Solar Energy, Vol. 19, pp. 325–329. Pergamon Press 1977. Printed in Great Britain

REVIEW PAPER

CALCULATION OF MONTHLY AVERAGE INSOLATION ON TILTED SURFACES

S. A. KLEIN
Solar Energy Laboratory, University of Wisconsin-Madison, Madison, WI 53706, U.S.A.

(*Received* 16 *June* 1976; *in revised form* 28 *October* 1976)

Abstract—Several simplified design procedures for solar energy systems require monthly average meteorological data. Monthly average daily totals of the solar radiation incident on a horizontal surface are available. However, radiation data on tilted surfaces, required by the design procedures, are generally not available. A simple method of estimating the average daily radiation for each calendar month on surfaces facing directly towards the equator has been presented by Liu and Jordan[1]. This method is verified with experimental measurements and extended to allow calculation of monthly average radiation on surfaces of a wide range of orientations.

INTRODUCTION

Estimates of the monthly average solar radiation incident on surfaces of various orientations are required for solar energy design procedures, heating load calculations, and other applications. Monthly averages of the daily solar radiation incident upon a horizontal surface are available for many locations. However, radiation data on tilted surfaces are generally not available.

A simple method of estimating the average daily radiation for each calendar month on surfaces facing directly towards the equator has been developed by Liu and Jordan[1]. Their method is described here and compared with the work of Page[2] and with additional experimental measurements. The method is then extended so that it is applicable for surfaces oriented east or west of south.

ESTIMATION OF AVERAGE DAILY RADIATION ON SURFACES FACING DIRECTLY TOWARDS THE EQUATOR

The average daily radiation on a horizontal surface, $\bar{H}$, for each calendar month can be expressed by defining $\bar{K}_T$, the fraction of the mean daily extraterrestrial radiation, $\bar{H}_0$.

$$\bar{K}_T = \bar{H}/\bar{H}_0 \quad (1)$$

$$\bar{H}_0 = \frac{1}{(m_2 - m_1)} \sum_{n=m_1}^{m_2} (H_0)_n \quad (2)$$

where m_1 and m_2 are, respectively, the days of the year at the start and end of the month and $(H_0)_n$ is the extraterrestrial radiation on a horizontal surface on day n of the year which is given by

$$(H_0)_n = \frac{24}{\pi} I_{sc} \left[1 + 0.033 \cos\left(\frac{360n}{365}\right)\right] \times [\cos\phi \cos\delta \sin\omega_s + (\omega_s 2\pi/360) \sin\phi \sin\delta] \quad (3)$$

where I_{sc} is the solar constant, n is the day of the year given for each month in Table 1, ϕ is the latitude, and δ is the solar declination which can be approximately expressed

$$\delta = 23.45° \sin[360(284 + n)/365] \quad (4)$$

ω_s is the sunset hour angle

$$\cos\omega_s = -\tan\phi \tan\delta. \quad (5)$$

$\bar{H}_0$ can be conveniently estimated from eqn (3) by selecting for each month, the day of the year for which the daily extraterrestrial radiation is nearly the same as the monthly mean value. Using the 16th day of each month can lead to small errors in $\bar{H}_0$, particularly for June and December. Recommended days for each month are given in Table 1. $\bar{H}_0$ is tabulated for each month as a function of latitude in Table 2. The value of the solar constant used in the construction of Table 2 is 4871 kJ hr^{-1} m^{-2}, Thekaekara and Drummond[3]], which is approximately 3 per cent lower than the value used by Liu and Jordan[1, 4] and Page[2].

The average daily radiation on a tilted surface, $\bar{H}_T$, can

Table 1. Recommended average day for each month

Month	Day of the year	Date
Jan.	17	17 Jan.
Feb.	47	16 Feb.
Mar.	75	16 Mar.
Apr.	105	15 Apr.
May	135	15 May
June	162	11 June
July	198	17 July
Aug.	228	16 Aug.
Sept.	258	15 Sept.
Oct.	288	15 Oct.
Nov.	318	14 Nov.
Dec.	344	10 Dec.

Table 2. Monthly average daily extraterrestrial radiation, kJ/m^2

Lat.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
25	23,902	28,115	32,848	37,111	39,356	40,046	39,606	37,832	34,238	29,413	24,909	22,669
30	21,034	25,679	31,141	36,436	39,569	40,706	40,071	37,534	32,917	27,213	22,161	19,714
35	18,069	23,072	29,200	35,497	39,530	41,129	40,292	36,976	31,348	24,820	19,296	16,687
40	15,403	20,319	27,040	34,303	39,247	41,328	40,281	36,166	29,542	22,255	16,344	13,626
45	11,998	17,448	24,677	32,869	38,737	41,322	40,055	35,118	27,515	19,541	13,344	10,579
50	8987	14,490	22,131	31,209	38,025	41,147	39,644	33,851	25,283	16,705	10,342	7605
55	6082	11,486	19,423	29,345	37,152	40,863	39,100	32,391	22,863	13,778	7396	4791

be expressed

$$\bar{H}_T = \bar{R}\bar{H} = \bar{R}\bar{K}_T\bar{H}_0 \tag{6}$$

where $\bar{R}$ is defined to be the ratio of the daily average radiation on a tilted surface to that on a horizontal surface for each month. $\bar{R}$ can be estimated by individually considering the beam, diffuse, and reflected components of the radiation incidence on the tilted surface. Assuming diffuse and reflected radiation to be isotropic, Liu and Jordan[1] have proposed that $\bar{R}$ can be expressed

$$\bar{R} = (1 - \bar{H}_d/\bar{H})\bar{R}_b + \bar{H}_d/\bar{H}(1 + \cos s)/2 + \rho(1 - \cos s)/2 \tag{7}$$

where $\bar{H}_d$ is the monthly average daily diffuse radiation, $\bar{R}_b$ is the ratio of the average beam radiation on the tilted surface to that on a horizontal surface for each month, s is the tilt of the surface from horizontal, and ρ is the ground reflectance. Liu and Jordan[4] suggest that ρ varies from 0.2 to 0.7 depending upon the extent of snow cover. $\bar{R}_b$ is a function of the transmittance of the atmosphere (except during times of equinox) which depends upon the atmospheric cloudiness, water vapor and particulate concentration. However, Liu and Jordan suggest that $\bar{R}_b$ can be estimated to be the ratio of extraterrestrial radiation on the tilted surface to that on a horizontal surface for the month. For surfaces facing directly towards the equator,

$$\bar{R}_b = \frac{\cos(\phi - s)\cos\delta\sin\omega_s' + \pi/180\,\omega_s'\sin(\phi - s)\sin\delta}{\cos\phi\cos\delta\sin\omega_s + \pi/180\,\omega_s\sin\phi\sin\delta} \tag{8}$$

where ω is the hour angle which is 15°×(hours from solar noon), afternoons, positive, mornings negative and ω_s' is the sunset hour angle for the tilted surface which is given by

$$\omega_s' = \min[\omega_s, \arccos[-\tan(\phi - s)\tan\delta]]. \tag{9}$$

Page has calculated values of $\bar{R}_b$ for five surface orientations of several latitudes by integrating the direct radiation on the tilted and horizontal surface calculated at hourly intervals for a standard direct radiation curve. Values of $\bar{R}_b$ calculated from eqn (8) are in reasonably good agreement with the values tabulated by Page as seen in Table 3. Page's values are slightly more conservative, i.e. closer to unity.

Since measurements of $\bar{H}_d$, the monthly average daily diffuse radiation are rarely available, $\bar{H}_d$ must be estimated from measurements of the average daily total radiation. A number of investigators have found that the diffuse radiation fraction, $\bar{H}_d/\bar{H}$, is a function of $\bar{K}_T$. Shown in Fig. 1 are the relationships reported by Liu and Jordan, and Page which can be expressed

$$\frac{\bar{H}_d}{\bar{H}} = \begin{cases} 1.390 - 4.027\bar{K}_T + 5.531\bar{K}_T^2 - 3.108\bar{K}_T^3 & \text{[Liu \& Jordan]} \quad (10a) \\ 1.00 - 1.13\bar{K}_T & \text{[Page].} \quad (10b) \end{cases}$$

Table 3. Comparison of values of $\bar{R}_b$ from Page[2] and eqn (8)

	$\phi = 30°$				$\phi = 40°$			
	$\phi - s = 0$		Vertical		$\phi - s = 0$		Vertical	
	Page	Eqn (8)	Page	Eqn (8)	Page	Eqn (8)	Page	Eqn (8)
Jan.	1.61	1.66	1.49	1.59	2.15	2.26	2.11	2.32
Feb.	1.40	1.43	1.06	1.13	1.72	1.79	1.50	1.59
Mar.	1.18	1.20	0.64	0.67	1.35	1.38	0.93	0.96
Apr.	0.99	1.00	0.29	0.30	1.07	1.06	0.48	0.48
May	0.89	0.87	0.13	0.11	0.90	0.88	0.27	0.25
June	0.84	0.87	0.06	0.05	0.84	0.80	0.19	0.17
July	0.85	0.84	0.09	0.08	0.85	0.83	0.22	0.21
Aug.	0.94	0.94	0.21	0.21	0.98	0.98	0.37	0.37
Sept.	1.09	1.12	0.45	0.50	1.20	1.24	0.69	0.74
Oct.	1.30	1.35	0.88	0.97	1.57	1.64	1.24	1.36
Nov.	1.53	1.60	1.33	1.46	1.98	2.12	1.86	2.10
Dec.	1.67	1.74	1.61	1.74	2.30	2.42	2.36	2.58

Page's relationship, which was derived from experimental measurements at 10 stations, tends to agree more closely with the additional measurements reported by Choudhury[5], Stanhill[6] and Norris[7]. The discrepancy is apparently due, at least in part, to the fact that a shade ring correction factor was applied to all reported diffuse radiation measurements except those taken at Blue Hill, Massachusetts, which Liu and Jordan used to derive their relationship. Page's relationship probably results in a more accurate estimate of the diffuse radiation fraction; however, values of $\bar{R}$ estimated from eqn (7) with $\rho = 0.2$ tend to agree more closely with experimental measurements when the Liu and Jordan relationship is used, as shown in the next section.

COMPARISON WITH EXPERIMENTAL RESULTS

Long-term measurements of the radiation incident on both tilted and horizontal surfaces are scarce. Measurements of the radiation incident on a horizontal and a south-facing vertical surface in Blue Hill, Massachusetts (lat. 42.2°N) for the yr 1952–56 have been presented by Liu and Jordan. In Table 4 experimental values of $\bar{R}$ are compared with values estimated from eqn (7) with $\rho = 0.2$ using the diffuse radiation fraction relationships of Liu and Jordan (eqn 10a) and Page (eqn 10b). In Table 5, similarly calculated values of $\bar{R}$ are compared with experimental values for a 38° north-facing surface at Highett, Victoria, Australia (lat. 37.9°S) for the years 1966–68[8]. Based on this experimental data, it appears that Liu and Jordan's relationship for the diffuse radiation fraction (eqn 10a) results in more accurate values of $\bar{R}$ than does Page's (eqn 10b). It is possible that the "underestimated" diffuse radiation fraction arising from eqn (10a) tends to cancel errors caused by the conservative assumptions of isotropic diffuse radiation and a ground reflectance of 0.2.

ESTIMATION OF AVERAGE DAILY RADIATION ON SURFACES ORIENTED EAST OR WEST OF SOUTH

Liu and Jordan's method of calculating $\bar{R}_b$ can be extended so that it is applicable for surfaces which are not oriented directly towards the equator by integrating the rate of extraterrestrial radiation on the surface for the period during which the sun is both above the horizon and in front of the surface and then dividing this

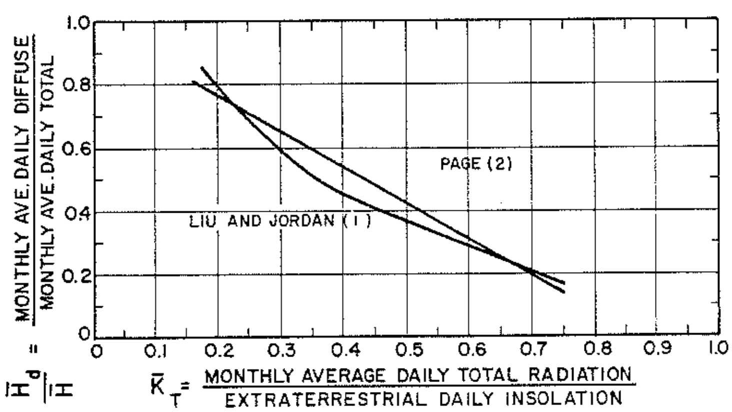

Fig. 1. Relationship of $\bar{H}_d/\bar{H}$ to $\bar{K}_T$.

Table 4. Comparison of experimental† and estimated values of $\bar{R}$ for a vertical surface facing south at Blue Hill, Mass., lat. 42°13′N

Month	K_T	$\bar{R}$ Experimental† (1952–56)	$\bar{R}$ Estimated from eqns (7) and (10a)	$\bar{R}$ Estimated from eqns (7) and (10b)
Jan.	0.411	1.80	1.72	1.55
Feb.	0.445	1.38	1.31	1.22
Mar.	0.445	0.93	0.91	0.87
Apr.	0.440	0.61	0.62	0.62
May	0.481	0.44	0.47	0.48
June	0.524	0.39	0.41	0.42
July	0.528	0.42	0.42	0.44
Aug.	0.485	0.54	0.55	0.55
Sept.	0.485	0.79	0.79	0.77
Oct.	0.466	1.23	1.18	1.11
Nov.	0.421	1.60	1.61	1.46
Dec.	0.422	1.94	1.91	1.72

†Source: Liu and Jordan (1962)[1].

Table 5. Comparison of experimental and estimated values of $\bar{R}$ for a 38° surface facing north at Melbourne, Australia, lat. 37.9°S

Month	K_T	$\bar{R}$ Experimental† (1966–68)	$\bar{R}$ Estimated from eqns (7) and (10a)	$\bar{R}$ Estimated from eqns (7) and (10b)
Jan.	0.46	0.85	0.88	0.89
Feb.	0.46	0.94	0.96	0.96
Mar.	0.41	1.10	1.09	1.06
Apr.	0.40	1.29	1.27	1.21
May	0.34	1.37	1.41	1.33
June	0.34	1.50	1.54	1.44
July	0.37	1.50	1.55	1.42
Aug.	0.39	1.34	1.34	1.28
Sept.	0.38	1.15	1.14	1.10
Oct.	0.39	0.98	0.99	0.98
Nov.	0.41	0.88	0.90	0.90
Dec.	0.42	0.84	0.86	0.87

result by $\bar{H}_0$. In this case

$$\begin{aligned}\bar{R}_b = \{&[\cos s \sin\delta \sin\phi]\pi/180[\omega_{ss}-\omega_{sr}]\\ &-[\sin\delta\cos\phi\sin s\cos\gamma]\pi/180[\omega_{ss}-\omega_{sr}]\\ &+[\cos\phi\cos\delta\cos s][\sin\omega_{ss}-\sin\omega_{sr}]\\ &+[\cos\delta\cos\gamma\sin\phi\sin s][\sin\omega_{ss}-\sin\omega_{sr}]\\ &-[\cos\delta\sin s\sin\gamma][\cos\omega_{ss}-\cos\omega_{sr}]\}/\\ &\{2[\cos\phi\cos\delta\sin\omega_s+\pi/180\omega_s\sin\phi\sin\delta\}\end{aligned} \qquad (11)$$

where γ is the surface azimuth angle, i.e. the deviation of the normal to the surface from the local meridan, the zero point being due south, east negative, and west positive. ω_{sr} and ω_{ss} are the sunrise and sunset hour angles on the tilted surface, given by

if $\gamma \not> 0$

$$\omega_{sr} = -\min[\omega_s, \arccos[(AB+\sqrt{A^2-B^2+1})/(A^2+1)]] \qquad (12)$$
$$\omega_{ss} = \min[\omega_s, \arccos[(AB-\sqrt{A^2-B^2+1})/(A^2+1)]]$$

if $\gamma \not< 0$

$$\omega_{sr} = -\min[\omega_s, \arccos[(AB-\sqrt{A^2-B^2+1})/(A^2+1)]]$$

~~if γ > 0~~ (13)

$$\omega_{ss} = \min[\omega_s, \arccos[(AB+\sqrt{A^2-B^2+1})/(A^2+1)]]$$

$$A = \cos\phi/[\sin\gamma\tan s]+\sin\phi/\tan\gamma \qquad (14)$$
$$B = \tan\delta[\cos\phi/\tan\gamma-\sin\phi/[\sin\gamma\tan s]]. \qquad (15)$$

An example demonstrating this method of estimating radiation on tilted surfaces follows.

EXAMPLE

Estimate the monthly averages of daily radiation incident on a surface tilted 43° from horizontal facing due south in Madison, Wisconsin (43°N lat.) and compare them with those incident if the surface were oriented 15° west of south.

Daily averages value of $\bar{H}$, the radiation incident on a horizontal surface can be found in Ref. [9]. The mean daily extraterrestrial radiation, $\bar{H}_0$, for each month can be determined from eqn (3) (using the days of the year in Table 1) or from Table 2 with interpolation. The ratio $\bar{H}/\bar{H}_O$ determines $\bar{K}_T$ for each month which can be used to calculate $\bar{H}_d/\bar{H}$ from eqn (10a) or (10b); (eqn (10a) is used in this example.) $\bar{R}$ is calculated from values of $\bar{R}_b$ for each month for both the south (eqn 8) and the 15° west of south (eqn 11) surfaces. The average daily radiation on each of the surfaces is the product of $\bar{R}\bar{H}$ for each month. These results are displayed in Table 6.

REFERENCES

1. B. Y. H. Liu and R. C. Jordan, Daily insolation on surfaces tilted toward the equator. *Trans. ASHRAE* 526–541 (1962).

Table 6. Calculation of daily average radiation on a 43° surface in Madison

Month	$\bar{H}$ Kj m² day⁻¹	$\bar{H}_0$ KJ m² day⁻¹	$\bar{K}_T$	$\bar{H}_d/\bar{H}$	$\bar{R}_b$ $\gamma=0°$	$\bar{R}_b$ $\gamma=15°$	$\bar{R}$ $\gamma=0°$	$\bar{R}$ $\gamma=15°$	$\bar{H}_T$ $\gamma=0$ KJ m² day⁻¹	$\bar{H}_T$ $\gamma=15°$ KJ m² day⁻¹
Jan.	6412	13,226	0.485	0.384	2.53	2.47	1.92	1.88	12,300	12,100
Feb.	9224	18,612	0.496	0.374	1.95	1.90	1.57	1.54	14,500	14,200
Mar.	13,992	25,762	0.543	0.337	1.46	1.44	1.28	1.27	18,000	17,800
Apr.	16,527	33,429	0.494	0.376	1.09	1.09	1.03	1.03	17,100	17,100
May	19,821	39,011	0.508	0.365	0.88	0.89	0.90	0.91	17,900	18,000
June	23,073	41,348	0.558	0.325	0.80	0.81	0.85	0.86	19,600	19,700
July	23,241	40,136	0.579	0.310	0.84	0.85	0.87	0.88	20,300	20,400
Aug.	19,762	35,552	0.556	0.327	0.99	1.00	0.98	0.98	19,400	19,400
Sept.	16,397	28,499	0.575	0.313	1.30	1.30	1.19	1.18	19,500	19,400
Oct.	11,277	20,684	0.545	0.335	1.77	1.73	1.49	1.47	16,800	16,500
Nov.	6311	14,472	0.436	0.428	2.36	2.30	1.75	1.71	11,000	10,800
Dec.	5632	11,785	0.478	0.390	2.75	2.68	2.04	2.00	11,500	11,300

2. J. K. Page, The estimation of monthly mean values of daily total short-wave radiation on vertical and inclined surfaces from sunshine records for latitudes 40°N–40°S. *Proc. UN Conf. on New Sources of Energy*, Paper No. 35/5/98 (1961).
3. M. P. Thekaekara and A. J. Drummond, Standard values for the solar constant and its spectral components. *Nat. Phys. Sci.* **229**(6) (1971).
4. B. Y. H. Liu and R. C. Jordan, The interrelationship and characteristic distribution of direct, diffuse, and total solar radiation. *Solar Energy* 4(3) (1960).
5. N. K. O. Choudhury, Solar radiation at New Delhi. *Solar Energy* 7(2), 44–52 (1963).
6. G. Stanhill, Diffuse sky and cloud radiation in Israel. *Solar Energy* **10**(2), 96–101 (1966).
7. D. J. Norris, Solar radiation on inclined surfaces. *Solar Energy* **10**, 72–77 (1966).
8. J. W. Bannister, *Solar Radiation Records*. Division of Mech. Engng, Commonwealth Scientific and Industrial Research Organization, Highett, Victoria, Australia (1966–1969).
9. B. Y. H. Liu and R. C. Jordan, The long-term average performance of flat-plate solar energy collectors. *Solar Energy* 7, 53–74 (1963).

Solar Energy. Vol. 20, p. 441. Pergamon Press 1978. Printed in Great Britain

ERRATUM

S. A. KLEIN: Calculation of monthly average isolation on tilted surfaces. *Solar Energy* **19**(4), 325 (1977).

A significant error in calculation has come to light in the information used to construct Table 5. A corrected version of this table is given below. In view of these corrected results, the sentences on p. 327 which read "...values of $\bar{R}$ estimated from eqn (7) with $\rho = 0.2$ tend to agree more closely with experimental measurements..." and "Based on this experimental data, it appears that Liu and Jordan's relationship for the diffuse radiation fraction (eqn 10a) results in more accurate values of $\bar{R}$ than does Page's (eqn 10b)" are now not justified. Thus the question of which correlation for the diffuse radiation fraction is best is still unsettled. The methods for calculating $\bar{R}$ (eqns 7–9 and 11–15) are not affected by the above error.

Table 5. Comparison of experimental and estimated values of $\bar{R}$ for a 38° surface facing North at Melbourne, Australia, lat. 37.9°S

Month	$\bar{K}_T$	$\bar{R}$ Experimental† (1966–68)	$\bar{R}$ Estimated from eqns. (7) and (10a)	$\bar{R}$ Estimated from eqns. (7) and (10b)
Jan.	0.60	0.85	0.87	0.88
Feb.	0.60	0.94	0.97	0.97
Mar.	0.53	1.10	1.12	1.10
Apr.	0.51	1.29	1.34	1.30
May	0.44	1.38	1.54	1.46
June	0.44	1.50	1.69	1.59
July	0.45	1.49	1.63	1.54
Aug.	0.50	1.34	1.43	1.38
Sept.	0.50	1.15	1.19	1.16
Oct.	0.51	0.98	1.01	1.00
Nov.	0.53	0.88	0.90	0.90
Dec.	0.54	0.84	0.85	0.86

†Source: Liu and Jordan (1962)[1].