O. M. Ibrahim
Mechanical Engineering Department,
University of Rhode Island,
Kingston, RI 02881

S. A. Klein
Mechanical Engineering Department,
University of Wisconsin-Madison,
Madison, WI

High-Power Multi-Stage Rankine Cycles

This paper presents an analysis of the multi-stage Rankine cycle aiming at optimizing the power output from low-temperature heat sources such as geothermal or waste heat. A design methodology based on finite-time thermodynamics and the maximum power concept is used in which the shape and the power output of the maximum power cycle *are identified and utilized to compare and evaluate different Rankine cycle configurations. The maximum power cycle provides the upper-limit power obtained from any thermodynamic cycle for specified boundary conditions and heat exchanger characteristics. It also provides a useful tool for studying power cycles and forms the basis for making design improvements.*

Introduction

A distinction must be made between operation at maximum efficiency and maximum power. A reversible engine (e.g., Carnot cycle) provides an upper limit on efficiency, but it does not provide power, and therefore it is not a realistic design goal. The existence of a maximum power point for the heat transfer limited (HTL) Carnot cycle and the limitation of the efficiencies of real processes resulting from finite heat transfer rate constraints were recognized by Novikov (1958), El-Wakil (1962), and by Curzon and Ahlborn (1975). They have shown that finite power can only be achieved by operating at less than the optimum efficiency and at the maximum power the efficiency is given by $1 - \sqrt{T_L/T_H}$, where T_H and T_L are the temperatures of the heat source and heat sink, respectively.

The cycle considered by Novikov (1958), El-Wakil (1962), and Curzon and Ahlborn (1975) operated between an isothermal heat source and sink. Practical power plants do not operate between constant temperature thermal reservoirs, but rather, they transfer heat to and from flowing streams which have finite thermal capacitance rates (i.e., mass flow rate-specific heat product). Ibrahim and Klein (1989) and Lee et al. (1990) analytically present the maximum power and the efficiency of a finite time Carnot heat engine operating between two reservoirs with finite heat-capacity rates. They show that the maximum power efficiency in this case is given by $1 - \sqrt{T_{L,\text{in}}/T_{H,\text{in}}}$ where $T_{H,\text{in}}$ and $T_{L,\text{in}}$ are the inlet temperatures of the hot and cold streams.

Curzon and Ahlborn's study has inspired many following studies on the relation between power and efficiency in thermal-mechanical conversion cycles. Leff (1987a) shows that the Brayton, Otto, Diesel, and Atkinson cycles also produce maximum power at an efficiency of $1 - \sqrt{T_L/T_H}$. Gordon (1988) has investigated the maximum-power-efficiency relations for solar-driven cycles. Bejan (1988) considered an irreversible power plant model, accounting for the heat loss through the plant to the surroundings. Several papers deal with maximum power of the Carnot cycle operating between finite thermal capacitance rates heat source and sink (e.g., Ondrechen et al., 1983; Wu, 1988). Some economic aspects of the maximum power problem have been discussed by Curzon and Ahlborn (1975), Bejan (1988), and Ibrahim et al. (1992). They have noted that the efficiencies of actual power plants are reasonably close to the maximum power efficiency of internally reversible Carnot-like heat engines operating over the same temperature extremes. Bejan (1982, 1988, 1995) studied the optimum way to allocate the total heat exchanger conductance to achieve maximum power. He concluded that the total heat exchanger conductance should be split evenly between the hot and cold-side heat exchangers.

A specific objective of this work is to extend the earlier efforts to identify the 'best' cycle that will result in the upper limit of the maximum power for specified external conditions. There exist thermodynamic cycles that produce more power than an equivalent Carnot cycle when heat transfer constraints are considered (Ibrahim and Klein, 1989). Ondrechen et al. (1983) and Leff (1987b) investigate the nature of a new cycle based on ideal gas behavior which provides the maximum efficiency for heat source of finite capacity. Ibrahim et al. (1991) have identified maximum power cycles for specified boundary conditions using sequential Carnot cycles.

A case study is considered in which the heat source and sink for the power cycle are fluid streams with inlet temperatures of 455 K and 286 K, receptively. Heat transfer to and from the power cycle occurs through heat exchangers which are described with traditional heat exchanger relationships. In all cycles considered in this study, the total heat exchanger conductance is split evenly between the hot and cold-side heat exchangers, i.e., $UA_H = UA_L$. Similar results are obtained for other heat exchanger conductance ratios. The results are presented for wide ranges of heat exchanger sizes and thermal capacitance rates ratio of the external stream.

Maximum Power Cycle Model

The purpose of this section is to identify the shape of the cycle which provides the maximum amount of power for given heat source and sink streams and heat exchanger characteristics. The maximum power (MP) cycle is necessarily irreversible since zero power results from perfectly reversible cycles. The thermodynamic irreversibilities occurring in real power cycles are primarily the result of heat transfer processes. To account for these irreversibilities, the MP cycle is modeled as an internally reversible power cycle coupled to heat source and sink streams through conventional counterflow heat exchangers.

An MP cycle model is found by recognizing that an internally reversible thermodynamic cycle can be broken into a sequence of Carnot cycles (Fig. 1) having the same total heat interactions with the heat source and the heat sink and has the same power output as the original cycle. As the number of cycles in sequence increases, the performance and shape of such a sequence approaches the performance and shape of the MP cycle.

Contributed by the Petroleum Division for publication in the JOURNAL OF ENERGY RESOURCES TECHNOLOGY. Manuscript received by the Petroleum Division, August 4, 1994; revised manuscript received March 25, 1995. Associate Technical Editor: G. M. Reistad.

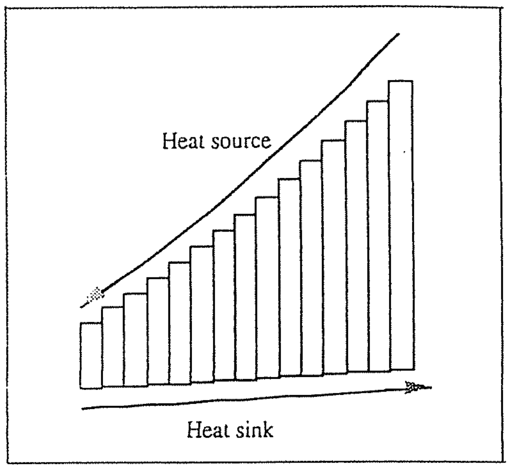

Fig. 1 A thermodynamic cycle broken into a sequence of Carnot cycles

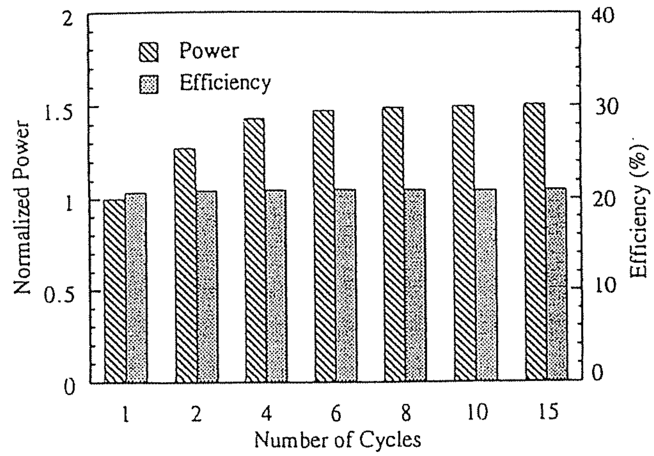

Fig. 2 Maximum power and corresponding efficiency versus the number of Carnot cycles in sequence

When the sequential Carnot cycles are coupled to a heat source and sink with finite thermal capacitance rates, the power from the N-Carnot cycles is given by

$$\dot{W} = \sum_{i=1}^{N} [\dot{C}_H \epsilon_H (T_{H,\text{in},i} - T_{h,i}) - \dot{C}_L \epsilon_L (T_{l,i} - T_{L,\text{in},i})] \quad (1)$$

$T_{H,\text{in},i}$ and $T_{L,\text{in},i}$ are the source and sink inlet temperatures for a Carnot cycle in the sequence where $T_{H,\text{in},l}$ and $T_{L,\text{in},N}$ are specified inlet source and sink temperatures. $\epsilon_{H,i}$ and $\epsilon_{L,i}$ are the effectivenesses of the hot side and cold-side heat exchangers of each cycle. $T_{h,i}$ and $T_{l,i}$ are the high and low temperatures of a Carnot cycle in the sequence.

The shape of the MP cycle is determined by maximizing $\dot{W}$ with respect to $T_{h,i}$ and $T_{l,i}$ for $i = 1, N$ subject to the entropy balance constraints

$$\frac{\dot{C}_H \epsilon_H (T_{H,\text{in},i} - T_{h,i})}{T_{h,i}} - \frac{\dot{C}_L \epsilon_L (T_{l,i} - T_{L,\text{in},i})}{T_{l,i}} = 0 \quad (i = 1, N) \quad (2)$$

An analytical solution is not apparent for this optimization problem. However, the optimum heat power cycle with finite thermal capacitance rate heat source and heat sink can be determined numerically.

The required number of Carnot cycles in sequence which will sufficiently identify the cycle is considered for the case where $\text{NTU}_H = 10$ and $\dot{C}_L/\dot{C}_H = 10$. Figure 2 shows the efficiency at maximum power and the maximum power as the number of Carnot cycles in sequence increases from 1 to 15. The efficiency at maximum power is nearly independent of the number of Carnot cycles in the sequence. The efficiency of the optimum heat power cycle is almost the same as the maximum power efficiency of a single Carnot cycle operating between the same external streams. (The 1-percent difference noted in this case can be due to numerical techniques and round-off error.) However, the maximum power increases as the number of Carnot cycles in sequence increases in an asymptotic manner. The difference between the maximum power obtained from 10 cycles in sequence and 15 cycles in sequence is very small (less than 1 percent), which indicates that the 15 cycles approximate the optimum heat power cycle.

Figure 3 shows the shape of the optimum heat power cycles in a T-$\dot{S}$ diagram. The shape varies with capacitance rate (mass flow specific heat product) of the external heating and cooling streams. Figure 3(a) shows the optimum shape where the capacitance rates of the source and sink streams are equal. In Fig. 3(b), the capacitance rate of the sink is 10 times that of the source. As shown in Figs. 3(a) and (b), the heat transfer processes for the optimum cycle are not isothermal, but rather occur over a range of temperatures. Nonisothermal processes allow the MP cycle to match the heat source and heat sink temperature distributions and reduce the irreversibility due to heat transfer. The slope of the temperature change of the optimum working fluid in the heat exchanger should be nearly parallel to the external stream flowing through the heat exchanger. The result corresponds to equal heat capacitance rates of the working fluid and the external stream flowing through the heat exchanger.

Figure 4 shows the effects of heat exchanger conductances on the MP cycle shape. As the heat exchanger conductances increase, the temperature gap between the heat transfer processes for the optimum cycle and the heat source and heat sink decreases, which reduces the external irreversibilities.

Performance Indices for the Comparison of Power Cycles

The maximum power from alternative power cycles for the same boundary conditions, i.e., the same heat source/sink inlet temperatures, thermal capacitance rates, and heat exchanger

Nomenclature

$\dot{C}$ = thermal capacitance rate, kW/K
h = enthalpy, kJ/kg
$\dot{m}$ = mass flow rate, kg/s
NTU = no. of transfer units, NTU = $UA/\dot{C}$
P = pressure, bar
$\dot{Q}$ = rate of heat transfer, kW
s = entropy, kJ/kg K
T = temperature, K
$\dot{S}$ = entropy transport rate, kW/K
UA = heat exchanger conductance, kW/K
$\dot{W}$ = power, kW
η = thermal efficiency
ω = power ratio, $\dot{W}/\dot{W}_{\max}$

Subscripts

H = heating fluid; heat source
h = high
in = in, inlet
L = cooling fluid; heat sink
l = low
max = maximum

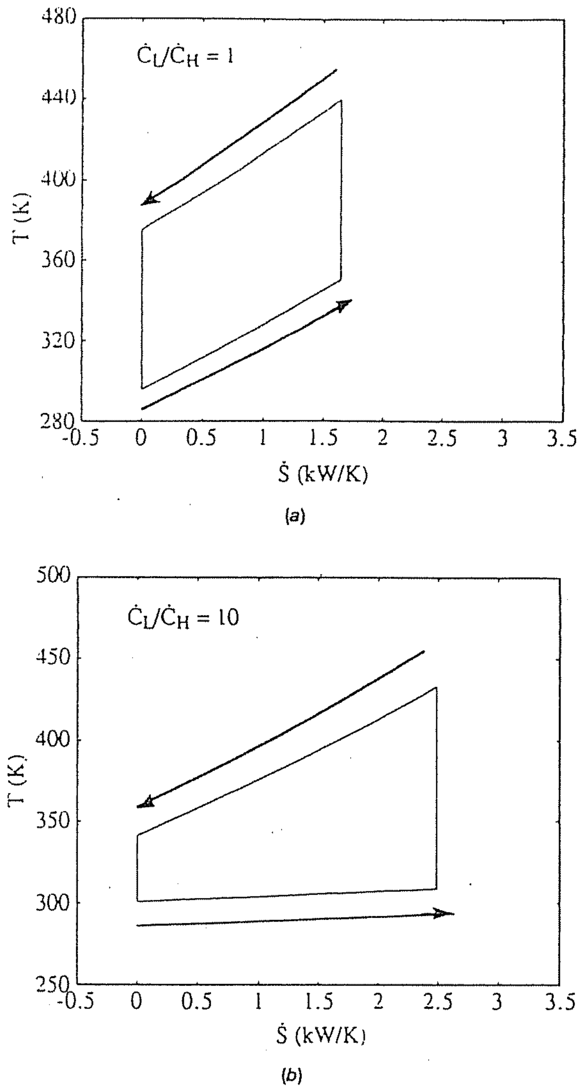

Fig. 3 Variation of the maximum power cycle shape with thermal capacitance rates ratio

conductances, provides a criterion to compare the thermodynamic performance of alternative power cycles. The upper limit for the power output for specified boundary conditions is provided by the MP cycle. The *power ratio*, ω, is defined as the ratio between the power output of a particular cycle to the power output of the MP cycle, $\omega = \dot{W}/\dot{W}_{max}$. In addition to the power output, the thermal efficiency of power cycles, $\eta = \dot{W}/\dot{Q}_{in}$, is a traditional measure of thermodynamic performance.

Single-Stage Rankine Cycle

A constant temperature heat transfer process, as required in the Carnot cycle, is easily achieved in practice by boiling or condensing a pure fluid at constant pressure. Most of energy transfer occurring in a single-stage Rankine cycle occurs this way. In this study, a single-stage Rankine cycle is defined as a cycle with constant boiling and condensing pressures and no vapor superheating. In the regenerative-Rankine cycle, feedwater heaters are used to raise the temperature of the liquid working fluid before it enters the boiler; accordingly, heat is supplied at higher average temperature and a higher efficiency is achieved.

Figure 5 shows the relation between power and efficiency for the simplified power cycle, the Rankine cycle, and the regenerative-Rankine cycle employing one and three-feedwater heaters. Along any curve, the power and efficiency vary as a result of changes in boiler pressure, and accordingly, changes in condenser pressure. Figure 5 shows that the efficiency trade-offs for the Rankine and the regenerative-Rankine are similar to those of the HTL Carnot cycle. The regenerative modification of the Rankine cycle is a technique for increasing the cycle efficiency; however, regeneration decreases the maximum power output. As the number of feedwater heaters increase, the performance of the regenerative Rankine cycle approaches the performance of HTL Carnot cycle.

Multi-Stage Rankine Cycle

The multi-stage Rankine cycle consists of a number of single-stage Rankine cycles operating in series. Each single-stage Rankine cycle can operate at a different boiling temperature, which allows variable temperatures during heat transfer, therefore, adding more flexibility to match the nonisothermal processes of the MP cycle. In this section, two and three-stage Rankine cycles are considered. Flow and *T-s* diagrams of the two-stage Rankine cycle are shown in Figs. 6 and 7. The cycle has two boiling pressures, i.e., two boiling temperatures.

The shapes of single and three-stage cycles are compared with the MP cycle in a *T-Ṡ* diagram. As shown in Fig. 8, the single-stage cycle which has constant temperature condensing and boiling processes nearly matches the heat transfer process on condenser side, but it does not match the heat transfer process

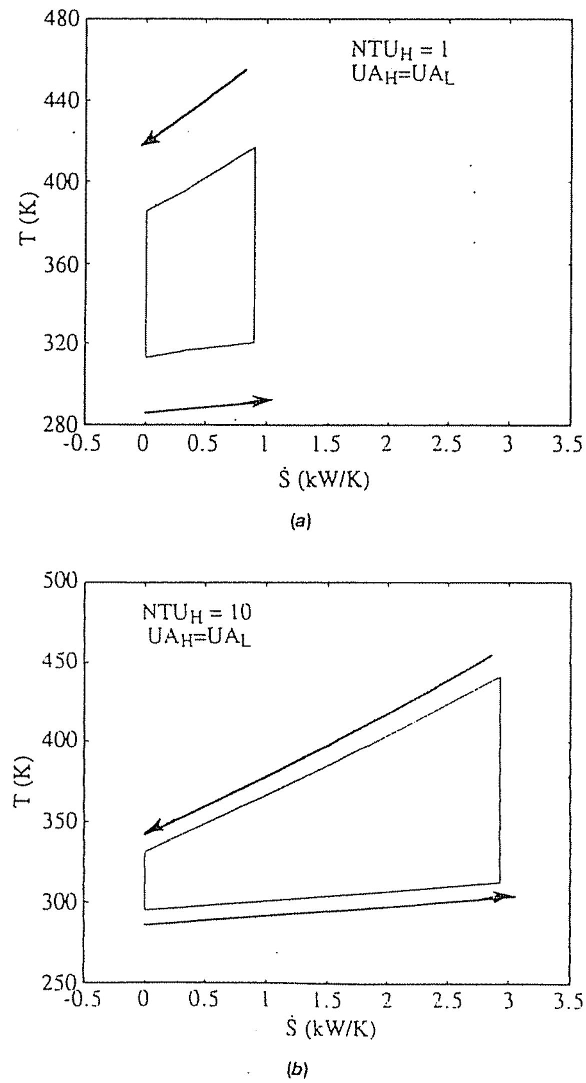

Fig. 4 Variation of the maximum power cycle shape with heat exchanger sizes

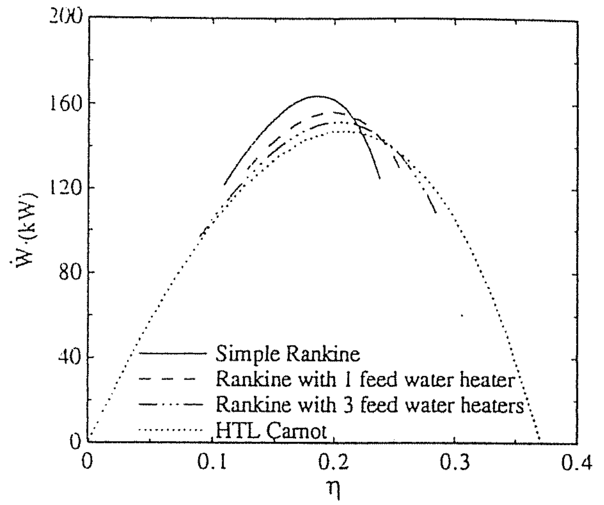

Fig. 5 Power efficiency trade-offs for a simple Rankine and Rankine cycle with regeneration

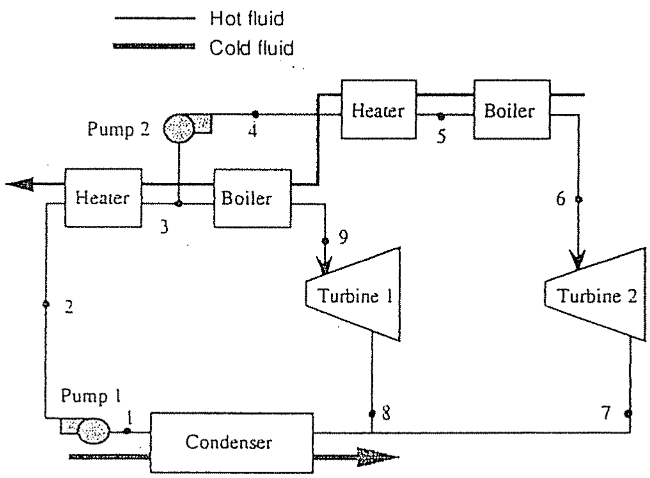

Fig. 6 Flow diagram for a two-stage Rankine cycle

on the boiler side. In Fig. 9, the shape of the three-stage cycle is compared with the MP cycle. The three-stage cycle has three boiling temperatures, which add more flexibility to match the nonisothermal processes of the MP cycle. As the number of the stages increase, the cycle shape more closely matches the maximum power cycle.

Power Optimization. The power optimization of the multistage Rankine cycles is done numerically using an engineering-solving program with built-in thermodynamic property data for pure fluid (Klein and Alvarado, 1994). Figure 10 shows the power efficiency trade-offs of single, two, and three-stage Rankine cycles. Along any curve, the power and efficiency vary by changes in boiler and condenser temperatures. Figure 10 also shows the existence of the maximum power points for the single, two, and three-stage Rankine cycles. The results also show a significant increase in power ratio as the number of Rankine cycles in series increase from 1 to 3. The efficiency trade-offs for all the cycles show that the maximum power efficiency is well approximated by $1 - \sqrt{T_{L,\mathrm{in}}/T_{H,\mathrm{in}}}$.

The effect of the thermal capacitance rates ratio on the single, two, and three-stage Rankine cycles is shown in Fig. 11 for the case where $\mathrm{NTU}_H = 10$. As the thermal capacitance rates ratio increases, the power ratio increases rapidly first and then levels off for thermal capacitance rates ratios greater than 10. The power output increases significantly as the number of Rankine cycles in series increases from 1 to 3.

The effect of the heat exchanger sizes on the power output of the power ratio is shown in Fig. 12, for the case where $\dot{C}_L/\dot{C}_H = 5$. For small heat exchangers (NTU < 5), the difference in power ratios of all cycles are relatively small. However, at

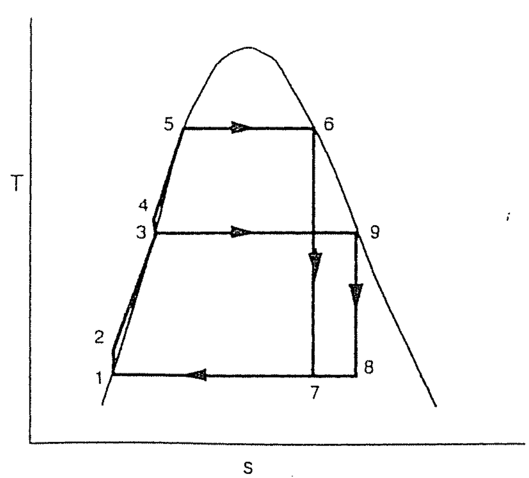

Fig. 7 *T-s* diagram for a two-stage Rankine cycle

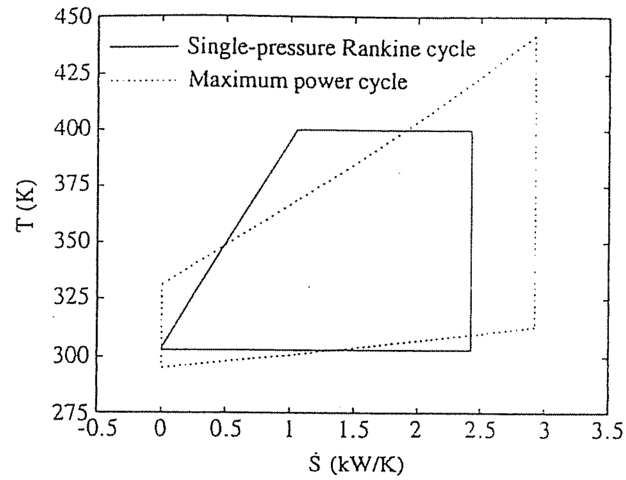

Fig. 8 Single-stage Rankine cycle and the maximum power cycle

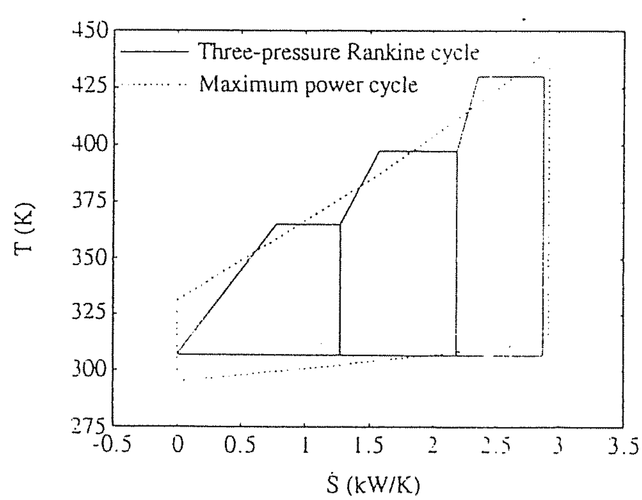

Fig. 9 Three-stage Rankine cycle and the maximum power cycle

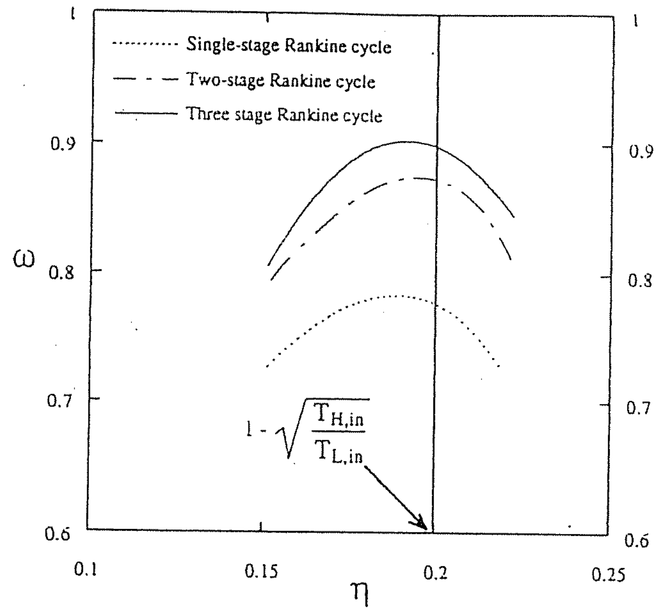

Fig. 10 Power ratio efficiency trade-offs of multi-stage Rankine cycles

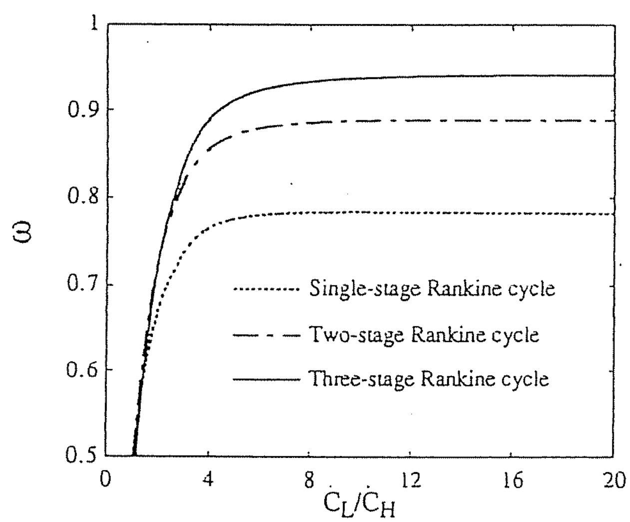

Fig. 11 Effect of the thermal capacitance rates ratio on the power output of multi-stage Rankine cycles

large heat exchangers, the three-stage cycle is superior. Compared to the single and two-stage cycles, the three-stage cycle produces the highest power output for a wide range of heat exchanger sizes. With power ratios higher than 0.9, it also provides a fairly close approximation of the MP cycle.

Conclusions

The design of heat power cycles which attain the maximum power possible for specified thermal boundary conditions are of significant practical importance. The MP cycle provides a useful tool for studying power cycles and forms the basis for making design improvements. When a power cycle interacts with source and sink streams having finite thermal capacitance rates, the maximum power results when the heat transfer processes occur at variable temperature paralleling the temperature change of the external streams. The varying temperature during the heat transfer processes reduces the thermodynamic irreversibility of heat exchange and the effect of the thermal pinch in the boiler, compared to the isothermal boiling/condensing processes occurring in the Rankine cycle. The variable tempera-

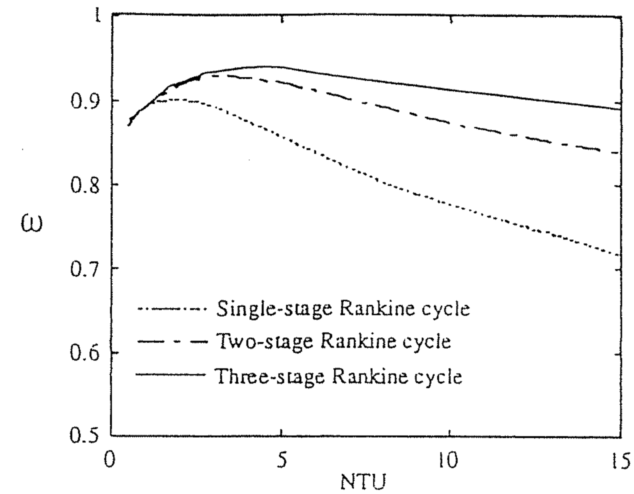

Fig. 12 Effect of heat exchanger size on the power output of multi-stage Rankine cycles

ture of the heat transfer processes of the MP cycle can be achieved by varying the pressure during the phase change of a pure fluid. The multi-stage Rankine cycle can have a higher performance compared to the single-stage Rankine cycle for the same boundary conditions, i.e., the same heat source/sink inlet temperatures, thermal capacitance rates, and heat exchanger conductances. This high performance of the multi-stage Rankine cycle is due to its flexibility in matching the MP cycles' heat transfer processes by adjusting the boiler pressures, and accordingly, the boiler temperatures.

A simple way to evaluate alternative power cycles during preliminary power cycle design is to compare the performance of any new proposed cycle to the cycle which produces maximum power for the same external conditions. The ratio of the power output of a proposed cycle to the maximum power is an important criterion for evaluation of new power cycles.

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