

A METHOD FOR ESTIMATING THE PERFORMANCE OF PHOTOVOLTAIC SYSTEMS

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(Received 14 June 1982; revision received January 1984; accepted 27 April 1984)

Abstract—A method is presented for predicting the long-term average performance of photovoltaic systems having storage batteries and subject to any diurnal load profile. The monthly-average fraction of the load met by the system is estimated from array parameters and monthly-average meteorological data. The method is based on radiation statistics, and utilizability, and can account for variability in the electrical demand as well as for the variability in solar radiation.

1. INTRODUCTION

This paper is devoted to a design procedure for predicting the performance of photovoltaic systems, using monthly-average meteorological data and easily measured design parameters. The method places as few restrictions as possible on the precise configuration of the photovoltaic system and on the time distribution of the load, and should, therefore, be suitable for a wide variety of applications.

The simplest type of system to analyze is one in which all electricity produced can be used immediately for the task at hand. Evans[1] has developed both computational and graphical design methods for determining the average electrical output of a photovoltaic array, taking into account the temperature dependence of photocell efficiency.

The problem becomes more difficult if the photovoltaic array sometimes produces energy in excess of the load. In addition to finding the total energy produced by the array, the designer must estimate how much of this energy can be applied directly to the load. Clearly this will depend on the time distributions of the load and the energy produced by the array.

Depending on the system, electrical energy in excess of the load may be dissipated, sold to a utility, or stored for later use. For systems incorporating a dedicated storage battery, an additional problem arises. Knowledge of the amount of energy available for storage is insufficient, since some of this energy may have to be dissipated when the storage battery is fully charged. In this case, the useful fraction of the excess energy must also be estimated.

Evans *et al.*[2, 3] have addressed all of the problems outlined above by preparing a set of graphs for predicting the solar load fraction supplied by a photovoltaic system with any energy storage capacity. Separate graphs are provided for each of 41 diurnal load profiles. The accuracy of this method appears to be quite satisfactory. However, given the widespread availability of microcomputers, an analytical method suitable for computer implementation

is desirable. The purpose of this paper is to present such a method.

2. SYSTEM PERFORMANCE WITHOUT STORAGE

Solar radiation utilizability, designated by ϕ , is a solar radiation statistical term defined as the fraction of the total radiation incident on a surface which exceeds a specified intensity called the critical level. The utilizability approach was originally developed [4, 5] as a method for predicting the long-term average performance of flat plate solar collectors. In this context, the critical radiation level is defined as the radiation intensity at which thermal losses from the collector are equal to thermal gains, and the net useful energy collection rate is zero. The daily utilizability function, designated by $\bar{\phi}$, is defined as the fraction of the total daily incident radiation which exceeds the critical level. For flat plate collectors, $\bar{\phi}$ represents the useful fraction of incident radiation. Several algorithms exist for calculating $\bar{\phi}$.

Siegel *et al.*[6] have applied the daily utilizability method to the analysis of photovoltaic systems. When the critical level is defined as the radiation intensity at which electrical production precisely matches the load, $\bar{\phi}$ represents the fraction of power production which exceeds the load. However, the daily utilizability method requires a constant daily critical level, so the method is applicable only to systems which experience a constant load during daylight hours.

A method published recently[7] allows the utilizability function to be evaluated on a monthly-average hourly basis, rather than monthly-average daily basis. This hourly utilizability algorithm allows the method of Siegel *et al.* to be extended to accommodate loads which vary from hour to hour. The procedure is outlined below.

For the i th hour of the day, the average electrical output of the array is

$$E_i = A_c \bar{J}_{T,i} \bar{\eta} \quad (1)$$

where A_c is the area of the photocells in the array, $\bar{I}_{T,i}$ is the monthly-average hourly radiation incident on the array, and $\bar{\eta}$ is the average efficiency of the array, including the transmittance of any protective cover and the efficiency of any power conditioning equipment. Procedures for estimating \bar{I}_T from monthly-average daily weather data and geometric considerations are given by Duffie and Beckman[8]. It is possible to estimate a monthly-average hourly array efficiency using monthly-average hourly values of solar radiation and ambient temperature. However, sensitivity of array efficiency to array temperature is such that this additional complexity is not justified. It is assumed here that the monthly-average daily photovoltaic conversion efficiency, as calculated by the method of Evans[1], can be used for each hour of the day.

A critical level can be defined as the radiation intensity, $I_{c,i}$, at which the rate of electrical energy production is equal to L_i , the monthly-average load for that hour

$$I_{c,i} = \frac{L_i}{A_c \bar{\eta}} \quad (2)$$

The fraction of insolation received at a rate exceeding this level, ϕ_i , can be estimated by the correlation of Clark *et al.*[7]. The monthly-average hourly electrical energy in excess of the load, $D_{o,i}$, is expressed as

$$D_{o,i} = E_i \phi_i \quad (3)$$

and the energy sent directly to the load is

$$E_{L,i} = E_i(1 - \phi_i) \quad (4)$$

Monthly-average daily results are obtained by summing hourly quantities over all hours of the day

$$\bar{D}_o = \frac{1}{24} \sum_i D_{o,i} \quad (5)$$

and

$$\bar{E}_L = \frac{1}{24} \sum_i E_{L,i} \quad (6)$$

The monthly-average fraction of the load supplied by the system without storage is

$$f_o = \bar{E}_L / \bar{L} \quad (7)$$

The hourly utilizability method implicitly assumes that the instantaneous electrical demand is always equal to the monthly-average hourly electrical demand (i.e. that $I_{c,i}$ is constant). In actuality the demand may vary from minute to minute within an hour, and from day to day within a month. Load variability will cause ϕ_i to be underpredicted. However, investigations by the authors and by others[2] reveal that the effects of fluctuations in the load are

surprisingly slight. For most purposes, knowledge of the average hourly loads is quite sufficient to characterize system performance.

3. ESTIMATION OF THE EFFECT OF ELECTRICAL STORAGE

The performance of a photovoltaic system without energy storage, given any load profile, can be estimated as described above. \bar{D}_o , defined by eqn (5), represents that energy which cannot be sent directly from the array to the load, but must be dissipated, sold or stored. In this section, a correlation is developed for estimating Δf_s , defined as the increase in the solar load fraction due to the addition of storage

$$\Delta f_s = f - f_o \quad (8)$$

where f is the solar load fraction met by the system with storage, and f_o is the load fraction met by a equivalent system with no storage.

If all of the excess energy of a system without storage could be stored, the resulting value of Δf_s would be \bar{D}_o / \bar{L} multiplied by the battery storage efficiency. This combination of parameters is designated by d_o .

$$d_o = \eta_b \bar{D}_o / \bar{L} \quad (9)$$

Consider the physical constraints which limit the possible values of Δf_s . If d_o is much less than B_c / \bar{L} , the ratio of the storage capacity to the average load, then the battery is never filled, and energy dissipation from the system with storage is zero. Regardless of the storage capacity, this limiting case occurs as d_o approaches zero.

$$\lim_{d_o \rightarrow 0} \Delta f_s = 0 \quad (10)$$

A quantity, Δf_{\max} , can be defined for very large values of d_o where the energy available for storage becomes very large relative to the load.

$$\Delta f_{\max} = \lim_{d_o \rightarrow \infty} \Delta f_s \quad (11)$$

Δf_s cannot exceed $1 - f_o$, since the load fraction supplied by the system cannot exceed unity.

$$\lim_{d_o \rightarrow \infty} \Delta f_s \leq 1 - f_o \quad (12)$$

For sufficiently large d_o , all of the daytime portion of the load will be met directly from the array. The battery will then be discharged only at night, and Δf_s may be limited by the effective daily storage capacity of the battery relative to the load.

$$\lim_{d_o \rightarrow \infty} \Delta f_s \leq B_c / \bar{L} \quad (13)$$

Combining eqns (12) and (13), the limiting value of Δf_s as d_o becomes very large is

$$\Delta f_{\max} = \min(1 - f_o, B_c/\bar{L}). \quad (14)$$

An equation for Δf_s which satisfies the constraints described above for both very large and very small values of d_o is

$$\Delta f_s = \frac{1}{2A} \{d_o + \Delta f_{\max} - [(d_o + \Delta f_{\max})^2 - 4Ad_o\Delta f_{\max}]^{1/2}\} \quad (15)$$

The parameter, A , which is the only degree of freedom in this equation, can be used to vary the rate at which Δf_s approaches Δf_{\max} as d_o increases. When $A = 1$, eqn (15) reduces to

$$\Delta f_s|_{A=1} = \min(d_o, \Delta f_{\max}) \quad (16)$$

which is precisely the result expected for infinite storage capacity (neglecting energy carryover from month to month). By adjusting the value of A , eqn (15) is suitable for all battery sizes as well as for all values of d_o .

In order to correlate the parameter A , values of Δf_s were calculated using TRNSYS[9], an hourly simulation program, in conjunction with photovoltaic array, regulator-inverter, and battery models similar to those used by Evans *et al.*[10]. These models treat the battery efficiency, η_b , the array thermal loss coefficient, U_L , the array cover transmittance, τ , and the array absorptance, α , as constants. The array efficiency is treated as a linear function of cell temperature.

The control strategy gives priority to meeting the load; the battery is charged only from the array, and only when the entire load is met by the array and excess energy is available. Based on 73 yr of hourly simulations, using 15 diurnal load profiles in Seattle, Madison and Albuquerque climates, the following empirical correlation for the parameter A was developed for battery storage capacities ranging from 0 to $2\bar{L}$ [11].

$$A = 1.315 - 0.1059 \frac{f_o \bar{L}}{B_c} - \frac{0.1847}{\bar{K}_T} \quad (17)$$

where \bar{K}_T is the monthly-average clearness index defined as the ratio of monthly radiation on a horizontal surface to the extraterrestrial radiation.

System performance calculated from monthly-average meteorological data, using the procedures described in this paper, agrees with TRNSYS simulation results with a standard deviation of less than 3 per cent on an annual basis. The accuracy of the design method relative to simulation results for 56 of the simulation years used in developing the correlation is summarized in Table 1. These results are

Table 1. Accuracy of design method relative to simulation results

672 Monthly Load Fractions	Average Error (%)	Standard Deviation of Error (%)
f_o	0.3	1.7
Δf_s	-0.8	3.7
f_s	-0.5	3.9
56 Annual Load Fractions		
f_o	0.3	0.7
Δf_s	-0.8	2.5
f_s	-0.5	2.4

all based on Madison weather data. A comparison of results for a system not included in the correlation development is provided by the example problem.

4. EXAMPLE

As an example of the procedure developed in this paper, the performance of a photovoltaic system in Boston, Massachusetts will be determined. The characteristics of the system are given in Table 2. The load profile used is appropriate for an average residential application (i.e. several houses).

The average array efficiency, $\bar{\eta}$, for the month of January is calculated to be 0.0942 in the manner described by Evans[1], with an assumed transmittance-absorptance product of 0.88 and a power conditioning efficiency of 0.88.

In January, the first hour after sunrise is from 8:00 to 9:00. The clearness index, the average horizontal radiation, and the average radiation incident on the array for this hour are estimated as described in Ref.[8], and are found to be

$$\bar{K}_T = 0.335$$

$$\bar{I}_T = 150.8 \text{ W/m}^2.$$

The average electrical output of the array for this hour, from eqn (1), is

$$E = (600 \text{ m}^2)(150.8 \text{ W/m}^2)(0.0942) = 8520 \text{ W}.$$

Next, the utilizability function must be evaluated. The average load for the hour from 8 to 9 is 10614 W. From eqn (2), the critical insolation level for the hour is

$$I_c = \frac{10614 \text{ W}}{(600 \text{ m}^2)(0.0942)} = 187.8 \text{ W/m}^2.$$

The hourly utilizability, ϕ , for this critical level is found to be 0.343 using the procedure in Ref.[7]. The energy dissipated by a system without storage is the product of E and ϕ

$$D_o = (8520 \text{ W})(0.343) = 2922 \text{ W}.$$

Table 2. Photovoltaic system characteristics for sample calculation

Boston MA	Latitude = 42.37°	January Weather Data
Area = 600 m ²	$\eta_r = 0.10$	$\bar{I} = 62.45 \text{ W/m}^2$
Slope = 50°	$T_r = 28 \text{ C}$	$\bar{T}_a = -1^{\circ}\text{C}$
$U_L = 40 \text{ W/m}^2 \text{ C}$	$\beta = 0.0039 \text{ C}^{-1}$	
$\tau = 1.0$	$\eta_{pc} = 0.88$	$\bar{K}_T = 0.396$
$\alpha = 0.88$	$\eta_b = 0.87$	
$\rho = 0.2$		
$B_c = 140 \text{ kW-hr}$	$\bar{L} = 12.5 \text{ kW}$	

[Load profile: sinusoidal, peak at 17:00,
amplitude/average = 0.25.]

The energy received directly by the load is

$$E_L = E(1 - \phi) = (8520 \text{ W})(1 - 0.343) = 5600 \text{ W}.$$

These calculations are repeated for each hour between sunrise and sunset. Results of these calculations are summarized in Table 3.

Summing the hourly results and averaging over a 24-hour day,

$$\bar{E}_L = \frac{1}{24} \sum E_i = 2912 \text{ W}$$

and

$$\bar{D}_o = \frac{1}{24} \sum D_o = 2426 \text{ W}.$$

The solar load fraction without storage is

$$f_o = \frac{E_L}{L} = \frac{2921 \text{ W}}{12500 \text{ W}} = 0.233.$$

The effect of the storage battery can now be calculated. From Table 2,

$$\frac{B_c}{L} = \frac{140 \text{ kW-hr}}{(12.5 \text{ kW})(24 \text{ hr})} = 0.467.$$

From eqn (9)

$$d_o = (0.87)(2426 \text{ W})/12500 \text{ W} = 0.169$$

and from eqn (14),

$$\Delta f_{\max} = \min[(1 - 0.233), (0.467)] = 0.467.$$

Equation (17) yields

$$A = 1.315 - \frac{(0.106)(0.233)}{(0.88)(0.467)} - \frac{0.1847}{0.396} = 0.79.$$

The increase in the solar load fraction due to storage, from eqn (15) is

$$\Delta f_s = \frac{1}{2(0.79)} \{0.169 + 0.467 - [(0.169 + 0.467)^2 - 4(0.79)(0.169)(0.467)]^{1/2}\} = 0.153.$$

The fraction of the load met by the system in January is then

$$f = 0.233 + 0.153 = 0.386.$$

Table 3. Results from sample calculation for January

Time	Load(W)	\bar{k}	$\bar{I}_T(\text{W/m}^2)$	X_c	ϕ	$E_L(\text{W-hr})$	$D_o(\text{W-hr})$
8-9	10614	.335	150.8	1.246	.343	5600	2922
9-10	11314	.378	259.2	0.782	.465	7746	6572
10-11	12096	.408	348.9	0.621	.528	9195	10052
11-12	12904	.424	399.5	0.579	.545	10147	11879
12-13	13686	.424	399.5	0.614	.524	10623	11414
13-14	14386	.408	348.9	0.739	.463	10464	8812
14-15	14958	.378	259.2	1.034	.354	9339	5014
15-16	15362	.335	150.8	1.835	.191	6774	1563

Table 4. Solar load fraction

Month	Design	Evans	TRNSYS
January	.39	.45	.40
February	.49	.54	.51
March	.56	.60	.58
April	.62	.64	.61
May	.66	.70	.68
June	.68	.73	.75
July	.67	.72	.70
August	.63	.64	.68
September	.64	.66	.68
October	.56	.60	.54
November	.37	.43	.40
December	.34	.40	.35

The entire procedure is repeated for each month. In Table 4, the results of these calculations are compared with results from the graphical method of Evans[3] and with TRNSYS simulation results. Both design methods agree well with the hourly simulation.

Acknowledgement—This work has been supported by the Solar Heating and Cooling Research and Development Branch, Office of Conservation and Solar Applications, U.S. Department of Energy.

NOMENCLATURE

A	storage correlation parameter (eqn 17)
A_c	array area
B_c	battery storage capacity
$D_{o,i}$	monthly-average hourly electrical energy in excess of the load
\bar{D}_o	monthly-average daily electrical energy in excess of the load
E_i	monthly-average hourly array output
$E_{L,i}$	monthly-average hourly energy delivered directly to load from array
\bar{E}_L	monthly-average daily energy delivered directly to load from array
f	monthly-average solar load fraction
f_o	monthly-average solar load fraction
F	annual solar load fraction
\bar{H}	monthly-average daily radiation on horizontal surface
$I_{c,i}$	critical insolation level for utilizability calculations
\bar{I}_T	monthly-average hourly radiation on tilted surface
\bar{k}_T	monthly-average hourly clearness index (ratio of hourly horizontal to extraterrestrial radiation)

\bar{K}_T	monthly-average daily clearness index (rate of daily horizontal to extraterrestrial radiation)
L_i	monthly-average hourly load
\bar{L}	monthly-average daily load
\bar{T}_a	monthly-average ambient temperature
T_r	array reference temperature corresponding to η_r
U_L	array thermal loss coefficient
X_c	dimensionless critical level
α	solar absorptance of array
β	temperature coefficient for cell efficiency (a cell material property)
Δf_{\max}	limiting value of Δf_i
Δf_i	increase in solar load fraction due to storage
$\bar{\eta}$	monthly-average photovoltaic array efficiency
η_b	battery storage efficiency
η_{pc}	efficiency of power conditioning equipment
η_r	array reference efficiency at temperature T_r
τ	solar transmittance of array cover
ϕ	utilizability function (fraction of incident radiation exceeding critical level $I_{c,i}$)

Subscript

i refers to hourly period i

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