

$$\left[ \frac{h}{2} + \frac{1n\{(\exp(-1.698h) + \exp(1.698h))/2\}}{3.396} + 0.2041 \right]$$

Equation 10 can also be used for the estimation of monthly cooling degree-days,  $D_c$ , by replacing  $h$  with  $h^*$ , where  $h^*$  is defined as

$$h^* \equiv (\bar{T}_a - T_b)/(\sqrt{(N/24)\sigma_m}) \quad (11)$$

#### Determination of $\sigma_m$

The long-term monthly-average temperature and the standard deviation of the monthly-average temperature are both required to use the relationships presented for the estimation of bin data and degree-days. Long-term values of monthly-average temperature are available for many locations from the Department of Commerce.<sup>2</sup> Values of the standard deviation are also available, but are in the form of monthly maps for the U.S.<sup>6</sup> Interpolation is usually required to use the maps.

During the colder months of the year, ambient temperature is more variable than during the warmer months. This is true of both daily and monthly-average temperatures (the two being statistically related) for all nine U.S. locations investigated. The variability of  $\bar{T}_a$  for a month from year to year is also related to the variation in  $\bar{T}_a$  from month to month over the course of a year. The standard deviation of the monthly-average temperatures from the annual average temperature,  $\sigma_{yr}$ , provides a convenient measure of the annual variation of  $\bar{T}_a$ . The following equation was fit to calculated values of  $\sigma_{yr}$ , and is shown in Figure 3.

$$\sigma_m = 1.45 - 0.0290\bar{T}_a + 0.0664\sigma_{yr} \quad (12)$$

where  $\bar{T}_a$  and  $\sigma_{yr}$  are in degrees Celsius. This regression equation allows the estimation of degree-days and bin data using only long-term monthly-average temperature. The use of Equation 12 in place of measured values of  $\sigma_m$  results in degree-days and bin data estimates of comparable accuracy.

#### Comparing the data

Annual bin data were developed from the hourly temperature measurements for each of the nine SOLMET locations. After sorting the measurements into temperature intervals of 5°F, the number of hours in each interval was divided by the total number of hours in the data set, yielding the fraction of the year that the ambient temperature was in each bin. This was done for each location, resulting in nine sets of annual bin data. The choice of 5°F as the bin size was made to allow comparisons between the SOLMET data and bin data published by the Air Force.<sup>12</sup>

The expression for  $Q(T_b)$  given in Equation 8 was used to generate monthly bin data for each location, which were then summed over all 12 months to create nine sets of generated annual bin data. The values of  $\bar{T}_a$  used were obtained from the measured data, while  $\sigma_m$  was both calculated from the data and estimated from Equation 12. The measured bin data (heavy solid lines), the bin data estimated using values of  $\sigma_m$  calculated from the data (solid lines), and the bin data estimated using values of  $\sigma_m$  obtained from Equation 12 (dashed lines) for three of the locations are compared in the left side of Figure 4.

Although the errors are significant for some bins, the overall agreement is quite good. The generated distributions follow the shape of the actual distributions fairly well. For the nine SOLMET locations, the use of values of  $\sigma_m$  estimated with Equation 12 produces results similar to those obtained when values of  $\sigma_m$  calculated from the measured data were used.

Annual ambient temperature bin data were also obtained from the Air Force<sup>12</sup> for three cities not contained

in the SOLMET data set. Since these data were not used in the development of Equations 8 or 12, they provide an independent test of these relationships. Values of  $\bar{T}_a$  were obtained from the National Weather Service<sup>13</sup> and were used with Equations 8 and 12 to generate monthly bin data for the three locations. The monthly data were then summed to yield annual bin data; the measured bin data (solid lines) and the estimated bin data (dashed lines) are compared in the right side of Figure 4.

The estimated bin data do not reproduce the maximum which occurs at 32°F in the measured data for many of the colder locations. Although the errors are still fairly small for these locations, the systematic nature of the error is evidence of some model inadequacy. Inspection of the monthly measured bin data revealed that during the late fall, winter, and early spring, the maxima of the monthly distributions were always near or at 32°F. This skewing of the distributions is not duplicated in the generated data, which on a monthly basis are always symmetric about  $\bar{T}_a$ . The cause of the skewness is believed to be connected with the solid-liquid phase change of water in the soil, lakes and snow on the ground.

The hourly temperature records for the nine SOLMET locations were also used to calculate annual heating and cooling degree-days. The base temperatures considered were between 1 and 20°C for heating and between 10 and 29°C for cooling. Values of  $\bar{T}_a$  and  $\sigma_m$  were obtained from the hourly data, and  $\sigma_m$  was also estimated from Equation 12. The relationship for degree-days given by Equation 10 was used to estimate annual heating and cooling degree-days (by summing monthly values) for both sets of  $\sigma_m$  at each location. In addition, the degree-day relationship developed by Thom was used with values of  $\sigma_m$  calculated from the data to estimate annual degree-days for each location. The annual bias error, the difference between the measured annual degree-days and the estimated annual degree-days, was determined for each location as a function of base temperature.

In Figure 5, the annual heating degree-days calculated directly from the hourly data are shown by the heavy solid line for two of the SOLMET locations which have a large number of heating degree-days. Also included in the figure are the absolute values of the annual bias errors for degree-days estimated with Equation 10 and values of  $\sigma_m$  calculated from the data (solid line), degree-days estimated with Equation 10 using values of  $\sigma_m$  estimated from Equation 12 (long dashes), and degree-days estimated with Thom's relationship and values of  $\sigma_m$  calculated from the data (short dashes). Comparisons are also shown in Figure 5 for cooling degree-days at two locations with large cooling degree-day totals.

The estimated annual heating degree-days are within 175°C-days of the measured values at all nine locations for the range of base temperature considered. Although Thom's relationship is more accurate than Equation 10 for Seattle, Equation 10 has a smaller combined error for the nine locations. The differences between heating degree-days found using Equation 10 and  $\sigma_m$  calculated from the data and heating degree-days found using Equation 10 and values of  $\sigma_m$  estimated from Equation 12 are always less than 50°C-days. For the combined data of all locations, the results are slightly better when  $\sigma_m$  is estimated using Equation 12. For cooling degree-days, the largest bias error is 150°C-days. Equation 10 is generally more accurate than Thom's relationship for cooling degree-days. Although the cooling degree-days obtained from Equation 10 using measured values of  $\sigma_m$  and those obtained from Equation 10 using estimates of  $\sigma_m$  differ by more than the corresponding sets of heating degree-days, the results are again more accurate on the average if  $\sigma_m$  is estimated from Equation 12.

# Estimation Of Degree-Days And Ambient Temperature Bin Data From Monthly-Average Temperatures

A method is provided which only needs the monthly-average ambient temperature to obtain estimates of daily bin data and the monthly heating or cooling degree-days for any base temperature.

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THE energy required to heat a building for a month can be estimated as the product of an overall building conductance, UA, and the number of heating degree-days evaluated at an appropriate base temperature. Well-established methods exist for calculating UA. Traditionally, 18.3°C has been used as a base temperature. However, higher levels of insulation and passive solar heating in modern construction, conservation measures such as night setback thermostats and lower thermostat settings, and a dramatic rise in residential electric usage have caused 18.3°C to be too high.

Tabulated degree-days for a number of base temperatures between 26.7°C (80°F) and -3.9°C (25°F) are available for several hundred locations in the U.S.<sup>2</sup> However, only the degree-day values for a base temperature of 18.3°C (65°F) were calculated from daily temperature records (for the period 1941-1970). Degree-day statistics for the other base temperatures were estimated from long-term monthly-average ambient temperatures for the period 1941-1970. Tape Deck 1440, available from the National Climatic Center (NCC), contains long-term (over 20 years) ambient temperature records for over 600 locations in the U.S. and other countries. Degree-days can be calculated for any base temperature of interest using these tapes, although the large number of data makes it costly and time consuming.

Degree-day values published by the NCC<sup>2</sup> for base temperatures other than 18.3°C were estimated using a relationship developed by Thom.<sup>3-5</sup> Thom's relationship makes possible the estimation of degree-days above or below any base temperature from the long-term monthly-average temperature and the standard deviation of the monthly-average temperature from its long-term average. Long-term monthly-average temperatures are available for most locations, and maps have been published<sup>6</sup> which give isolines of the required standard deviation for each month in the U.S.

The estimation of heat pump system performance requires more information about the ambient temperature distribution than the monthly degree-days provide. Ambient

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temperature bin data can be used along with heat pump performance data to estimate the energy required to run the heat pumps system.<sup>1</sup> Bin data are the number of hours that the ambient temperature was in each of a set of equally sized intervals of ambient temperature. Bin data are also used in the estimation of heating and cooling loads for buildings.<sup>1,7</sup> Since as many as 200 bin data values are required for 12 months at a single location, the use of bin data involves more computational effort and data storage than the use of degree-days.

In this article, a relationship is established between ambient temperature bin data and degree-days. Long-term temperature data for nine U.S. locations are used to develop a relationship for estimating bin data. This equation is then used to derive a relationship for estimating monthly degree-days. Both relationships are compared to data from 12 U.S. locations. Additional correlations are developed to allow the estimation of bin data or degree-days for any hour of the day.

#### Calculating degree-days

The heating degree-days for a month,  $D_H$ , is defined by the following equation:

$$D_H(T_b) = \frac{N}{24} \int_{T_{min}}^{T_b} (T_b - T) P(T) dT \quad (1)$$

where  $N$  is the total number of hours in the month,  $T_{min}$  is the lowest ambient temperature occurring during the month,  $T_b$  is the base temperature, and  $P(T)$  is the unit density distribution function for ambient temperature. The factor of 24 is a conversion factor between  $D_H$ , which has units of degree-days, and the integral, which has units of degree-hours. Although the National Weather Service uses daily maximum and minimum temperatures to compile monthly degree-days, hourly temperature data provide a more accurate estimate of monthly degree-days. All of the relationships and comparisons presented here use hourly data. A more complete discussion of degree-days and base temperature can be found in several references.<sup>1,8,9</sup>

Figure 1a shows a typical discretized distribution of ambient temperature developed using measured data for March in Madison, Wisconsin. The ordinate of each bar in the figure represents the fraction of the total hours the temperature was within the temperature interval enclosed by the bar on the abscissa. This figure can be interpreted as bin data for bins of 1°F. Figure 1a is an approximation to  $P(T)$ , the probability distribution of ambient temperature for this data, which is shown in Figure 1b. The quantity  $NP(T)\Delta T$

is the number of hours the ambient temperature was within an interval of temperature of width  $\Delta T$  centered on  $T$ .<sup>\*2</sup>

The heating degree-days for a month,  $D_H$ , can be calculated from ambient temperature bin data by summing the product of the number of hours the temperature was in a temperature interval (bin) and the temperature difference between the base temperature ( $T_b$ ) and the midpoint of the temperature interval. Given enough data, the accuracy of this approximation will improve as the bin size becomes smaller. A more exact relationship between heating degree-days and the distribution of ambient temperature can be obtained by differentiating Equation 1. Since the upper limit on the integral is a variable, it is necessary to use Leibnitz' rule.<sup>10</sup> Differentiation of Equation 1 then yields

$$\frac{d}{dT_b} D_H(T_b) = \frac{N}{24} \int_{T_{min}}^{T_b} P(T) dT \quad (2)$$

The integral on the right side of Equation 2 is the cumulative distribution of ambient temperature,  $Q(T_b)$ .  $NQ(T_b)$  is the number of hours that the ambient temperature was less than  $T_b$ . The number of hours the ambient temperature was within a temperature interval of width  $\Delta T$  centered on  $T$  is given by the difference between  $NQ(T + \Delta T/2)$  and  $NQ(T - \Delta T/2)$ . If an analytic expression were available for monthly heating degree-days as a function of base temperature, Equation 2 could be used to obtain a relationship for  $Q(T_b)$ . Once  $Q(T_b)$  is known, bin data can be generated for a month using any bin size desired.

#### Existing relationships

The relationship developed by Thom for the estimation of degree-days is represented by the following equation:

$$D_H = \frac{N}{24} (T_b - \bar{T}_a + \ell \sqrt{N/24} \sigma_m) \quad (3)$$

where  $\ell$  is given by

$$\ell = 0.34 \exp(-4.7h) - 0.15 \exp(-7.8h) \text{ for } h \geq 0$$

$$\ell = 0.34 \exp(4.7h) - 0.15 \exp(7.8h) - h \text{ for } h < 0 \quad (4)$$

and  $h$  is defined as

$$h \equiv (T_b - \bar{T}_a) / (\sqrt{N/24} \sigma_m) \quad (5)$$

$\sigma_m$  is the standard deviation of the monthly-average temperature,  $\bar{T}_a$ .

Thom's expression for degree-days can be differentiated to obtain a relation for  $Q(T_b)$ . Since the form of Equation 4 depends on the sign of  $h$ , two expressions must be derived for  $Q(T_b)$ . For  $h$  greater than 0,

$$Q(T_b) = 1 - 1.60 \exp(-4.7h) + 1.17 \exp(-7.8h) \quad (6)$$

For  $h$  less than 0,

$$Q(T_b) = 1.60 \exp(4.7h) - 1.17 \exp(7.8h) \quad (7)$$

When  $T_b$  is less than  $T_{min}$ ,  $Q(T_b)$  must equal 0, since no temperatures occurred below this value. When  $T_b$  is greater than  $T_{max}$ , the largest value of ambient temperature,  $Q(T_b)$  must equal 1, since no ambient temperatures occurred above this value. Although Equations 6 and 7 exhibit correct asymptotic behavior, there is a discontinuity in the relationship at a value of  $h$  equal to zero. Since the expression for degree-days developed by Thom assumes the distribution of ambient temperature is symmetric about the monthly-average temperature  $\bar{T}_a$ , the value of  $Q(T_b)$  calculated from Equation 6 or Equation 7 should equal 0.5 when  $T_b$  is equal to  $\bar{T}_a$  ( $h$  is zero). However, the values of  $Q(\bar{T}_a)$  calculated from Equations 6 and 7 are 0.43 and 0.57, respectively.

Thom fit an equation to the degree-day versus base temperature data, for which a "best fit" was one that

"The estimation of heat pump system performance requires more information about the ambient temperature distribution than the monthly degree-days provide. Ambient temperature bin data can be used along with heat pump performance data to estimate the energy required to run the heat pumps systems.<sup>1</sup> Bin data are the number of hours that the ambient temperature was in each of a set of equally sized intervals of ambient temperature. Bin data are also used in the estimation of heating and cooling loads for buildings.<sup>1,7</sup> Since as many as 200 bin data values are required for 12 months at a single location, the use of bin data involves more computational effort and data storage than the use of degree-days."

minimized the sum of the squared differences between the measured degree-days and the degree-days predicted by Equation 3. However, there is relatively poor agreement between the slope of the data and the slope of the fitted equation, particularly at the endpoints. Consequently, Thom's relationship, although reasonably accurate for the estimation of degree-days, is not in a form useful for the estimation of bin data. The accuracy of Thom's method for degree-day evaluation is discussed later.

#### A distribution function

Rather than developing a relationship for heating degree-days from the data and then using Equation 2 to derive an expression for  $Q(T_b)$ , a relationship for the cumulative distribution of ambient temperature will be developed directly from the data. This expression for  $Q(T_b)$  can then be integrated to obtain a relationship for heating degree-days. The advantage of this approach is that integration tends to smooth out irregularities in the fit to the data, whereas differentiation tends to amplify irregularities.

Hourly SOLMET data<sup>11</sup> for Madison, Wisconsin; Washington, DC; Albuquerque, New Mexico; Miami, Florida; Fort Worth, Texas; Columbia, Missouri; New York City; Phoenix, Arizona; and Seattle, Washington, were used to examine the cumulative distribution of ambient temperature for monthly periods. An average of 22 years of data were used for each location. A correlation was developed between the cumulative distribution and  $h$  (defined by Equation 5) for each calendar month in the nine locations. The cumulative distributions for four of the locations are shown in Figure 2.

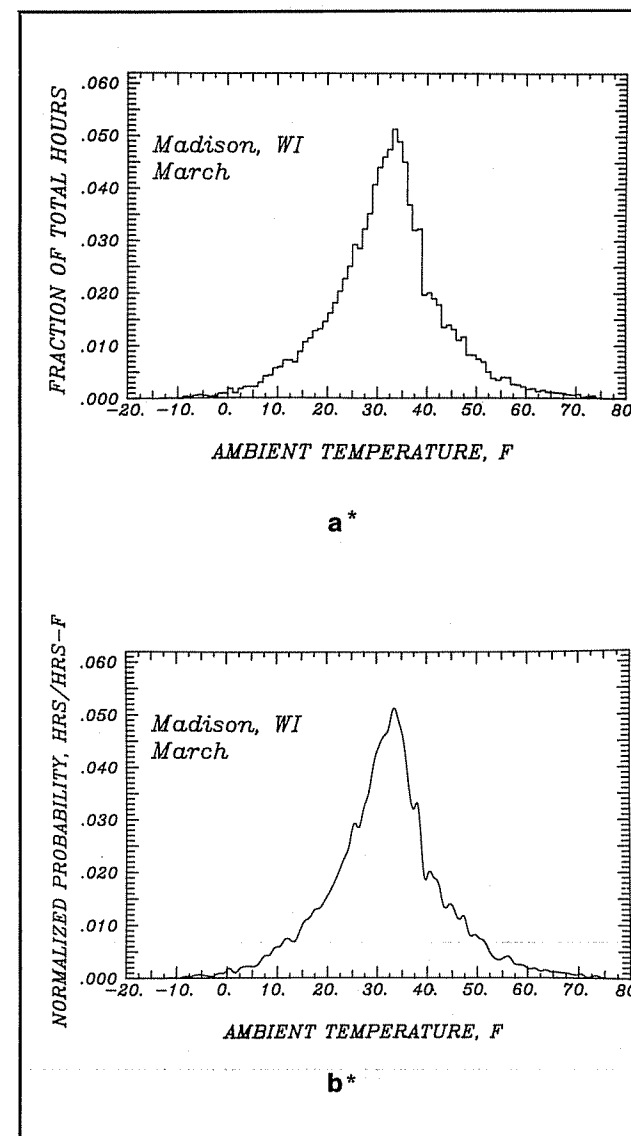


Figure 1a Figure 1b Standardized distributions of ambient temperature for March in Madison, Wisconsin. a) Discretized distribution (bin data). b) Unit probability distribution.

The solid line is the average for all 12 months and the dashed lines represent plus and minus one standard deviation of the monthly distributions from the average.

The mean and standard deviation of the monthly distribution of ambient temperature are, in general, quite variable for the different months and locations. The use of  $h$  as the independent variable in place of  $T$  eliminates this problem. Subtraction of the mean ( $\bar{T}_a$ ) centers the distributions about zero, while division by the standard deviation standardizes the temperature scale. The cumulative distribution data for the nine SOLMET locations were used to develop the following single parameter, continuous relationship between  $Q(T_b)$  and  $h$ :

$$Q(T_b) = \frac{1 + \tanh(1.698h)}{2} = \frac{1}{1 + \exp(-3.396h)} \quad (8)$$

Equation 8, like Equations 6 and 7, assumes that the distribution of ambient temperature is symmetric about the average. While symmetry is observed for some of the distributions, others are noticeably skewed. The strength and sea-

\* The ordinates in Figures 1a and 1b are the same because the bin width is 1°F. The ordinate in Figure 1a depends on the bin size, while the ordinate in Figure 1b is independent of the bin size.

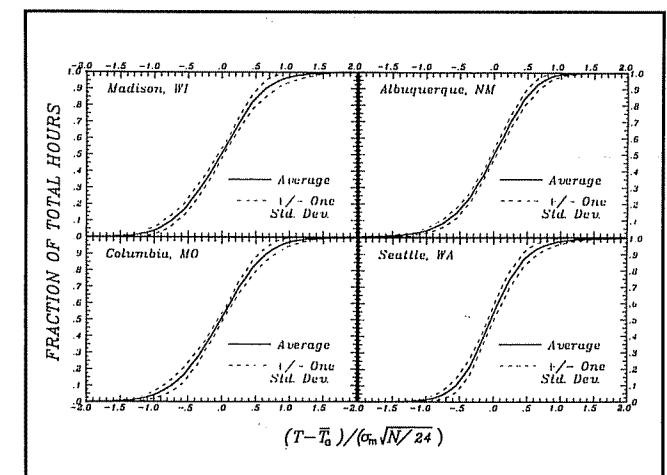


Figure 2 Annual average cumulative distributions and standard deviations of monthly curves from annual averages for four SOLMET locations.

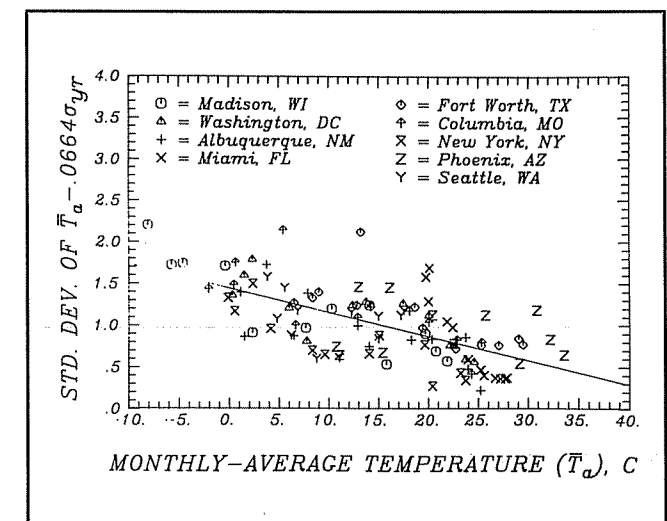


Figure 3 The relationship between the standard deviation of the monthly-average ambient temperature and the monthly-average ambient temperature.

sonality of the skewness vary among locations, and an attempt to correlate the skewness with the available measurements resulted in no identifiable trends. Since the distributions are skewed, both to the right and to the left of the average temperature, the use of a symmetric monthly distribution function is one way of representing a "typical" distribution of ambient temperature suitable for the calculation of annual degree-day totals.

#### Estimating degree-days

If Equation 2 is integrated with respect to base temperature, the result is

$$D_H(T_b) = \frac{N}{24} \int_{T_{min}}^{T_b} Q(T_b) dT_b \quad (9)$$

Combining Equations 8 and 9, changing the independent variable from  $T_b$  to  $h$ , and performing the integration yields

$$D_H(T_b) = \sigma_m \left( \frac{N}{24} \right)^{3/2} \left[ \frac{h}{2} + \frac{1n(\cosh(1.698h))}{3.396} + 0.2041 \right] \quad (10)$$

$$= \sigma_m \left( \frac{N}{24} \right)^{3/2}$$

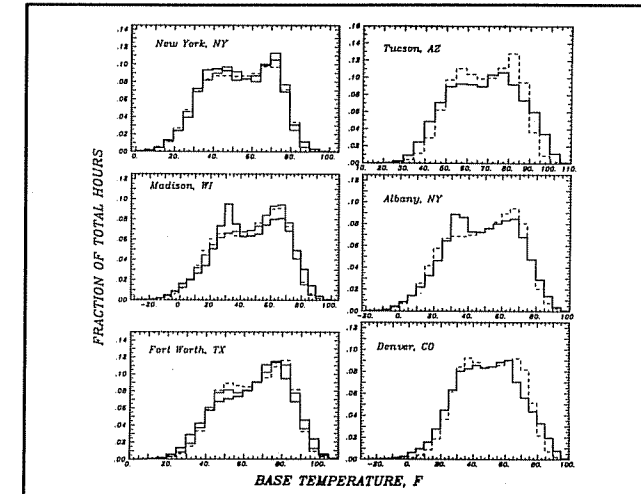


Figure 4 Comparisons of measured annual bin data (—) to bin data generated using Equation 8; measured values of  $\sigma_m$  (---) and bin data generated using Equation 8; and  $\sigma_m$  estimated from Equation 12 (· · ·).

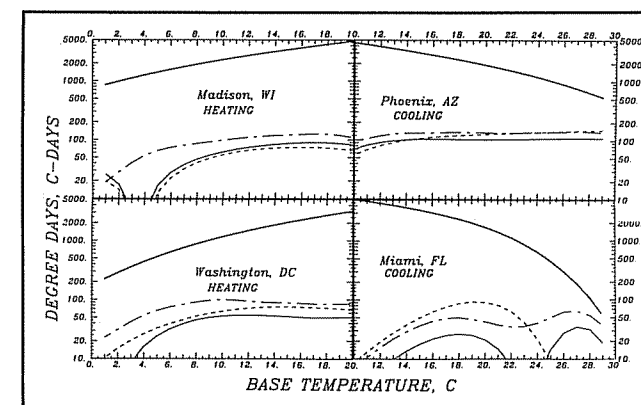


Figure 5 Annual heating and cooling degree-days (—) and annual bias errors for degree-days estimated from Equation 10 with measured values of  $\sigma_m$  (---); degree-days estimated from Equation 10 with  $\sigma_m$  estimated from Equation 12 (· · ·); and degree-days estimated with the relationship of Thom and measured values of  $\sigma_m$  (— · — · —).

#### Use on an hourly basis

The comparisons presented in the previous section are for the entire 24-hour day. Since the ambient temperature is not constant throughout the day, the number of degree-days in each hour will vary. The balance temperature of the building may also be a function of time of day. As a result, the heating and cooling loads calculated using hourly degree-day values will, in general, be more accurate than those calculated using the daily average of the hour balance temperatures and daily degree-days. Similarly, the performance estimates for heat pump systems will be more accurate when hourly ambient temperature bin data are used because of the nonlinear dependence of the heat pump performance on ambient temperature.

The relationships provided for the estimation of bin data and degree-days (Equations 8 and 10) were developed from hourly temperature data and daily values of  $\sigma_m$  ( $\sigma_m$  is always very close to the average of the hourly values of  $\sigma_{m,h}$ ). Although the monthly-average ambient temperature for each hour changes, the monthly temperature distributions for each hour have the same shape. This allows the bin data and degree-day expressions to be used either on an hourly or a daily basis.

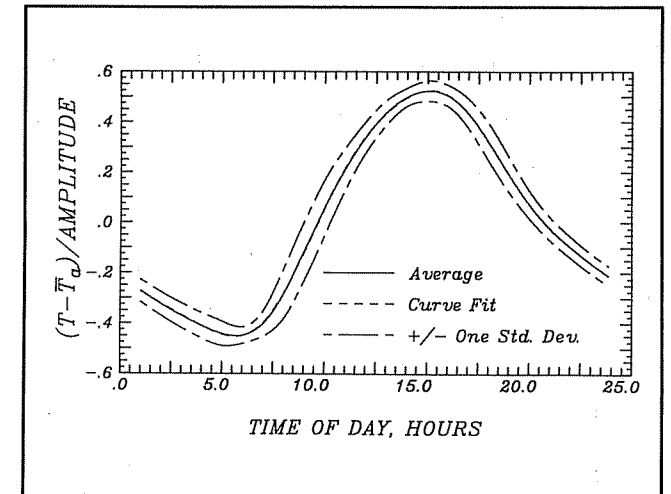


Figure 6 The average normalized diurnal temperature variation for all months at nine SOLMET locations, the standard deviation of the monthly curves from the average, and Equation 13.

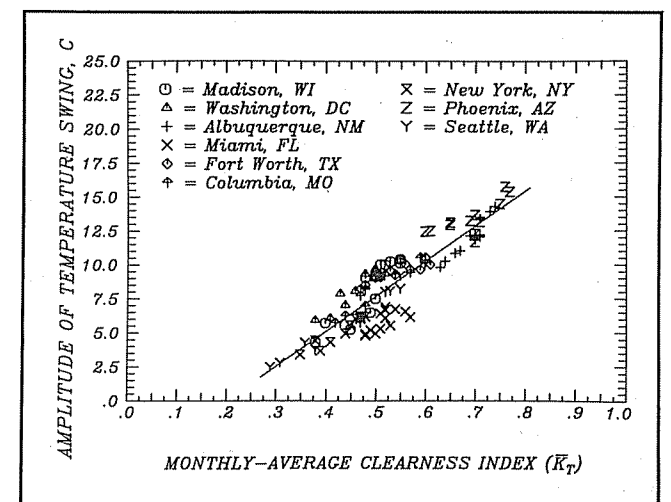


Figure 7 The relationship between the amplitude of the monthly-average diurnal temperature swing and the monthly-average solar radiation reaching the ground.

Hourly values of the monthly-average ambient temperature,  $\bar{T}_{a,h}$ , and its standard deviation,  $\sigma_{m,h}$ , are required to use the bin data and degree-day relationships on an hourly basis. For most locations only average daily values are available, and a method is needed to estimate hourly values from the daily averages. The hourly SOLMET data were used to find  $\bar{T}_{a,h}$  and  $\sigma_{m,h}$  at each hour of the day for each calendar month.  $\bar{T}_{a,h}$  and  $\sigma_{m,h}$  were plotted as a function of hour of the day for each month resulting in 108 curves (12 months at each nine locations) for each of these variables. The diurnal variation of  $\sigma_{m,h}$  is substantially different for different months at a single location, and even more variable for the same month at different locations. However, the hourly variation in  $\sigma_{m,h}$  is small, allowing the daily value of  $\sigma_m$  from Equation 12 to be used for each hour, with no significant effect on the accuracy of the methods.

In strong contrast to the diurnal variation of  $\sigma_{m,h}$ , the nature of the diurnal variation of  $\bar{T}_{a,h}$  is only a weak function of location and time of year.<sup>14</sup> Although the amplitudes of the 108 average monthly curves vary considerably, the hours at which the minimum and maximum temperatures occur and the shapes of the curves are nearly constant. To facilitate direct comparison of the diurnal variation of  $\bar{T}_{a,h}$

for different months, the curves were standardized by first subtracting the monthly-average daily temperature from each of the hourly values and then dividing by the amplitude of the curve.

The average of the 108 standardized curves is represented by the solid line in Figure 6. The long dashes are plus and minus one standard deviation of the individual monthly curves from the average. The small size of the standard deviation is an indication of the similarity of the diurnal temperature variations for different months and locations when presented in this manner. Also shown in the figure (short dashes) is the curve fit to the average diurnal temperature variation given by

$$\bar{T}_{a,h} - \bar{T}_a / A = 0.4632 \cos(t^* - 3.805) + 0.0984 \cos(2t^* - 0.360) + 0.0168 \cos(3t^* - 0.822) + 0.0138 \cos(4t^* - 3.513) \quad (13)$$

where  $\bar{T}_{a,h}$  is the hourly monthly-average ambient temperature,  $A$  is the amplitude of the diurnal variation (peak to peak), and  $t^*$  is given by

$$t^* = 2\pi(t-1)/24 \quad (14)$$

where  $t$  is in hours with 1 corresponding to 1 am and 24 corresponding to midnight.

If  $\bar{T}_a$  and  $A$  are known for a month, Equation 13 can be used to find the long-term average ambient temperature for any hour.  $\bar{T}_a$  is generally known, but  $A$  is less readily available. The amplitude was found to be related to  $K_T$ , the ratio of the total solar radiation striking a horizontal surface in a month to the monthly extraterrestrial radiation on a horizontal surface. Values of  $K_T$  are available for a large number of locations.<sup>13</sup> A relationship was found between  $A$  and  $K_T$  using the SOLMET data. The relationship, shown with the data in Figure 7, is given by

$$A = 25.8K_T - 5.21 \quad (15)$$

where  $A$  is in degrees Celsius.

Monthly heating and cooling degree-days were calculated for each hour of the day using the hourly SOLMET data for nine locations. Equations 10 and 12 were used to estimate the degree-days for each hour in two ways. For one set of estimated degree-days, the daily values of  $\sigma_m$  and  $\bar{T}_a$  were used for all hours. For the second set of estimates, the daily value of  $\sigma_m$  was used for all hours, but hourly estimates of  $\bar{T}_{a,h}$  were obtained from Equations 13 and 15. The RMS error was found for each set of degree-day estimates by first summing up the squared differences between the estimated and actual degree-day values over all hours, base temperatures, months, and locations, then dividing by the number of differences, and taking the square root. The RMS error for the heating degree-day estimates using calculated hourly values of  $\bar{T}_{a,h}$  is less than 10 percent of the average hourly heating degree-day value, while the RMS error for heating degree-day estimates using the daily value of  $\bar{T}_a$  is nearly 30 percent of the average. For cooling degree-days, the RMS errors for estimates obtained using calculated hourly values of  $\bar{T}_{a,h}$  and the daily value of  $\bar{T}_a$  are less than 10 percent and greater than 40 percent of the average, respectively.

#### Discussion

The relationships presented for the estimation of ambient temperature bin data and for the estimation of heating and cooling degree-days were developed using  $\sigma_m$  as an independent variable. It is possible to describe distributions of hourly (or daily) temperature using the standard deviation of the monthly-average temperature ( $\sigma_m$ ) because of the statistical relationship that exists between  $\sigma_m$  and  $\sigma$ , the standard deviation of the hourly (or daily) temperature from

the long-term average. Bin data and degree-day relationships were also developed using  $\sigma$  as an independent variable, along with a correlation of  $\sigma$  to  $\bar{T}_a$ . However, these relationships were found to be no more accurate than those developed using  $\sigma_m$ , even though  $\sigma$  is more directly related to the daily temperature distribution. The variability of the ambient temperature distributions is large enough that the small error introduced by approximating  $\sigma$  with  $\sigma_m$  has no effect.

While the degree-days calculated from the data in the present study used hourly values of temperature, the degree-days published by the National Climatic Center for a base of 65°F are found using the average of the daily maximum and daily minimum temperatures. Heating degree-days were also calculated for the nine SOLMET locations using both the average daily temperature and the average of the maximum and minimum hourly temperatures. For the nine locations on an annual basis, the differences between the degree-days calculated from daily average temperatures and those calculated from the averages of the daily minimum and maximum temperatures range from 5 to 50°C-days. The differences between the degree-days calculated from hourly data and those calculated from the average daily temperatures are somewhat larger, ranging from 50 to 200°C-days.

#### Conclusions

The contribution of this article is a method in which it is only necessary to supply a single piece of information, the monthly-average ambient temperature, to obtain estimates of daily bin data and the monthly heating or cooling degree-days for any base temperature. If the monthly-average solar radiation is also known for the month, bin data and degree-days can be estimated for any hour of the day.

#### Acknowledgement

This work has been supported by the Solar Heating and Cooling Research and Development Branch, Office of Conservation and Solar Applications, U.S. Department of Energy.

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