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Parameter Estimation for Multiresponse Nonlinear Chilled-Water Plant Models

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ABSTRACT

Development of optimal control strategies for complex heating, ventilating, and air-conditioning (HVAC) systems often requires a model of the system. Models of conventional heating or cooling plant equipment are nonlinear with multiple input and output variables. Model parameter values (e.g., an overall heat transfer coefficient, UA, *in a heat exchanger model) need to be assigned so that the model adequately predicts the performance of the actual system. Parameters for a model of an operating system can be determined with regression techniques using measurements of model variables. The work presented in this paper considers the multiresponse nonlinear parameter estimation problem associated with a system of interconnected components in a chilled-water plant. Methods studied for solving the estimation problem include ordinary least squares, weighted least squares, and a determinant criterion derived from Bayesian estimation theory. Potential pitfalls in multivariate regression, such as linear dependencies among responses, are identified.*

The application and analysis of the parameter estimation methods are directed toward building a predictive model for use in optimal supervisory control strategies. Various solution methods are reviewed and applied to a simulated chilled-water plant for comparison and analysis. The parameter estimation techniques are then used to create a predictive model of an operational chilled-water plant. The plant has both electric and steam-driven chillers and cooling towers with multispeed fans. The goal of the predictive model is to predict variables associated with operational costs such as electric motor power and steam consumption. The relative merits of the different regression techniques are judged (compared) according to how well the model can predict these cost-related variables.

INTRODUCTION

Generally, optimization studies of chilled-water facilities require a model of the plant. A search of the prediction of plant performance over a range of operation allows the minimum or maximum value of the objective function (e.g., energy costs) to be obtained. The model must predict system performance accurately to obtain valid results. In general, a system model will have parameters that can be set such that the model approximates actual system behavior. Finding good values or estimates of these parameters can be cast as a nonlinear, multivariate minimization problem in which a set of parameters that minimize the difference between model predictions and observed system measurements is sought. The difference between predictions and observations in terms of a scalar objective function requires consideration of the statistical properties of the data. In this paper, methods of estimating chilled-water plant model parameters are presented and used with an example chilled-water plant model where true parameter values are known and are also used to estimate parameters for a model of an actual chilled-water plant.

MODELING METHODS

The following vector equation can be used to describe a general model of a multivariate system:

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \boldsymbol{\beta}) \tag{1}$$

where $\mathbf{y}$ is a vector of dependent variables associated with costs, $\boldsymbol{\beta}$ is a vector of parameters, and $\mathbf{x}$ is a vector of independent or forcing variables. The equation could represent any given functional form such as a set of polynomial equations (a curve fit) or an artificial neural network. Alternatively, the equation could represent a set of mechanistic equations, such as conservation laws and transport equations derived from theory. The different functional forms may have individual advantages in predictive abilities or the ease with which the

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Box and Draper (1990) approached the multiresponse estimation problem using probabilistic arguments (Bayesian estimation) and developed a criterion for the determinant. It was demonstrated that given a data set Y, the probability that a set of parameter estimates, β, is the set of true parameter values is inversely proportional to a measure of the residuals as follows:

$$p(\beta|Y) \propto \left|R^{\mathrm{T}}R\right|^{-n/2} \tag{7}$$

The probability is maximized when the determinant of $R^{\mathrm{T}}R$ is minimized. Bates and Watts (1988) derive the same criterion using a maximum likelihood argument.

Potential Problems in Parameter Estimation

Although the determinant criterion provides a means for weighing residuals according to variance, its computation can sometimes be problematic. A number of potential problems that can arise include linear dependencies among the equations, overparameterization of the model, and variance in the parameters. These problems are discussed in this section.

If one vector of residuals (a column of R) happens to be linearly dependent (or nearly linearly dependent) with another residual vector, $R^{\mathrm{T}}R$ will not be full rank and the determinant will be zero (or a very small number). Any minimization algorithm will either fail or halt at this apparent minimum. Similar problems are faced when using the two-step weighted least-squares method where the covariance matrix is estimated from the inverse of $R^{\mathrm{T}}R$. If not of full rank, the inverse of $R^{\mathrm{T}}R$ is not defined.

Linear dependencies among residuals can be caused by linear dependencies in the responses. Such dependencies among the measured variables can occur in many HVAC system components, for example, a counterflow heat exchanger. The inlet and outlet temperatures are measured and an estimate of the overall heat transfer parameter, UA, is to be determined from the temperature measurements. The mass flow rates and fluid-specific heats are known and assumed to be constant. An NTU-effectiveness model can be used to represent the heat exchanger and an estimate of the heat exchanger's overall heat transfer coefficient, UA, is desired.

The overall heat transfer coefficient, UA, can be estimated using either measurements of T_{ho} or measurements of T_{co}. Qualitatively, one might expect that using both measured outlet temperatures in a multiresponse regression technique would result in a better estimate of UA. However, if the overall heat transfer coefficient, UA, does not vary with temperature, the residuals associated with T_{co} and T_{ho} are linearly dependent by virtue of the energy balance.

Energy and mass balances are often used in HVAC component models to predict dependent variables. Care must be used to find and remove any linearly dependent responses from the residual matrix when using multiresponse regression techniques to determine system parameters.

In some cases of nonlinear estimation, the parameter values at a problem solution are not unique. This problem will occur if the model function is specified with too many parameters (overparameterized). Overparameterization is not always the result of a bad model. Parameter identification problems can occur in exact models when a fit is made with limited data.

Another problem with parameter identification is the presence of large variances in the parameter estimates. Parameter estimates are functions of random variables and possess a certain distribution. The distribution of the estimates depends both upon the model and the distribution of the random variables. For models with multiple parameters, a variance-covariance matrix, V_β, of the parameter estimates can be defined. If multiple sets of data taken at equivalent values of independent or regressor variables are available, the parameter estimation problem can be solved with each data set and variance in the resulting parameter estimates computed. The set of resulting parameter estimates from the multiple (replicated) data will not be equal but will demonstrate some random error and an associated variance.

The occurrence of parameter identifiability problems does not necessarily negate the use of the parameter estimates. Whether or not the parameter estimates result in a model that can predict the measured data with accuracy can be judged by evaluating the residuals associated with each response. The estimated parameters may still yield a model with adequate predictive capabilities. However, the individual component parameters possessing identifiability problems should be limited to use with the original composite system model used in the regression. If nearly "true" parameter estimates are required for use with individual component models in different composite systems, parameter variance should be examined. Large variance in parameters may also bring into question the ability of mechanistic models to extrapolate outside the range of the measured data used in the fit.

EXAMPLE OF PARAMETER ESTIMATION

Plant Description

A chilled-water plant is used as an example to illustrate multiresponse regression techniques. The model consists of four components as shown in Figure 1. The component models are linked and solved using the TRNSYS (Klein et al. 1988) simulation program. This example system represents a steam-driven chiller, a cooling tower, and a surface condenser. Separate chiller and steam turbine components represent the steam-driven chiller. The system function is to cool a flow of water, $\dot{m}_{chw}$, from some relatively warm chilled-water return temperature, T_{chwr}, to a prescribed chilled-water supply temperature, T_{chws}. The chiller compressor is driven by a steam turbine that exhausts to a surface condenser. A cooling tower is used to reject heat from the chiller's refrigerant condenser and the steam condenser to the environment.

The chiller component model predicts the compressor power consumption, P_{comp}, and leaving temperature of the condenser water, T_{cwr}. Input variables to the chiller model include entering chilled-water temperature, T_{chwr}; a setpoint for application, as the condenser water flow is nearly constant and the condensing temperature does not vary widely. The effectiveness is the ratio of actual heat exchanged to the maximum possible heat exchange if the entering condenser water were heated to the saturated steam temperature. However, the steam exhaust enthalpy rather than the saturated steam temperature is available as an input from the steam turbine component. The steam saturation temperature, T_{sat}, is determined from an energy balance,

$$\varepsilon \dot{m}_{cw} c_p (T_{sat} - T_{cdwi}) = \dot{m}_{steam}(h_{exh} - h_f(T_{sat})) \tag{16}$$

where $h_f(T_{sat})$ is the enthalpy of saturated liquid steam at a temperature of T_{sat}.

The condensate may be subcooled in the hot well by heat exchange through the vessel walls. Separate measurements of hot well temperatures are made at a nearby plant, and a model quantifying the temperature drop due to subcooling as a function of water flow rate and condensing temperature was used:

$$T_{sat} - T_{hw} = f_{hw} \exp\left(-\frac{\dot{m}_{steam}}{\dot{m}_{steam,\,norm}}\right)(T_{sat} - T_{cwi}). \tag{17}$$

Although the condensate in the hot well will lose heat to the ambient environment, the ambient conditions are unknown. As an approximation, it is assumed that the subcooling in the hot well will be some fraction of the difference between the saturated temperature in the condenser, T_{sat}, and the entering condenser water, T_{cwi}. At high steam flow rates, the condensate leaving the hot well will approach the saturation temperature and at low steam flow rates will tend toward the entering condenser water temperature.

The TRNSYS cooling tower component model was developed by Braun (1988) and incorporates an effectiveness/NTU relationship to model heat and mass transfer transport. Cooling tower effectiveness, ε_a, is defined as the ratio of change in air enthalpy, η_a, to the maximum possible change in air enthalpy:

$$\varepsilon_a = \frac{h_{a,\,out} - h_{a,\,in}}{h_{sat}(T_{cwr}) - h_{a,\,in}}. \tag{18}$$

The maximum possible change in enthalpy would occur if the airflow left the cooling tower saturated at the entering water temperature, T_{cwr}. For the counterflow cooling tower used in this work, the air-side effectiveness is calculated in terms of the NTU and a ratio of heat capacitance rates, C_r:

$$\varepsilon_a = C_r^{-1}(1 - \exp\{-C_r[1 - \exp(-\mathrm{NTU})]\}). \tag{19}$$

The heat capacitance ratio, C_r, is given in terms of the air and entering water mass flow rates and specific heats:

$$C_r = \frac{\dot{m}_a c_{sat}}{\dot{m}_{cw} c_{p,\,cw}}. \tag{20}$$

The specific heat associated with the moist airstream, c_{sat}, is given as the change in saturated air enthalpy with respect to temperature. An average value is used where

$$c_{sat} = \frac{h_{sat}(T_{cwi}) - h_{sat}(T_{cwo})}{T_{cwi} - T_{cwo}}. \tag{21}$$

A general correlation is given for NTU in terms of the flow rates and two parameters c and n:

$$\mathrm{NTU} = c\left(\frac{\dot{m}_w}{\rho_{air} V_{max} f_{air}}\right)^{n+1}. \tag{22}$$

The air mass flow rate represented in the denominator of Equation 22 includes another parameter, V_{max}, the maximum volumetric flow rate through the tower. The fraction of the maximum airflow rate, f_{air}, that corresponds to fractional fan speed is a model input.

Fifteen parameters are used in the four component models representing the example plant: α_0, α_1, α_2, α_3, α_4, α_5, η_m, β_0, β_1, β_2, UA, f_{hw}, c, n, and V_{max}. The objective in this regression problem is to determine estimates for these parameters using a set of simulated measured data. Both unweighted and weighted least-squares methods as well as the determinant criterion are used to estimate the parameters. A simulated measured data set was generated by the composite model with a set of selected parameter values ("true" parameters). These "true" data were then contaminated with noise following a multivariate normal distribution with a zero mean and selected covariance matrix V.

Measurements for all dependent variables would rarely be available in an actual plant. Of the eight dependent variables defined for this example (T_{cws}, T_{cdwo}, P_{comp}, $\dot{m}_{steam}$, h_{exh}, T_{cwr}, T_{hw}, and p_{exh}) it is assumed that two are not measurable: the compressor power, P_{comp}, and the enthalpy of the turbine exhaust, h_{exh}. The multivariate regression is performed with the remaining six dependent variables. The set of test data consists of 240 (hourly) responses. All independent variables were varied with periodic functions to simulate periodic weather changes. A separate data set (not used in the regression analysis) was used to evaluate the predictive capabilities. The independent variables in the separate set were varied over ranges not encountered in the data set used for the fit.

Solution of the parameter estimation problem involves minimizing some quantity such as a sum of squared residuals or the determinant of the $R^T R$ matrix. For this example study, three different algorithms were used to minimize either a sum-of-squares objective function or the determinant objective function.

A sequential quadratic programming (SQP) minimization algorithm included in NAG (1991) was used on both the sum-of-squares and determinant formulations. Sequential quadratic programming is a frequently used method for solving nonlinear constrained optimization problems. A software package that uses the Levenburg-Marquardt algorithm was also used on the sum-of-squares problem formulation. Since the computation of a least-squares objective function is internal to the software spond to residuals (measured minus predicted) over all the observations used in the regression (240 observations). The sets indicated by "P" correspond to differences between the predicted values and the true values (errors rather than residuals) over another 240 observations not used in the fit.

The first entries in each table are reference values for comparing the residual (and error) quantities and for comparing the parameter estimates. The parameter values are those that were used to generate the original data set and the additional 240 observations used for predictive comparisons. The data used in the fit (the first 240 observations), however, were "contaminated" with noise from a multivariate normal distribution to produce a set of simulated data measurements.

Ordinary least squares was used with three different minimization algorithms and three different sets of responses for comparison. Regression results using six, four, and three responses in the sum of squared residual objective functions are given in Table 1. The reduced number of responses is used to evaluate the effect of possible collinearity among the responses upon the regression results.

Collinearity among the responses causes degeneracy in the R^TR matrix used in the weighted least-squares and determinant criterion methods and subsequently results in an unsuccessful regression. While not representing the proper objective function with respect to statistical theory, collinearity when using the ordinary least-squares method will at most cause uneven weighting among the responses. Ideally, in a least-squares formulation, responses should be weighted according to their variance, but disproportionate weighing caused by collinearity may not strongly the influence the regression results.

The two responses removed for the four response runs were the temperature leaving the hot well (T_{hw}) and the temperature

TABLE 2 Example System Regression Results—Weighted Least Squares

Estimation Method[1]	No. Responses Used[2]	Min. Algorithm[3]	Data Set[4]	Sum of Squared Residuals, All Responses (Errors) (Scaled)	Sum of Squared Residuals (Errors), Steam Flow Rate (kg/h × 10^6)	Standard Deviation of Steam Flow Residuals (Errors) (kg/h)	Mean of Steam Flow Residuals (Errors) (kg/h)
Residual "True" Values			F	184.4	2.46	101.2	–8.28
			P	~0	~0	~0	~0
Weighted Least Squares	Six	SQP	F	30590.8	427.99	1316.7	238.33
			P	43492.3	608.72	1517.17	492.55
		LM	F	358.9[c]	4.72[c]	140.31[d]	–8.29
			P	1049.2	14.47	232.66	–80.04
	Four	SQP	F	6219.8	86.89	589.8	–125.24
			P	8513.0	119.07	669.8	–222.25
		LM	F	353.4[b]	4.47[a]	136.5[b]	–7.75[d]
			P	973.7[d]	13.22[d]	222.2	–77.26
	Three	SQP	F	430.8	5.81	155.4	12.78
			P	965.1	13.35	215.2	–97.53
		LM	F	403.9	5.45	150.9	–5.26[b]
			P	1071.3	14.91	232.0	–92.26

[1]Estimation Method: Ordinary least squares, weighted least squares, or determinant criterion

[2]Number of responses used in objective function:

Six – p_{exh}, $\dot{m}_{steam}$, T_{cdwo}, T_{hw}, T_{cwi}, T_{cwo}

Four – p_{exh}, $\dot{m}_{steam}$, T_{cdwo}, T_{cwi}

Three – $\dot{m}_{steam}$, T_{cdwo}, T_{cwi}

[3]Minimization algorithm used: Sequential quadratic programming (SQP), Levenburg-Marquardt (LM), Nelder-Mead (NM).

[4]Data set: F – Residuals over points used in the fit. P – Residuals over data points used to compare predictive capabilities.

[a, b, c, d]Indicates "best" four results in category F or P over all 18 runs listed in Tables 1-3.

lem, as evidenced by the results given in Table 3. Any region in the parameter space that may give rise to collinearity in the responses will correspond to a local minimum of the determinant objective function. The Nelder-Mead minimization algorithm (a derivative-free downhill search technique) performed much better than the SQP algorithm when using the determinant criterion. The resulting residual measures were on the same order as those from using the Nelder-Mead algorithm and the ordinary least-squares objective function. However, the SQP algorithm outperformed the Nelder-Mead algorithm when using the sum-of-squares objective function. This difference gives some indication that the multidimensional objective function surface of the determinant criterion is more troublesome to minimization algorithms that evaluate derivatives than the corresponding sum-of-squares surface.

Overall, the ordinary least-squares method and either the Levenburg-Marquardt or the sequential quadratic programming method yielded the best results for the example regression problem. Weighted least squares did not demonstrate any significant improvement over ordinary least squares. The determinant criterion was comparatively unsuccessful in solving the example problem.

Residuals resulting from a successful regression should support the assumptions made in the analysis. The normally distributed errors for steam flow rate added to the true steam flow rate values to produce simulated measured data with instrument noise are plotted against the true values in Figure 2. The steam flow residuals from the regression run having the smallest sum of squares demonstrate good results and are plotted in Figure 3. The residuals have a standard deviation of 136.5 kg/h—very close to the standard deviation in the error (100 kg/h)—and a mean of −7.8 kg/h corresponding to distribution centered close to an error of zero.

Plots of measured and predicted values vs. time of steam flow rate or other response variables can also aid in comparing the regression results. However, for most of the runs showing relatively small standard deviation in steam mass flow rates,

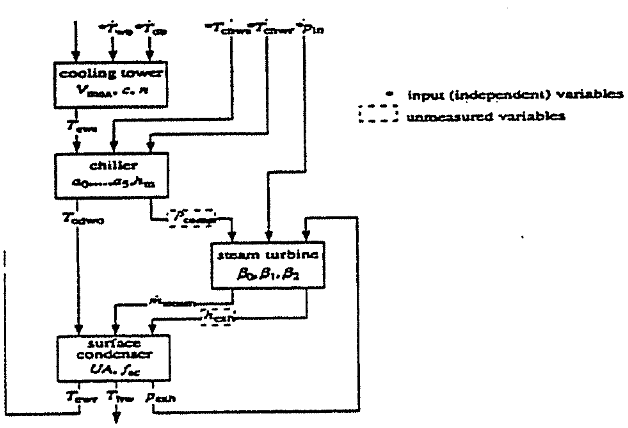

Figure 2 *Error vs. "true" steam flow rate values for the 240 points of simulated measured data used in the regression analyses. The random error has a standard deviation of 100 kg/h.*

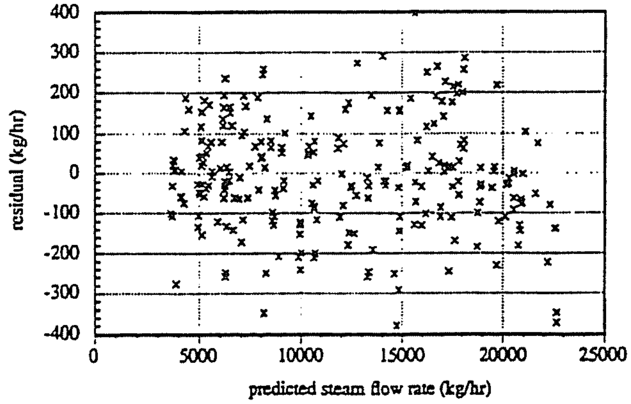

Figure 3 *Residuals (measured - predicted values) of steam flow rate for 240 data points using weighted least squares with the Levenburg-Marquardt minimization algorithm and four responses in the objective function. The residuals have a standard deviation of 136.5 kg/h.*

curves of predicted and measured values overlap each other and are indistinguishable in a plot scaled over the range of steam flow rate. Any run resulting in a standard deviation of steam flow residuals less than about 300 kg/h could be considered successful, yielding reasonably accurate steam flow rate predictions.

Qualitatively, this multiresponse, nonlinear parameter estimation problem is almost certain to have some level of parameter identification problems. It is difficult for any minimization algorithm to arrive at precisely a local minima when the objective function surface exhibits "flat" regions in any of the 13 dimensions. Some statistical measures of parameter variance for the model and minimization method are presented below.

Parameter Variance

One hundred sets of simulated replicated data were used to obtain 100 sets of parameter estimates. The same ordinary least-squares method with six responses using the SQP algorithm was used in each case, with the same starting values for the parameters. Each data set contains errors drawn from a multivariate pseudorandom number generator where the errors follow a multivariate normal distribution with a given covariance matrix. The resulting statistical properties calculated are a function of both the model and the estimation procedure. It has been demonstrated that, in general, nonlinear estimation problems will nearly always have biased parameters. The mean, bias, and standard deviation (square root of variance) of the example problem parameter estimates are given in Table 4.

The parameter means demonstrate significant bias, and most parameters demonstrate significant standard deviation. The two parameters showing relatively small standard deviation are the overall heat transfer coefficient, UA, and the coefficient c used in the cooling tower NTU. These two parameters can be estimated with a single observation and cannot contribute to overparameterization. For example, the coefficient c is

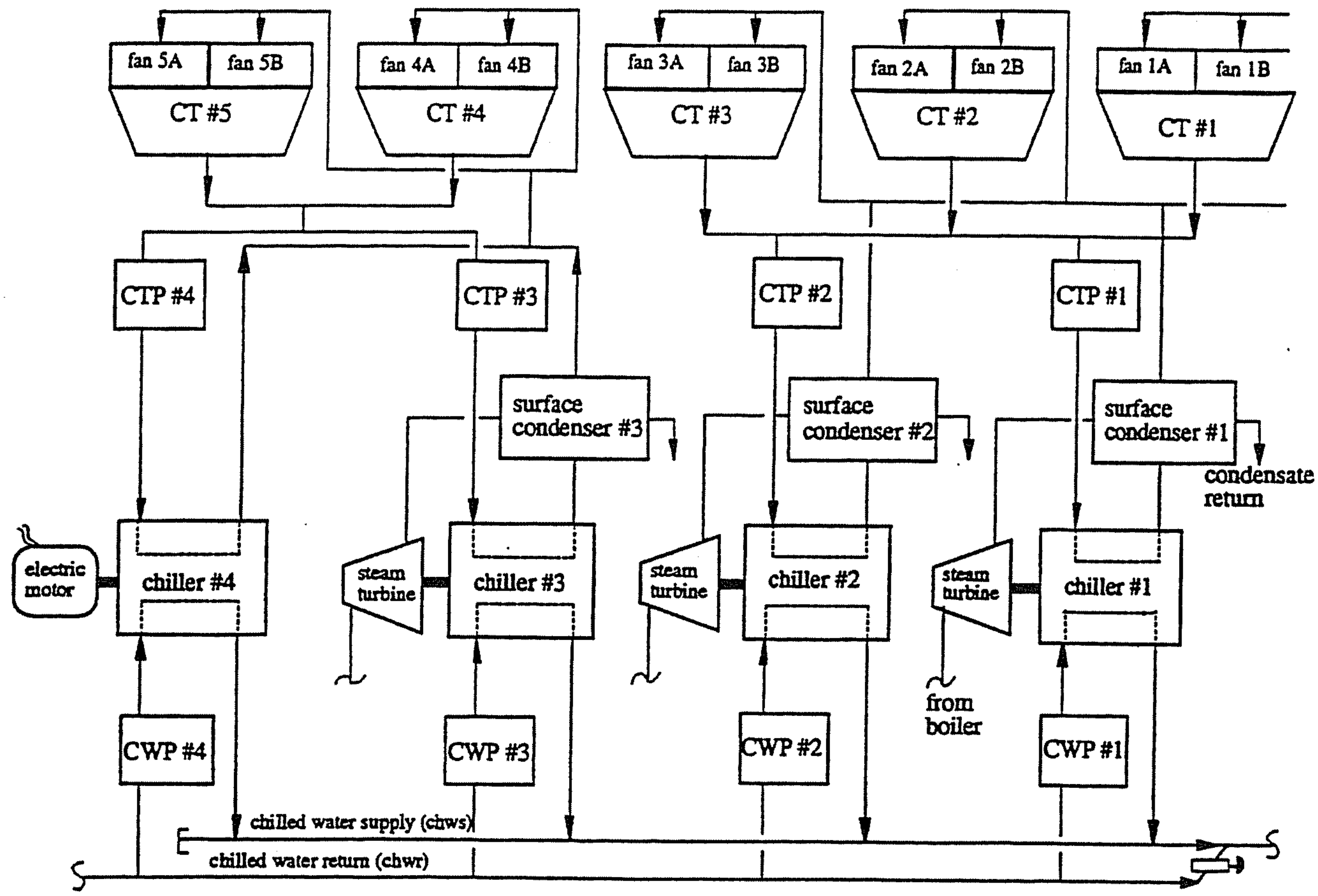

Figure 4 Schematic of Walnut Street chilled-water plant.

System Model

The system was modeled in TRNSYS, with the individual component models of chillers, heat exchangers, and a cooling tower combined in a TRNSYS "deck" of the plant, where the connections between outputs of one component and inputs of others are defined. A wiring diagram showing the connections defined in the deck is given in Figure 5. Independent (regressor) variables include wet- and dry-bulb temperatures, T_{wb} and T_{db}; chilled-water return temperatures entering the chiller evaporators, $T_{chws\ 3}$ and $T_{chws\ 4}$; the chilled and condenser water mass flow rates, $\dot{m}_{chw\ 3}$, $\dot{m}_{chw\ 4}$, $\dot{m}_{cw\ 3}$, and $\dot{m}_{cw\ 4}$; and the turbine inlet steam pressure, p_{in}. The independent variables are indicated by a bold line and the dependent variables by a thin line. The dashed lines indicate control variables, defined separately from independent and dependent variables when used in optimal supervisory control analysis. However, for the purposes of parameter estimation, the control variables are considered independent variables. Control variables include the fan speeds for the individual cooling tower fans γ_1, γ_2, γ_3, and γ_4 and the chilled-water supply temperatures, $T_{chws\ 3}$ and $T_{chws\ 4}$.

Each output of each component is a dependent variable. Chiller output variables include compressor power (P_{comp}) and condenser water outlet temperature (T_{cwo}). Dependent variables from the steam turbine component include steam mass flow rate and steam exhaust enthalpy, h_{exh}. Surface condenser outputs include condenser water outlet temperature (T_{cdwo}), hot well water temperature (T_{hw}), and exhaust pressure (p_{exh}). The electric chiller condenser outlet flow and surface condenser outlet flow are mixed before entering the cooling tower, and their mixed temperature is designated as the condenser water return temperature, T_{cwr}, and the combined condenser water flow rate is $\dot{m}_{cw}$. Cooling tower outputs include fan motor power, $P_{ct\ fans}$, and the temperature of the cooling tower outlet water supplied to the chiller condensers, T_{cws}.

Parameter Estimation

Data taken during July and August 1994 were used for this study. A number of observations were not recorded or incomplete and were removed from the data set. The data set represents approximately 54 days of operating data. There are 512 observations, made at two-hour intervals. The data set was partitioned into two parts, which will be referred to as periods I and II. The first 240 observations (period I) are used in the regression analysis to determine parameter estimates. The next 272 observations (period II) are used to compare the predictive capabilities over data not used in the fit.

The sequential quadratic programming (SQP) minimization algorithm was used for parameter estimation studies with

TABLE 5 Walnut Street Regression Results

Estimation Method[1]	No. Responses Used[2]	Data Set[3]	Sum of Squared Residuals, All Responses (Scaled)	Standard Deviation of Steam Flow Residuals (Kg/h)	Standard Deviation of Electric Power Residuals (Kw)
Ordinary Least Squares	Two	F	19764[b]	1170.3[c]	219.0[b]
		P	29060[b]	1160.9	249.6[b]
	Four	F	26960	1450.2	258.8
		P	31957[c]	1434.9	296.8
	Six	F	31785	1576.7	278.7
		P	39226	1615.7	313.8
	Eight	F	28144	1469.9	243.7
		P	36792	1568.7	264.2[c]
Determinant Criterion	Two	F	22662[c]	1301.9	243.0[c]
		P	32994	1380.0	264.7
	Four	F	30184	1154.9[b]	345.7
		P	38009	1089.2[b]	363.9
	Six	F	27732	1319.7	353.2
		P	44282	1122.7[c]	581.0
	Eight	F	195325	1831.7	608.7
		P	294856	1599.4	900.3
[4]Subsystem		F	17558[a]	1129.0[a]	185.3[a]
		P	11063[a]	867.3[a]	127.3[a]

[1]Estimation Method: Ordinary least squares or determinant criterion

[2]Number of responses used in objective function:

Eight – p_{exh}, $\dot{m}_{steam}$, T_{cdwi}, T_{cdwo}, T_{hw}, T_{cws}, T_{cwo4}, $P_{comp\#4}$

Six – $\dot{m}_{steam}$, T_{cdwi}, T_{cdwo}, T_{cws}, T_{cwo4}, $P_{comp\#4}$

Four – $\dot{m}_{steam}$, T_{cdwo}, T_{cws}, $P_{comp\#4}$

Two – $\dot{m}_{steam}$, $P_{comp\#4}$

[3]Data set: F – Residuals over points used in fit. P - Residuals over points used to compare predictive capabilities.

[4]Subsystem – Results from using parameter estimates found in separate subsystem runs.

[a, b, c] Indicates "best" three results in category F or P over all eight runs and the separate subsystem results listed in Table 5.

rion were applied and are discussed separately below. Both methods produced similar results with neither method demonstrating any clear advantage in producing a better fit in all cases. The determinant criterion did not demonstrate poor performance when compared to least squares as in the example problem. The only significant difference between the two applications is the use of real rather than simulated measured data in the parameter estimation runs. Measurement noise in the simulated measured data set was relatively small compared to estimated noise in the real data set. For example, steam flow rate residual standard deviation in the example data set was set at 100 kg/h (0.95% of the mean steam flow rate) and is estimated to be 749 kg/h (7.0% of the mean steam flow rate) in the real data set (the estimate is from results of best fit for the steam flow rate). With small errors, the objective function surface given by the determinant criterion may have a less well-defined minima than the objective function surface given by the sum of squared errors and may represent a more difficult problem for the minimization algorithm.

Using the determinant criterion and four responses yielded the smallest standard deviation in steam flow residuals (1,089 kg/h) for period II. The steam flow standard deviation increased using both fewer and more responses. The increase in standard deviation using six and eight responses (1,123 and 1,599 kg/h, respectively) indicates some amount of collinearity with the added responses. As expected, known collinearities among all 10 measured responses caused the run using the determinant criterion to fail. The minimization algorithm never moved from the starting point. Using only steam flow rate and electric chiller power to fit the parameters resulted in a steam flow residuals standard deviation of 1,380 kg/h.

example system and an actual operational chilled-water plant. The influence of using different sets of response variables in the estimation problem was investigated. Using more responses than just those for which predictions are needed (e.g., cost-associated responses) can result in a better predictive model. However, adding responses that can be collinear with other responses is detrimental. If significant differences in response variance are suspected, weighted least squares or the determinant criterion formulation should be used rather than ordinary least squares.

For a multiresponse plant model, it is beneficial to reduce the dimension of the problem by breaking the system into the smallest subsystems possible before estimating parameters. The smallest subsystems may be individual components where all input variables are available as measurements or a group of components when some input measurements are unavailable.

The methods presented were used to build and fit a predictive model of a chilled-water plant at a U.S. university. A comparison of model predictions with measured data not used in the parameter estimation problem demonstrates a good fit.

NOMENCLATURE

$\boldsymbol{\beta}$ = vector of parameters

$\mathbf{r}$ = vector of residuals

$\mathbf{x}$ = vector of independent (regressor) variables

$\mathbf{y}$ = vector of dependent (response) variables

m = number of responses

$\dot{m}_{chw}$ = chilled-water mass flow rate

$\dot{m}_{cw}$ = condenser water mass flow rate

$\dot{m}_{steam}$ = steam mass flow rate

n = number of observations

p_{exh} = turbine steam exhaust pressure

p_{inlet} = turbine steam inlet pressure

P_{comp} = chiller compressor power

$P_{ct\ fans}$ = cooling tower fan power

q = number of regressor variables

R = matrix of residuals

T_{cdwi} = inlet cooling water temperature, steam (surface) condenser

T_{cdwo} = outlet cooling water temperature, steam (surface) condenser

T_{chwr} = chiller chilled-water inlet temperature (chilled-water return)

T_{chws} = chiller chilled-water outlet temperature (chilled water supply)

T_{ci} = heat exchanger cold fluid inlet temperature

T_{co} = heat exchanger cold fluid outlet temperature

T_{cwi} = chiller condenser inlet water temperature

T_{cwo} = chiller condenser outlet water temperature

T_{cwr} = cooling tower inlet water temperature (condenser water return)

T_{cws} = cooling tower outlet water temperature (condenser water supply)

T_{db} = dry-bulb temperature

T_{hi} = heat exchanger hot fluid inlet temperature

T_{ho} = heat exchanger hot fluid outlet temperature

T_{hw} = outlet steam condensate (hot well) temperature, steam (surface) condenser

T_{wb} = wet-bulb temperature

UA = heat exchanger overall heat transfer coefficient-area product

V = variance-covariance matrix associated with the response variables

W = matrix of weighting factors

X = matrix of measured regressor variables

Y = matrix of measured response variables

γ = cooling tower fan, fraction of full speed

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