Use of Linear Predictive Control

for a Solar Electric Generating System

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ABSTRACT

In a Solar Electric Generating System (SEGS), it is important to maintain a specified set point for the collector outlet temperature. Currently, a skilled plant operator adjusts the volume flow rate of the heat transfer fluid circulating through the collectors to achieve this goal. In this paper, a linear model predictive controller that approximates the behaviour of the operator and that can be used to control the plant is described. The development of the plant model and controller are presented. The performance of the controller is evaluated and the influence of the control on the gross output of the plant is examined. The use of the linear model predictive controller to control building energy systems that have significant capacitance and fluid time delays is discussed.

INTRODUCTION

A solar electric generating system (SEGS), shown in Figure 1, refers to a class of solar energy systems that use parabolic troughs in order to produce electricity from sunlight. The parabolic troughs are long parallel rows of curved glass mirrors that focus the sun’s energy on an absorber pipe located along its focal line. These collectors track the sun by rotating around a north-south axis. The heat transfer fluid (HTF) is oil circulated through the pipes. Under normal operation the heated HTF leaves the collectors with a specified collector outlet temperature and is pumped to a central power plant area. There, the HTF is passed through several heat exchangers where its energy is transferred in a boiler to water, which is the working fluid of the power plant. The steam is used in turn to drive a turbine generator to produce electricity. The facility modeled in this paper is the 30 MWe SEGS VI plant, constructed in 1988 by Luz International Ltd., and is located in the Mojave Desert of southern California.

In operation, the temperature of the HTF leaving the parabolic trough collector is controlled by a skilled operator. He maintains a specified set point for the outlet temperature by adjusting the volume flow rate of the HTF within upper and lower bounds. The collector outlet temperature is mainly affected by changes in the sun intensity, collector inlet temperature volume flow rate of the HTF. The ambient temperature and the wind speed have a small influence of the outlet temperature. Operator knowledge of the daily path of the sun and observation of clouds together with many years of experience and training give him the
ability to accomplish this task. However, there are limitations to the performance of a human controller. For the next generation of plants, automatic control is desirable. A control algorithm that approximates the operator behavior can be used to design and operate plants.

The solar power plant is characterized by significant thermal capacitance of the HTF and significant time for the fluid to flow through the collector array. Automatic control of the HTF in a parabolic trough collector through proportional control has been attempted but is not always satisfactory due to time delays. In the study reported here, a linear model predictive controller was developed for the plant. The essential idea behind model predictive control (MPC) is to optimize forecasts of process behavior to meet specified objectives. Forecasting is accomplished with a process model, and therefore the model is the essential element. Linear optimization techniques are employed, requiring that the non-linear model be linearized. Constraints on both the collector outlet temperature and the volume flow rate need to be included.

An accurate model of the plant is necessary to develop and test the controller. The SEGS plant is shown schematically in Figure 1. It can be divided into two subsystems: the solar collector field and the power plant. Models for each subsystem were developed.

**Figure 1 Flow Diagram of the 30 MWe SEGS Plant for Pure Solar Mode**

**PLANT MODEL**

The model for the solar subsystem is developed first. The heat collection element is shown in
Figure 2. There is an absorber pipe in which the HTF flows. A glass envelope covers the absorber pipe, which is assumed to have no radial temperature gradients. A partial vacuum exists in the annulus between the absorber pipe and the glass envelope. The time for the HTF to go from the inlet and outlet varies from approximately 3 minutes at the minimum flow rate to 0.5 hour at the maximum flow rate, and thus a transient model is needed.

A transient energy balance for the HTF leads to the following partial differential equation for the HTF temperature:

\[
\rho_{HTF} c_{HTF} A_{ABS,i} \frac{\partial T_{HTF}}{\partial t} = -\rho_{HTF} c_{HTF} \dot{V}_{HTF} (t) \frac{\partial T_{HTF}}{\partial z} + \pi D_{ABS,j} h_{ABS-HTF} (T_{ABS} - T_{HTF})
\]  

(1)

The HTF volume flow rate is \( \dot{V}_{HTF} \). The distance along the collector is \( z \) and \( t \) is the time. The boundary condition for equation (1) is

\[ T_{HTF}(0,t) = T_{HTF,\text{inlet}}(t) \]  

(2)

with \( T_{HTF,\text{inlet}} \) is the HTF collector field inlet temperature. The initial condition for equation (1) is

\[ T_{HTF}(z,0) = T_{HTF,0} \]  

(3)

The differential equation for the absorber temperature is

\[
\rho_{ABS} c_{ABS} A_{ABS} \frac{\partial T_{ABS}}{\partial t} = Q_{\text{absorbed}} - Q_{\text{internal}} - \pi D_{ABS,j} h_{ABS-HTF} (T_{ABS} - T_{HTF})
\]  

(4)

The absorbed solar energy is \( Q_{\text{absorbed}} \) and \( Q_{\text{internal}} \) is the heat transfer between the absorber and the envelope. The initial condition for equation (4) is

\[ T_{ABS}(z,0) = T_{ABS,0} \]  

(5)

The glass envelope is assumed to have no radial temperature gradients. The differential equation for the envelope temperature is given through

\[
\rho_{ENV} c_{ENV} A_{ENV} \frac{\partial T_{ENV}}{\partial t} = Q_{\text{internal}} - Q_{\text{external}}
\]  

(6)

The heat transfer between the envelope and the environment is \( Q_{\text{external}} \). The initial condition for equation (6) is
The heat transfer coefficient, $h_{ABS,HTF}$, is calculated through the Dittus-Boelter equation for turbulent flow in circular tubes. The heat transmitted between the absorber and the envelope, $Q_{internal}$, is calculated from free convection flow in the annular space between long, horizontal, concentric cylinders and radiation. The heat transfer between the envelope and the environment, $Q_{external}$, is estimated through relations for a circular cylinder in cross flow and radiation.

The absorbed solar energy, $Q_{absorbed}$, is the direct normal solar radiation that is absorbed by the absorber after accounting for optical losses. Because of the north-south tracking of the collectors, only the direct normal solar radiation times the cosine of the angle of incidence is available as thermal energy. This energy is further reduced through mirror reflectivity, dirt on the mirrors, transmissivity of the envelope, absorptivity of the absorber, the mutual shading of the collectors during the sunrise and the sunset, end losses and additional losses due to shading by the HCE arms and bellows. The parameters to calculate these losses were taken from an experimentally verified steady-state model developed at Sandia with empirical coefficients obtained experimentally on a test facility at Sandia.

Figure 3 shows the predicted and measured solar collector field outlet temperature for December 14, 1998. Although this day has clouds and the solar insolation varies over the day, the temperature rise through the collector field was between 50 and 100 K. The predicted temperature matches the measurement quite well. The solar collector field model was found to predict the outlet temperature quite well.

The power plant, shown in Figure 1, is a Rankine cycle with reheat and feedwater heating. In the model development, each heat exchanger network (preheater, steam generator, and superheater) was treated as a single heat exchanger. The two high-pressure feedwater heaters were modeled as one high-pressure feedwater heater and the three low-pressure feedwater heaters were modeled as one single low-pressure feedwater heater. The power plant model is
a steady-state model with the effectiveness and the heat transfer coefficients in the heat exchangers functions of the steam/water mass flow rate. The pump and turbine efficiencies were assumed to be constant, with values taken from $^4$.

**LINEAR MODEL PREDICTIVE CONTROL (MPC)**

The linear model predictive controller employs a linear optimal control strategy to minimize the difference between predicted and target outputs $^6$. To use linear optimization, the nonlinear model developed for the plant needs to be linearized. In developing this linear model predictive control concept, it is useful to think of the plant model as a block with inputs and outputs as it is shown in Figure 4.

The uncontrolled forcing functions to the plant model are the measured values of cooling water inlet temperature at the condenser, steam or water mass flow rate in the power plant, and environmental data (solar radiation, ambient temperature, and wind speed). The controlled variable is the HTF volume flow rate and the control variable is the collector outlet temperature. The MPC controller senses the collector outlet temperature and calculates a HTF volume flow rate that will allow the outlet temperature setpoint to be met.

![Figure 4 Block Diagram with Plant Model and Controller](image)

The states of the collector model, $T$, are given through the differential equations (1), (4) and (6). These are discretized in the $z$ direction to transform the partial differential equations into a set of nonlinear ordinary differential equations

$$\frac{dT}{dt} = h(T_{HTF}, T) + g(S_{amb}, v_{wind})$$  \(\text{(8)}\)

with initial conditions

$$T(0) = T_0$$  \(\text{(9)}\)

The outlet temperature measurement is given by

$$T_{out} = h(T)$$  \(\text{(10)}\)

The set of ordinary differential equations given by equation (8) is then linearized and
transformed into a time discrete form at any time $k$.

$$\Delta T_{k+1} = A\Delta T_k + B\Delta \dot{V}_{HTF,k} + G_d \Delta d_k$$ \hspace{1cm} (11)

An additional linear differential equation for the collector inlet temperature with respect to the collector outlet temperature, the steam mass flow rate and the heat exchanger water inlet temperature was added to equation (11). The disturbance vector, $\Delta d_k$, is of the form

$$\Delta d_k = \begin{bmatrix} \Delta S_k \\ \Delta T_{amb,k} \\ \Delta \dot{V}_{Wind,k} \\ \Delta \dot{V}_{Steam} \\ \Delta T_{Water,k} \end{bmatrix}$$ \hspace{1cm} (12)

The initial condition for equation (11) is $\Delta T_0$ and the measurement is

$$\Delta T_{out,k} = C\Delta T_k$$ \hspace{1cm} (13)

Set points for the plant model are defined for the collector outlet temperature, $T_{out,set}$, for the HTF volume flow rate as the input, $\dot{V}_{HTF,set}$, and for the states, $T_{set}$. For the linearized model, these set points become $\Delta T_{out,set}$, $\Delta \dot{V}_{HTF,set}$ and $\Delta T_{set}$.

**Figure 5 MPC Controller**

The structure of a MPC controller is shown in Figure 5. The controller consists of the plant model, the state estimator, the target calculation, and a receding horizon regulator. The receding horizon regulator is based on the minimization of the following objective function at time $k$.

$$\min_{\Delta \dot{V}_{HTF}} \sum_{j=0}^{\infty} \left( Q \left( \Delta T_{out,k+j} - \Delta T_{out,set} \right)^2 + P \Delta \dot{V}_{HTF,k+j}^2 \right)$$ \hspace{1cm} (14)

where $Q$ is a penalty parameter for the difference between the actual collector outlet temperature and the set point temperature. The parameter $P$ is a penalty parameter on the rate of change of the HTF volume flow rate as the input in which

$$\Delta \dot{V}_{HTF,k+j} = \Delta \dot{V}_{HTF,k+j} - \Delta \dot{V}_{HTF,k+j-1}.$$ \hspace{1cm} (15)
Penalizing the rate of change of the input can be useful for a better attenuation of possible oscillations, which might occur in the controlled collector outlet temperature. The vector $\Delta V_{HTF}^N$ contains $N$ future optimal open-loop control moves where the first input value in $\Delta V_{HTF}, \Delta V_{HTF,k}$, is injected into the power plant model. In addition, constraints on the collector outlet temperature, on the HTF volume flow rate and on the rate of change of the HTF volume flow rate can be considered.

The state estimator is a linear observer that estimates the states of the system from the input (the HTF volume flow rate), the measured disturbances (environmental data, steam mass flow rate, heat exchanger water inlet temperature), and the measurement of the collector outlet temperature. Ideally the linear model should predict the same states as the actual process (in this case the non-linear detailed plant model). The differences between the collector outlet temperatures as predicted by the linear model and the detailed model are multiplied by an observer gain and fed back to the linear model to minimize the difference. The observer gain is calculated as the discrete steady-state Kalman filter gain with the intention to minimize the mean-square error of the state estimate.

The regulator with the estimator described above would not be able to control the collector outlet temperature to the set point without exhibiting an offset. Integral action was introduced to eliminate the offset. It was assumed that the difference between the collector outlet temperature prediction of the estimator and the measurement is caused by an input step disturbance, which in turn is estimated. In some cases, integral action in the control can decrease stability due to increasing differences in the dynamic between the linear model used in the controller and the nonlinear plant model on which the controller acts.

To eliminate offset during control, the set point used in the receding horizon regulator has to be updated with respect to the measured disturbance and the estimated difference between the collector outlet temperature prediction and the measurement. The latter represents the second part of the integral action implementation. The target calculation is formulated as a mathematical program to determine the new set point.

The receding horizon regulator, the target calculation and the state estimator are appropriately linked together to form the MPC controller as shown in Figure 5. The MPC controller was implemented in MATLAB, which was chosen as the controller language since its control and optimization toolboxes provide the procedures (e.g. the quadratic program) needed to calculate the adjustment. The plant model was implemented in EES using its great feature of built-in thermodynamic fluid property functions. Since the interface between MATLAB and EES is not defined, a communication between these two programs was established through Dynamic Data Exchange (DDE) under the Windows operating system. MATLAB as the client initiates the DDE communication and requests EES, the server, to solve the plant model equations. The actual data (e.g. the collector outlet temperature) is transferred through data files between the two communicating processes.

RESULTS

The performance of the controller is shown in Figures 6, 7, and 8 for two different days. Figure 6 shows the collector outlet temperature and the HTF volume flow rate for June 20, 1998. For the HTF volume flow rate, the dashed line represents the flow as controlled by the human operator on that day. The related collector outlet temperature, calculated through simulation with the plant model, is the dashed line in the left figure. The HTF volume flow rate can be considered.
rate shown as solid line represents the input calculated through model predictive control. The solid line in the left figure is the corresponding collector outlet temperature. The automatic controller is turned on at 8.00 hr in the morning and turned off at 18.8 hr. The start up and shut down is assumed to be done by the operator. The automatic controller has the ability to hold the collector outlet temperature at a constant set point (653.9 K) for most of the time throughout the day. The performance of the linear model predictive controller is better than that of the human operator. The occurrence of oscillations at the start of automatic control and just before the controller is turned off are due to differences between the linear model used in the controller and the nonlinear model that represents the plant.

Figure 6 Simulated Collector Outlet Temperature and HTF Volume Flow Rate for June 20, 1998. Dashed line is measurement and the solid line is simulation.
In Figure 7, the collector outlet temperature and the HTF volume flow rate are shown for December 16, 1998. Because of the different level of the solar forcing function, the nonlinear model was linearized around a different operating point on the winter day. The dashed lines represent the human operator and the solid lines represent automatic control. During winter days, when the energy in the system is relatively low, the model behavior tends to become more nonlinear. Integral action is then excluded on that day in the automatic controller. The automatic controller is turned on at 9:00 hr and turned off at 16:00 hr. Although a small offset between the automatically controlled collector outlet temperature and the set point (597.3 K) can be seen, the automatic control action results in a collector outlet temperature much closer to the set point compared to the human controlled one.

In Figure 8, the gross output for June 20, 1998 and December 16, 1998 is shown. The small-dashed line represents the gross output for human operator, the solid line represents automatic control, and the long-dashed line represents useful energy. The left hand figure in Figure 8 shows the calculated gross output for June 20, 1998. The small-dashed line and solid line represent the gross output for human operator and automatic control, respectively. The useful energy is plotted. As can be seen from the two plots, the
fact that the automatic controller shows a better performance than the human controller in generating a constant set point collector outlet temperature does not improve the gross output significantly.

The right hand figure in Figure 8 shows the calculated gross output for December 16, 1998. Also in this case, there is no significant improvement in the gross output through automatic control. This is a result of the use of the measured steam/water flow rate in the plant model, which is not optimal. An optimization of this flow rate would be necessary to determine whether there is an increase in the gross output.

CONCLUSIONS

A nonlinear model of the 30 MWe SEGS VI parabolic trough plant has been established. The model consists of a dynamic model for the collector field and a steady-state model for the power plant. The model was used to examine the use of a linear model predictive controller to maintain a specified constant collector outlet temperature. The approach was evaluated on a summer day and a winter day when the power plant was operating in pure solar mode. The controller showed the ability to maintain the collector outlet temperature close to the specified set point most of the day. The automatic controller demonstrated better control of the collector outlet temperature than that of a human operator.

For this power plant application, further studies should include a model predictive control strategy that maximizes the gross output rather than the collector outlet temperature. Controlling both the HTF volume flow rate and the steam mass flow rate in the power plant is expected to increase the daily gross output of the parabolic trough plant.

The model linear predictive controller described in this paper is applicable to systems with significant thermal capacitance and transient times for the fluid flows. In situations with these characteristics, conventional control techniques such as proportional control are not satisfactory. A predictive method such as described here is one approach to control.

ACKNOWLEDGEMENTS

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REFERENCES


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NOMENCLATURE

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<th>Symbol</th>
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<tbody>
<tr>
<td>A</td>
<td>cross-sectional area</td>
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<tr>
<td>c</td>
<td>specific heat</td>
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<td>$\Delta d_k$</td>
<td>disturbance</td>
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<td>$h_{\text{ABS,HTF}}$</td>
<td>heat transfer coefficient between absorber and heat transfer fluid</td>
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<td>$\dot{m}_{\text{steam}}$</td>
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Subscripts

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