

## AN IMPROVED DESIGN METHOD FOR SOLAR WATER HEATING SYSTEMS

J. E. BRAUN, S. A. KLEIN and K. A. PEARSON  
 Solar Energy Laboratory, University of Wisconsin-Madison, WI 53706, U.S.A.

(Received 26 April 1982; accepted 30 September 1982)

**Abstract**—In this paper, the  $\bar{\phi}$ ,  $f$ -chart method is extended to open-loop solar hot water heating systems. This method is an improvement over the  $f$ -chart method since it does not impose any restrictions on the water set temperature, water mains temperature, or the preheat tank loss coefficient. The procedure is general for both one-tank and two-tank systems. The modified  $\bar{\phi}$ ,  $f$ -chart method is still applicable to the closed-loop systems for which it was originally developed.

### INTRODUCTION

The most widely used design method for sizing solar water heating systems is the  $f$ -chart method[1]. The  $f$ -charts are correlations of hundreds of detailed simulations performed for a particular system configuration using the computer program TRNSYS[2]. As a result of the assumptions involved in its development, there are restrictions on the application of the method. The original  $f$ -chart for solar water heating was developed for systems with (1) separate preheat and auxiliary tanks, (2) a preheat tank loss coefficient of  $0.42 \text{ W/m}^2 \text{ }^\circ\text{C}$ , (3) no auxiliary tank losses, (4) water set temperatures between  $50$  and  $70^\circ\text{C}$ , and (5) water mains temperatures between  $5$  and  $20^\circ\text{C}$ . Klein and Buckles[3] presented a modification to the method for considering auxiliary tank losses.

Overall, the  $f$ -chart method is an accurate tool for sizing domestic water heating systems with well-insulated tanks. However, it is not valid for comparing the performance of systems with differing amounts of storage tank insulation. The method is also not applicable to many process water heating systems where make-up water enters the system above  $20^\circ\text{C}$  and/or the desired set temperature lies above  $70^\circ\text{C}$ . Table 1 illustrates a comparison between

TRNSYS and  $f$ -chart results for systems characteristic of domestic hot water heating and industrial process heating. The  $f$ -chart method provides very accurate estimates of the monthly solar fractions for the domestic application, but not for the higher temperature system.

A more general and fundamental design procedure called the  $\bar{\phi}$ ,  $f$ -chart method[4] has been developed for closed-loop liquid-based solar heating systems. The term closed-loop implies that a heat exchanger exists between the solar system and load. The  $\bar{\phi}$ ,  $f$ -charts are very similar to the  $f$ -charts with the exception that the temperature requirement of the system is considered through the use of the utilizability (or  $\bar{\phi}$ ) concept[5-7]. The method is general for both low temperature (e.g. space heating) and high temperature (e.g. absorption air conditioning) systems. In addition, storage losses are handled explicitly so that the whole range of practical loss coefficients can be considered.

The  $\bar{\phi}$ ,  $f$ -chart design method in its original form is not applicable for typical open-loop water heating systems that require high temperature delivery water and receive relatively low temperature makeup water. In this paper the  $\bar{\phi}$ ,  $f$ -chart method is extended to these typical water heating systems. The resultant

Table 1. Comparison between TRNSYS and  $f$ -chart solar fractions

Domestic Hot Water Heating (40 C Set and 11 C make-up)			Industrial Process Heating (65 C Set and 50 C make-up)		
	TRNSYS	$f$ -chart		TRNSYS	$f$ -chart
Jan	0.45	0.47	Jan	0.56	0.55
Feb	0.61	0.64	Feb	0.77	0.68
Mar	0.81	0.81	Mar	0.83	0.75
Apr	0.83	0.81	Apr	0.98	0.84
May	0.84	0.90	May	0.99	0.85
Jun	0.93	0.95	Jun	0.98	0.85
Jul	0.96	0.99	Jul	0.99	0.83
Aug	0.97	0.99	Aug	0.98	0.84
Sep	0.93	0.91	Sep	0.98	0.82
Oct	0.76	0.71	Oct	0.94	0.80
Nov	0.51	0.45	Nov	0.76	0.68
Dec	0.36	0.30	Dec	0.60	0.57
Year	0.75	0.74	Year	0.86	0.75

method does not impose any restrictions on the water set temperature, the water mains temperature, the preheat tank loss coefficient or the auxiliary tank loss coefficient. The procedure is further generalized to consider either one-tank or two-tank systems. The modified  $\bar{\phi}, f$ -chart method overcomes the major limitations of the  $f$ -chart method and is still applicable to the closed-loop systems for which it was developed.

#### REVIEW OF THE $\bar{\phi}, f$ -CHARTS

The  $\bar{\phi}, f$ -chart design method was originally developed to allow monthly performance predictions for a class of liquid-based solar systems termed "closed-loop". As shown in Fig. 1, these systems consist of solar collectors, sensible energy storage, and a closed-loop flow circuit in which thermal energy is supplied across a heat exchanger to a load above a specified minimum temperature,  $T_{\min}$ . Initially, it was necessary to assume that the load heat exchanger could transfer energy at an infinite rate whenever the storage temperature was above  $T_{\min}$  and that there were no storage losses. For this situation, the  $\bar{\phi}, f$ -chart correlation expresses the useful energy collection in terms of the system parameters and the maximum energy collection if the collector inlet temperature were equal to  $T_{\min}$ . The resulting expression (eqn (10) of Ref. [3]) is given here in slightly modified form as

$$Q_u = Q_{\max} - a (\exp(bf) - 1)(1 - \exp(cX))L \quad (1)$$

where,

$$Q_{\max} = AF'_R(\bar{\tau}\alpha)\bar{H}_T N \bar{\phi}_{\max}$$

$$a = 0.015(C_s / (350 \text{ kJ/m}^2 \text{ } ^\circ\text{C}))^{-0.76}$$

$$b = 3.85$$

$$c = -0.15$$

$$X = AF'_R U_L (100^\circ\text{C}) \Delta t / L.$$

The first term in eqn (1),  $Q_{\max}$ , is the maximum energy that could be collected by the solar system. The second part of eqn (1) corrects for the fact that energy is not always collected with an inlet temperature equal to the minimum useful value. The degree of correction depends upon several factors related to the system. The parameter  $a$  is an inverse function of the storage capacity. As the storage

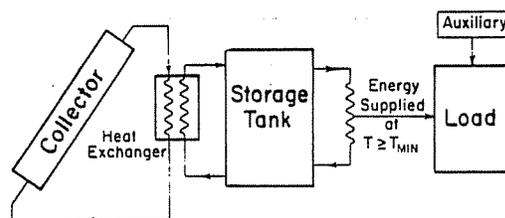


Fig. 1. Schematic of a closed-loop system.

capacity increases, the collector inlet temperature approaches the minimum useful value and the useful gain approaches a maximum.  $X$  is proportional to the collector loss coefficient and is a measure of the sensitivity of system performance to the collector inlet temperature. A collector for which  $X$  is zero (a thermally perfect collector) is unaffected by an increase in collector inlet temperature and the useful energy collection is always maximum. The relationship between the actual and maximum energy collections also depends upon the fraction of the load that is met by solar. The higher the solar fraction, the greater the difference between the collector inlet temperature and  $T_{\min}$ .

In addition to the factors already discussed, energy collection would also be reduced by inefficiencies associated with a finite heat exchanger and storage losses. A finite heat exchanger was handled in the  $\bar{\phi}, f$ -chart method by defining an effective minimum useful temperature,  $T'_{\min}$ , which satisfies the following equation.

$$f = \epsilon C_{\min}^* (T'_{\min} - T_{\min}) / L. \quad (2)$$

$T'_{\min}$  is used in place of  $T_{\min}$  for the evaluation of  $Q_{\max}$  in eqn (1). It is defined by eqn (2) as the temperature necessary to meet the fraction  $f$  of the average load rate. Physically,  $T'_{\min}$  represents a lower limit on the average collector inlet temperature,  $T_c$ , for a particular solar fraction  $f$ . The minimum average collector inlet temperature for a solar fraction,  $f$ , would occur if the solar energy collection and load were both distributed evenly throughout each hour of every day of the month. In this situation, the storage temperature would be a constant equal to  $T'_{\min}$ . In reality,  $T_c$  is generally higher than  $T'_{\min}$  for two reasons. The storage temperature is directly influenced by the level of radiation, which is not uniform over time. There may also be periods when the rate at which energy can be delivered by the solar system exceeds the load. During these periods, energy delivery must be tempered in some way. For a space heating system, this is accomplished by cycling the system on and off to maintain a fixed room temperature. In a process water heating application in which water is demanded at a particular temperature, a mixing valve might be employed. Regardless of the method, energy tempering generally results in higher storage temperatures during the hours of energy collection than if energy were always removed at its maximum rate. The difference between the minimum temperature necessary to meet a fraction  $f$  of the average load rate and the average collector inlet temperature is related to the amount of tempering required. In other words,  $T'_{\min}$  is fixed by the value of  $f$  and  $T_c$  increases with the degree of tempering. This effect is in turn related to value of  $f$ . For a system that meets a high fraction of the load, tempering will increase the difference between  $T'_{\min}$  and  $T_c$  (i.e. the difference between the actual energy collection and  $Q_{\max}$ ).

Storage losses were dealt with in the  $\bar{\phi}, f$ -chart method by defining an average storage temperature for losses that satisfies

$$f = (Q_u - (UA)(T_i - T_{env})\Delta t)/L \quad (3)$$

Equation (3) is simply a monthly energy balance on the storage medium neglecting the monthly change in internal energy. The average storage temperature used for losses is taken to be the mean of the average collector inlet temperature and  $T'_{min}$  or

$$T_i = (T_c + T'_{min})/2 \quad (4)$$

The average collector inlet temperature,  $T_c$ , is determined by calculating  $\bar{\phi}$ , the average daily collector utilizability as

$$\bar{\phi} = Q_u / (AF'_R(\bar{\tau}\alpha)\bar{H}_T N) \quad (5)$$

The average critical level and thus average inlet temperature can be found from correlations for  $\bar{\phi}$  [5-7].

The  $\bar{\phi}, f$ -chart method in its final form involves the solution of eqns (1)-(5) for  $T'_{min}$  and therefore solar fraction,  $f$ . This requires an iterative technique such as direct substitution or a secant method.

#### APPLICATION OF THE $\bar{\phi}, f$ -CHARTS TO OPEN-LOOP SYSTEMS

An "open-loop" system differs from a closed-loop system in that heat is extracted from storage to the load via a mass flow of fluid that ultimately leaves the system. Figure 2 illustrates an example of a typical open-loop water heating system. Water is drawn from the system at a desired temperature,  $T_{set}$  and replaced with make-up water at a temperature  $T_{mains}$ . An auxiliary tank provides additional energy, if necessary, to bring the solar preheated water up to the set temperature. If the preheated water temperature is greater than the desired temperature, a mixing valve tempers the delivered water to  $T_{set}$ .

It is not obvious why the  $\bar{\phi}, f$ -chart design method is not applicable to water heating systems. First of all, energy is transferred to a load above a minimum temperature equal to the water make-up temperature,  $T_{mains}$ . Secondly, an open flow of fluid is thermally equivalent to having a heat exchanger with an effectiveness of 1 and equal hot side and cold side capacitance rates. Thus, one could theoretically construct closed-loop and open-loop systems which were thermally identical.

The problem in applying the  $\bar{\phi}, f$ -chart method to open-loop systems stems from the nature of the energy required by the load. The load of a system can be characterized by its energy requirement, the minimum useful temperature, and the temperature necessary to fully meet the load (e.g. the set temperature for water heating). For a closed-loop system with a heat exchanger that can transfer heat at an infinite rate, the minimum useful temperature and the tem-

perature necessary to meet the load are one and the same. This is the situation for which the original method was developed. Water heating systems, however, generally require high delivery temperatures for relatively low make-up temperatures. The load of the water heating system is therefore characterized by two temperatures, rather than the one temperature,  $T_{min}$ , for which the  $\bar{\phi}, f$ -charts were developed.

The difference between the temperature necessary to meet the load and the minimum useful temperature affects the degree of energy tempering a system requires. For a system with an infinite load flow rate (i.e. no temperature rise required by the load), the system must temper delivered energy whenever the storage temperature is above the minimum useful value. As the nature of the load changes toward lower flow rates with higher temperature requirements, the storage reaches the required delivery temperature less of the time.

Since the nature of the load requirement affects the degree of tempering, it also affects the relationship between the actual energy collection and  $Q_{max}$ . For a particular solar fraction, the difference between  $Q_u$  and  $Q_{max}$  will be greater for a system that must temper delivered energy more often. In order to include this effect in the  $\bar{\phi}, f$ -chart method and make the procedure general for both open- and closed-loop systems, a dimensionless parameter,  $Z$ , will be defined as

$$Z = L / (C_L(100^\circ\text{C})) \quad (6)$$

For an open-loop water heating system,  $C_L$  is the product of the monthly mass of water usage and the specific heat,  $MC_p$ . For a closed-loop system with a load heat exchanger,  $C_L$  is the product of the heat exchanger effectiveness and the minimum monthly capacitance flowing through the heat exchanger,  $\epsilon C_{min}^*$ .  $Z$  is a measure of the temperature rise required by the load. A system for which  $Z$  is zero has an infinite load flow rate requiring a zero temperature rise.

Following the formulations of Ref. [4], the relationship between the actual energy collection and  $Q_{max}$  was recorrelated using the form of eqn (1) to include the parameter  $Z$ . The resulting expression for  $Q_u$  is general for both open- and closed-loop systems and can be represented as

$$Q_u = Q_{max} - a(\exp(bf) - 1)(1 - \exp(cX)) \times \exp(dZ)L \quad (7)$$

where  $a, b, c, f, X$  and  $L$  are defined as in eqn (1) and  $d$  is a dimensionless constant equal to  $-1.959$ . The solar fraction for a closed-loop system was given in eqn (2), while for an open-loop water heating system it is

$$f = MC_p(T'_{min} - T_{mains})/L \quad (8)$$

As  $Z$  approaches zero (i.e. infinite load flow rate) eqn (7) reduces to the original  $\bar{\phi}, f$ -chart correlation given

in eqn (1).  $Z$  only becomes an important parameter for systems that require temperatures significantly greater than  $T_{\min}$  to meet the load. Nonetheless, eqn (7) is general over the entire range of values of  $Z$ .

In order to consider storage losses, it is necessary to calculate the average storage temperature,  $T_s$ . In the original method  $T_s$  was taken to be the average of  $T_c$  and  $T'_{\min}$ . This is not necessarily a good estimate of  $T_s$ , but if storage losses are small relative to energy collection, then only a minor error is introduced. However, the error may be significant if the tank is not well insulated or if a very high temperature application is being considered. In an effort to improve the  $\bar{\phi}, f$ -chart method, a correlation for  $T_s$  was developed. The form of the correlation is very similar to eqn (7) and is given as

$$T_s = T'_{\min} + g (\exp(kf) - 1) \exp(hZ) \quad (9)$$

where,

$$g = (0.2136^\circ\text{C})(C_s/(350 \text{ kJ/m}^2 \text{ }^\circ\text{C}))^{-0.704}$$

$$h = -4.002$$

$$k = 4.702$$

This expression eliminates the need for calculating the average collection temperature from a  $\bar{\phi}$  correlation.

The "modified"  $\bar{\phi}, f$ -chart method for open-loop systems involves the iterative solution of eqns (3), (7)–(9) for  $T'_{\min}$ . The solar fraction,  $f$  is then given by eqn (8). The new method is also general for closed-loop systems except that the solar fraction is defined by eqn (2) rather than eqn (8). This procedure is demonstrated in the Appendix.

#### COMPARISONS

Results of hundreds of TRNSYS simulations of the system depicted in Fig. 2 were compared with design method results for a variety of different loads, collector types, storage sizes, and preheat tank insulation thicknesses. Table 2 gives the upper and lower bounds on the system parameters employed. The collector areas and loads used in these comparisons are characteristic of small domestic systems. The method, however, is also applicable to large systems if the collector area, storage volume, load, and all conductances in the system are equally scaled. The simulations incorporated a daily usage profile devel-

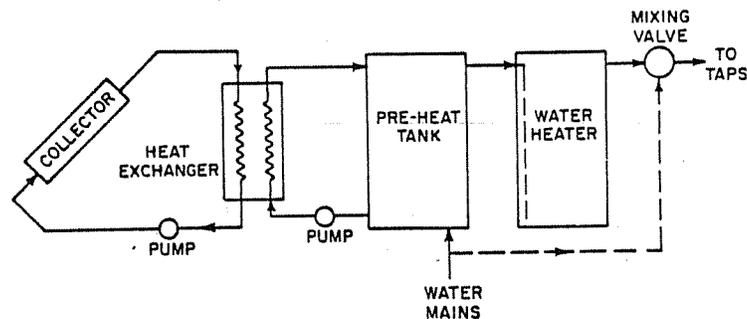


Fig. 2. Schematic of an open-loop system.

Table 2. Range of parameters for TRNSYS and design method comparisons

Parameter	Lower Bound	Upper Bound
<b>Collector:</b>		
$F_R(\bar{\tau}\alpha)$	0.70	0.85
$F_R U_L$	2.78 $\text{W/m}^2\text{-C}$	8.33 $\text{W/m}^2\text{-C}$
Orientation	slope=latitude	slope=latitude
<b>Storage:</b>		
$C_s$	175 $\text{kJ/m}^2\text{-C}$	700 $\text{kJ/m}^2\text{-C}$
$U_t$	0.25 $\text{W/m}^2\text{-C}$	1.5 $\text{W/m}^2\text{-C}$
$T_{\text{env}}$	20 C	20 C
<b>Load:</b>		
$T_{\text{mains}}$	5 C	75 C
$T_{\text{set}}$	25 C	90 C
Water Usage	20 liters/day- $\text{m}^2$	1800 liters/day- $\text{m}^2$

oped by Rand Corporation Survey[8] for a "typical" residence. Previous studies[3, 4] have shown that the effect of the daily load profile on system performance is small. The preheat tank was assumed to be fully mixed. This assumption was also employed in the development of the  $f$ -chart,  $\bar{\phi}$ ,  $f$ -chart, and modified  $\bar{\phi}$ ,  $f$ -chart design methods. It is possible to design a system that maintains a significant degree of stratification if the collector-fluid flowrate is low. For typical flowrates, on the order of  $0.0151/s\text{-m}^2$ , stratification improves the performance of a water heating system by at most 5 per cent. The degree of improvement is specific to the particular tank design. Auxiliary tank losses were set to zero for these comparisons. In the next section, a method is presented for handling these losses. The meteorological data used in the analyses were for typical years (TMY) in Madison (WI), Charleston (SC), Albuquerque (NM) and Seattle (WA). The design method used incident radiation and utilizability determined using the methods of Klein[7, 9].

Excellent agreement was found between TRNSYS and the modified  $\bar{\phi}$ ,  $f$ -chart design method for the range of parameters given in Table 2. The total standard deviation for a comparison of monthly solar fractions was equal to 0.030, while the standard deviation for annual results was 0.019. The maximum difference between annual results was 0.05. In contrast, comparisons between the original  $f$ -chart method with the same simulation results yield monthly and annual standard deviations of 0.065 and 0.055. The maximum difference between annual results was 0.15. Figures 3 and 4 give the annual comparisons between the design methods and TRNSYS.

These results are not meant to imply that  $f$ -chart is not an accurate tool for predicting the performance of water heating systems. The accuracy of this method suffers when applied outside the range of parameters for which it was developed. The modified

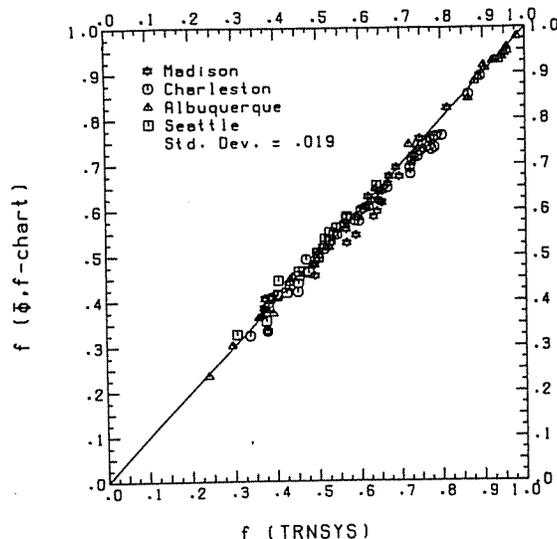


Fig. 3.  $\bar{\phi}$ ,  $f$ -chart and TRNSYS comparisons.

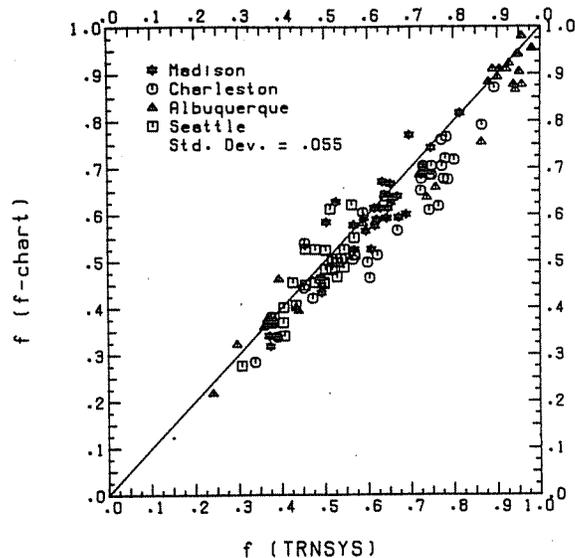


Fig. 4.  $f$ -chart and TRNSYS comparisons.

$\bar{\phi}$ ,  $f$ -chart method, on the other hand, is accurate over a much wider range of system parameters. This method, however, is more complicated to apply than the  $f$ -chart method. When applicable, the  $f$ -chart method can still be used to predict the performance of domestic water heating systems.

#### AUXILIARY TANK LOSSES

The comparisons in the previous section were all performed assuming no energy losses from the auxiliary tank. Buckles and Klein[3] have shown that these losses can be considered as part of the load. In other words, the total load to use in a design method is assumed to be the sum of the energy requirement due to the water usage and the auxiliary tank losses. Table 3 illustrates a comparison between the results of two simulations which supports the validity of this assumption. In one case (Case A), auxiliary losses were considered, representing 25 per cent of the energy requirement associated with the water usage. For Case B there were no auxiliary losses and the water usage was increased such that the load requirement was equal to the load requirement plus auxiliary losses from Case A. The agreement between the two cases is good, even though the auxiliary losses represent a significant fraction of the load. For more well-insulated auxiliary tanks, the differences would be less. In general, to consider auxiliary tank losses in the modified  $\bar{\phi}$ ,  $f$ -chart or  $f$ -chart methods, the water usage should be adjusted such that the load is equal to the sum of the energy due to usage and auxiliary tank losses.

#### SINGLE TANK SYSTEMS

All of the results and discussion thus far have been concerned with solar water heating systems that utilize separate preheat and auxiliary tanks. Buckles and Klein[3] have shown that single tank systems that incorporate an auxiliary heater in the upper part

Table 3. Comparison between the performance of systems with and without auxiliary tank losses

System Parameters		
<b>Case A</b>		
$A = 4 \text{ m}^2$	$F'_R U_L = 4.17 \text{ W/m}^2\text{-C}$	
	$F'_R(\bar{\tau}\alpha) = 0.75$	
	$T_{\text{env}} = 20 \text{ C}$	
$C_s = 200 \text{ kJ/m}^2\text{-C}$	$(UA)_c = 0.80 \text{ W/C}$	
	$(UA)_a = 4.85 \text{ W/C}$	
Water Usage = 300 liters/day	$T_{\text{set}} = 50 \text{ C}$	
	$T_{\text{mains}} = 10 \text{ C}$	
<b>Case B</b>		
same as Case A except; Water Usage = 375 liters/day		
	$(UA)_a = 0 \text{ W/C}$	
Solar Fractions		
	Case A	Case B
Jan	0.27	0.28
Feb	0.36	0.38
Mar	0.52	0.54
Apr	0.53	0.55
May	0.58	0.61
Jun	0.65	0.68
Jul	0.71	0.74
Aug	0.70	0.73
Sep	0.62	0.65
Oct	0.46	0.48
Nov	0.30	0.31
Dec	0.21	0.22
Year	0.49	0.52

Table 4. Design parameters for industrial water heating system

Collector:	
A	60 m <sup>2</sup>
$F'_R(\bar{\tau}\alpha)$	.75
$F'_R U_L$	4.17 W/m <sup>2</sup> -C
Orientation	slope = latitude
Storage:	
$C_s$	350 kJ/m <sup>2</sup> -C
$(UA)_c$	10 W/C
$(UA)_a$	5 W/C
$T_{\text{env}}$	20 C

of the tank may perform slightly better than two-tank systems. This is primarily due to a reduction in losses as a result of the smaller surface area associated with one tank. It is possible to consider one-tank systems in either the  $f$ -chart or modified  $\bar{\phi}$ ,  $f$ -chart design methods by assuming an effective two-tank system. The auxiliary tank of this imaginary system is assumed to have a  $(UA)$  of the portion of the single tank above the heating element. The preheat tank is considered to have the same total capacitance and loss coefficient as the single tank. The difference in the simulated performance of one-tank and effective two-tank (as defined above), systems was found to be less than 2 per cent. This indicates that both the  $f$ -chart and the modified  $\bar{\phi}$ ,  $f$ -chart methods can be used in this manner to size single-tank systems.

### CONCLUSIONS

The  $\bar{\phi}$ ,  $f$ -chart method has been extended to apply to open-loop water heating systems. The resulting procedure overcomes the major limitations of the  $f$ -chart method concerning set temperature, mains temperature and preheat tank loss coefficient. The method is applicable for either one-tank or two-tank systems. The results obtained with this procedure agree with detailed TRNSYS simulation results within 2 per cent on an annual basis and 3 per cent for monthly comparisons. Although more general, this method is also more complicated to apply than the  $f$ -chart method. It is therefore recommended that the  $f$ -charts be used to estimate the performance of solar water heating systems when applicable. For high temperature industrial applications or if it is of interest to study the effect of the storage tank loss coefficient on system performance, the modified  $\bar{\phi}$ ,  $f$ -chart method should be employed.

### NOMENCLATURE

- $A$  collector area
- $C_L$   $eC_{\text{min}}^*$  for closed-loop systems and  $MC_p$  for open-loop systems
- $C_s$  the capacitance per unit effective collector area ( $AF_R$ ) of storage
- $f$  fraction of the load,  $L$ , that is met by the solar system
- $F'_R$  collector overall heat removal efficiency factor corrected for the presence of a collector-storage heat exchanger as outlined in Ref. [10]
- $F'_R U_L$  negative of the slope of the collector efficiency curve corrected for the presence of a collector-storage heat exchanger as outlined in Ref. [10]
- $F'_R(\bar{\tau}\alpha)$  intercept of the collector efficiency curve corrected for the presence of a collector-storage heat ex-

changer and non-normal solar incidence as outlined in Ref. [10]

- $\bar{H}$  monthly-average daily horizontal radiation  
 $\bar{H}_T$  monthly-average daily incident radiation on the collector surface  
 $L$  monthly total energy load; for a water heating system this is defined as the sum of the energy requirement due to water usage and the energy lost by the auxiliary tank  
 $MC_p$  monthly total load capacitance of a water heating system; product of monthly mass of water usage and the specific heat of water  
 $N$  number of days in the month  
 $T_s$  monthly-average storage temperature  
 $T_c$  monthly-average collector inlet temperature  
 $T_{env}$  environmental temperature for storage losses  
 $T_{mains}$  temperature of the make-up water  
 $T_{min}$  minimum useable temperature for a solar system; for an open-loop water heating system this is equal to  $T_{mains}$   
 $T'_{min}$  temperature necessary to meet a fraction  $f$  of the average load rate  
 $T_{set}$  set temperature for a water heating system  
 $Q_{max}$  maximum monthly energy collection if the collector inlet temperature were always equal to  $T'_{min}$  ( $T_{min}$  for infinite heat exchange)  
 $Q_u$  monthly useful energy collection  
 $U_i$  preheat tank loss coefficient  
 $(UA)_a$  auxiliary tank conductance for heat loss  
 $(UA)_c$  preheat tank conductance for heat loss  
 $\Delta t$  number of hours in the month  
 $\epsilon C^*_{min}$  product of the heat exchanger effectiveness and the minimum of total monthly capacitances flowing through the heat exchanger  
 $\bar{\phi}$  monthly-average daily utilizability; fraction of the total radiation absorbed by the collector that is useful  
 $\bar{\phi}_{max}$  maximum monthly-average daily utilizability occurring for a collector inlet temperature equal to  $T'_{min}$

## REFERENCES

- W. A. Beckman, S. A. Klein and J. A. Duffie, A design procedure for solar heating systems. *Solar Energy* 18, 113 (1976).
- S. A. Klein *et al.*, *TRNSYS—A Transient Simulation Program*. Users Manual, Engineering Experiment Station Report No. 38-11, Solar Energy Laboratory, University of Wisconsin-Madison (1981).
- W. E. Buckles and S. A. Klein, Analysis of solar domestic hot water heaters. *Solar Energy* 25, 417 (1980).
- S. A. Klein and W. A. Beckman, A general design method for close-loop solar energy systems. *Solar Energy* 22, 269 (1979).
- S. A. Klein, Calculation of flat-plate collector utilizability. *Solar Energy* 21, 393 (1978).
- M. Collares-Pereira and A. Rabl, Simple procedure for predicting long-term average performance of non-concentrating and concentrating solar collectors. *Solar Energy* 23, 235 (1979).
- J. C. Theilacker and S. A. Klein, Improvements in the utilizability relations. *Proc. AS/ISES Meet.*, Phoenix, Vol. 3.1, p. 271 (1980).
- J. J. Mutch, *Residential Water Heating, Fuel Consumption, Economics and Public Policy*. RAND Report R1498 (1974).
- S. A. Klein and J. C. Theilacker, An algorithm for calculating monthly-average radiation on inclined surfaces. *J. Solar Energy Engng* 103, 29 (1981).
- J. A. Duffie and W. A. Beckman, *Solar Engineering of Thermal Processes*. Wiley-Interscience, New York (1980).

## APPENDIX

## Example

A solar energy system is to be designed to provide hot water for cleaning purposes in an industrial meat packing plant located in Madison-Wisconsin. A schematic of the proposed system appears in Fig. 2. Energy is available from other processes in the plant allowing a make-up water temperature of 40°C. The desired temperature and daily usage of the cleaning water are 50°C and 10,000 l. The proposed design parameters are given in Table 4. Estimate the performance of this system during the month of May.

The monthly-average horizontal radiation,  $\bar{H}$ , and ambient temperature are 19.32 MJ/day/m<sup>2</sup> and 16.1°C. The monthly-average incident radiation,  $\bar{H}_T$  is determined from Ref. [9] as 17.86 MJ/day/m<sup>2</sup>. The load is the sum of the energy requirement due to the water usage and the auxiliary tank losses. Thus,

$$\begin{aligned} L &= MC_p(T_{set} - T_{mains}) + (UA)_a(T_{set} - T_{env})\Delta t \\ &= (10000 \text{ l./day})(1 \text{ kg/l.})(0.00419 \text{ MJ/Kg } ^\circ\text{C}) \\ &\quad \times (10^\circ\text{C})(31 \text{ days}) + (5\text{W/C})(30^\circ\text{C})(24 \text{ hr/day}) \\ &\quad \times (31 \text{ days})(0.0036 \text{ MJ/hr/W}) = 13,391 \text{ MJ.} \end{aligned}$$

The water usage that should be used in this method is that which would yield a load of 13,391 MJ with no auxiliary losses or

$$MC_p = L/(T_{set} - T_{mains}) = 1339 \text{ MJ/}^\circ\text{C.}$$

The design procedure involves the iterative solution of eqns (3), (7)–(9) for  $T'_{min}$ . The most straight-forward method for solving these equations (although not necessarily the most efficient) is by direct substitution. The following steps illustrate this procedure.

Step 1. Initial guess of  $T'_{min}$  and  $f$ 

A good initial guess for  $T'_{min}$  would be the solution from the previous month. Since this is not available for this example, a guess of  $T'_{min}$  will be taken as 48°C. For this value, eqn (8) gives a solar fraction of 0.80.

Step 2. Estimation of  $Q_u$ 

In order to estimate  $Q_u$  using eqn (7), it is first necessary to perform some preliminary calculations. From Ref. [7], with  $T'_{min}$  equal to 48°C and an ambient temperature of 16.1°C,  $\bar{\phi}_{max}$  is found to be 0.604. The values of  $Q_{max}$ ,  $Z$  and  $X$  are

$$\begin{aligned} Q_{max} &= (60 \text{ m}^2)(0.75)(17.86 \text{ MJ/day})(31 \text{ days})(0.604) \\ &= 15048 \text{ MJ} \end{aligned}$$

$$\begin{aligned} X &= (60 \text{ m}^2)(4.17 \text{ W/m}^2 \text{ } ^\circ\text{C})(100^\circ\text{C})(744 \text{ hr}) \\ &\quad \times (0.0036 \text{ MJ/hr/W})/(13391 \text{ MJ}) \\ &= 5.0 \end{aligned}$$

$$Z = (13391 \text{ MJ})/(1339 \text{ MJ/C}(100^\circ\text{C})) = 0.10$$

With these values eqn (7) gives an estimate of  $Q_u$  of

$$Q_u = 15048 \text{ MJ} -$$

$$0.015 (\exp(3.85(0.8)) - 1)(1 - \exp(-0.15(5)))$$

$$(\exp(-1.959(0.1)))(13391 \text{ MJ}) = 13,239 \text{ MJ.}$$

Step 3. New estimate of  $f$  and  $T'_{min}$ 

The average storage temperature to be used for losses is computed using eqn (9) as

$$\begin{aligned} T &= 48^\circ\text{C} + (0.2136^\circ\text{C})(\exp(4.702(0.8)) - 1) \\ &\quad \times \exp(-4.002(0.1)) \\ &= 54.01^\circ\text{C.} \end{aligned}$$

A new estimate of the solar fraction is determined with eqn (3) as

$$f = (13,239 \text{ MJ} - (10 \text{ W/C})(0.0036 \text{ MJ/hr/W})(34.01^\circ\text{C}) \\ \times (744 \text{ hr})) / (13391 \text{ MJ}) = 0.921.$$

From eqn (8) a new estimate of  $T'_{\min}$  can be determined as

$$T'_{\min} = T_{\text{mains}} + fL / (MC_p) = 49.21^\circ\text{C}.$$

Steps 2 and 3 could be repeated with these new values of  $f$  and  $T'_{\min}$ . A better estimate of  $T'_{\min}$ , however, is the average of the most recent determination of  $T'_{\min}$  (i.e.  $49.21^\circ\text{C}$ ) and the previous estimate ( $48^\circ\text{C}$ ) or  $48.61^\circ\text{C}$ . This value gives an  $f$  of 0.861. Using these values, the next iteration (repeating steps 2 and 3) yields values of  $T'_{\min}$  and  $f$  of  $48.62^\circ\text{C}$  and 0.862. These values are not significantly different than the previous ones, so the procedure may end here.