from the last iteration is greater than a specified tolerance, then go to step 1.

The complete optimization algorithm is implemented in a computer program that simulates the optimal operation of a system through time. The system is described through input data that specify the components, their parameters, and their interconnections in a manner similar to that of the TRNSYS (1984) simulation program. At each simulation time step, data for the uncontrolled variables (e.g., weather, schedules) are read and the constrained nonlinear optimization of the continuous control variables is performed for each feasible combination of discrete controls with the combination giving the minimum being the optimal control. Not all possible combinations of discrete controls are feasible. For instance, the operation of more than one condenser or chilled water pump might be nonoptimal under all conditions when only one chiller is on, so these combinations would not be worth considering. In the implementation of the optimization algorithm for this study, the feasible combinations of discrete modes are specified as input data. In the event that the optimization algorithm were implemented for on-line optimal control, a better approach for determining the optimal discrete control modes at each time interval would be to order the feasible combinations of modes and only allow a single change (up or down) between combinations within the list.

The complete mathematical description of a specific optimization problem depends upon the choice of the independent controls and the constraints imposed upon the system. Generally the supply air temperature for each air handler is considered to be an independent (adjustable) control variable. In this case, the local loop controller varies the air and water flow through the cooling coil in order to maintain that setpoint, along with the desired temperature in the zone. However, in terms of the optimization algorithm, it is more straightforward to consider the relative air and water flow rates to the cooling coil as control variables, with an equality constraint that forces the zone temperature to be maintained. One advantage of this formulation is that the power consumption of the air handler may be represented as a quadratic function of the speed in a simple manner. It is also easier to handle bounds on the fan speed as compared with the supply air setpoint, since the fan speed has a natural upper and lower bound, while physical constraints on the setpoint vary according to the coil entering air and water conditions and design.

A SYSTEM-BASED ALGORITHM FOR NEAR-OPTIMAL CONTROL

The methodology for determining optimal values of control variables described in the previous section could be used for on-line optimal control of chilled water systems. However, in order to calibrate the models for a specific plant, measurements would be required for inputs, outputs, and power consumptions for each component in the system. Also, depending upon the number of control variables, the computational requirements may be restrictive. An alternative approach for near-optimal control described in this section involves correlating the overall system power consumption with a single function that allows for rapid determination of optimal control variables and requires measurements of only total power over a range of conditions.

System Cost Function

The concept of utilizing quadratic functions for the power consumptions of individual components can be extended to the system as a whole. In the vicinity of any optimal control point, system power consumption may be approximated with a quadratic function of the continuous control variables according to Equation 8. However, the quadratic relationship changes with changes in the operating modes and uncontrolled variables (e.g., load and ambient conditions). It has been found that a quadratic function also correlates power consumption in terms of the uncontrolled variables over a wide range of conditions, so that the following cost function may be applied for determining optimal control points.

$$J(\boldsymbol{f}, \boldsymbol{M}, \boldsymbol{u}) = \boldsymbol{u}^T\hat{\boldsymbol{A}}\boldsymbol{u} + \hat{\boldsymbol{b}}^T\boldsymbol{u} + \boldsymbol{f}^T\hat{\boldsymbol{C}}\boldsymbol{f} + \hat{\boldsymbol{d}}^T\boldsymbol{f} + \boldsymbol{f}^T\hat{\boldsymbol{E}}\boldsymbol{u} + \hat{g} \qquad (17)$$

where $\hat{\boldsymbol{A}}$, $\hat{\boldsymbol{C}}$, and $\hat{\boldsymbol{E}}$ are coefficient matrices; $\hat{\boldsymbol{b}}$ and $\hat{\boldsymbol{d}}$ are coefficient vectors; and $\hat{g}$ is a scalar constant. The empirical coefficients of the above cost function depend upon the operating modes so that it is necessary to determine these constants for feasible combinations of discrete control modes. Once again, many mode combinations may be unfeasible or clearly non-optimal under all conditions and therefore need not be considered. Some advantage may also be taken of the symmetry in the quadratic terms of Equation 17. Both $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{C}}$ may be expressed as symmetric matrices, so that only the upper (or lower) triangular coefficients need be determined.

Near-Optimal Control Algorithm

One advantage of the cost function of Equation 17 is that a solution for the optimal control vector that minimizes the cost may be determined analytically by applying the first-order condition for a minimum. Equating the Jacobian of Equation 17 with respect to the control vector to zero and solving for the optimal control (with symmetric $\hat{\boldsymbol{A}}$) gives

$$\boldsymbol{u}^* = \boldsymbol{k} + \boldsymbol{K}\boldsymbol{f} \qquad (18)$$

where

$$\boldsymbol{k} = -\tfrac{1}{2}\hat{\boldsymbol{A}}^{-1}\hat{\boldsymbol{b}}$$

$$\boldsymbol{K} = -\tfrac{1}{2}\hat{\boldsymbol{A}}^{-1}\hat{\boldsymbol{E}}$$

The cost associated with the unconstrained control defined by Equation 18 is

$$J^* = \boldsymbol{f}^T\Theta\boldsymbol{f} + \sigma^T\boldsymbol{f} + \tau \qquad (19)$$

where

$$\Theta = \boldsymbol{K}^T\hat{\boldsymbol{A}}\boldsymbol{K} + \hat{\boldsymbol{E}}\boldsymbol{K} + \hat{\boldsymbol{C}}$$

$$\sigma = 2\boldsymbol{K}\hat{\boldsymbol{A}}\boldsymbol{k} + \boldsymbol{K}\hat{\boldsymbol{b}} + \hat{\boldsymbol{E}}\boldsymbol{k} + \hat{\boldsymbol{d}}$$

$$\tau = \boldsymbol{k}^T\hat{\boldsymbol{A}}\boldsymbol{k} + \hat{\boldsymbol{b}}^T\boldsymbol{k} + \hat{g}$$

The control defined by Equation 18 results in a minimum (rather than maximum) power consumption only if the Hessian of the cost function is a positive-definite matrix, or, in the case of Equation 17, if $\hat{\boldsymbol{A}}$ is a positive-definite matrix. If this condition holds and if the system power consumption correlates with Equation 17, then Equation 18 dictates that the optimal continuous control variables vary as a near-linear function of the uncontrolled variables. However, a different linear relationship applies for each feasible combination of discrete control modes. In the implementation of the algorithm, the minimum cost associated with each mode combination is computed from Equation 19. The costs for each combination are compared in order to identify the minimum. Simple bounds on the continuous control variables may be handled as previously outlined.

Parameter Estimation

The total number of empirical coefficients in Equation 17 that need to be determined for each feasible set of modes is

$$N_{coef} = N_u^2 - N_u(N_u - 1)/2 + N_u + N_f^2 + N_f(N_f - 1)/2 + N_f + N_fN_u + 1 \qquad (20)$$

where N_u is the number of continuous control variables and N_f is the number of uncontrolled variables.

One approach for determining these constants would be to apply regression techniques directly to measured total power consumption. Since the cost function is linear with respect to the empirical coefficients, linear regression techniques may be utilized. A set of experiments could be performed on the system over the expected range of operating conditions. In some cases, the quadratic cost function may only be an accurate index near the optimal control points, so that it would be necessary to repeat the experiments in the vicinity of the control defined by Equation 18. Possibly, the regression could be performed on-line using least-squares recursive parameter updating (Ljung 1983).

Another approach for estimating coefficients of the empirical system model would involve regression to results of a simulation of the system. By using mechanistic models for the individual components, data over a limited range of conditions would be sufficient to calibrate the coefficients of the models. The use of the component-based optimization algorithm as a simulation tool would ensure a good fit near the optimal control points.

Rather than fitting empirical coefficients of the system cost function of Equation 17, the coefficients of the optimal control Equation 18 and the minimum cost function of Equation 19 could be determined directly with regression applied to optimal control results. At a given set of conditions, optimal values of the continuous control variables could be estimated through trial-and-error variations in the system or with the component-based optimization algorithm. Only $(N_f + 1)$ independent conditions would be necessary to determine coefficients of the linear control law given by Equation 18. The coefficients of minimum cost function could then be determined from system measurements with the linear control law in effect. The total number of coefficients to determine with this approach is less than that for direct regression to power measurements.

$$N_{coef} = N_u(N_f + 1) + N_f^2 - N_f(N_f - 1)/2 + N_f + 1 \qquad (21)$$

The disadvantage of this approach is that there is no direct way to handle constraints on the controls.

Application to Chilled-Water Systems

In order to apply this technique, it is necessary to identify both the control variables for which the optimization is to be performed and the uncontrolled variables that affect the system performance through time. Using the component-based optimization algorithm described in this paper, Braun (1988) has shown that the important uncontrolled variables are the total chilled water load and ambient wet-bulb temperature. Additional secondary uncontrolled variables that could be important if varied over a wide range would be the individual zone latent to sensible load ratios and the ratios of individual sensible zone loads to the total sensible loads for all zones.

Braun (1988) has also identified control simplifications that reduce the number of independent control variables and simplify the optimization. These simplifications and their implications are summarized as follows.

1. *Variable-Speed Tower Fans:* Operate all tower cells at identical fan speeds. The only tower control variable is fan speed, which is equivalent to air flow relative to the maximum possible flow.
2. *Multi-Speed Tower Fans:* Increment lowest tower fans first when adding tower capacity. Reverse for removing capacity. With this sequencing, a single independent tower control variable is the relative tower air flow.
3. *Variable-Speed Pumps:* The sequencing of variable-speed pumps should be directly coupled to the sequencing of chillers to give peak pump efficiencies for each possible combination of operating chillers. Multiple variable-speed pumps should be controlled to operate at equal fractions of their maximum speed. With this sequencing arrangement, a single independent control variable for the condenser pump is the flow relative to the maximum possible flow.
4. *Chillers:* Multiple chillers should have identical chilled water set temperatures and the evaporator and condenser water flows for multiple chillers should be divided according to the chillers' relative cooling capacities. The independent chiller control variables are a single chilled water temperature and the number of chillers operating.
5. *Air Handlers:* All parallel air handlers should have identical supply air setpoint temperatures. As a result, only a single setpoint control variable applies to all air handlers.

Using these general results, a reduced set of independent control variables is: 1) supply air set temperature, 2) chilled water set temperature, 3) relative tower air flow, 4) relative condenser water flow, and 5) the number of operating chillers.

The supply air and chilled water setpoints are continuously adjustable control variables. However, since the chilled water flow requirements are dependent upon these controls, there may be discrete changes in power consumption associated with varying these controls, if there are discrete control changes in the pump operation. For the same total flow rate, the overall pumping efficiency changes with the number of operating pumps. However, this has a relatively small effect upon the overall power consumption, so that the discontinuity may be neglected in fitting the overall cost function to changes in the control variables.

For variable-speed cooling tower fans and condenser water pumps, the relative tower air and condenser water flows are continuous control variables. Analogous to the chilled water flow, the overall condenser pumping efficiency changes with the number of operating pumps, so that there may be a discontinuity in the power consumption associated with continuous changes in the overall relative condenser water flow. This discontinuity may be neglected in fitting the overall cost function to changes in this control variable.

With variable-speed pumps and fans, the only significant discrete control variable is the number of chillers operating. A chiller mode defines which of the available chillers are to be on-line. The optimization problem involves determining optimal values of only four continuous control variables for each of the feasible chiller modes. The chiller mode giving the minimum overall power consumption represents the optimum. In order for a chiller mode to be feasible, it must be possible to operate the specified chillers safely within their capacity and surge limits. In practice, abrupt changes in the chiller modes should also be avoided. Large chillers should not be cycled on or off, except when the savings associated with the change are significant.

For fixed-speed cooling tower fans and condenser water pumps, there are only discrete possibilities for the relative flows. One method of handling these variables is to consider each of the discrete combinations as separate modes. However, for multiple cooling tower cells with multiple fan speeds, the number of possible combinations may be large. A simpler approach that works well in this case is to treat the relative flows as continuous control variables and to select the discrete relative flow that is closest to that determined with the continuous optimization. At least three relative flows (discrete flow modes) are necessary for each chiller mode in order to fit the quadratic cost function. The number of possible sequencing modes for fixed-speed pumps is generally much more limited than that for cooling tower fans, with at most two or three possibilities for each chiller mode. In fact, with many current designs, individual pumps are physically coupled with chillers and it is impossible to operate greater or fewer pumps than the number of chillers operating. Thus, it is generally best to treat the control of fixed-speed condenser water pumps with a set of discrete control possibilities, rather than using a continuous control approximation.

The methodology for near-optimal control of a chilled water system may be summarized as follows:

1. Change the chiller operating mode if at the limits of chiller operation (surge or capacity).
2. For the current set of conditions (load and wet-bulb), estimate the feasible modes of operation that would avoid limits on chiller and condenser pump operation.
3. For the current operating mode, determine optimal values of the continuous controls with Equation 18.
4. Determine constrained optimum if controls exceed their bounds.

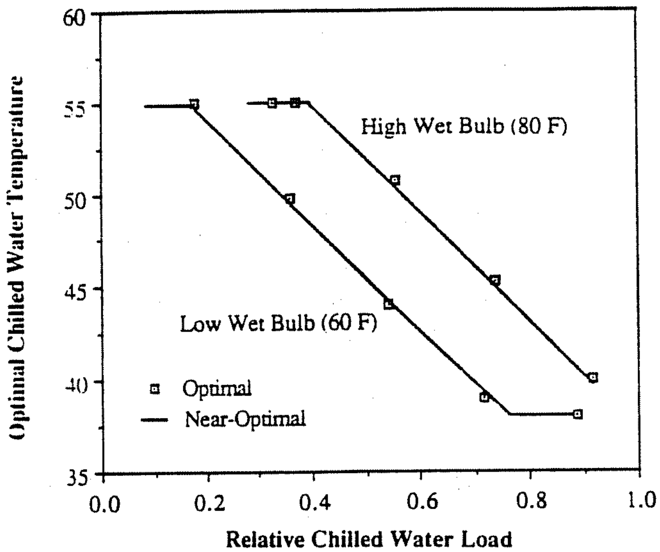

Figure 3 Comparisons of optimal chilled water temperature

5. Repeat steps 3 and 4 for each feasible operating mode.
6. Change the operating mode if the optimal cost (Equation 19) associated with the new mode is significantly less than that associated with the current mode.
7. Change the values of the continuous control variables. When treating multiple-speed fan control with a continuous variable, use the discrete control closest to the optimal continuous value.

If the linear optimal control Equation 18 is directly fit to optimal control results, then there is no direct way of handling the constraints. A simple solution is to constrain the individual control variables as necessary and neglect the effects of the constraints on the optimal values of the other controls and the minimum cost function. The variables of primary concern with regard to constraints are the chilled water and supply air set temperatures. These controls must be bounded for proper comfort and safe operation of the

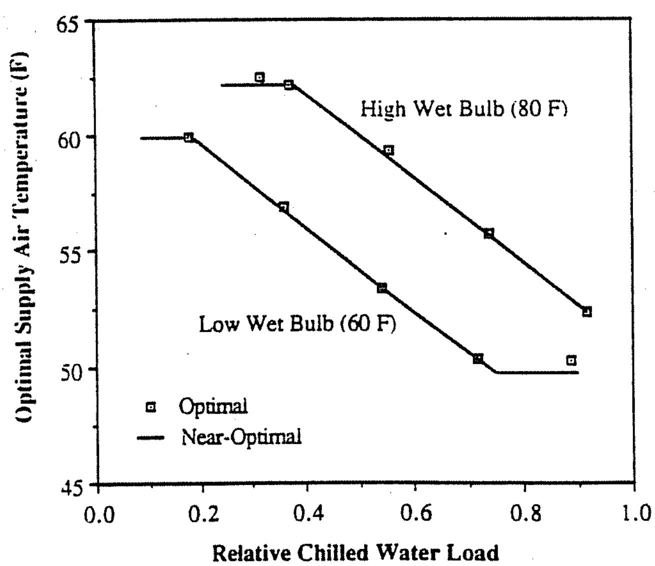

Figure 4 Comparisons of optimal supply air temperature

equipment. On the other hand, the cooling tower fans and condenser water pumps should be sized so the system performs efficiently at design loads and constraints on control of this equipment should only occur under extreme conditions.

There is a strong coupling between optimal values of the chilled water and supply air temperature, so that decoupling these variables in evaluating constraints is generally not justified. However, when either control is operating at a bound, optimization results indicate that the optimal value of the other "free" control is approximately bounded at a value that depends only upon the ambient wet-bulb temperature. As a result, the optimal value of this "free" control (either chilled water or supply air setpoint) is estimated at the load at which the other control reaches its limit. The coupling between optimal values of the chilled water and condenser water loop controls is not as strong, so that interactions between constraints on these variables may be neglected.

COMPARISONS

Braun (1987) correlated the power consumption of the Dallas/Fort Worth (D/FW) airport chiller, condenser pumps, and cooling tower fans with the quadratic cost function given by Equation 17 and showed good agreement. Since the chilled water loop control was not considered, the chilled water setpoint was treated as a known uncontrolled variable. Control of the four tower cells with two-speed fans and the three condenser pumps was treated with continuous control variables. The optimal control determined by the near-optimal Equation 18 also agreed well with that determined using a nonlinear optimization applied to a detailed simulation of the system.

In order to evaluate results of the system-based methodology for a complete system that includes the air handlers, the component-based optimization was applied to an example system, described by Braun (1988). Coefficients of the optimal control and minimum system cost function were fit to results of the component-based optimization over a range of conditions. Figures 3 through 6 show some comparisons between the controls as determined with the component-based and system-based methods for a range of loads, for a relatively low and high ambient wet-bulb temperature (60°F and 80°F).

In Figures 3 and 4, optimal values of the chilled water and supply air temperatures are compared. The near-optimal control equation provides a good fit to the optimization results for all conditions considered. The chilled water temperature was constrained between 38°F and 55°F, while the supply air setpoint was allowed to float freely. Figures 3 and 4 show that for the conditions where the chilled water temperature is constrained, the optimal supply air temperature is also nearly bounded at a value that depends upon the ambient wet-bulb.

Optimal relative cooling tower air and condenser water flow rates are compared in Figures 5 and 6. Although the optimal controls are not exactly linear functions of the load, the linear control equation provides an adequate fit. The differences in these controls result in insignificant differences in overall system power consumption, since, as Braun (1988) has shown, the optimum is extremely flat with respect to these variables. The nonlinearity of the con-

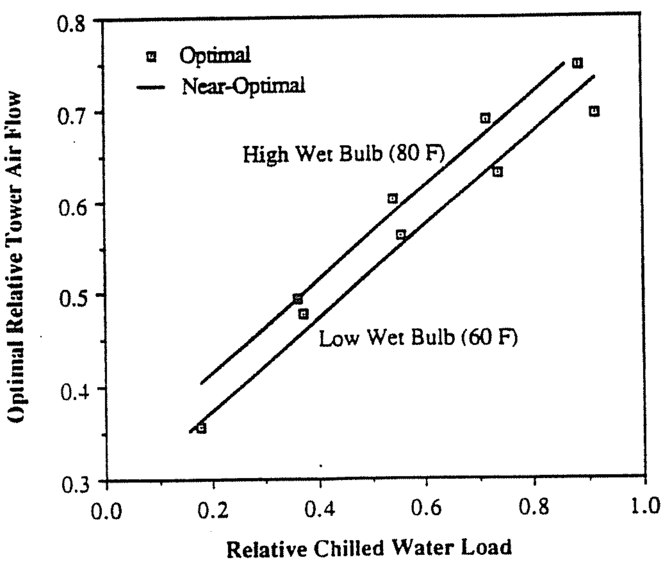

Figure 5 *Comparisons of optimal tower control*

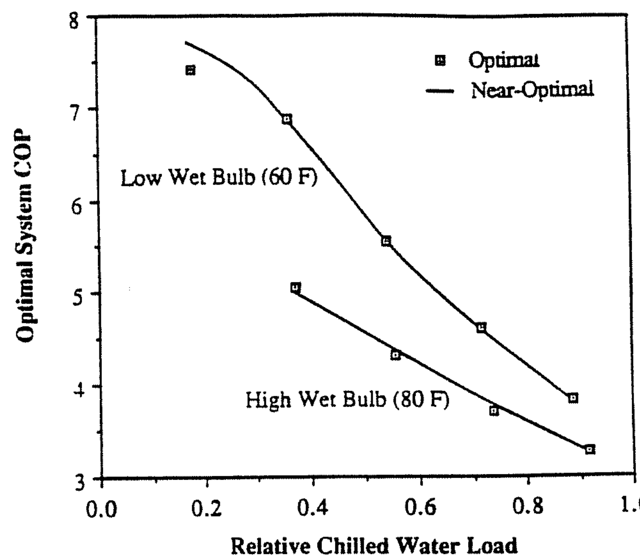

Figure 7 *Comparisons of optimal system performance*

denser loop controls is partly due to the constraints imposed upon the chilled water set temperature. However, this effect is not very significant. Figures 5 and 6 also suggest that the optimal condenser loop control is not very sensitive to the ambient wet-bulb temperature.

In order to determine the optimal discrete mode of operation, it is necessary to have a reasonably accurate model of the minimum cost of operation for each mode. Figure 7 shows a comparison between the optimal system coefficient of performance (COP) determined with the component-based optimization algorithm and the near-optimal quadratic cost function of Equation 19. The differences between the results are very small over a wide range of chilled water loads and ambient wet-bulbs. This model works well, even though it does not explicitly consider the constraints on the chilled water temperature that are exhibited at low and high loads in Figure 3.

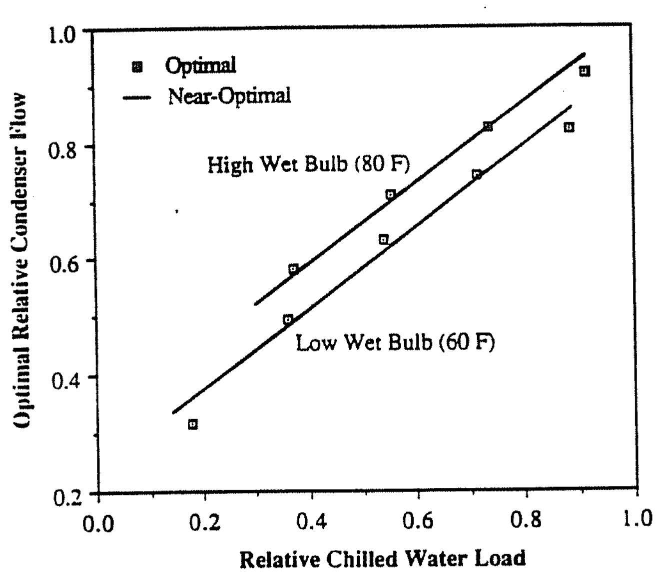

Figure 6 *Comparisons of optimal condenser pump control*

CONCLUSIONS

Two methodologies have been presented for determining optimal control points of chilled water systems. A component-based nonlinear optimization algorithm was developed as a simulation tool for investigating optimal system performance. Results of this algorithm, implemented in a computer program, led to the development of a simpler system-based methodology for near-optimal control.

The advantage of the component-based algorithm over the system-based approach is that it provides a "true" solution to the optimization problem, including any nonlinear constraints. Each of the components in the system is represented as a separate subroutine with its own parameters, controls, inputs, and outputs. Models of components may be either mechanistic or empirical in nature, so that the methodology is useful for evaluating both system design or control characteristics. Braun (1988) applied this methodology to typical chilled water systems to study both design and control issues.

The component-based algorithm takes advantage of the quadratic cost behavior of the components found in chilled water systems in order to solve the optimization problem in an efficient manner. However, in order to utilize this methodology as a tool for on-line optimization, it is necessary to have detailed performance data for each of the individual system components. Results of detailed optimizations identified simplifications that reduced the number of control variables to five and uncontrolled variables to two. The system-based, near-optimal control methodology presented in this paper utilizes an overall system cost function in terms of these variables. This cost function leads to a set of linear control laws for the continuous control variables in terms of the total chilled water load and ambient wet-bulb temperature. Separate control laws are required for each feasible combination of discrete controls and the costs associated with each combination are compared to identify the optimum. The overall procedure is simple enough so as to be implementable either manually or on-line using microcomputers. For manual control applications, charts such as those that appear in Figures 3 through 6 could be used to determine optimal control as a function of load and wet-bulb.

Additional work is necessary in order to apply either of these methodologies to on-line optimal control. In particular, methods for determining parameters of the models need to be investigated. The performance characteristics of the system may change over time, so that it could be necessary to update the model parameters. It is also important to identify an appropriate time interval for making control decisions. There may be inefficiencies associated with changing controls too frequently in response to small changes in the uncontrolled variables. The next step is to test these methodologies as part of an energy management system for controlling an actual system.

NOMENCLATURE

f = vector of uncontrolled variables that affect the operating cost (e.g., load and ambient conditions)

g = vector of equality constraints (e.g., satisfy the zone loads)

h = vector of inequality constraints (e.g., simple bounds on control variables)

J = instantaneous system operating cost (power for all-electric system)

M = vector of discrete control variables (e.g., number of chillers and pumps operating)

u = vector of continuous control variables (e.g., chilled water and supply air temperatures)

x = vector of input stream variables to components (e.g., temperature and mass flow rate)

y = vector of output stream variables from components (e.g., temperature and mass flow rate)

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DISCUSSION

Z. Cumali, Principal, CCB/Cumali Associates, San Francisco, CA: Some of the conclusions presented in this paper do not agree with the results we have observed in our work.

The first impression we wish to correct is the fact that solution of optimal system operation problems does not require drastic specification of models to be applied in real time.

The building in which the simplified rules will result in more energy consumption is described in the reference paper. In summary the system has four cooling towers, three chillers and corresponding pumps, and four major variable-volume systems. All units have variable speed control except the chillers.

1. Setting all supply air temperatures the same does not lead to even near-optimal results when the coil characteristics and loads are quite different from unit to unit, when any of the flows reach minimum or maximum on the air or water side, or if the relative cost of pumping water is larger than moving the air. An example of minimum air condition in one unit shows that power input

Load in Tons	117	42	75	83	kW
Supply air temperature, °F					
Optimal	52	59	52	52	216
Based on rule	59	59	59	59	247

If all temperatures are reduced to 52°F, then the second unit will have less than minimum required air or result in significant overcooling. The 12.6% change is typically what one expects to save from optimization.

2. Setting chilled water temperatures to be the same does not produce near-optimal results again when the chiller characteristics are different and minimum or maximum flow conditions are reached. Using the same cases as above but running two chillers, one large and old and one small and new, we have the following results:

Chilled water temperatures		Power input kW
42	54	343
42	42	363
49	54	375

In the last case we have reached the maximum flow condition in the first chiller and therefore have to limit the supply temperature to 49°F.

As these examples show, the conclusions stated in the two papers quite often result in considerable increases in energy input. It is therefore very important that the authors emphasize the conditions and the assumptions which significantly limit the applicability of their results, e.g., the similarity of performance characteristics of equipment and effects of operational constraints, etc.

Unfortunately, this level of simplification appears to be counterproductive and possibly misleading in that the casual reader is left with the impression that the problem may be solved with a few simple rules. Quite to the contrary, this field of study is complex and will require much research and many more Ph.D. theses from people of the caliber of Dr. Braun and his advisors, who are to be commended and encouraged for continuing the work presented in these papers.

Reference: "Global Optimization of HVAC System Operations in Real Time," Zulfikar Cumali, ASHRAE Transactions, presented in Dallas, TX, 1988.

J.E. Braun: We agree that the "best" solution for determining the optimal control for a given system is to have a detailed model of the complete process that operates in parallel with the actual system. An optimization algorithm is then applied to this model in order to determine the optimal control. The component-based optimization methodology presented in our paper addresses this goal. However, this type of approach requires detailed measurements for each component within the system in order to update parameters of the models so that they adequately match the real performance. Often these measurements are not available or are inaccurate. In addition, the description of the system to be modeled and optimized requires considerable expertise and the effort is beyond the capabilities of most installers of energy management control systems.

Our results indicate that this level of effort is not necessary in order to accomplish near-optimal control of these systems. Our paper develops a set of heuristic rules for good control, along with a system-based approach for determining near-optimal control set points. Mr. Cumali brings up an important point concerning the effects of operational constraints on the applicability of broad-based "rules of thumb." However, common sense heuristics for handling these constraints should give near-optimal results under most circumstances. Mr. Cumali questions two of the rules that were established and provides examples that he believes contradict our results. However, he appears to have misinterpreted and misapplied these rules.

The first example presented by Mr. Cumali concerns the rule of utilizing identical supply air set temperatures for all air handlers with variable-air-volume (VAV) control. The results of his optimal control analysis show three out of four air handlers operating with identical set temperatures with the fourth set at the minimum value necessary to maintain the minimum flow requirement. He then compares the power requirements for this case with setting all set temperatures equal to the one for the air handler that is constrained at its minimum flow. This is not a correct comparison. The rule does not state that the supply air temperatures should be set to the value for the minimum flow air handler. This rule provides a simplification for determining a single optimal discharge air temperature for air handlers with VAV control, rather than having to treat all discharge air temperatures as separate variables. When an air handler is at minimum flow, then it is no longer utilizing VAV control. The flow rate is constant and the discharge air temperature should be adjusted to maintain the room condition. The discharge air temperatures of the remaining VAV controlled air handlers may be considered identical in the optimization. It is interesting to note that the optimal values of discharge air temperature for air handlers under VAV control in Mr. Cumali's example are identical.

The system-based methodology should be applied assuming that no constraints are violated. However, only those air handlers that are not operating at minimum flow should have their setpoints adjusted to the optimized value. Correct application of this procedure will give results that are much closer to the optimum than that for Mr. Cumali's example.

In Mr. Cumali's second example, he compares the optimal power consumption with that for identical chilled water set temperatures when operating two chillers. However, the basis for these comparisons is incorrect. Mr. Cumali compares his optimal (42° F and 54° F) with 1) identical setpoints equal to his lower optimized value of 42° F and 2) identical setpoints equal to his upper optimized value of 54° F (although the first chiller was constrained to operate at 49° F). The choice of identical setpoints for the comparisons is incorrect in that they are not optimized values. Each of the three cases results in significantly different overall chilled water supply temperatures. The advantage of utilizing identical set temperatures is that the optimization process is simplified. The value of this single setpoint cannot be arbitrarily established but must be estimated utilizing an optimization methodology (e.g., as outlined in our paper). For Mr. Cumali's example, an optimal single setpoint would fall somewhere between 42° F and 54° F. In order to handle flow constraints, the system-based methodology should be applied assuming that no constraints are violated. However, only those chillers that are not operating at minimum (or maximum) flow should have their setpoints adjusted to the optimized value.

Mr. Cumali has pointed out that a detailed approach yields reductions in power consumption over simplified procedures. However, this requires a considerable investment of time and money. We disagree with Mr. Cumali's assertion that the level of simplification of our methodology is counterproductive. This field has not evolved to a point where on-line optimal control of chilled water systems using detailed system models is widely applied. Appropriate application of the heuristics and simplified methodology developed for near-optimal control in this study provide significant improvements over current practice.