A Design Method for Thermosyphon Solar Domestic Hot Water Systems

M. P. Malkin

S. A. Klein

J. A. Duffie

A. B. Copsey

Solar Energy Laboratory,
University of Wisconsin-Madison,
Madison, WI 53706

A modification to the f*-Chart method has been developed to predict monthly and annual performance of thermosyphon solar domestic hot water systems. Stratification in the storage tank is accounted for through use of a modified collector loss coefficient. The varying flow rate throughout the day and year in a thermosyphon system is accounted for through use of a fixed monthly "equivalent average" flow rate. The "equivalent average" flow rate is that which balances the thermosyphon buoyancy driving force with the frictional losses in the flow circuit on a monthly average basis. Comparison between the annual solar fraction predited by the modified design method and TRNSYS simulations for a wide range of thermosyphon systems shows an RMS error of 2.6 percent.*

Introduction

Thermosyphon solar domestic hot water systems are widely used in Australia and Israel, and are gaining popularity in Japan, the United States, and elsewhere. The collector fluid in a thermosyphon system is circulated by natural convection, eliminating the need for a pump and controller. The flow rate in a natural circulation thermosyphon system varies throughout the day and year, depending on the absorbed radiation, fluid temperatures, system geometry, and other factors.

Numerous studies have been conducted to investigate the transient temperatures and flow rates throughout the day in thermosyphon systems [1-11]. A detailed model of a thermosyphon system comprising collector, storage tank, and connecting piping has been developed for use with TRNSYS [12]. Comparisons between TRNSYS simulation results and measurements taken at the National Bureau of Standards from January-December 1980 show excellent agreement between the simulations and experiments [2]. The detailed computer simulations, which require hourly meteorological data, are useful for understanding the process dynamics of the system. However, due to the complexity of the models, and large amount of required computing time, detailed simulations are not practical for estimating the monthly and annual performance of a variety of thermosyphon system configurations.

The f-Chart method [13] is a widely used design tool for estimating long term performance of forced circulation solar heating systems. It requires only monthly average meteorological data and system parameters as inputs. In its present form, the f-Chart method is not appropriate for estimating the performance of thermosyphon systems for two reasons. First, the f-Chart method was developed for active systems with a fixed known flow rate of fluid through the collector. Second, the f-Chart method assumes that the storage tank is in a fully mixed state (uniform temperature at any time), which is a reasonable (but conservative) assumption for systems operating at conventional high collector flow rates. Recent experiments and simulations have shown that optimum performance for active systems may be achieved with a flow rate on the order of 1/5 of the conventional rates [8, 14-20]. The enhanced performance at low flow rates is due to the thermal stratification in the storage tank, enabling low temperature fluid to enter the collector, thereby reducing collector losses. Thermosyphon systems usually operate in the low flow range, and hence exhibit thermally stratified tanks.

The f-Chart method may be modified to enable prediction of the improved performance of systems exhibiting stratified storage tanks. Furthermore, the varying flow through a thermosyphon system may be approximated by an "equivalent average" fixed flow rate for each month in an active system. The active system operating at this fixed flow rate will yield similar results for monthly solar fraction as the thermosyphon system. Thus, the long-term performance of a thermosyphon system may be predicted using a modified form of the f-Chart method, as described below.

Stratified Tank Modification

A thermally stratified storage tank returns fluid to the collector at a temperature below that of the average temperature in the storage tank. The lower return temperature from a stratified storage tank increases collection efficiency by reducing thermal losses from the collector. This is shown by the Hottel-Whillier collector equation, where T_i is the temperature of fluid returning from the tank to the collector [21].

$$Q_u = A_c F_R [I_T(\tau\alpha) - U_L (T_i - T_a)] \tag{1}$$

Copsey [22] shows that the long-term solar fraction of a stratified tank system can be obtained by analysis of an other-

Contributed by the Solar Energy Division for publication in the JOURNAL OF SOLAR ENERGY ENGINEERING. Manuscript received by the Solar Energy Division, March, 1986.

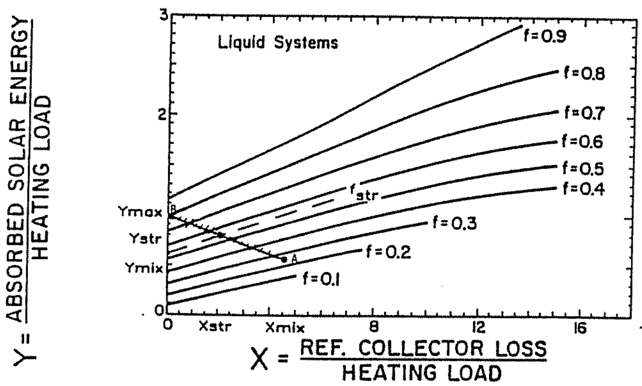

Fig. 1 Liquid system *f*-Chart from reference [22]

wise identical fully mixed tank system with a reduced collector loss coefficient (U_L). The collector heat removal factor (F_R) is a function of the collector loss coefficient and the collector flow rate, hence a modification to the *f*-Chart method that is based on the collector loss coefficient will also require modification of F_R. The *f*-Chart method for DHW systems includes the collector losses in the X parameter, and the heat removal factor in both the X and Y parameters.

$$X = \frac{A_c F_R U_L (11.6 + 1.18 T_{set} + 3.86 T_{mains} - 2.32 \bar{T}_a)\Delta t}{L} \quad (2)$$

$$Y = \frac{A_c F_R (\overline{\tau\alpha}) \bar{H}_T N}{L} \quad (3)$$

The cross-hatched line on the liquid *f*-Chart in Fig. 1 illustrates the path taken by decreasing the collector losses, while simultaneously modifying F_R. The point to the lower right is the location on the liquid system *f*-Chart of a fully mixed tank system. The mixed tank solar fraction can be obtained from the *f*-Chart with coordinates X_{mix} and Y_{mix}. If the collector had no thermal losses, then the X parameter would be zero, and the Y parameter would be

$$Y_{max} = \frac{A_c (\overline{\tau\alpha}) \bar{H}_T N}{L} \quad (4)$$

where $F_R = 1$. The solar fraction for a stratified tank system will always be between the limits of the solar fraction for a fully-mixed tank system with the actual collector loss coefficient, and a fully-mixed tank system with $U_L = 0$.

If the path shown in Fig. 1 is approximated by a straight line, a relationship between the *f*-Chart parameters can be expressed as

$$\frac{\Delta X}{\Delta X_{max}} = \frac{X_{mix} - X_{str}}{X_{mix}} = \frac{Y_{str} - Y_{mix}}{Y_{max} - Y_{mix}} \quad (5)$$

The factor $\Delta X/\Delta X_{max}$ is shown by Copsey to be a function of the monthly average collector to load flow ratio ($\bar{M}_C/M_L$) and fully-mixed tank solar fraction, F.

$$\frac{\Delta X}{\Delta X_{max}} = \frac{C_1(\bar{M}_C/M_L)}{[C_2(\bar{M}_C/M_L) + C_3F + F + C_4F^2]^2 + 1} \quad (6)$$

The coefficients $C_1 - C_4$ that minimize the RMS error between TRNSYS simulations of a stratified tank active system and the *f*-Chart method modified for stratified storage are $C_1 = 1.040$, $C_2 = 0.726$, $C_3 = 1.564$, and $C_4 = -2.760$. Equation (6) is valid for values of $\bar{M}_c/M_L$ greater than 0.3. Due to the nature of equation (6), a high solar fraction combined with $\bar{M}_C/M_L$

Nomenclature

- A = coefficient in equation (7)
- A_c = collector area (m^2)
- B = coefficient in equation (7)
- C_j = coefficient j defined below equation (6)
- C_p = specific heat of water (kJ/kg-K)
- d = diameter (m)
- E = collector effectiveness defined by equation (12)
- f = friction factor
- F = solar fraction
- F_R = collector heat removal factor
- g = gravitational acceleration (m/s^2)
- h_F = friction head loss (m)
- h_T = thermosyphon head (m)
- H = height of storage tank (m)
- H_j = height above reference at point j(m)
- H_T = daily radiation on collector plane per unit area (kJ/m^2-day)
- I_c = critical radiation level (W/m^2)
- I_T = instantaneous radiation per unit area on collector (W/m_2)
- k = friction factor for bends in connecting pipe
- K = thermal conductivity of water
- K_s = stratification coefficient
- ℓ = length of collector risers, headers, or piping (m)
- L = monthly hot water heating load (kJ)
- $\dot{m}$ = collector fluid flow rate (kg/hr)
- M = mixing number defined by equation (11)
- $\bar{M}_C/M_L$ = ratio of average daily collector flow to daily load flow
- N = days per month
- N_p = collector pump operating time (hr)
- Q_u = rate of useful collector energy gain (kJ/hr)
- Re = Reynold's number
- S = specific gravity of water
- T_a = ambient temperature (C)
- T_{mains} = mains supply water temperature (C)
- T_{set} = auxiliary heating set temperature (C)
- T_{tank} = average temperature of water in storage tank (C)
- u = fluid velocity (m/s)
- U_L = collector overall heat loss coefficient (W/C-m^2)
- X = *f*-Chart correlation parameter
- Y = *f*-Chart correlation parameter
- ϕ = daily utilizability
- $(\tau\alpha)$ = transmittance-absorptance product
- Δt = number of seconds per month (s/month)
- $\Delta X/\Delta X_{max}$ = correction for stratified storage
- μ = fluid dynamic viscosity (kg/m^2-s)

Subscripts

- i = inlet
- max = maximum possible value
- mix = applied to fully-mixed storage tank
- str = applied to stratified storage tank
- o = outlet

An overbar indicates monthly average values.

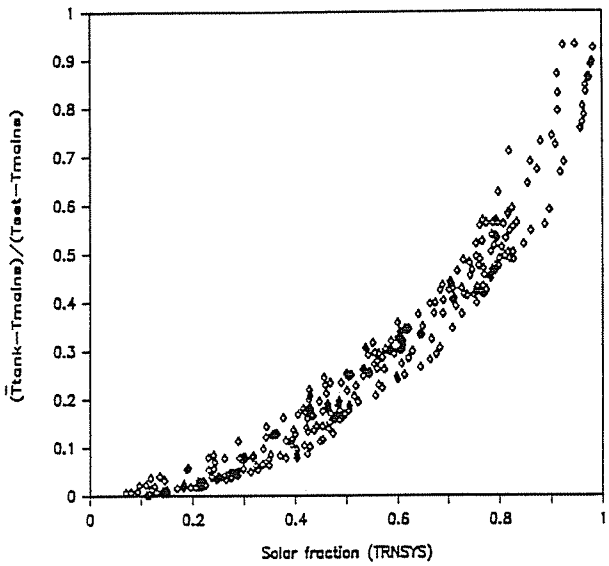

Fig. 2 Monthly average storage tank temperature (nondimensionalized) as a function of monthly fraction energy savings by solar

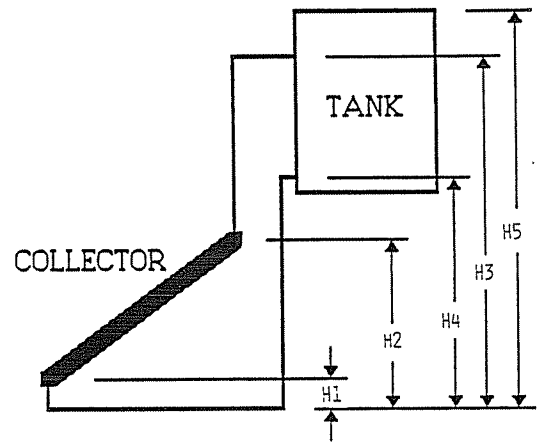

Fig. 3 Thermosyphon system schematic (adapted from [1])

near one can give $\Delta X/\Delta X_{max} > 1$. For these cases $\Delta X/\Delta X_{max}$ should be set equal to one. $\bar{M}_C/M_L$ is estimated using the Evans et al. [23] utilizability correlation in the relation for collector pump operating time [24] for an active system. In this case, $\bar{M}_c$ is the product of the collector flow rate and N_p is the monthly-average daily number of hours of operation where

$$N_P = -\bar{H}_T(A + 2B\bar{I}_c) \tag{7}$$

The constants, A and B, are functions of the monthly average clearness index, collector slope, and latitude and are given in [23]. I_C is the monthly-average critical radiation level defined as

$$\bar{I}_c = \frac{F_R U_L (\bar{T}_i - \bar{T}_a)}{F_R(\tau\alpha)} \tag{8}$$

Once the stratified tank is accounted for, the problem of varying flow rate in a thermosyphon system may be addressed.

Equivalent Average Flow Rate

The varying flow rate in a thermosyphon system may be approximated by an "equivalent average" fixed flow rate in an active system. An iterative scheme has been developed for estimating this flow rate for use with the f-Chart method modified for stratified storage, to allow prediction of the long-term performance of thermosyphon systems.

Using an initial estimated value for flow rate, the solar fraction of a stratified tank active system is evaluated using the f-Chart method with Copsey's modification for stratified storage. The collector parameters $F_R U_L$ and $F_R(\tau\alpha)$ are corrected for the estimated flow at other than test conditions, as outlined in Duffie and Beckman [21]. Thermal losses from the connecting pipes may also be accounted for as outlined in Duffie and Beckman, in which the combination of pipes plus solar collector is equivalent in thermal performance to a solar collector with parameters $F_R U_L'$ and $F_R(\tau\alpha)'$.

The average temperature in the storage tank is calculated using a correlation developed between solar fraction of a thermosyphon system and a nondimensional form of the monthly average tank temperature, deduced from numerous TRNSYS simulations, shown in Fig. 2. A variety of locations (Albuquerque, NM; Madison, WI; Seattle, WA); collector areas (1.4–4.2 m^2), load draws (300–600 ℓ/day), tank sizes (125–500 ℓ), and collector parameters ($F_R U_L$ 3.6–8.6 W/m^2-C, $F_R\tau\alpha$ 0.7-0.8) were included in the correlation. The correlation was developed under the assumption of a constant (UA) value for the storage tank of 1.46 W/C. A cubic equation for the data from a least squares regression routine is

$$\frac{\bar{T}_{tank} - T_{mains}}{T_{set} - T_{mains}} = 0.117F + 0.356F^2 + 0.424F^3 \tag{9}$$

The temperature at the bottom of the storage tank will be between the mains temperature (T_{mains}) and the average tank temperature ($\bar{T}_{tank}$), depending on the degree of thermal stratification present. An approximate measure of the stratification may be obtained using the stratification coefficient, K_s, defined by Phillips and Dave [25].

$$K_s = \frac{A_c(I_T F_R(\tau\alpha) - F_R U_L(T_i - T_a))}{A_c(I_T F_R(\tau\alpha) - F_R U_L(\bar{T}_{tank} - T_a))} \tag{10}$$

Although the Phillips and Dave study assumes zero load draw on the system and more than one tank turnover per day, it will provide a rough estimate of the temperature profile in the tank, and hence of the temperature of fluid returned to the collector. The stratification coefficient is a function of two dimensionless variables, the mixing number (M) and the collector effectiveness (E).

$$M = \frac{A_s K}{\dot{m} C_p H} \tag{11}$$

$$E = \frac{F_R U_L}{\dot{m} C_p} \tag{12}$$

Physically, the mixing number is the ratio of conduction to convection in the storage tank. In the limit, as conduction in the tank becomes negligible, and M approaches zero, Phillips and Dave show that:

$$K_s = \frac{\ell n(1/1-E)}{E(1 + M\ell n(1/1-E)} \tag{13}$$

The temperature of return fluid from tank to collector may be found from equation (10):

$$T_i = K_s T_{tank} + (1 - K_s)\left(\frac{F_R(\tau\alpha)}{F_R U_L} I_T + T_a\right) \tag{14}$$

Using the estimate of pump operating time from equation (7), the monthly average temperature of return fluid is approximated as:

$$\bar{T}_i = K_s \bar{T}_{tank} + (1 - K_s)\left(\frac{F_R(\overline{\tau\alpha})}{(F_R U_L)(N_P)} \bar{H}_T + \bar{T}_a\right) \tag{15}$$

Since the thermal losses from the tank-collector connecting pipe have already been accounted for by the modified collector parameters, $\bar{T}_i$ is also the monthly-average collector inlet temperature. For values of $\bar{T}_i$ calculated by equation (15) which are less than T_{mains}, $\bar{T}_i$ is set equal to T_{mains}. The collector outlet temperature at the estimated flow rate is found by equating the Hottel-Whillier equation with an energy balance across the collector:

$$\dot{m}C_p(T_o-T_i)=A_cF_R[I_T(\tau\alpha)U_L(T_i-T_a)] \qquad (16)$$

Integrating equation (16) for a monthly period results in the monthly average collector fluid outlet temperature, $\bar{T}_o$.

$$\bar{T}_o=\bar{T}_i+\frac{A_c}{\dot{m}C_pN_p}[\bar{H}_TF_R(\overline{\tau\alpha})-F_RU_LN_p(\bar{T}_i-\bar{T}_a)]$$

$$=T_i+\frac{A_c}{\dot{m}C_pN_p}[\bar{\phi}\bar{H}_TF_R(\overline{\tau\alpha})] \qquad (17)$$

Once the monthly average collector fluid inlet and output temperatures are known, an estimate of the thermosyphon head may be found based on the relative positions of the tank and collector. Close [1] has shown that the thermosyphon head generated by the differences in density of fluid in the system may be approximated by making the following assumptions: (1) the temperature distribution in the tank is linear; (2) water from the collector rises to the top of the tank; (3) there are no thermal losses in the connecting pipes. In this case:

$$h_T=\frac{1}{2}(S_i-S_o)\left[2(H_3-H_1)-(H_2-H_1)-\frac{(H_3-H_4)^2}{(H_5-H_4)}\right] \qquad (18)$$

where S_i is the specific gravity of the fluid at the collector inlet, S_o the specific gravity at the collector outlet, and the positions (H_1)-(H_5) are as shown in Fig. 3. The design method described in this paper considers only direct thermosyphon systems where water is the collection fluid. A parabolic relationship between specific gravity of water and temperature in degrees Celsius is used to calculate S_i and S_o:

$$S=1.00026-3.906\times10^{-5}T-4.05\times10^{-6}T^2 \qquad (19)$$

The "equivalent average" flow rate is that which balances the thermosyphon buoyancy force with the frictional resistances in the flow circuit. The flow circuit comprises the collector headers and risers, connecting pipes, and storage tank. For each component of the flow circuit, the Darcy-Weisbach equation for friction head loss is employed [1]:

$$h_F=\frac{f\ell u^2}{2gd}+\Sigma\frac{ku^2}{2g} \qquad (20)$$

where ℓ is the length of the component and f is the friction factor. For laminar flow in pipes where $\mathrm{Re}<2000$, $f=64/\mathrm{Re}$. For turbulent flow, f depends on surface roughness and Reynold's number as described in most heat transfer textbooks.

The Reynolds number, Re, at the estimated flow rate is calculated using a correlation for dynamic viscosity of water (μ in kg/m-s) as a function of temperature (in C) [12]:

$$\mu=\frac{0.1}{2.1482[T-8.435+\sqrt{8078.4+(T-8.435)^2}]-120} \qquad (21)$$

The term $(\Sigma[ku^2/2g])$ is included in the friction loss equation to account for losses associated with bends, tees, and other restrictions in the piping. Although the majority of the pressure drop in the flow circuit usually occurs across the collector risers, the minor frictional losses are included to enhance the accuracy of the flow rate estimate. The pressure drop across the optional backflow prevention check valve should also be included. For entry from the tank to connecting pipe, $k=0.5$. For right-angle bends in the connecting pipe, the equivalent length of pipe is either increased by $30d$ for laminar flow, or $k=1$ for turbulent flow. Cross-sectional changes at junctions of connecting pipes and collector headers, and headers and risers, are accounted for in the following way [12]:

Sudden Expansion:

$$k=0.667\left(\frac{d_i}{d_o}\right)^4-2.667\left(\frac{d_i}{d_o}\right)^2+2 \qquad (22)$$

Table 1 Range of parameters studied in comparison between design method and TRNSYS

Location:	Albuquerque, NM; Madison, WI; Seattle, WA; Sterling, VA
Collector Area:	1.4 m^2-5.6 m^2 (each panel 1.4 m^2)
Collector Slope:	30°-90°
(F_RU_L):	3.6 W/m^2-C-8.6 W/m^2-C
$F_R(\tau\alpha)$:	0.7-0.8
Riser Diameter:	5 mm-20 mm
Number of Risers in Each Panel:	3-15
Connecting Pipe Diameter:	19 mm-38 mm
Connecting Pipe Length:	4 m-12 m
Number of Bends in Connecting Pipe:	4-12
Connecting Pipe Thermal Losses:	0 W/m^2-C - 11.1 W/m^2-C
Height of Storage Tank Above Collector:	0 m-2 m
Storage Tank Size:	100ℓ -500ℓ
Horizontal Storage Tank Length/Diameter Ratio:	2.7-5.4
Vertical Storage Tank Height/Diameter Ratio:	1.0-2.7
Daily Load Draw:	150ℓ -500ℓ

Sudden Contraction:

$$k=-0.3259\left(\frac{d_o}{d_i}\right)^4-0.1784\left(\frac{d_o}{d_i}\right)^2+0.5 \qquad (23)$$

where d_i and d_o are the inlet and outlet pipe diameters, respectively. For losses at the entry of the connecting pipe to tank, $k=1$. Friction in the storage tank is neglected. Developing flow in the collector risers, headers, and connecting pipes is accounted for by adjusting the friction factor as recommended by Morrison and Ranatunga [3, 4]:

$$f=f+\left(1+\frac{0.038}{[\ell/(d\mathrm{Re})]^{0.964}}\right) \qquad (24)$$

All the components of the friction head loss in the flow circuit at the estimated flow rate are combined and a comparison is made with the previously calculated thermosyphon head. If the thermosyphon head does not balance the frictional losses to within one percent, a new guess of the flow rate through the connecting pipes is made by successive substitution. The procedure is repeated with the new estimate of flow rate until convergence to within on percent is reached. Convergence is usually obtained within three iterations. The resulting single value for monthly flow rate is that which balances the thermosyphon driving force with the frictional losses in the flow circuit. The solar fractions are calculated assuming a fixed flow rate operating in an active system. The procedure is carried out for each month of the year, with the previous months' "equivalent average" flow rate as the initial guess of flow rate for the new month. The fraction of the annual heating load supplied by solar energy is the sum of the monthly solar energy contributions divided by the annual load,

$$F=\frac{\sum_{i=1}^{12}F_iL_i}{\sum_{i=1}^{12}L_i} \qquad (25)$$

Comparison Between the Design Method and TRNSYS Simulations

The design procedure outlined above was compared to detailed simulations using the TRNSYS simulation program. The range of system configurations and locations investigated are outlined in Table 1. Comparison between the monthly solar fractions calculated by the modified f-Chart method outlined above, and TRNSYS simulations with 1/4-hour timesteps, for all cases listed in Table 1 is shown in Fig. 4. The

Table 2 Description of thermosyphon system for example calculation

Monthly average daily horizontal radiation:	11,591.0 kJ/m^2-day
Monthly average ambient temperature:	10.0°C
Monthly average clearness index:	0.61
Collector slope:	33.4°C
Number of collector panels:	2.0
Collector area per panel:	1.4 m^2
Collector test parameter $F_R U_L$:	17.0 kJ/hr-m^2-°C
Collector test parameter $F_R(\tau\alpha)$:	0.80
Collector test flow rate:	71.5 kg/hr-m^2
Number of risers per panel:	10.0
Riser diameter:	0.005 m
Combined header length per panel:	1.6 m
Header diameter:	0.02 m
Tank-collector connecting pipe length:	4.0 m
Collector-tank connecting pipe length:	3.0 m
Connecting pipe diameter:	0.02 m
Number of bends in connecting pipe:	5.0
Connecting pipe heat loss coefficient:	10.0 kJ/hr-m^2-°C
Storage tank volume:	250.0 ℓ
Storage tank diameter:	0.49 m
Daily load draw:	300.0 ℓ
Mains water temperature:	12.0°C
Auxiliary set temperature:	60.0°C
Reference height (H_1:)	0.0 m
Height of collector outlet above reference (H_2):	1.0 m
Height of pipe inlet to tank above reference (H_3):	2.2. m
Height of tank return to collector above reference (H_4):	1.0 m
Height of tank top above reference (H_5):	2.32 m

Table 3 Summary of calculations for example

Month	$\bar{H}_T$ [kJ/m^2]	X_{mix}	Y_{mix}	F_{mix}	$\dot{m}$ [kg/hr]	$\bar{M}_c/M_L$	X_{str}	Y_{str}	F_{str}
Jan	17879	1.77	0.56	0.40	31.8	0.95	0.78	0.63	0.51
Feb	21354	1.69	0.69	0.50	36.3	1.16	0.61	0.77	0.62
Mar	24332	1.65	0.81	0.58	43.3	1.40	0.45	0.90	0.71
Apr	27087	1.52	0.92	0.66	51.0	1.68	0.26	1.01	0.80
May	27178	1.32	0.92	0.67	51.6	1.76	0.20	1.02	0.80
Jun	26304	1.10	0.89	0.67	50.3	1.76	0.18	0.99	0.78
Jul	24607	0.97	0.82	0.63	45.9	1.69	0.21	0.91	0.74
Aug	24993	1.01	0.84	0.64	47.2	1.67	0.21	0.93	0.75
Sep	25436	1.14	0.86	0.64	48.2	1.66	0.22	0.95	0.76
Oct	23636	1.37	0.78	0.58	43.7	1.42	0.37	0.87	0.70
Nov	19789	1.60	0.64	0.46	35.4	1.09	0.63	0.71	0.57
Dec	16902	1.72	0.53	0.37	31.1	0.90	0.79	0.59	0.48
Year				0.57					0.69

monthly RMS error is 5.2 percent, and the monthly bias error is -1.4 percent. On an annual basis the RMS error is 2.6 percent, and the bias error is -1.5 percent, for all locations and system configurations studied (Fig. 5). It should be noted that for comparison purposes, values of $\bar{H}_T$, the monthly average radiation incident on a tilted surface, are obtained from integrating the TRNSYS hourly radiation calculations. Conduction between the fluid segments in the storage tank is not considered in the TRNSYS simulations. Morrison and Braun [2] suggest that conduction should be included for horizontal tanks with an in-tank electrical auxiliary heater. However, for tanks without an in-tank auxiliary heater, simulations have shown that conduction may be neglected for all reasonable tank geometries [26]. Since the design method was developed for thermosyphon syphon systems acting as preheat for in-line heaters, storage tank conduction is neglected.

This design method, based on the original f-Chart formulation, may be easily programmed on a microcomputer. It requires only monthly average weather data and description of the system component geometry as inputs. By the iterative scheme outlined above, the monthly average and thus yearly performance of a thermosyphon solar domestic hot water system may be predicted, enabling design choices to be made for the sizing of the components of a system.

Example Calculation

To illustrate the necessary calculations, the performance of the thermosyphon system described in Table 2 will be estimated for January in Phoenix, Arizona. Monthly-average meteorological data for Phoenix can be found in Duffie and Beckman [21]. The monthly-average solar radiation on the collector surface, $\bar{H}_T$, can be estimated from the horizontal data using the method described by Duffie and Beckman. Using the diffuse fraction correlation proposed by Erbs et al. [27], $\bar{H}_T = 17879$ kJ/m^2-day.

An initial estimate of the "equivalent average" collector flow is 15 kg/hr-m^2 (42 kg/hr for the 2.8 m^2 collector.) The collector parameters, $F_R U_L$ and $F_R(\tau\alpha)$ must be corrected for flow rates other than the 71.5 kg/hr-m^2 used in the ASHRAE 93-77 test [28] by use of equation 7.5.9 in Duffie and Beckman. The corrected values of $F_R U_L$ and $F_R(\tau\alpha)$ are 15.28 kJ/hr-m^2-C and 0.719, respectively. A further correction to these parameters to account for thermal losses from the connecting pipes is made using equations 10.3.9. and 10.3.10 in Duffie and Beckman. The resulting values of $F_R U_L$ and $F_R(\tau\alpha)$ are 16.45kJ/hr-m^2-C and 0.711.

The ratio $(\overline{\tau\alpha}/(\tau\alpha)$ can be calculated for each month in the

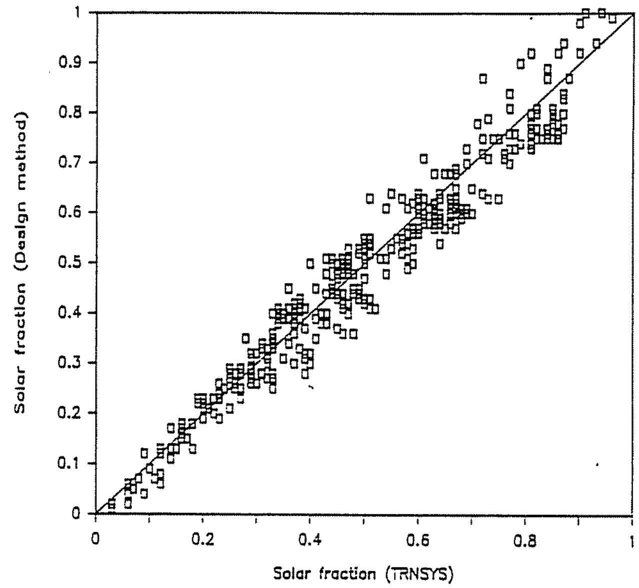

Fig. 4 Comparison between design method and TRNSYS simulations (monthly fractional energy savings by solar)

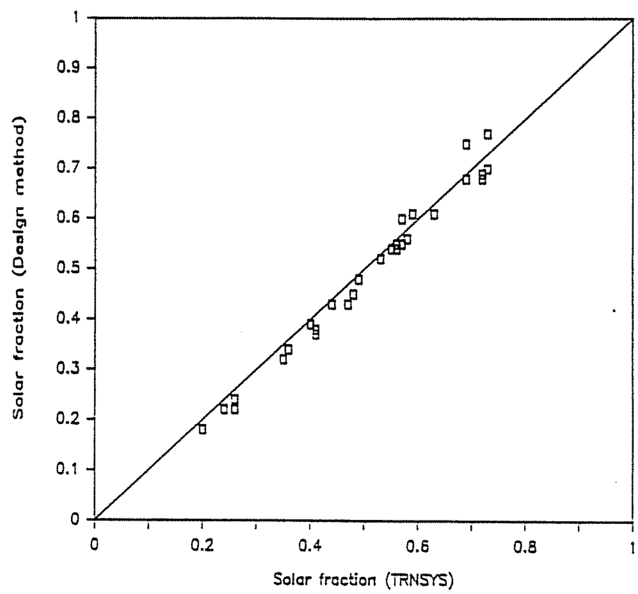

Fig. 5 Comparison between design method and TRNSYS simulation (annual fractional energy savings by solar)

manner described by Klein [29], although for simplicity it is assumed to be unity in this example. The monthly load, L, is the product of the daily draw, the number of days in the month, the specific heat of water, and the difference between the set and mains temperatures. For the system in Table 2, $L=1.87$ GJ/day. The f-Chart parameters, X and Y, calculated with equations (2) and (3), are 1.85 and 0.59, respectively. The solar fraction obtained from the f-Chart in Fig. 1 with these parameters is 0.41.

Assuming $\bar{T}_i$ to be the mains temperature (12°C) as a first guess, the critical level, I_c, is 12.85 W/m^2 from equation (8). (An improved estimate of $\bar{T}_i$ is available during successive iterations.) Using this value in equation (7), the average daily collector pump operating time is 8.9 hours where $A=-0.00182$ and $B=9.29\text{E-}7$ using the relations given by Evans et al. [23]. The ratio of $\bar{M}_c/M_L$ is then 42 kg/hr × 8.9 hr/day/300 kg/day = 1.25. This ratio is used in equation (6) to yield $\Delta X/\Delta X_{max}=0.60$. Using equation (5), X_{str} is 0.74. To estimate Y_{str} it is first necessary to calculate Y_{max} from equation (4). $(\tau\alpha)$ is not known. The product, $F_R(\tau\alpha)$ at the ASHRAE 93-77 test conditions is 0.80. Assuming $(\tau\alpha)$ to be 0.825, Y_{max} is 0.68. Thus $Y_{str}=0.644$ and the solar fraction of an active system with stratified storage is found from the f-Chart with X_{str} and Y_{str} to be 0.52.

An estimate of the average tank temperature is provided by equation (9). With $T_{set}=60°C$, $\bar{T}_{mains}=12°C$ and $F=0.52$, $T_{tank}=22.4°C$. The stratification coefficient, K_s, is 1.16 from equation (13) with $M=4.9\text{E-}4$ and $E=0.26$ from equations (11) and (12). $\bar{T}_i$ is now calculated from equation (15) to be 10.6°C. Since this temperature is lower than T_{mains}, $\bar{T}_i$ is set to 12°C. Equation (17) provides an estimate to $\bar{T}_o$ equal to 34.2°C where $\bar{\phi}$ is 0.98 from the Evans et al. correlation at the critical level calculated in equation (8).

The specific gravity of water at the collector inlet and outlet is 0.999208 and 0.994196, respectively, from equation (19). The thermosyphon head for the geometry described in Table 2 is 0.005787 m from equation (18). The frictional resistance at the assumed collector flow rate is calculated using equation (20) for each component in the flow circuit as summarized below.

Connecting Pipes

$\dot{m}=42$ kg/hr
$u=0.0372$ m/sec through the 0.02 m diameter pipe
$\text{Re}=783$
$f=0.089$ (after correcting for developing flow with equation (24))
$h_F=0.003231$ m (including the effects of pipe bends and entry and exit cross-sectional changes with equations (22) and (23))

Risers

$\dot{m}=2.1$ kg/hr
$u=0.0298$ m/s through the 0.005 m diameter risers
$\text{Re}=157$
$f=0.415$
$h_F=0.006903$ m

Headers

$\dot{m}=22.0$ kg/hr
$u=0.0195$ m/sec through the 0.02 m diameter headers
$\text{Re}=411$
$f=0.17$
$h_F=0.00054$ m

The sum of the frictional head terms is 0.01065 m which is larger than the thermosyphon head. A new guess of the collector flowrate is made and the calculations are repeated. The thermosyphon and frictional heads are within 1 percent at an "equivalent average" flowrate of 31.9 kg/hr. The January solar fraction at this flowrate is 0.51. The calculations for other months are summarized in Table 3. The annual solar fraction for this system is 0.69. A listing of a computer program which does these calculations is available in [26].

Acknowledgment

This work was supported by the Solar Heating and Cooling Research and Development Branch, Office of Conservation and Solar Applications, US Department of Energy.

References

1 Close, D. J., "The Performance of Solar Water Heaters with Natural Circulation," *Solar Energy*, Vol. 6, 1962, pp. 33–40.
2 Morrison, G. L., and Braun, J. E., "System Modeling and Operation Characteristics of Thermosyphon Solar Water Heaters," *Solar Energy*, Vol. 34, 1985, pp. 389–405.
3 Morrison, G. L., and Braun, J. E., "Thermosyphon Circulation in Solar Collectors," *Solar Energy*, Vol. 24, 1980, pp. 191–198.
4 Morrison, G. L., and Ranatunga, D. B. J., "Transient Response of Thermosyphon Solar Collectors," *Solar Energy*, Vol. 24, 1980, pp. 51–61.
5 Gupta, C. L., and Garg, H. P., "System Design in Solar Water Heaters with Natural Circulation," *Solar Energy*, Vol. 12, 1968, pp. 163–182.
6 Ong, K. S., "A Finite Difference Method to Evaluate the Thermal Performance of a Solar Water Heater," *Solar Energy*, Vol. 16, 1974, pp. 137–147.
7 Ong, K. S., "An Improved Computer Program for the Thermal Performance of a Solar Water Heater," *Solar Energy*, Vol. 18, 1976, pp. 183–191.
8 Mertol, A., et al., "Detailed Loop Model (DLM) Analysis of Liquid Solar Thermosiphons with Heat Exchangers," *Solar Energy*, Vol. 27, 1981, pp. 367–386.
9 Huang, B. J., "Similarity Theory of Solar Water Heater with Natural Circulation," *Solar Energy*, Vol. 25, 1980, pp. 105–116.
10 Sodha, M. S., and Tiwari, G. N., "Analysis of Natural Circulation Solar Water Heating Systems," *Energy Con. and Mgmt.*, Vol. 21, 1981, pp. 283–288.
11 Baughn, J. A., and Dougherty, D. A., "Experimental Investigation and Computer Modeling of a Solar Natural Circulation System," AS-ISES Annual Meeting, 1977.
12 Klein, S. A., et al., *TRNSYS 12.1 User's Manual*, University of Wisconsin, Solar Energy Laboratory, 1983.
13 Klein, S. A., et al., "A Design Procedure for Solar Heating Systems," *Solar Energy*, Vol. 18, 1976, pp. 113–127.
14 Tabor, H., "A Note on the Thermosyphon Solar Hot Water Heater," *COMPLES*, Vol. 33, No. 17, 1969.
15 Van Koopen, C. W. F., "The Actual Benefits of Thermally Stratified Storage in a Small and a Medium Size Solar System," *Proceedings of the ISES Meeting*, Atlanta, GA, May 1979.
16 Veltkamp, W. B., "Thermal Stratification in Heat Storage," in C. den Ouden, *Thermal Storage of Solar Energy*, Vol. 2, Martinus Nijhoff, The Haugue, No. 6, 1980.
17 Gordon, J. M., and Zarmi, Y., "Thermosyphon Systems: Single versus Multi-Pass," *Solar Energy*, Vol. 27, No. 5, 1981.
18 Collares-Pereira, M., et al., "Design and Optimization of Solar Industrial Hot Water Systems with Storage," *Solar Energy*, Vol. 32, 1984, pp. 121–133.
19 Fanney, A. H., and Klein, S. A., "Thermal Performance Comparisons for Solar Hot Water Systems Subjected to Various Collector and Heat Exchanger Flow Rates," *Proceedings ASES Conference*, Boulder, CO, 1986, pp. 256–260.
20 Wuestling, M. D., et al., "Investigation of Promising Control Alternatives for Solar Water Heating Systems," *ASES Journal of Solar Energy Engineering*, Vol. 107, No. 3, 1985.
21 Duffie, J. A., and Beckman, W. A., *Solar Engineering of Thermal Processes*, Wiley, New York, 1980.
22 Copsey, A. B., *A Modification of the f-Chart Method for Solar Domestic Hot Water Systems with Stratified Storage*, M.S. thesis, University of Wisconsin-Madison, 1984.
23 Evans, D. L., et al., "A New Look at Long Term Collector Performance and Utilizability," *Solar Energy*, Vol. 28, 1982, pp. 13–23.
24 Mitchell, J. C., Theilacker, J. C., and Klein, S. A., "Calculation of Monthly Average Collector Operating Time and Parasitic Energy Requirements," *Solar Energy*, Vol. 26, 1981, pp. 555–558.
25 Phillips, W. F., and Dave, R. N., "Effects of Stratification on the Performance of Liquid-Based Solar Heating Systems," *Solar Energy*, Vol. 29, 1982, pp. 111–120.
26 Malkin, M. P., *Design of Thermosyphon Solar Domestic Hot Water Systems*, M.S. thesis, University of Wisconsin-Madison, 1985.
27 Erbs, D. G., Klein, S. A., and Duffie, J. A., "Estimation of the Diffuse Radiation Fraction for Hourly, Daily and Monthly-Average Global Radiation," *Solar Energy*, Vol. 28, 1982, pp. 293–302.
28 ASHRAE, Standard 93-77, "Methods of Testing to Determine the Thermal Performance of Solar Collectors," American Society of Heating, Refrigeration, and Air Conditioning Engineers, Atlanta, GA, 1977.
29 Klein, S. A., "Calculation of the Monthly-Average Transmittance-Absorptance Product," *Solar Energy*, Vol. 23, 1970, pp. 547–551.