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DIFFUSE FRACTION CORRELATIONS

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Abstract—The influence of climatic and geometric variables on the hourly diffuse fraction has been studied, based on a data set with 22,000 hourly measurements from five European and North American locations. The goal is to determine if other predictor variables, in addition to the clearness index, will significantly reduce the standard error of Liu- and Jordan-type correlations ($I_d/I = f(k_t)$). Stepwise regression is used to reduce a set of 28 potential predictor variables down to four significant predictors: the clearness index, solar altitude, ambient temperature, and relative humidity. A piecewise correlation over three ranges of clearness indices is developed to predict the diffuse fraction as a function of these four variables. A second piecewise correlation is developed for predicting the diffuse fraction as a function of the clearness index and solar altitude, for use when temperature and relative humidity are not available. A third piecewise correlation of the Liu- and Jordan-type is developed from the same data set. Comparing this correlation with the new correlations provides a direct measure of the value of added predictor variables. The full diffuse fraction correlation reduced the residual sum squares by 14% when compared to the correlation that is a function of the clearness index only. The correlation including the clearness index and solar altitude diminished the residual sum squares by 9%. The correlations exhibited some degree of location dependence. This is expected, as the climates are quite different. The correlations also showed some seasonal dependence; the errors are higher in the fall and winter than on an annual basis.

1. INTRODUCTION

A crucial input required in the transient simulation of solar energy systems is hourly radiation incident on the collecting surface. Actual measurements of hourly solar radiation data would be desirable for input but probably are not available for the site and collector orientation under consideration. However, hourly global radiation on a horizontal surface is one of the most widely available measurements in addition to other climatic variables (ambient temperature, dew point, etc.). Extensive databases exist for a variety of locations including SOLMET[1], McKay[2].

If only global horizontal radiation is measured, two problems exist: first, determining the fraction of the global which is diffuse (or beam); second, estimating the respective beam, diffuse, and ground reflected components on a tilted surface of any orientation. The first of these two problems will be addressed in this paper. (Both are addressed in [3].)

Early work by Liu and Jordan[4] showed a relationship between daily diffuse and daily total radiation on a horizontal surface. Although their original correlation was developed for daily values, it has been used for computing the hourly diffuse fraction as a function of the hourly clearness index, k_t (ratio of hourly global horizontal to hourly extraterrestrial radiation)[5]. Other authors have developed diffuse fraction correlations specifically for hourly intervals. Orgill and Hollands[6] and Erbs *et al.*[7] correlate the diffuse fraction with the hourly clearness index. Stauter and Klein[8] use a clearness index, k_c, where a "clear sky" radiation, I_c, replaces extraterrestrial radiation in the definition of k_t. Iqbal[9] built on the work of Bugler[10] to develop a correlation which predicts hourly diffuse radiation (in the form I_d/I_o) as a function of k_t and solar altitude. The models based on k_t (referred to as Liu- and Jordan-type models) are convenient because the only required input is hourly global horizontal radiation, I.

A drawback with using the Liu- and Jordan-type models is the high standard error associated with estimating the hourly diffuse fraction. The scatter plot of data from Cape Canaveral, FL in Fig. 1 graphically illustrates the problem of estimating the hourly diffuse fraction as a function of k_t only. For example, at $k_t = 0.5$, the measured diffuse fraction ranges from 0.2 to nearly 1.0. It is clear that the hourly diffuse fraction is not a function of k_t only. Models such as Orgill and Hollands[6] and Erbs[7] provide a single deterministic value of the hourly diffuse fraction for a given k_t. In an effort to mimic the variation of the diffuse fraction at a particular value of k_t, Hollands and Chra[11] developed a probability density function which allows the diffuse fraction to vary about its mean value at a given value of k_t. Other authors suggest that the variation of the diffuse fraction for a particular value of k_t is due to other unidentified variables[6,9,10,12,13].

Garrison[12] uses post-1976 SOLMET data from 33 U.S. sites to graphically illustrate the dependence of diffuse fraction on surface albedo, atmospheric precipitable moisture, atmospheric turbidity, solar elevation, and global horizontal radiation. Without further statistical analysis, the relative significance of the variables suggested by Garrison remains unknown. Skartveit and Olseth[13] and Iqbal[9] suggest that the second most important variable after k_t is the solar altitude.

This paper focuses on assessing the influence of commonly measured climatic variables on the diffuse fraction and correlating the significant variables to reduce the standard error of Liu- and Jordan-type models. First, the data used in this study are introduced. Second, the influence of commonly measured climatic

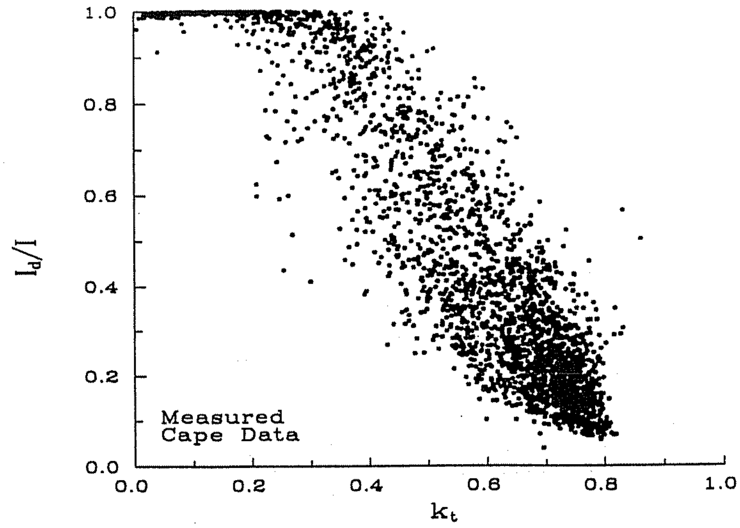

Fig. 1. Measured diffuse fraction vs. clearness index for Cape Canaveral, FL.

variables on the diffuse fraction are investigated. Third, a new hourly diffuse fraction correlation is presented. Fourth, the relative improvement of the new correlation over current Liu- and Jordan-type models is quantified.

2. DATA BASE

Data from five locations (with a minimum of one year of data from each location) is used to investigate the influence of climatic and geometric predictors on the diffuse fraction. An additional year of data from an independent location is used to test the derived correlations. Table 1 lists a summary of the sites included in the data base.

The data from Albany, NY were taken under the Solar Energy Meteorological Research and Training Sites (SERMTS)[14] program at the State University of New York, Albany. Data from Cape Canaveral, FL were provided by the Florida Solar Energy Center (FSEC). (This data set will be referred to as the "Cape" data.) Three sites with one year of data from each location comprise the European data sets. All sites provide measured values of global horizontal, diffuse (or direct normal), ambient temperature, and wet bulb temperature (or dew point). The U.S. sites use Eppley pyranometers and pyrheliometers to monitor radiation. While the European sites use Kipp solarimeters (shade rings were used to monitor diffuse radiation). An additional year of data from Oslo, Norway was provided by SINTEF and maintained as independent for comparing the performance of derived correlations.

The integrity and validity of any empirically derived correlation relies on the quantity and quality of data used in the model development. It is assumed that a sufficient quantity of data exists but the quality of the data need to be examined.

Three types of data checks were performed to identify data missing, data which clearly violate physical limits, and extreme data. When the data were known to be "bad" or "missing," the data fields were filled with a key sequence of numbers (by the reporting location) to clearly indicate the erroneous observation. Any hour with data flagged as bad or missing was omitted. Second, any hour with an observation that violated a physical limit or conservation principle was eliminated from the data set including: reported hours with negative values of radiation, diffuse fraction greater than 1, beam radiation exceeding the extraterrestrial beam radiation, and dew point temperature greater than dry bulb temperature. In other circumstances, reported data did not exceed physical limits

Table 1. Data base site summary

	United States		European			
Site Location	Albany	Cape Canaveral	Copenhagen	Hamburg	Valentia	Oslo
Latitude	42.7° north	28.4° north	55.7° north	53.5° north	51.95° north	59.56° north
Longitude	73.8° west	80.6° west	12.6° east	10.0° east	10.22° west	10.41° east
Standard Meridian	75.0° west	75.0° west	15.0° east	15.0° east	0.0°	15.0° east
Data Period (From)	1/1/79	1/1/80	1/1/*	1/1/*	1/1/*	1/1/79
Data Period (To)	12/31/82	12/31/80	12/31/*	12/31/*	12/31/*	12/31/79

* Year of dataset unknown.

but were categorized as "extreme" and had to be edited manually. There are various combinations of the diffuse fraction and clearness index values which would produce questionable data points. The limits below were used to identify such particular cases. (Similar limits were used by Erbs[7].) Under cloudy overcast sky conditions (low values of k_t), it is reasonable to expect that a large portion of the incoming radiation will be scattered by the clouds in the atmosphere resulting in a large diffuse fraction. Case 1 places a limit on the diffuse fraction under the cloudy overcast sky conditions. If an hour had a measured diffuse fraction that was less than 0.90 for a clearness index less than 0.20, it was eliminated from the data set. Similarly, case 2 places a limit on the diffuse fraction under clear sky conditions.

$$\text{Case 1:} \quad I_d/I < 0.90 \quad \text{and} \quad k_t < 0.20$$

$$\text{Case 2:} \quad I_d/I > 0.80 \quad \text{and} \quad k_t > 0.60.$$

The quality tests discussed will help eliminate spurious data and minimize any impact that suspect data would have on a derived correlation. The final data set constructed from the measured data that passed all of the quality control checks discussed above produced approximately 22,000 hours of data for this study. The Oslo dataset provided approximately 3000 hours of data for independently comparing derived correlations.

3. DEVELOPMENT OF NEW HOURLY DIFFUSE FRACTION CORRELATIONS

The motivation behind investigating hourly diffuse fraction correlations is to determine if incorporating additional predictor variables will significantly reduce the standard error of the current Liu- and Jordan-type models. The goal is to find an hourly diffuse fraction model which is more accurate than current Liu and Jordan type models but remains computationally simple for use in hourly simulation programs such as TRNSYS[5]. Also, the inputs to the correlation should be limited to commonly observed climatic variables or quantities that can be calculated from commonly observed climatic variables, e.g., ambient temperature, wet bulb temperature, dew point temperature, relative humidity, etc.

Several steps are necessary to develop an empirical model which improves the prediction capabilities of the current Liu- and Jordan-type models. The approach used in this study included the following four basic steps: (i) assemble a set of predictor variables; (ii) identify a potential model form; (iii) adopt a predictor selection procedure; and (iv) fit the model.

Predictor variables are independent variables that may affect the response. It is clear that the response, the diffuse fraction, is influenced by k_t. The set of predictors used in this study was limited to k_t and other commonly measured climatic data. Factors such as atmospheric turbidity and ground reflectance were not included because they are not commonly measured at radiation monitoring sites. The full set of predictor variables used in this investigation is given in Table 2.

Monthly average hourly quantities and ratios of the hourly to monthly average hourly values of the climatic predictors and k_t were included in an attempt to account for possible predictor location dependence. For example, the range of ambient temperature over a year at Albany will be much larger than the range of ambient temperature for Cape Canaveral, but each location experiences a similar range of measured diffuse fractions over the year. However, the ratio of hourly ambient temperature to monthly average hourly ambient for both locations may be approximately the same magnitude. This method of scaling the predictors may improve the correlations. Exponential terms are included to represent atmospheric extinction. Geometric terms such as solar altitude and optical air mass are included based on the findings of other authors.

With the set of predictors identified, a model form must be established, e.g., linear, nonlinear, first order, second order, etc. The Liu- and Jordan-type models are all linear models. Liu and Jordan[5], Orgill and Hollands[6], and Erbs[7] use piecewise fitted models with varying degree order polynomials. For this study, a piecewise (in k_t) first order linear model of the following form will be used to fit the data:

Table 2. Diffuse fraction predictor variables

$T_a(hr)$	$T_{wb}(hr)$
$\overline{T_a}(hr)$	$\overline{T_{wb}}(hr)$
$T_a(hr)/\overline{T_a}(hr)$	$T_{wb}(hr)/\overline{T_{wb}}(hr)$
$T_{dp}(hr)$	$k_t(hr)$
$\overline{T_{dp}}(hr)$	$\overline{k_t}(hr)$
$T_{dp}(hr)/\overline{T_{dp}}(hr)$	$k_t(hr)/\overline{k_t}(hr)$
$\omega(hr)$	$\phi(hr)$
$\overline{\omega}(hr)$	$\overline{\phi}(hr)$
$\omega(hr)/\overline{\omega}(hr)$	$\phi(hr)/\overline{\phi}(hr)$
$\sin(\alpha)$	$T_a(hr)\cdot\phi(hr)$
$m = 1/\cos(\theta_z)$	$\phi(hr)/[T_a + 273]$
$k_t(hr)\cdot m$	$\phi(hr)/\overline{\phi}(hr)\cdot T_a(hr)$
$\exp[\sin(\alpha)]$	$\exp[\phi(hr)/(T_a(hr))]$
$\exp[k_t\cdot m]$	$\exp[\phi(hr)/(\overline{\phi}(hr)\cdot T_a(hr))]$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_j x_j + \epsilon \quad (1)$$

where y is the true response, β_j is the j^{th} model parameter, x_j is the j^{th} predictor, and ϵ represents the model error.

In an effort to gain understanding of the diffuse fraction's association with each predictor variable, analysis was performed on the center interval of k_t (initially assuming the same interval as Orgill and Hollands, $0.35 < k_t < 0.75$). Stepwise regressions were performed on a monthly basis for each location to determine if location or seasonal bias existed in the selection of the best set of predictors. (For information regarding stepwise regression, see Draper and Smith[15].) The complete set of predictor variables listed in Table 2 was used in the stepwise procedure. On a monthly basis, there was not a great deal of consistency in the variable selection due to the relatively short time interval; therefore, stepwise regression was performed on a yearly basis for each location. The four best predictor variables to explain the deviations in the diffuse fraction are k_t, $\sin(\alpha)$, $\bar{T}_a$, $\phi/\bar{\phi}$.

Unfortunately, monthly average hourly data are not included in commonly used data sets[1,2]. A correlation with monthly average hourly variables would force users to derive the necessary quantities from existing data sets. At this point, a decision was made to eliminate the use of monthly average hourly predictors and pursue a correlation based only on hourly values of the predictors. The impact of this decision will be investigated when the new set of predictor variables are selected.

The stepwise selection procedure was applied to the center interval of k_t using the set of predictors with the monthly average hourly predictors removed. On an hourly basis, the top four predictors were selected; the best predictors to explain the deviations in the diffuse fraction are k_t, $\sin(\alpha)$, T_a, ϕ.

This set of predictors provides the foundation for the remaining development and analysis of a new diffuse fraction correlation.* As will be shown, substituting T_a for $\bar{T}_a$ and ϕ for $\phi/\bar{\phi}$ does not significantly effect the results.

The next step in the diffuse fraction correlation development is to determine the best k_t intervals for piecewise fitting. The appropriate k_t interval will minimize the standard error of the final correlation. A manual search technique was employed to find the clearness index interval which minimized the standard error of the correlation. The center interval which minimized the standard error is $0.3 < k_t < 0.78$.

The standard error for the correlation in the center interval is 0.129. The standard error for the correlation which included the monthly average predictors is also 0.129 (based on the interval $0.35 < k_t < 0.75$). Thus, it appears that there is not a significant loss (from an error standpoint) by not including the monthly average hourly predictor variables in the correlation.

The clearness index is the most important variable in the low and middle intervals but at the high interval, the significance of the clearness index decreases dramatically. The solar altitude effects are not as strong under cloudy skies (low values of k_t) but under clear skies (high values of k_t), the solar altitude becomes the dominant predictor variable. For clear sky conditions, the diffuse fraction increases for decreasing solar altitude angles due to the longer path length required for radiation to travel. These results are consistent with those found by Skartveit and Olseth[13].

The final version of the full correlation is given below.

Interval: $0 \le k_t \le 0.3$; Constraint: $I_d/I \le 1.0$.

$$I_d/I = 1.000 - 0.232k_t + 0.0239\sin(\alpha) - 0.000682T_a + 0.0195\phi \quad (2a)$$

Interval: $0.3 < k_t < 0.78$;

Constraint: $I_d/I \le 0.97$ and $I_d/I \ge 0.1$.

$$I_d/I = 1.329 - 1.716k_t + 0.267\sin(\alpha) - 0.00357T_a + 0.106\phi \quad (2b)$$

Interval: $0.78 \le k_t$; Constraint: $I_d/I \ge 0.1$.

$$I_d/I = 0.426k_t - 0.256\sin(\alpha) + 0.00349T_a + 0.0734\phi. \quad (2c)$$

Because the above piecewise correlation includes multiple predictor variables, it is possible that some combinations of predictors may produce unreasonable values of the diffuse fraction, e.g., greater than 1; therefore, subsequent constraints are placed on the correlation in each interval to assure reasonable predicted values. The constraint values on the middle and high interval of k_t are based on the observations from the data sets.

At this point, a simple piecewise correlation exists which gives the hourly diffuse fraction as a function of hourly clearness index, solar altitude, ambient temperature, and relative humidity. It would be desirable to provide a reduced form of the current correlation for use when hourly ambient temperature and/or relative humidity data are not available. The result is a piecewise model which provides estimates of the hourly diffuse fraction as a function of the clearness index and solar altitude angle.

* The reviewers suggested that precipitable water may be an important predictor variable and should have been included in Table 2. Precipitable water is not a commonly measured variable but several correlations of precipitable water vapor in terms of dew point temperature[16,17] are available. The Bolsenga[16] and Smith[17] correlations for precipitable water vapor were included in a subset of predictor variables (the subset included k_t, $\sin(\alpha)$, T_a, ϕ, and precipitable water vapor) and the stepwise selection procedure was repeated. Precipitable water vapor was not selected as one of the top four predictors in the interval $0 < k_t < 0.78$. In the interval $k_t > 0.78$, where only 3% of the data lie, precipitable water vapor was the second most important predictor variable. A correlation that explicitly includes k_t, precipitable water, and ambient temperature was derived but the standard error was slightly greater than the correlation derived based on the above four variables.

Interval: $0 \le k_t \le 0.3$; Constraint: $I_d/I \le 1.0$.

$$I_d/I = 1.020 - 0.254k_t + 0.0123\sin(\alpha) \quad (3a)$$

Interval: $0.3 < k_t < 0.78$;

Constraint: $I_d/I \le 0.97$ and $I_d/I \ge 0.1$.

$$I_d/I = 1.400 - 1.749k_t + 0.177\sin(\alpha) \quad (3b)$$

Interval: $0.78 \le k_t$; Constraint: $I_d/I \ge 0.1$.

$$I_d/I = 0.486k_t - 0.182\sin(\alpha). \quad (3c)$$

A final correlation which is a function of k_t only was also developed. This correlation (identified as "ktcorr") will allow direct comparison of the new correlations to the Liu- and Jordan-type correlations. The correlation, ktcorr, is given below.

Interval: $0 \le k_t \le 0.3$; Constraint: $I_d/I \le 1.0$.

$$I_d/I = 1.020 - 0.248k_t \quad (4a)$$

Interval: $0.3 < k_t < 0.78$

$$I_d/I = 1.45 - 1.67k_t \quad (4b)$$

Interval: $0.78 \le k_t$

$$I_d/I = .147. \quad (4c)$$

The derived correlation based on k_t is similar to Orgill and Hollands[6] and Erbs[7] as shown in Fig. 2. (Each of these three correlations was based on an entirely independent data set.)

4. MODEL PERFORMANCE

A simple composite residual sum squares (CRSS) comparison is used to quantify the improvement of the new hourly diffuse fraction correlation over current Liu- and Jordan-type models. In an effort to provide a fair comparison between the new hourly diffuse fraction correlation (eqns 2) and the current Liu- and Jordan-type models, the correlation based on k_t (eqns 4) derived from the existing data set will be included in all model comparisons. By comparing the ktcorr model with the new diffuse fraction correlation, the relative merit of added climatic and geometric terms in the new model will be directly assessed. The reduced correlation based on the clearness index and solar altitude (eqns 3) will also be included in the correlation comparison. The only other existing model that will be included in the model comparisons will be the Erbs[7] model. The Orgill and Hollands model is similar to the Erbs models so that conclusions about the Erbs model also apply to the Orgill and Hollands model. Location and seasonal effects are noted. The CRSS is calculated by the following relationship:

$$CRSS = \Sigma[(I_d/I)_{\text{pred}} - (I_d/I)_{\text{meas}}]^2. \quad (5)$$

Applying eqn (5) to the complete data set yielded the results listed in Table 3. On an overall basis, the new hourly diffuse fraction correlation reduces the residual sum squares of the correlation based on k_t only (derived from the same data set) by 14.4%. The reduced hourly diffuse fraction correlation shows a 9% improvement over ktcorr. Included in Table 3 are results for one year of independent data from Oslo, Norway. The correlations given by eqns (2) and (3) showed 17% and 13.5% reduction in residual sum squares when compared with ktcorr. An improvement of 26% and 23% is gained when compared with the Erbs' correlation at the same location.

The derived correlations exhibit slight location and seasonal dependencies. Location differences are noted in Fig. 3 by plotting the residual mean squares ($RSS/(n-p)$) for each location. The variation in the residual mean squares for each location suggests that the correlations are not entirely location independent. Copenhagen exhibits the largest residual mean square.

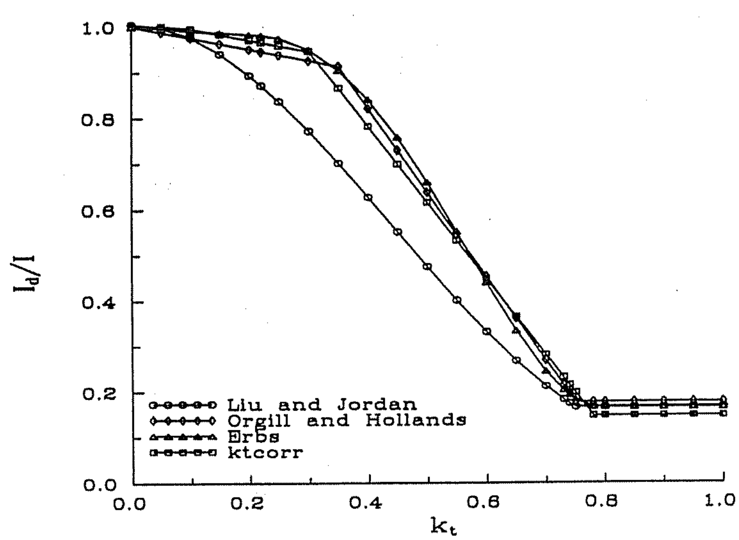

Fig. 2. Diffuse fraction correlations including ktcorr.

Table 3. Composite residual sum squares results

Location	n	Eqn. 2	Eqn. 3	Eqn. 4	Erbs
Albany, '79	2178	33.04	35.67	38.16	37.01
Albany, '80	2214	35.42	36.05	39.64	37.73
Albany, '81	2126	27.37	29.72	32.96	30.99
Albany, '82	1994	23.38	25.07	28.80	29.54
Cape	3596	45.85	50.63	55.29	57.97
Valentia	3386	37.95	41.04	46.37	47.24
Hamburg	3279	34.75	34.51	35.59	37.30
Copenhagen	3150	55.49	58.52	65.78	77.64
Total	21923	293.25	311.21	342.59	355.42
Oslo [a]	2927	35.27	36.76	42.52	47.74

[a] Oslo dataset was not used in deriving the correlations given by eqns (2)–(4).

However, the overall variation in residual mean square from location to location is of the same order as the yearly variation in residual mean square for Albany, 1979–1982. Also, residual mean squares are higher in the fall and winter months than on an annual basis. No attempt is made to account for location or seasonal effects. The authors feel that the current correlation is acceptable.

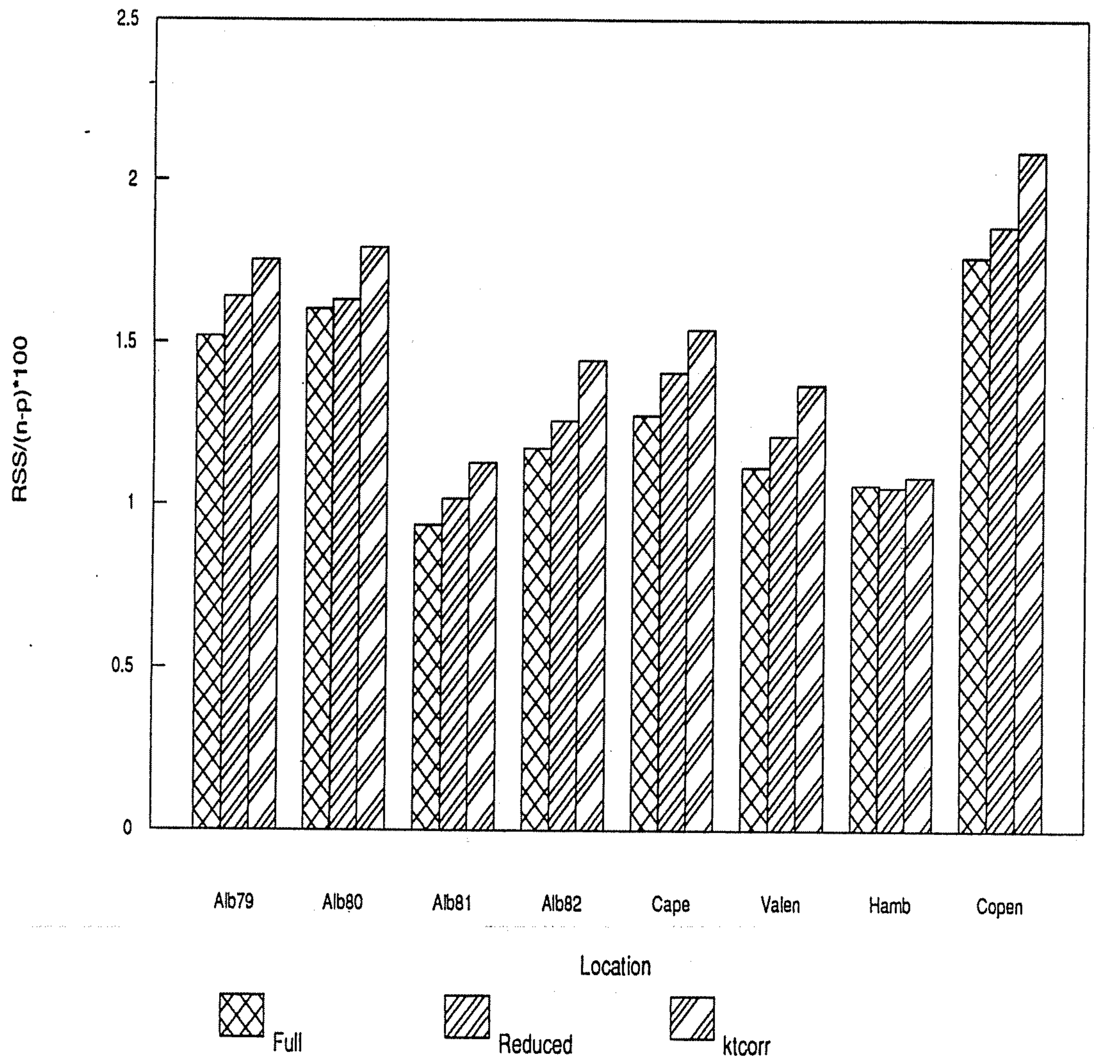

Fig. 3. Location residual mean squares.

5. CONCLUSIONS

The goal of this research was to reduce the standard error of the current Liu- and Jordan-type correlations by including additional predictor variables in the model. Stepwise regression is used to reduce a large set of potential predictor variables down to four significant predictors. The significant predictors include hourly values of clearness index, solar altitude, ambient temperature, and relative humidity. The final version of the piecewise correlation is given by eqns (2). In the event that hourly ambient temperature and/or hourly relative humidity data are not available, a reduced form of the correlation was derived to predict the diffuse fraction as a function of clearness index and solar altitude (eqns (3)).

The new correlation reduced the composite residual sum of squares by 14.4% when compared to a k_t correlation derived from the same data set. The reduced form of the correlation reduces the composite residual sum squares by 9.2%. When an independent data set is used, the new correlation reduced the residual sum of squares by 26% compared to the Erbs' correlation. Some location and seasonal dependencies were suggested but their effects are considered to be negligible to the correlation's overall performance.

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NOMENCLATURE

- *CRSS* Composite residual sum squares for all locations
- *RSS* Composite residual sum squares for one location
- S_c solar constant (a value of 1353 W/m^2 was used)
- hr hourly
- I hourly total radiation on a horizontal surface, kJ/m^2
- I_b hourly beam radiation on a horizontal surface, kJ/m^2
- I_d hourly diffuse radiation on a horizontal surface, kJ/m^2
- I_o hourly extraterrestrial radiation on a horizontal surface, kJ/m^2
- k_t hourly clearness index, I/I_o
- m optical air mass
- n number of observations
- p number of parameters in the model
- *RSS* Composite residual sum squares for one location
- *SS* Sum of squares
- T_a ambient temperature, °C
- T_{dp} dew point temperature, °C
- T_{wb} wet bulb temperature, °C
- $\bar{x}$ monthly average hourly value of x

Greek

- α solar altitude angle
- β_i true model parameter
- ϵ model error, residual
- ϕ relative humidity (fraction)
- ω humidity ratio

Subscripts

- i datum
- meas measured data
- pred model predicted

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