MODELING HEAT TRANSFER IN ROOMS
USING TRANSFER FUNCTION METHODS

by

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A thesis submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE
(Engineering)

at the

UNIVERSITY OF WISCONSIN-MADISON

1982

ACKNOWLEDGMENTS

I wish to thank Professors J.W. Mitchell, W.A. Beckman, J.A. Duffie, and S.A. Klein for their help, encouragement, and friendship during the past 16 months. Helping and encouraging students in their research is expected of professors, and these four have gone well beyond the call of duty. But, that friendship between teacher and student can be included in the list is rare, and it is a real tribute to them and to the Solar Energy Lab they have nurtured, that a spirit of friendship and concern can be so visible in their day to day interactions with the students.

My gratitude for help, encouragement, and friendship also goes out to all my fellow grad students in the Lab. I am particularly grateful to Daryl, a virtually unlimited source of information and amusing conversation. Thanks also to Brian, Carol, Andy, Fran, and Mike who made ERB 1335 such a special place. I would like to thank Jim Braun for his help with computing in general and TRNSYS in particular. I am indebted to Bob Howe for the loan of his TRNSYS model, and, of course, for his charming wit and friendly smile.

My deepest love and appreciation goes to my wife Rachel, who has so cheerfully given up a house and a job to follow me on my search for greener grass. My deepest apologies to her and my newborn twins that I have been so busy with research and writing these past two months at a time when I was needed most at home.

This project was partially funded by the U.S. Department of Energy and ASHRAE, and I thank those organizations for their support.

I now return to life as a high school science teacher grateful to have had this opportunity to learn more engineering, to meet new friends, and to realize that my place really is in the classroom helping young people discover the joys of science.

ABSTRACT

Two simplified methods for calculating one-dimensional transient conduction in multi-layer slabs, the response factor method and the z-transfer function method, are outlined. The effect of the assumptions of constant surface heat transfer coefficients is investigated in relation to the transfer function method.

A room model with three elements, external wall, glazing and a single interior partition-ceiling-floor element, is developed. The errors resulting from the use of a combined interior convection/radiation resistance by assuming the partition is at room temperature are investigated. The model uses wall transfer functions for both exterior and interior walls, and uses separate convection, long wave radiant exchange, and absorbed short wave radiation calculations at each surface. A deadband thermostat and "free" cooling with outside air are included in the model. The model is economical while including significant detail in the heat transfer mechanisms.

The concept of room transfer functions is reviewed, and a comprehensive room transfer function (CRTF) is proposed and tested. The CRTF correlates the heating and cooling loads with current and previous sol-air

temperatures, room temperatures, and heat gains to the room. This single function, when used with simple logic representing a thermostat with deadband and/or night setback, can be used to compute hourly loads during heating and cooling, or room temperature when the equipment is off. The transfer function is obtained by regression of a limited data set from a simulation. The method is tested with hourly simulation data generated by the three-element room model developed in this thesis, and by zone and building models constructed using TRNSYS, a transient simulation program.

TABLE OF CONTENTS

LIST OF FIGURES

Page

Figure

NOMENCLATURE

A	coefficient matrix
a	transfer function coefficient
B	column vector
b	transfer function coefficient
c	transfer function coefficient
d	transfer function coefficient
f	view factor, area fraction
h	heat transfer coefficient
I	hourly solar irradiation
k	conductivity
l''	irradiation from lights
P	previous transfer function contribution
$\dot{Q}$	heating or cooling load (energy per unit time)
q	heat gain (energy per unit time)
q''	heat flux
R	resistance
ΔR	net long wave black body radiation
s''	absorbed solar flux
T	temperature, surface temperature
U	conductance
v	room transfer coefficient
w	room transfer coefficient
Y	response factor

Z response factor, coefficient matrix

z z-transform variable

Subscripts

a ambient

c convection, constant

e equivalent (sol-air for e,o)

g glazing

o outside

p partition

r room air, radiation

s-a sol-air

t tilted

w wall

Greek

α thermal diffusivity

α^* solar absorbtance

Δ time step, difference

ε emittance

τ time, transmittance

σ Stephan-Boltzman constant

1.0 INTRODUCTION

1.1 Objectives

Calculation of hourly heating or cooling loads and space temperatures in buildings is very complex. New methods have been developed to simplify the calculations, however they introduce assumptions and restrictions. Even with these methods, the calculations can be extensive and expensive.

The objectives of this thesis are to outline these new methods and to investigate the effects of some of the assumptions, to develop a room model based on these methods with some modification, and to find a method to calculate hourly loads and temperatures using a single functional relationship which is simpler and more economical than the full hourly simulation calculations.

1.2 Organization

The research efforts of the past fifteen months have been concentrated in three major areas, and are reported in the following three chapters.

In the Chapter 2, two simplified methods for calculation of transient conduction in multi-layer slabs, the response factor method, and a later improvement, the z-transfer function method, are outlined. The effects of the assumption of constant surface resistances in relation to transfer functions are investigated.

In Chapter 3, a model for a room with simple geometry is developed. The common practice of combining radiation and heat transfer resistances at the inside surface by assuming the interior surfaces are at room temperature is investigated. The use of equivalent temperatures as a method for including long and short wave radiation at a surface in a transfer function approach is developed. The equations governing the heat transfer in a three element room are then developed using transfer functions for the interior as well as the exterior wall. Control is added with a model for a thermostat with deadband between the heating and cooling setpoints.

Finally, in Chapter 4, the concept of a comprehensive room transfer function is introduced and compared with the current ASHRAE methods for hourly load calculations. This function, when used with a control logic representing a thermostat, allows rapid calculations of heating or cooling loads and the room temperature, resulting in savings of computing costs. The comprehensive room transfer function is tested with data from the model developed in the Chapter 3 and with TRNSYS zone and building models.

1.3 Literature Survey

The transfer function methods for transient conduction in multi-layer slabs is introduced by Stephenson and Mitalas [1,2]. Mitalas and Arsenault [3] give a computer program to calculate the function coefficients. Ceylan [4] also presents a method for computing the coefficients. These transfer function coefficients are tabulated by ASHRAE [5]. Pawelski [6] and Kimura [7] give detailed outlines of the transfer function method. Horn [8] investigates the use of a constant room temperature transfer function for varying room temperature, and explores the use of an effective building capacitance. ASHRAE [5] gives a method for hourly load calculations using room and room air transfer functions in conjunction with conduction transfer functions. Chen [9] shows ways of combining the conduction transfer functions of all the surfaces in a room. Pawelski [6] uses regression fits to finite difference solutions for conduction in walls with parallel paths to obtain transfer function coefficients for the walls.

2.0 RESPONSE FACTOR AND TRANSFER FUNCTION METHODS FOR ONE-DIMENSIONAL TRANSIENT CONDUCTION

2.1 Introduction

Modeling the heat transfer and temperatures in buildings requires solution for transient conduction through walls, roof, and floor. Methods have been developed in the last fifteen years which allow determination of heat transfer and temperatures with very few algebraic calculations. The response factor method, and an improvement, the z-transfer function method, give sets of coefficients which are calculated only once for each type of wall, roof, or floor. With these coefficients and weather data, heat fluxes and temperatures can be rapidly calculated.

The one-dimensional transient conduction problem will be formulated for exterior walls, and the traditional method of solution by finite difference methods will be briefly summarized. Next, the response factor and the z-transfer function methods will be introduced with a summary of their origin. Finally, the problem of time-varying surface heat transfer coefficients in relation to the transfer function method is explored.

2.2 Governing Equations

For practical application in building load calculations, heat conduction through plane walls and slabs can be modeled as one-dimensional systems represented by the classical heat conduction equation for constant thermal properties.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \qquad (2.2.1)$$

At each boundary there may be convective and/or radiative heat transfer. The First Law requires that the conduction, convection, and radiation heat transfer at the surface balance. At the outside surface, with convection only, the balance is:

$$- k \frac{\partial T_o}{\partial x} = h_{c,o} (T_a - T_o) \qquad (2.2.2)$$

There is also the possibility of absorbtion of solar radiation and of long wave radiation exchange with the surroundings and sky. These will be addressed in Section 3.2.2.

At the inside surface, conduction must balance convection to the room air and long wave radiation exchange with the surroundings. For present it is assumed the interior surfaces are black for long wave radiation, and the surroundings seen by the wall are isothermal and at room air temperature. The balance then becomes:

$$- k \frac{\partial T_i}{\partial x} = h_{c,i} (T_i - T_r) + 4\sigma\overline{T}^3 (T_i - T_r) \quad (2.2.3)$$

A linearization of the radiation term is used. The range of temperatures inside a building is small enough that this is a good approximation. The possibilities of absorbtion of solar radiation through windows, absorbtion of radiation from lighting, and radiation exchange with multiple surfaces not at room temperature are also possible, and will be addressed in Section 3.2.2.

2.3 Finite Difference Methods

Traditionally, transient conduction problems have been solved by finite difference methods which may be found in any standard heat transfer text. The general idea is to divide the wall into thin sections, and to treat each of these sections as an isothermal element with a lumped thermal capacitance. Conservation of energy requires that the net energy transfer with neighboring elements must equal the change in energy storage within the elements. The equation for each element then includes temperatures of neighboring elements, and all of these equations must be solved simultaneously to find the current temperature of each element. Time is divided into small discrete time steps, and the temperatures are solved during each time step with a time-dependent forcing function at the boundaries.

The accuracy of these methods depends on the size of the elements and the time step. As these decrease, the numerical solution approaches the exact solution. However, the computing time and cost rapidly escalates as well. These numerical methods also require considerable effort to set up. Node size and placement has to be chosen, conductances and capacitances for each element calculated, and all input to the computer program.

2.4 Response Factor Method

The Response Factor Method of solution of transient one-dimensional conduction problems was developed by Stephenson and Mitalas [1] in 1967. The central ideas of the method are that if the system is linear, and if the forcing function can be adequately represented by a time series of triangular pulses, then if the system response to a unit triangular pulse can be determined, the actual response of the system will be the superposition or summation of the responses to all the triangular pulses which constitute the input forcing function. Each individual response is the response to the unit input scaled by the magnitude of the actual input pulse.

As seen in Figure 2.4.1, a forcing function such as outside temperature, can be represented by a summation of triangular pulses, and each pulse is just a scaled unit pulse. The resulting representation is a series of amplitudes at each time step connected by ramps.

A typical output, or response, to a unit input pulse is shown in Figure 2.4.2. The amplitudes of the response at each time step after the unit pulse are called response factors.

As an example, consider a temperature difference producing a heat flux at the inside surface of a wall. Figure 2.4.2 shows a typical response of heat flux to a unit temperature pulse for a time step of one hour. The attenuation, or gain, of the response is a result of the conductance of the wall, while the lag is a result of the capacitance. This wall shows negligible instantaneous heat flux to the room. The flux increases to a peak after about three hours, then decays slowly over the next several hours. In order to find the current heat flux to the room, a superposition of the responses to the current and all previous temperature pulses is made. For example, the temperature three hours ago is now resulting in a flux equal to the third hour response

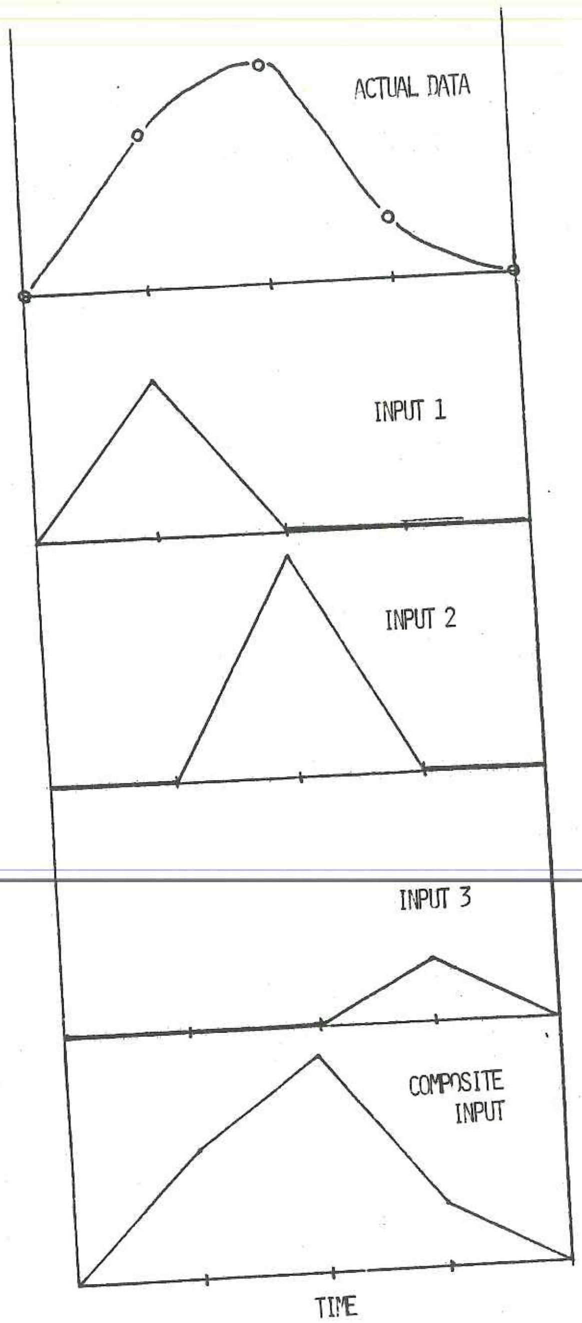


Figure 2.4.1 Decomposition Of Input Into Triangular Pulses.

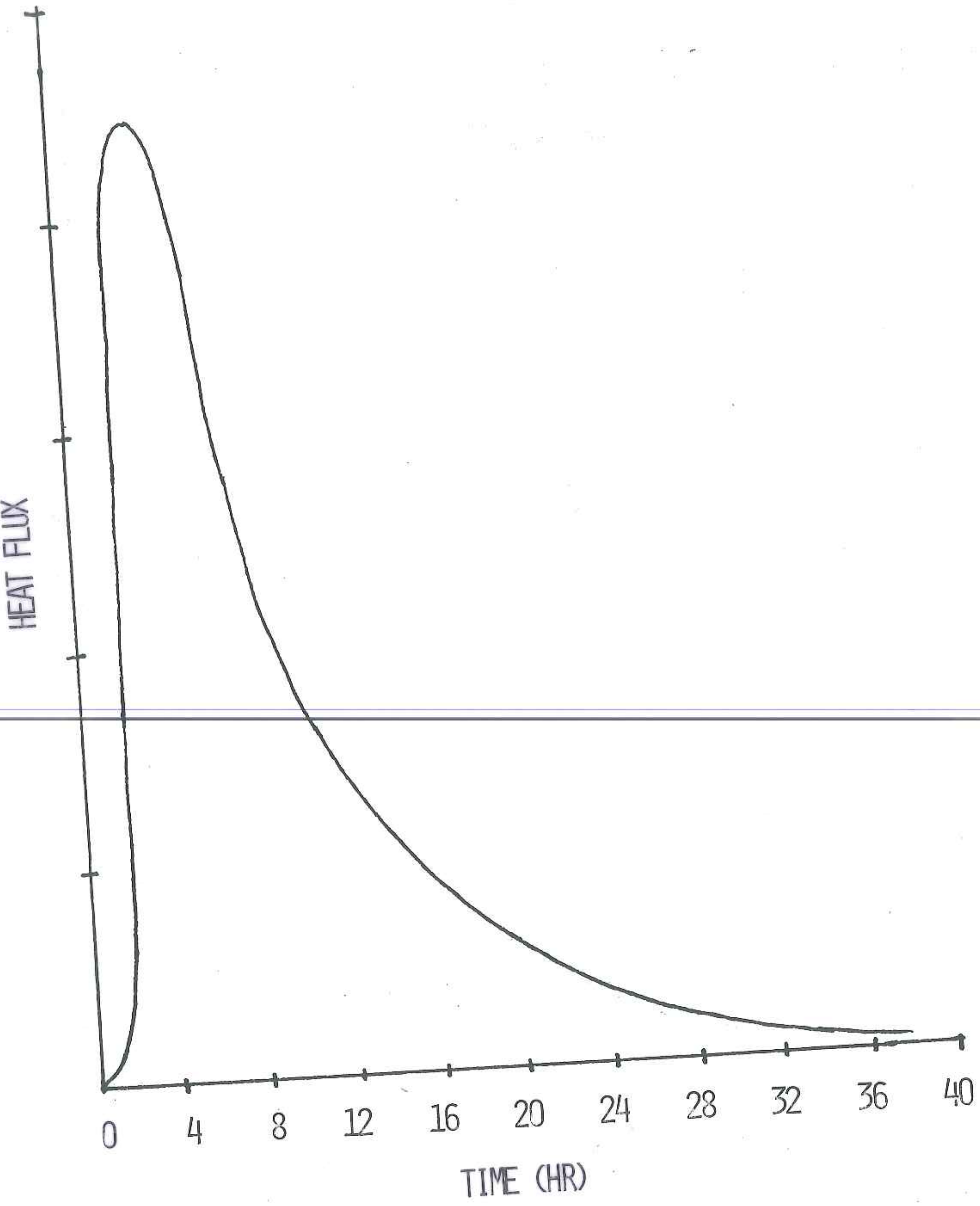


Figure 2.4.2 Heat Flux Response Of A Wall To A Unit Triangular Pulse

factor multiplied (scaled) by the temperature at that time. There is a similar contribution from every other previous time step, and all of these individual responses must be summed to find the total response at the current time step. Mathematically:

$$q_\tau = \sum_{n=o}^{\infty} T_{o,\tau-n\Delta} Y_n \tag{2.4.1}$$

Figure 2.4.3 shows hypothetical responses to the series of input pulses shown in Figure 2.4.1. The actual response of the wall is then equal to the sum of all the responses at a particular time, resulting in the composite response shown at the bottom of the figure. Though theoretically the response to an input pulse continues indefinitely, for conduction through standard construction walls with time steps of one hour, the response is negligible after 24 to 48 hours. However, since the response is the summation over all previous hours, the sum of many small numbers may become significant, and the summation may require even more than 48 terms, especially for massive construction.

A wall may have time-varying forcing functions on both sides. Since the system is linear, the two forcing functions can be treated separately, and the total response is just the sum of both sets of individual re-

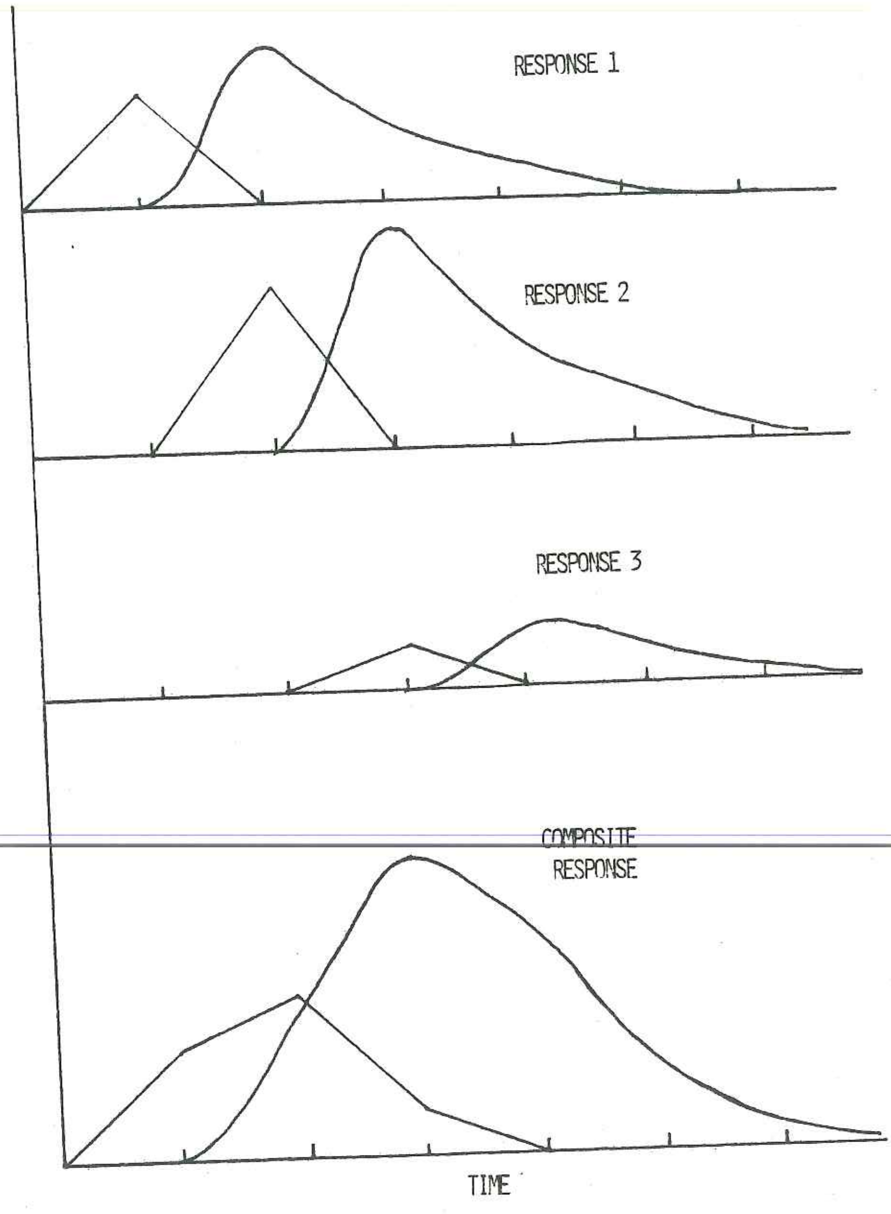


Figure 2.4.3 Superposition of Individual Responses.

sponses. A set of response factors for the heat flux at the inside surface for an inside temperature pulse can be derived. Then:

$$q_\tau = \sum_{n=o}^{\infty} T_{o,\tau-n\Delta} Y_n - \sum_{n=o}^{\infty} T_{i,\tau-n\Delta} Z_n \qquad (2.4.2)$$

Response factors are obtained by taking the Laplace transformation of the conduction equation (Equation 2.2.1), solving and expressing the solution as a series expansion, transforming back to the time domain and making special substitutions for the surface temperature and discrete time steps. This series expression is then compared term by term with the general form of the response factor equation (Equation 2.4.1) to give the response factors. The original derivation can be found in Stephenson and Mitalas [1]. A more detailed description of the derivation and the method can be found in Kimura [7] or Pawelski [6].

The response factor method, while offering potential savings in time and expense, still requires many algebraic calculations, and the transferal and storage of response factors and histories which may easily total over 100. It does lead, however, to a further improvement by Stephenson and Mitalas, the Z-Transfer Function Method.

2.5 Z-Transfer Function Method

Stephenson and Mitalas [2] found that if the previous outputs (heat fluxes) were used as well as the inputs (temperatures), large reductions could be made in the number of previous time steps required, and, therefore, in the number of calculations.

The key to this method is finding the ratio of the Z-transform of the output, as given by the solution of the differential equations, to the Z-transform of the input. This ratio is called the Z-transfer function, K(z).

$$K(z) = \frac{g(z)}{f(z)} = \frac{\text{Z-transform of output time series}}{\text{Z-transform of input time series}}$$

and can be expressed as:

$$K(z) = \frac{a_o + a_1 z^{-1} + a_2 z^{-2} + \ldots}{b_o + b_1 z^{-1} + b_2 z^{-2} + \ldots} \tag{2.5.2}$$

Then,

$$a_o f_\tau + a_1 f_{\tau-\Delta} + a_2 f_{\tau-2\Delta} + \ldots = b_o g_\tau + b_1 g_{\tau-\Delta} + b_2 g_{\tau-2\Delta} + \ldots \tag{2.5.3}$$

and the output at the current time step can be solved in terms of current and previous inputs and previous outputs, giving

$$g_o = \sum_{n=o} a_n f_{\tau-n\Delta} - \sum_{n=1} b_n g_{\tau-n\Delta} \tag{2.5.4}$$

For a detailed derivation, see Stephenson and Mitalas [2], Kimura [7], or Pawelski [6].

As with Response Factors, there can be more than one input driving the system, for example, time-varying outside and inside temperatures. The general form of the Z-transfer function for conduction in walls most widely used is:

$$q''_\tau = \sum_{n=o} b_n T_{e,\tau-n\Delta} - \sum_{n=o} c_n T_{r,\tau-n\Delta} \tag{2.5.5}$$

$$- \sum_{n=1} d_n q''_{\tau-n\Delta}$$

Note that if the room temperature is constant, or nearly constant, as in many commercial buildings, then:

$$q''_\tau = \sum_{n=o} b_n T_{e,\tau-n\Delta} - \sum_{n=1} d_n q''_{,\tau-n\Delta} \tag{2.5.6}$$

$$- T_{r,\text{constant}} \sum_{n=0} c_n$$

2.6 Obtaining Z-Transfer Functions

Mitalas and Arsenault [3] wrote a FORTRAN program which calculates transfer function coefficients given the properties of each of the massive layers of a multi-layer wall or slab, the thermal resistances of any air spaces, and the surface convection-radiation resistances. Using this program, transfer functions for any multi-layer wall or slab may be calculated.

ASHRAE [5] has tabulated the sets of coefficients for determining heat flux into the room for many standard walls, partitions, roofs, and floors for the case of constant room temperature only. The tables contain the b and d coefficients for each hour, but only the sum of the c coefficients is tabulated, since, for constant room temperature, the c coefficients can be combined as shown in Equation 2.5.6.

For many buildings, especially residential, where the room temperature is allowed to float through a fairly large range, the c coefficients are required for accurate results. Horn [7] investigated the effect of using the constant room temperature transfer function for non-constant room temperatures, and found that significant errors can occur even for total energy quantities, especially during fall and spring. It seems that using the c coefficients separately for the case of non-constant room temperature is required for accuracy, and therefore the c coefficients need to be calculated by a program such as that of Mitalas and Arsenault.

An alternative method for calculating the coefficients has been formulated by Ceylan [4]. His program is a bit more flexible for specifying boundary condition types. However, setting up the input to the program is more complicated than Mitalas and Arsenault.

An example using the Mitalas and Arsenault program is included in Appendix A.

2.7 Transfer Functions and Variable Surface Conductances

As presented in the previous sections, transfer functions can be obtained for outside and room temperature driving functions. These driving temperatures are air temperatures, so a surface conductance is included at each surface to give the resulting transfer function. It is necessary for these conductances to be constant. Changing the surface conductance requires calculation of a new set of coefficients.

The assumptions of constant interior surface conductance is probably fairly good, though they do certainly change with surface temperature. The calculation of this conductance is addressed in the next chapter. The assumption of constant exterior surface conductance is suspect, however, since wind speed, and therefore the convection coefficient, varies considerably. Since wind coefficients are not known very well for buildings, the assumption of constant average wind speed is often used.

For most buildings, the surface resistances are only a small fraction of the total resistance of the wall. However, these surface resistances can be very important in that the rate at which heat can be transferred to and from the mass of the wall is strongly dependent on them. Therefore, while an incorrect convection estimate will make little difference in the total amount of energy transferred to and from the room for a given ΔT, the time distribution of that heat transfer can differ significantly.

In Figure 2.7.1, the heat flux into the room for a block and face brick exterior wall is shown as calculated by two different transfer functions, but using the same outside sol-air temperature distribution typical of a south surface on a sunny March day in Madison. The sol-air temperatures are based on $h_o = 6$ Btu/hr-ft^2-F (34 W/m^2-C), however, the transfer functions use exterior

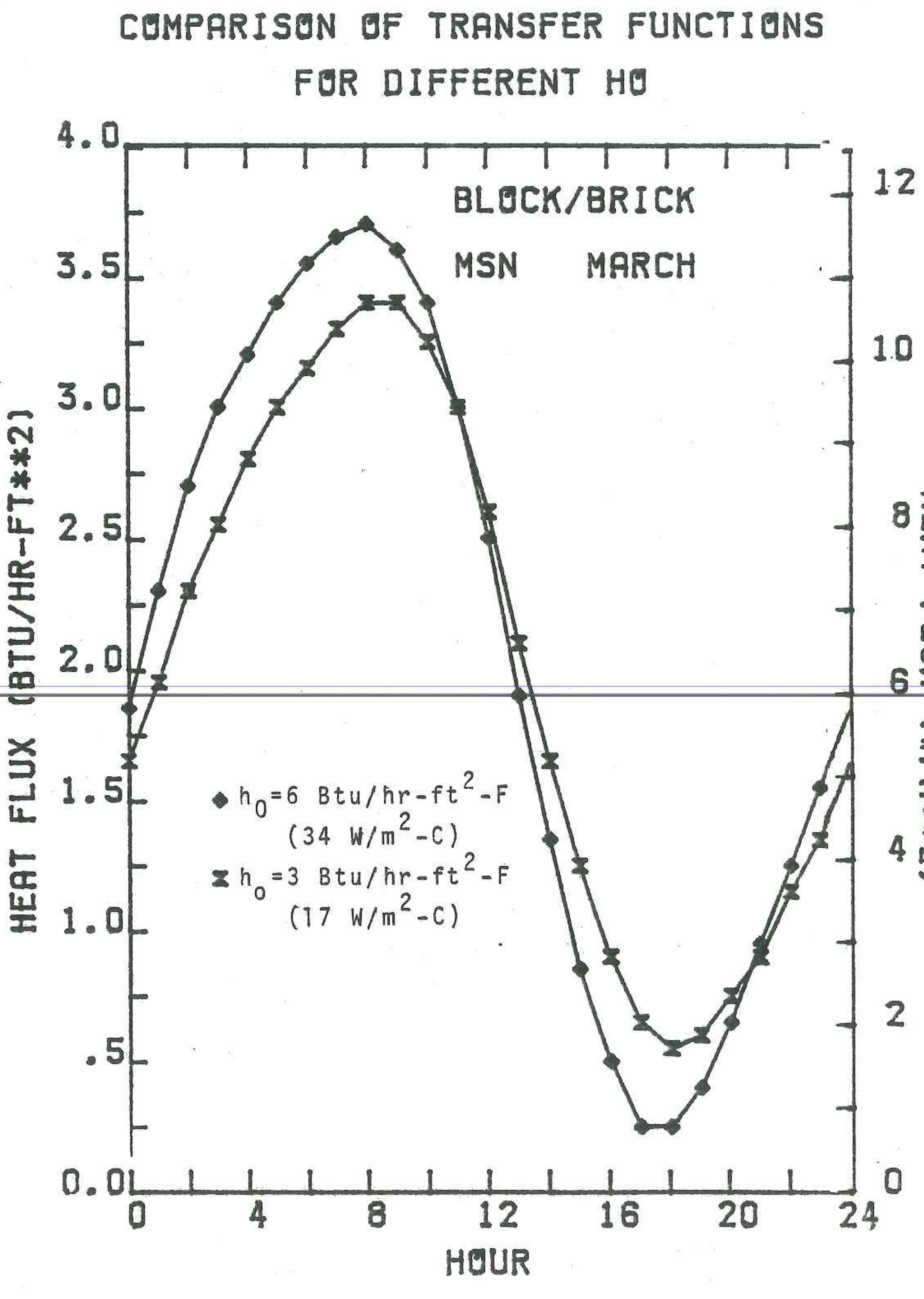


Figure 2.7.1 Comparison Of Transfer Functions For Different Heat Transfer Coefficients.

surface conductances of h_o = 3 Btu/hr-ft^2-F and h_o = 6 Btu/hr-ft^2-F (17 and 34 W/m^2-C). It can be seen that the lower convection rate at the exterior surface results in attenuation of the amplitude and some lag over the case of the higher convection rate. The integrated heat transfers are nearly identical, as they should be, since the overall conductances are not significantly different. It can be seen that significant errors in hourly loads can occur if a function derived for one h_o is used for a case in which the h_o is different.

A transfer function from surface to surface could be used if surface temperatures are known. Then the surface conductances are outside the transfer function. The two transfer function equations for inside and outside surface fluxes given surface temperatures can be solved simultaneously with the two equations relating surface and air temperatures, surface conductances, and heat fluxes at each surface. This gives the heat flux at the interior surface as a function of air temperatures and surface conductances. The complete derivation can be found in Appendix B.

There is a minor problem in such an approach, however. A transfer function solution is based on, and assumes, a ramp input. While the air temperatures are specified as a series of ramps, the corresponding surface temperature is determined by the thermal state of the wall, as well as the air temperature, and it may not follow a similar ramp, especially after maximum or minimum temperatures. The surface temperature is not truly an independent forcing function. It was found by comparison of solutions of air to air versus surface to surface transfer functions, differences up to 3% could occur just after maxima or minima in light construction walls while differences were negligible in heavy walls. Reducing the time step to one-half hour improved the agreement considerably.

In the work to follow, it was decided to use constant average values of the convection coefficients in order to keep the models somewhat simpler. Also, convection coefficients are not known very well for buildings, and since errors are so easily introduced via the sol-air temperatures, which are very sensitive to convection, the assumptions of a constant average value is probably safe.

2.8 Conclusion

The z-transfer function method provides a fast, inexpensive numerical solution of transient conduction through multi-layer walls. While changes in surface heat transfer coefficients make little difference in the total energy conducted through walls of standard construction, the hourly heat fluxes can be quite different. The transfer function method can be formulated to accomodate varying heat transfer coefficients, however, since the coefficients are not known very well for a particular building anyway, the standard transfer function method is probably sufficient for most applications.

3.0 DEVELOPMENT OF A 3-ELEMENT ROOM MODEL

3.1 Introduction

A model for the transient thermal response of a room should account for all heat transfer mechanisms at the surfaces, as well as the transient conduction and storage in the massive elements. In this chapter, a simplified room or zone model is developed to simulate a system consisting of an exterior massive wall with glazing area and an interior massive partition. It is assumed in this model that the floor, ceiling, and partitions are of a similar construction and at the same temperature, and can be treated as a single element. The derivation of the equations for the general case of each roof, wall, and ceiling being different is included in Appendix C. However, for the purposes of this research project, the 3-element model was considered adequate.

It was decided that a room model should include the effects of thermal storage in the partitions by using transfer functions, rather than by assuming some estimated value for a lumped thermal capacitance. Treating the system this way also allows interior surface temperatures to be calculated, and therefore long-wave radiant exchanges between the interior partitions and the exterior wall and window can be calculated.

It is also required that the model can handle solar radiation on the interior surfaces that enters by the glazing, and/or other radiant loads, such as that from artificial lighting. Convection and radiation are treated separately to avoid some of the problems and assumptions in using a combined heat transfer coefficient. Constant average convection coefficients are used, so transfer functions from outside air to inside air may be used.

In this chapter, simplifications in modeling the heat transfer mechanisms in a room will be considered. The use of equivalent temperatures is outlined, and the effects of the assumption of a combined inside convection/long wave radiation heat transfer coefficient are investigated, and conclusions drawn. Then the equations governing the heat transfer in a room with exterior wall, glazing, and a combined interior partition-ceiling-floor element are developed and combined into a single set of simultaneous equations. Control of the room temperature by a thermostat with a deadband between heating and cooling is incorporated, as well as provision for "free" cooling with outside air when possible. Finally, an example using the model is presented.

3.2 Considerations and Simplifications in Modeling Heat Transfer in a Room

3.2.1 Combined Heat Transfer Coefficients

Heat transfer at an inside surface can occur by convection of room air, long wave radiant exchange with other surfaces, and absorbtion of solar radiation through windows, and radiation from lights.

Convection and long wave radiant exchange are often combined in a single combined heat transfer coefficient, or conductance. If the surrounding surfaces are black and at room temperature, then in a network analogy, the convection and radiation resistances at the inside surface of a wall are in parallel as seen in Figure 3.2.1, where h_r is a linearized radiation coefficient.

$$h_r = 4\sigma f_{i,j}\overline{T}^3 \qquad (3.2.1)$$

For typical room temperatures,

$$h_r = 0.99 \text{ Btu/hr-ft}^2\text{-F } (5.62 \text{ W/m}^2\text{-C})$$

and for turbulent natural convection with a surface-air temperature difference of 10F (5.6 C)

$$h_c = 0.47 \text{ Btu/hr-ft}^2\text{-F } (2.67 \text{ W/m}^2\text{-C})$$

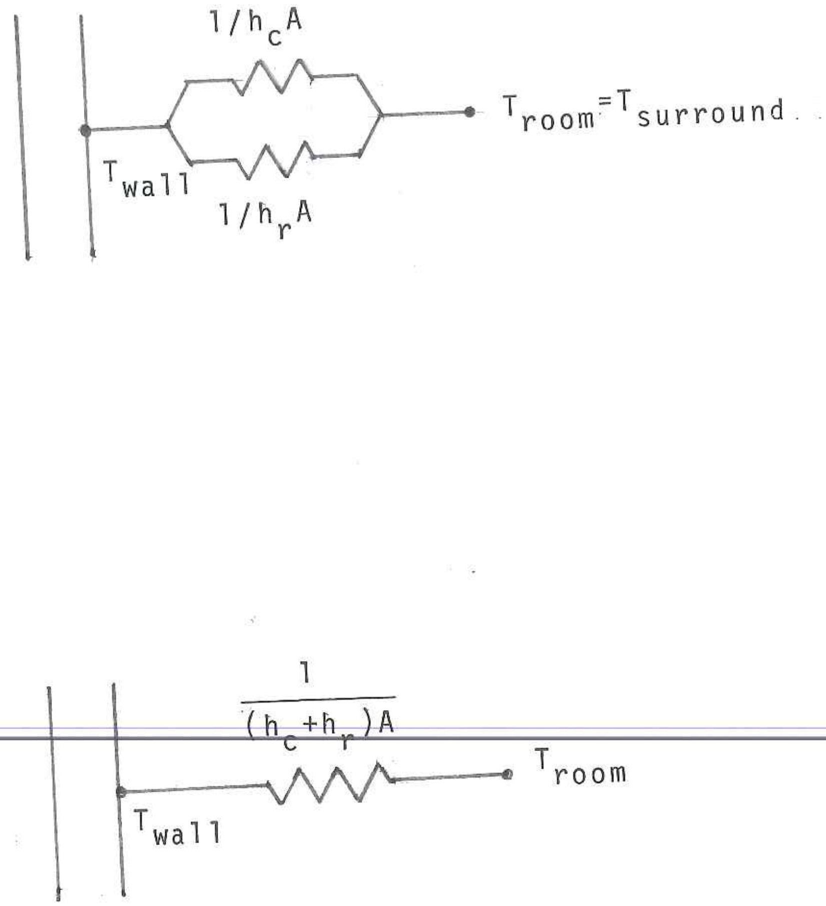


Figure 3.2.1 Combined Interior Radiation/ Convection Surface Resistance.

giving a combined inside heat transfer coefficient

$$h_i = 1.46\ \text{Btu/hr-ft}^2\text{-F}\ (8.29\ \text{W/m}^2\text{-C})$$

The assumption that the interior surfaces are black is fairly good for long wave radiation. However, the assumption that the interior surfaces are at room temperature is questionable under certain circumstances. For example, the interior walls opposite a large expanse of glass can significantly depart from room temperature in sub-freezing weather. Solar radiation on a wall may raise the surface temperature well above the ambient room air temperatures. In fact, the whole concept of thermal storage in massive walls depends on a temperature difference between the mass and the air.

For the simple case of two black surfaces facing each other, the thermal network is shown in Figure 3.2.2 and 3.2.3. Comparison of Figure 3.2.3 with Figure 3.2.1 shows that the addition of a convection resistance at the interior partition surface can significantly change the magnitude of the total resistance. If the sun is shining on the surface or if it is facing a cold, single-glazed window at night, the temperature of the surface will not be room temperature, and convection will occur at the partition.

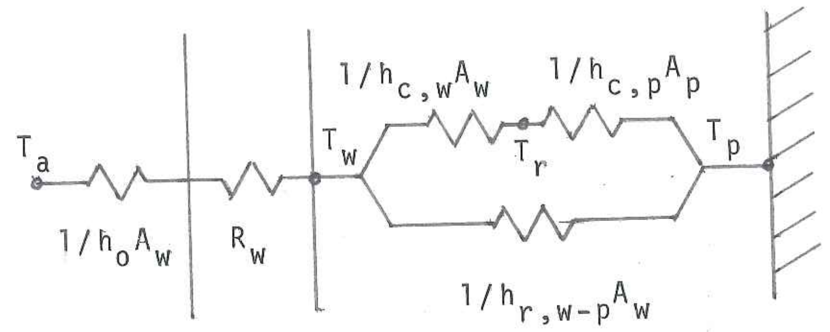


Figure 3.2.2 Radiation and Convection Network for Two Facing Black Walls.

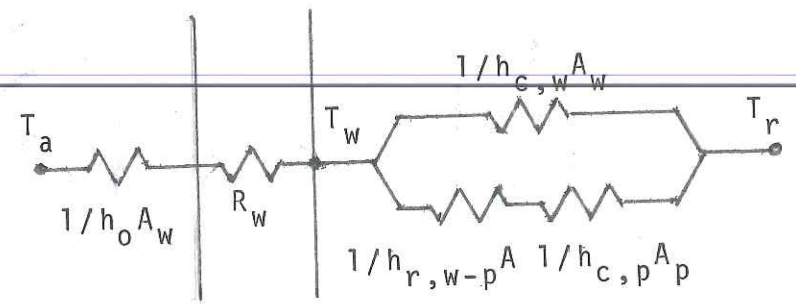


Figure 3.2.3 Rearrangement of Two Surface Network in Figure 3.2.2.

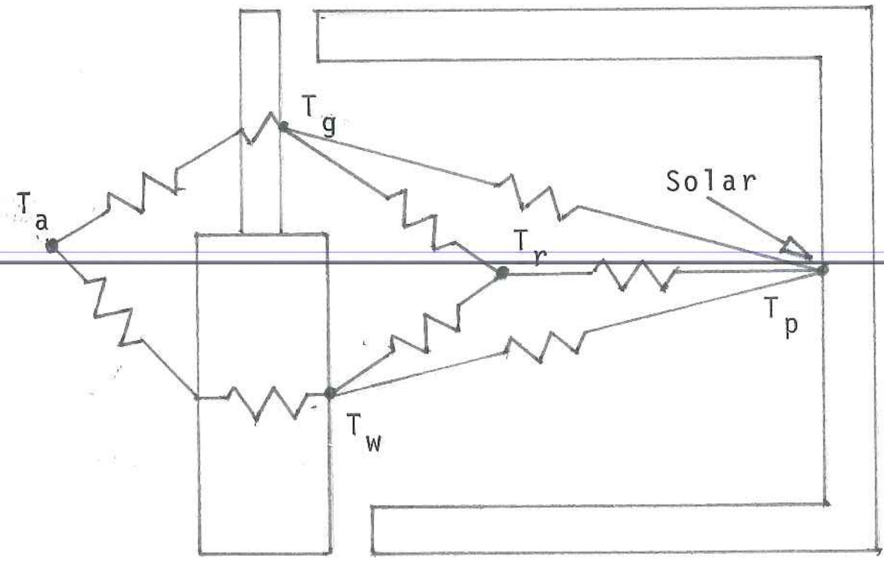


Figure 3.2.4 Network for Room with Exterior Wall, Glazing, and Interior Partition-Ceiling Floor Element.

The complete network for the case of an exterior wall, glazing, and interior adiabatic partition is shown in Figure 3.2.4. Comparisons between the solution of this network and one using constant, combined inside heat transfer coefficients for the exterior wall and glazing were made for several cases. For the network of Figure 3.2.4, the resistances were estimated and the network was solved for the surface temperatures given an applied room temperature and outside temperature. The resistances were then corrected using a convection correlation and the linearized radiation expression, and the network was solved again. For the simplified network, the ASHRAE recommendation for heat transfer coefficients given above for a temperature difference of 10F (5.6C) was used.

Figures 3.2.5 and 3.2.6 compare the heat flux to the room air by conduction from the outside as predicted by the constant combined conductance case versus the solution of the network in Figure 3.2.4 for temperature-dependent, separate heat transfer coefficients as a function of glazing fraction. Cases for single and double glazing, with and without solar gains into the room, for winter and summer are shown. The solar radiation used is the ASHRAE design solar heat gain through a

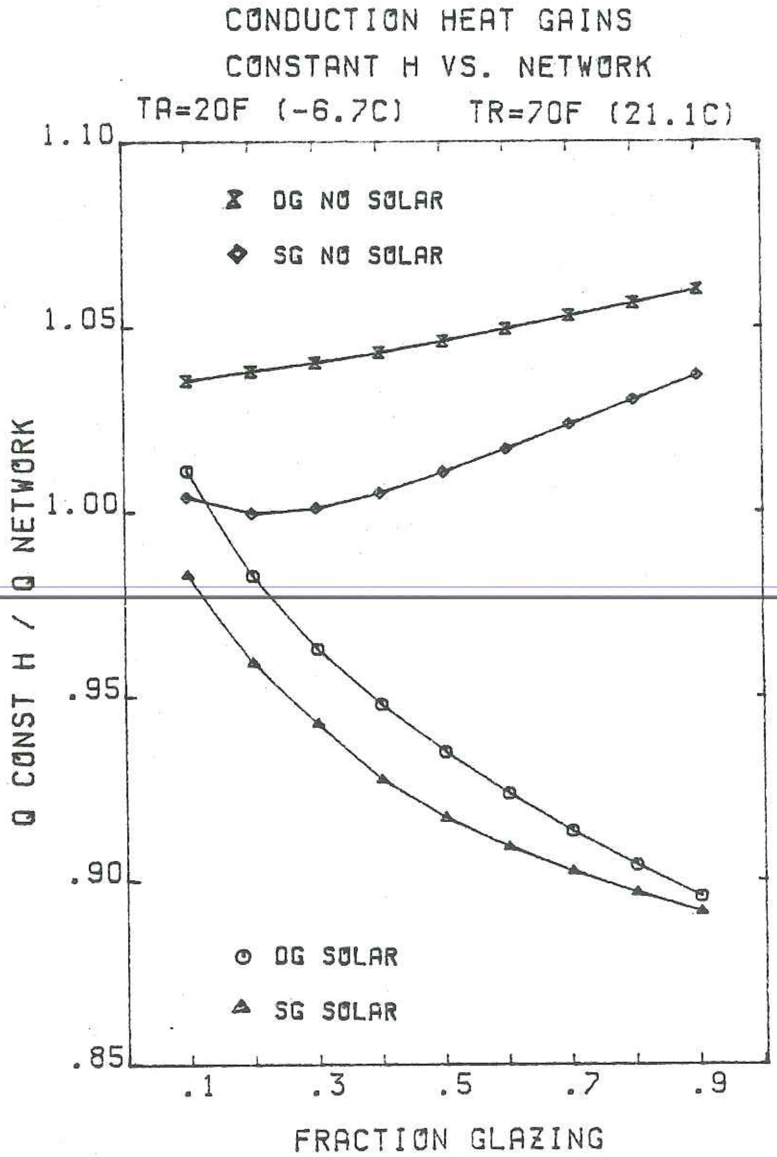


Figure 3.2.5 Comparison of Calculated Heating Loads Between a Network and (UA)ΔT Model Based on Constant Combined Radiation/Convection Coefficients.

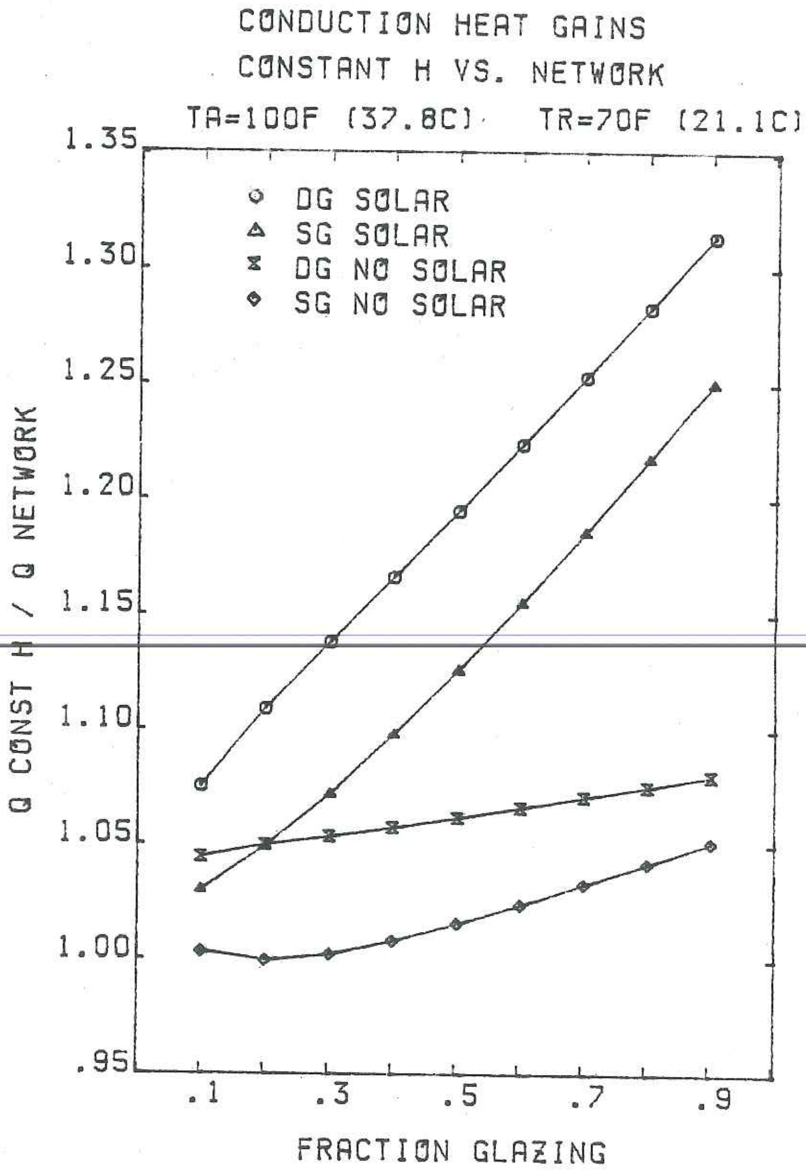


Figure 3.2.6 Comparison of Calculated Cooling Loads Between a Network and (UA)ΔT Model Based on Constant Combined Radiation/Convection Coefficients.

vertical south glazing at 40^{o} latitude on March 21 and August 21; the ambient temperatures are 20F and 100F (-6.7C to 37.8C), with a room temperature of 70F (21.1C). An interior to exterior surface area ratio of 5:1 is used, and the interior surroundings are considered isothermal and black.

The comparisons of the winter day, Figure 3.2.5, reflect that the convection losses at the cold glazing are higher than predicted by the constant combined h, however, the radiative losses from the cool partition to the outside walls are lower than predicted, and the total loss is nearly correct. However, for the double glazing, the glass and partition are warmer, but the underprediction of the radiation is not as great as the over prediction of convection, and the total loss is overpredicted. With solar gain, the partition is close to, or even above, room temperature, giving radiative losses equal to or above those predicted by a constant combined h. The glass is still quite cool, giving rise to higher convection losses than predicted. The result is a significant underprediction of total losses, especially at high glazing fractions. In all of the above analysis, the exterior wall acts similarly to the glazing, but because of the insulation, the wall temperature is much closer to room temperature than the glazing, and its contribution to the error is much smaller.

The analysis of the summer cases of Figure 3.2.6 is similar. In summer, the interior partitions get very warm and reduce the radiative gains from the outside wall and glass. This is usually a much greater factor than the increased convection at the glazing. Therefore, the total gain tends to be overpredicted significantly by a constant, combined h_i, especially when there is solar gain to the room.

In light of these findings, it was decided to separate convection and infrared radiation at interior surfaces. This can be handled fairly easily using the equivalent temperature concepts of the next section. The use of temperature-dependent heat transfer coefficients adds considerable expense because of the necessity of computing the inverse of the conductance matrix each time the coefficients change, and it would require the more cumbersome wall transfer functions for variable surface conductances. It was decided to use constant user-supplied heat transfer coefficients, which can be chosen depending on the expected temperatures.

3.2.2 Equivalent Temperatures

In addition to convection at the outside surface of an exterior wall, there is also the possibility of absorbtion of solar radiation and exchange of longwave radiation with the sky and surroundings. By using the concept of a sol-air or equivalent temperature, the system can still be modeled with a single surface conductance, and therefore, the transfer function approach can be used.

The concept is that there is an equivalent air temperature such that heat transfer by convection only from air at this temperature to the surface is exactly equal to the sum of all the actual heat transfer to the surface, by convection and radiation. This temperature, when used at an outside surface, is called the sol-air temperature. As an example, the heat transfer at a sunlit wall and an ambient air temperature 80F (26.7C) may be equal to the heat transfer for a shaded wall in ambient air of 100F (37.8C). The sol-air temperature of the sunlit wall is then 100F (37.8C).

The heat balance at the outside surface is

$$g'' = \alpha^* I_t + h_o (T_a - T_o) - \varepsilon \Delta R \qquad (3.2.2)$$

This form of the balance is that used by ASHRAE [8]. ΔR is the long wave radiative exchange term as defined by

ASHRAE. It is difficult to estimate for a particular wall since it depends on the sky condition as well as temperatures of the surroundings. ASHRAE recommends using ΔR = 20 Btu/hr - ft^2 (63 W/m^2) for horizontal surfaces and $\Delta R = 0$ for vertical surfaces.

With a sol-air temperature, T_e, the heat flux could be expressed by:

$$q'' = h_o (T_e - T_o) \tag{3.2.3}$$

Solving these two equations,

$$T_e = T_a + \frac{\alpha^* I_t}{h_o} - \frac{\varepsilon \Delta R}{h_o} \tag{3.2.4}$$

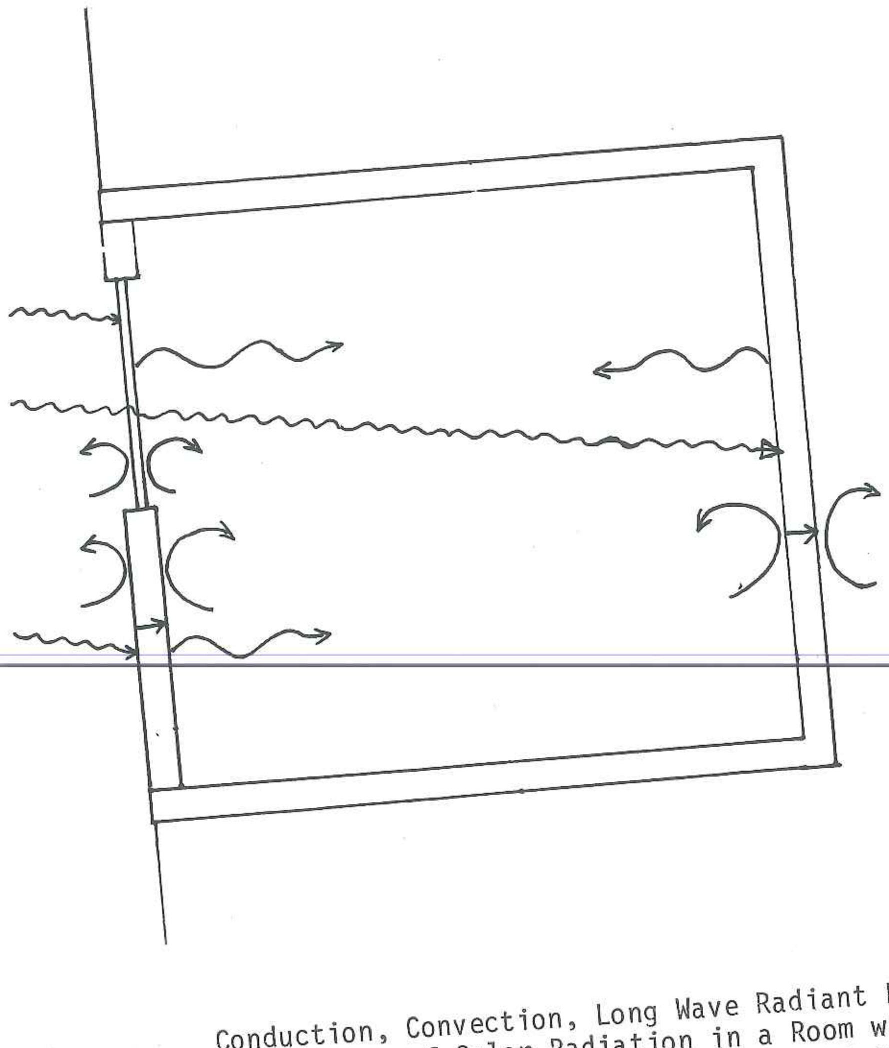
The sol-air temperature is strongly dependent on the value of the outside convection coefficient, which is not known very well.

A similar equivalent temperature can be found for inside surfaces with solar radiation through windows, radiation from lights, and long wave radiation with other interior surfaces. Then, the equivalent temperature for the ith surface is found to be:

$$T_{e,i} = T_r + \frac{s'' + l'' + \sum_{j=1}^{n} h_{r,j} (T_j - T_i)}{h_{c,j}} \tag{3.2.5}$$

where

$$h_r = 4\sigma f_{ij} \bar{T}^3$$



Conduction, Convection, Long Wave Radiant Exchange, and Absorbtion of Solar Radiation in a Room with Exterior Wall, Glazing, and Interior Partition-Ceiling-Floor Element.

It has been assumed that the surfaces are black. This assumption is good for long wave radiation by common construction materials.

For solar radiation through windows and for radiation from lights, the room should act as a black enclosure since very little of any reflected light will exit the room. Deciding how to distribute the solar radiation on the interior surfaces of a room is very difficult. For the 3-element model, the radiation is assumed to be spread uniformly on the interior partition-wall-ceiling surface. In the multi-element model in Appendix C, the user supplies the fraction of the total solar gain for each surface.

The sol-air or equivalent temperature concept gives a convenient method for combining radiation and convection at surfaces allowing use of the convenient transfer function method.

3.3 Derivation of Governing Equations

In this section, the equations which govern the heat transfer in a single room or zone with three elements--exterior wall, glazing, and interior partition-ceiling-floor are developed. Appendix C contains the derivation for a zone with more than three construction elements.

The following heat transfer occurs at the surfaces:

Solar radiation absorbed by outside wall

$$q'' = \alpha_w^* I_t \tag{3.3.1}$$

Solar radiation absorbed by glazing

$$q'' = \alpha_g^* I_t \tag{3.3.2}$$

Solar radiation transmitted by glazing and absorbed inside (f is the fraction of element to total exterior areas)

$$q'' = \tau_g I_t f_g/f_p \tag{3.3.3}$$

Convection at outside wall

$$q'' = h_{c,w,o} (T_a - T_{w,o}) \tag{3.3.4}$$

Convection at outside glazing surface

$$q'' = h_{c,g,o} (T_a - T_{g,o}) \tag{3.3.5}$$

Convection at inside wall

$$q'' = h_{c,w,i} (T_{w,i} - T_r) \tag{3.3.6}$$

Convection at inside glazing surface

$$q'' = h_{c,g,i} (T_{g,i} - T_r) \tag{3.3.7}$$

Convection at partition-ceiling-floor surface

$$q'' = h_{c,p} (T_p - T_r) \tag{3.3.8}$$

Net radiant exchange between glazing and p-c-f

$$q'' = h_{r,g-p} (T_p - T_{g,i}) \tag{3.3.9}$$

Net radiant exchange between wall and p-c-f

$$q'' = h_{r,w-p} (T_p - T_{w,i}) \tag{3.3.10}$$

Conduction through glazing

$$q'' = (T_{g,o} - T_{g,i}) / R_g \tag{3.3.11}$$

Conduction and storage in the wall and p-c-f are represented by transfer functions. By defining the following sol-air and equivalent temperatures, heat transfer modes at the surface can be combined to allow use of the transfer functions.

$$T_{e,w,o} = T_a + \alpha_w^* I_t / h_{o,w} \tag{3.3.12}$$

$$T_{e,g,o} = T_a + \alpha^* I_t / h_{o,w} \tag{3.3.13}$$

$$T_{e,p} = T_r + \frac{f_g h_{r,g-p}(T_g - T_p) + f_w h_{r,w-p}(T_w - T_p) + I_t \tau_g f_g}{f_p h_{c,p}} \tag{3.3.14}$$

$$T_{e,w,i} = T_r + \frac{h_{r,w-p} (T_p - T_{w,i})}{h_{c,w,i}} \tag{3.3.15}$$

If the conductance of the glazing from the sol-air temperature node to the inside surface temperature node is used,

$$U_g = \frac{1}{R_g + 1/h_{c,g,o}} \qquad (3.3.16)$$

the energy balance on the glazing is

$$h_{c,g,i}\, T_r + h_{r,g\text{-}p}\, T_p - (U_g + h_{c,g,i} + h_{r,g\text{-}p})\, T_g = -U_g\, T_{e,g,o} \qquad (3.3.17)$$

The transfer function for the heat flux at the inside surface of the wall is

$$q''^{0}_{w} = \sum_{n=0} b^n_w\, T^n_{e,w,o} - \sum_{n=0} c^n_w\, T^n_{e,w,i} - \sum_{n=1} d^n_w\, q''^{n}_{w} \qquad (3.3.18)$$

Combining Equations 3.3.18 and 3.3.15, and rearranging,

$$q''^{0}_{w} = b^0_w\, T^0_{e,w,o} - c^0_w\, (T_r + \frac{h_{r,w\text{-}p}}{h_{c,w}}\, (T_p - T_{w,i}) + P_w \qquad (3.3.19)$$

where P_w is the contribution to the flux from previous time steps

$$P_w = \sum_{n=1} b^n_w\, T^n_{e,w,o} - \sum_{n=1} c^n_w\, T^n_{e,w,i}$$

The flux at the inside surface is also given by

$$q''^{0}_{w} = h_{c,w,i}\, (T_{w,i} - T_r) + h_{r,w\text{-}p}\, (T_{w,i} - T_p) \qquad (3.3.20)$$

Combining Equations 3.3.19 and 3.3.20

$$(h_{c,w,i} - c^0_w)\ T_r + h_{r,w-p}\ (1 - c^0_w/h_{c,w})\ T_p$$

$$+ \left[h_{r,w-p}\ (c^0_w/h_{c,w,i} - 1) - h_{c,w,i}\right] T_w \qquad (3.3.21)$$

$$= -\ (b^0_w\ T^0_{e,w,o} + P_w)$$

For the partition-ceiling-floor, the transfer function is

$$q''^0_p = \sum_{n=o} (b^n_p - c^n_p)\ T^n_{e,p} - \sum_{n=1} d^n_p\ q''^n_p \qquad (3.3.22)$$

This form assumes the temperatures are the same on both sides, or, in other words, the construction is adiabatic at the centerline. It is also possible to use a known other side temperature very easily, though it is not done here. Combining Equations 3.3.22 and 3.3.14, and again separating out the contribution from previous time steps, the flux is

$$q''^0_p = (b^0_p - c^0_p) \cdot \left[T_r + P_p - \frac{f_g h_{r,g-p}(T_p - T_g) + f_w h_{r,w-p}(T_p - T_w) + I_t f_g \tau_g}{f_p h_{c,p}} \right] \qquad (3.3.23)$$

The flux at the surface is also given by

$$q''^{0}_{p} = h_{c,p}\,(T_p - T_r) + h_{r,w-p}\,(T_p - T_w)\,f_w/f_p$$

$$+ h_{r,g-p}\,(T_p - T_g)\,f_g/f_p + I_t f_g \tau_g / f_p \qquad (3.3.24)$$

Combining Equations 3.3.23 and 3.3.24

$$\left(1 + \frac{b_p^0 - c_p^0}{h_{c,p}}\right)\left(\frac{h_{r,g-p} f_g}{f_p}\right) T_g$$

$$+ \left(1 + \frac{b_p^0 - c_p^0}{h_{c,p}}\right)\left(\frac{h_{r,w-p} f_w}{f_p}\right) T_w + (b_p^0 - c_p^0 + h_{c,p})\,T_r$$

$$- \left[h_{c,p} + \left(\frac{f_g\,h_{r,g-p} + f_w\,h_{r,w-p}}{F_p}\right)\left(1 + \frac{b_p^0 - c_p^0}{h_{c,p}}\right)\right] T_p$$

$$= - P_p - \left(1 + \frac{b_p^0 - c_p^0}{h_{c,p}}\right)\left(\frac{f_g\,\tau_g\,I_t}{f_p}\right)$$

There are two possible solutions for the room air temperature. During periods of heating or cooling, it is assumed the room temperature is fixed at the setpoint and the correct amount of energy is added or removed from the room by the system to maintain the setpoint. In this case, the net energy transferred to the room air is calculated. This type of control will be called energy rate control. The thermostat control is discussed in the next section. If the room temperature is within the accepted comfort zone, or in the thermostat deadband, there is no auxiliary heating or cooling, and the room temperature "floats" within the allowed range depending on the loads on the room.

For the first solution,

$$T_r = T_{setpoint} \tag{3.3.26}$$

For the second case, since the room air has negligible thermal capacitance, the sum of the convective exchanges between all interior surfaces and the room air must equal zero.

$$f_p h_{c,p} T_p + f_w h_{c,w,i} T_{w,i} + f_g h_{c,g,i} T_g - (f_w h_{c,w,i} + f_g h_{c,g,i} + f_p h_{c,p}) T_r = 0 \tag{3.3.27}$$

Equations 3.3.17, 3.3.21, 3.3.25, and 3.3.26 or 3.3.27 form a system of four equations relating the three surface temperatures and the room air temperature, and can be expressed as a matrix equation.

$$[A] \times (T) = (B) \tag{3.3.28}$$

or, solving for the temperature vector,

$$(T) = [A]^{-1} \times (B) \tag{3.3.29}$$

Note that the A matrix contains only heat transfer coefficients and geometrical information, and therefore the inverse is only calculated once. There are two inverse matrices--one for the case of a constant room temperature during heating and cooling, and one for the case of a floating room temperature. The B vector includes the current sol-air temperatures and the P terms containing the historical contributions in the transfer functions, all of which need to be updated at each time step.

The simulation program must decide at each time step in which mode the room is operating. This control is described in the next section.

The thermal state of the room can thus be represented by a fairly simple system of equations by using the transfer function approach, even though the convection, long wave radiation, and solar gains are handled separately, rather than being lumped by combined heat transfer coefficients. This leads to greater accuracy with minimal additional computing cost.

3.4 System Control

3.4.1 Thermostat with Deadband

The room model outlined in the last section uses energy rate control. It is assumed that the HVAC system always exactly meets the instantaneous heating or cooling load unless the room temperature is within a deadband between the low and high thermostat setpoints. Otherwise, the room temperature floats within the deadband to satisfy a heat balance on the room air.

The procedure is to calculate the new room temperature with the floating mode matrix. If the room is within the deadband, no energy addition or removal is necessary and there is no load. Surface temperatures are then calculated so that equivalent temperatures can be calculated for use in the transfer functions during future time steps. If the new room temperature is greater than the cooling setpoint, the system is in the cooling mode, the room temperature is set to the setpoint, and the surface temperatures are calculated using the fixed temperature mode matrix. The load is then calculated by summing the convection from all the interior surfaces. New equivalent temperatures are calculated for future use. Similarly, if the calculated room temperature is below the heating setpoint, the room temperature is set, and temperatures and the load are calculated as in the cooling case.

3.4.2 "Free" Cooling

If the model calculates a cooling load, it will attempt to meet any or all of the load by positive ventilation of outside air, so-called free cooling. The possible reduction in the cooling load is:

$$\dot{Q}_{free\ cool} = (\rho c_p)_{air}\ \dot{V}\ (T_a - T_r) \qquad (3.4.1)$$

While theoretically any load could be handled if there is a temperature difference, actually it is limited by the supply air fan. One of the system parameters is the maximum volumetric flow rate for the ventilation system. It is assumed that at partial free cooling loads, the fan is cycled or modulated to provide the correct volume of outside air.

3.5 Weather Data, Subroutines, and Cost

The model calculates hourly loads and temperatures using SOLMET TMY (Typical Meteorological Year) data [10]. Solar irradiation on the vertical surface is calculated in a separate subroutine. This requires the azimuth angle of the wall, the month, day, and hour, total horizontal and normal beam radiation, and a flag indicating snowcover for use in the reflected solar calculation. Output of hourly loads and temperatures and/or periodic summaries is possible. The matrix inverion routine uses Gauss-Jordan Elimination.

An hourly simulation for one year requires about 10 seconds CPU time on a UNIVAC 1100, and costs less than $1.00

3.6 Example

The hourly heating loads per square foot of exterior area and the room temperatures for a south room in Madison on a typical sunny March day are shown in Figure 3.6.1. The building has a brick and block exterior wall with 10% double glazing. The interior is 4 inch (.10 m) poured concrete. The thermostat setpoints are 67F and 73F (19.4C and 22.8C).

The monthly summary showed a total heating load for March of 888 Btu/ft^2 (10.1 MJ/m^2) with a maximum hourly load of 7.54 Btu/hr-ft^2 (85.7 KJ/hr-m^2). There was a small cooling load of 11 Btu/ft^2 (125 KJ/m^2) which was met by outside air ventilation. No lights were included in the model.

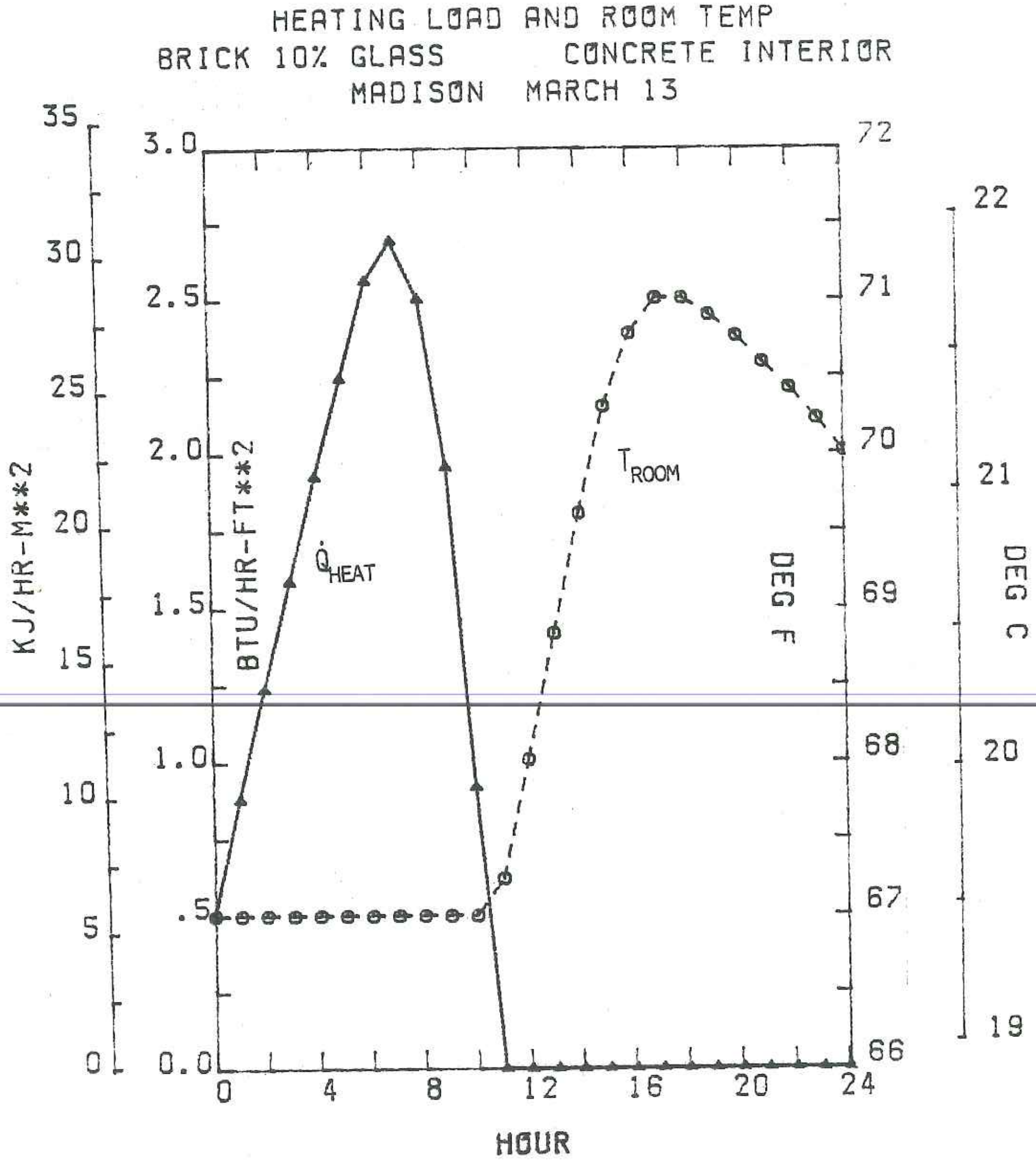


Figure 3.6.1 Hourly Heating Load and Room Temperature Calculated by the 3-Element Room Model.

Such hourly information and monthly summaries can be compared for cases in which various parameters are changed. One of the projects left undone was a factorial design study of the effect of the various heat transfer coefficients on the loads for different construction types and glazing fractions. Such a study would help answer the question of how much accuracy is required in knowing the real values of the heat transfer coefficients before it is meaningful to even include variable heat transfer coefficients in a model.

3.7 Conclusions

The model outlined in this chapter provides a convenient, fast, and inexpensive solution of the thermal state of a simplified room. While the geometry of the room has been oversimplified by the assumption of a single partition-ceiling-floor element, the physics of the heat transfer mechanisms is fairly detailed by use of separate long wave radiation calculations, the use of equivalent temperatures to include solar and lighting radiation, and the use of transfer functions to represent the transient conduction in the exterior wall and the p-c-f. However, this model, or one for more surfaces based on the equations in Appendix C, can be used as a good tool for investigating the sensitivity of heat transfer co-efficients at different surfaces, the effect of the in-terior construction, glazing and insulation combinations, or as used in the next chapter, to determine whether comprehensive room transfer functions can be obtained.

4.0 ROOM TRANSFER FUNCTIONS

4.1 Introduction

The concept of a Comprehensive Room Transfer Function (CRTF), a single function correlating heating/cooling loads with sol-air temperatures, solar gains, lighting and occupancy gains and room temperature will be proposed and tested. This function can be used with logic representing a deadband thermostat control to calculate hourly HVAC equipment loads and room temperatures. After outlining the current room transfer functions given by ASHRAE, the comprehensive room transfer function will be defined. Methods of selection of data used to obtain the function are explored. Finally the comprehensive room transfer function is tested, first for the room model of the last chapter, then for a multi-zone building modeled with TRNSYS.

4.2 The ASHRAE Room Transfer Function Method

The ASHRAE Handbook [5] gives a method for obtaining hourly cooling loads using room transfer functions. The method assumes a constant room temperature in calculating the load. If the temperature is not constant, the calculated load is modified by another transfer function.

Conduction loads are first calculated for each wall, roof, and floor using the constant room temperature conduction transfer functions (See Equations 2.5.5 and 2.5.6). These calculations are done at each time step. Also, the heat gains to the space from lights, solar radiation, equipment, and people are determined. Because of the thermal capacitance of the walls, floors, etc., and of the furnishings, only part of the energy gains are immediately transferred to the room air. Some of the energy will be stored in the mass, and will be released at a later time. The Room Transfer Function, Equation 4.2.1, gives the time distribution of the release of the energy for the different types of gains.

$$\dot{Q}_\tau = \sum_{i=o} v_i \, q_{\tau-i\Delta} + \sum_{j=1} w_i \, \dot{Q}_{\tau-i\Delta} \qquad (4.2.1)$$

There are separate sets of v coefficients for solar gains, for conduction gains, for lighting gains, etc. Again, by using previous values of the output (cooling load), the number of previous values required is greatly reduced. The v and w coefficients are tabulated in ASHRAE [5] for light, medium, and heavy construction with a very limited choice of lighting-ventilation-furnishings combination.

If the room temperature is not constant, then information about the terminal unit, operation schedules, and total conductance of the space to the surroundings must be combined with normalized coefficients of a room air transfer function given for light, medium, or heavy construction which relates room temperature, energy extraction rate, and the constant room temperature cooling load, to give the room temperature.

The ASHRAE method is intended for use in cooling situations. It requires extensive calculations for each timestep, using three levels of transfer functions. Finally, the user must decide which set of functions most nearly represents his building. However, the method is a fast inexpensive alternative to the very detailed hourly simulation programs available.

4.3 The Comprehensive Room Transfer Function

In the ASHRAE method, three separate levels of transfer functions are required to arrive at the final load and temperature of a room, and the use is restricted to the configurations tabulated. In this section, a single comprehensive transfer function will be proposed. This transfer function allows calculation of heating or cooling loads, or room temperature, and can incorporate different heating and cooling setpoints. Energy rate control, in which the equipment exactly meets the load at all times, is used here. However, the technique, with some modification, should be applicable in temperature level control.

Assume that the heating or cooling load, Q_τ, can be represented as:

$$\dot{Q}_\tau = \sum_{i=o} (a_i T_{s-a,\tau-i\Delta} + b_i T_{r,\tau-i\Delta} + c_i q_{s,\tau-i\Delta} + d_i q_{1,\tau-i\Delta} + e_i q_{p,\tau-i\Delta}) + \sum_{j=1} f_j \dot{Q}_{\tau-j\Delta} + q_{inst} \qquad (4.3.1)$$

There are two major differences between this equation and Equation 4.2.1. First, conduction gains do not appear explicitly; rather sol-air temperatures are used in conjunction with room temperatures. The question of evaluating sol-air temperatures is addressed later. Second, the room temperatures appear within the transfer function. Note also that any additional instantaneous heat gains, such as heat convected from equipment or people, are included within the transfer function.

If such a transfer function can be found, then by combining it with a control logic which determines whether the system is in the heat, cool, or off mode, then the load can be found when equipment is on and the room temperature is at a setpoint, or the room temperature can be found if the equipment is off and there is no load. In the latter case, the current load, $\dot{Q}_\tau$, is set to zero, and Equation 4.3.1 is rearranged to solve for the unknown current room temperature, $T_{r,\tau}$. Thus two different outputs can be determined from a single CRTF depending on whether the current room temperature or the current load is fixed and known.

The CRTF can perhaps be determined directly in a manner similar to that for wall transfer functions. However, it seems easier to assume a form, and then try to fit data from a detailed simulation over a limited time with a linear regression. Presumably, this method could also be used on measured data from an existing building to obtain the transfer function.

There seem to be two distinct advantages of this method. First, there are many fewer calculations once the coefficients are determined. Second, the user can find a transfer function customized for his building, rather than using a generic transfer function which may not apply to his case. The drawback is that he needs data for the linear regression, which means some limited simulation must be made. The following sections investigate methods of finding and using the comprehensive room transfer function.

4.4 Obtaining Data for the Regression

To obtain a fit which can accurately predict the loads and temperatures, there must be a representative distribution of data over the range for which the function is to be valid. Otherwise, there can be poor predictions near the "edges" and in regions where the data used in the regression are sparse.

Figure 4.4.1 shows a hypothetical load curve versus outside temperature for an exterior wall with no capacitance and no internal loads. In a real building, the load lags the heat gains and depends on the magnitude of several heat gains, not just conduction, and the load curve cannot be represented in a plane. At low outside temperatures, heating is required, and the room temperature is held at the heating setpoint by the heating plant. As the temperature rises, the heating load decreases toward zero. At this point, the heating equipment is off and the room temperature begins to rise through the deadband until it reaches the cooling setpoint. Then, cooling begins, and the load rises with outside temperature.

A set of data from a simulation over these conditions can then be fit to obtain a transfer function relating load, outside temperature, and room temperature.

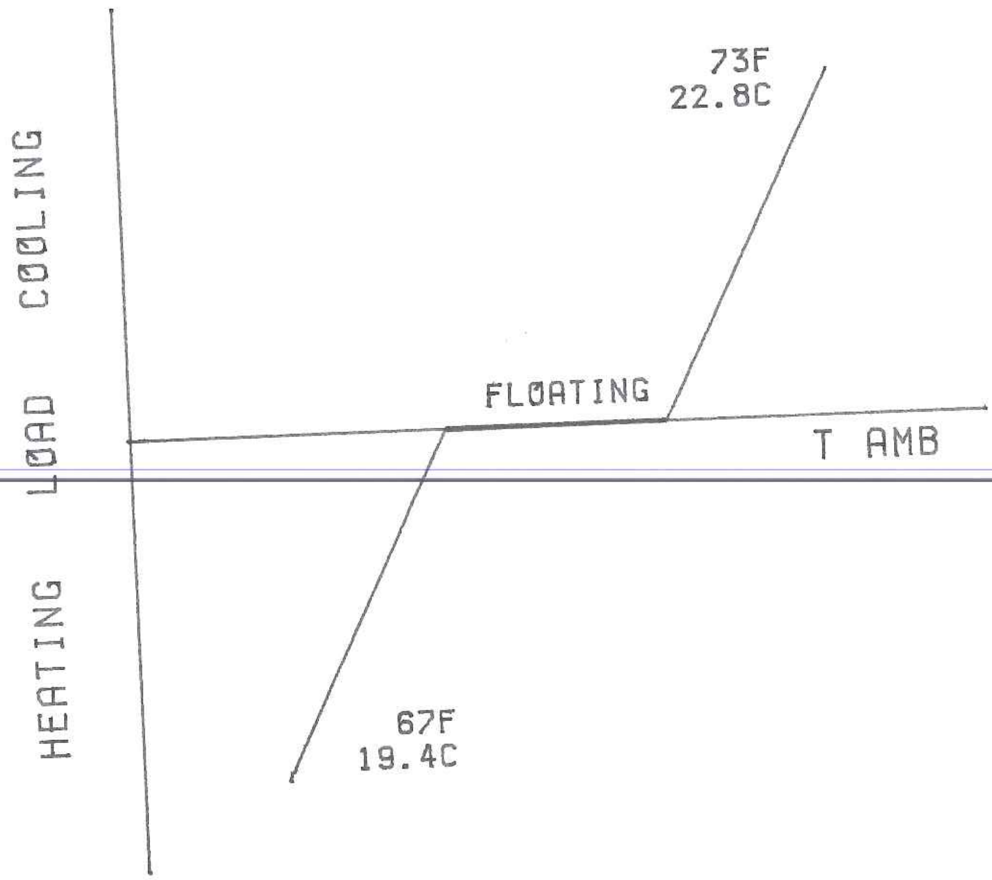


Figure 4.4.1 Typical Load Versus Ambient Temperature For a Room Without Capacitance With Deadband Thermostat.

If the system is truly linear, and if sufficient data are used in the regression, the resulting CRTF should be applicable to any deadband. However, a different approach using less data can be used to improve the accuracy of the CRTF when applied to different deadbands.

An alternative is to do a few different simulations for single setpoints, i.e., the heating setpoint equals the cooling setpoint and the room temperature is constant. For example, run the simulation for setpoints of 65F, 70F, and 75F (18.3C, 21.1C, and 23.9C). Next, switch off the heating/cooling, and do the simulation for a range of conditions to obtain the floating room temperature response to these conditions. Data for this simple case might look like that in Figure 4.4.2. Then use all four sets of data to obtain a single fit. Such a spread of data will then allow use of any deadband, as long as it is within the region covered by the data set.

A final way of obtaining a data set for a regression fit is to use measured data from an existing building during normal operation, or during an experimental operation done along the lines of the case above. Again, a representative distribution over the range of the fit is desirable. Unfortunately, a real data base was unavailable for this project.

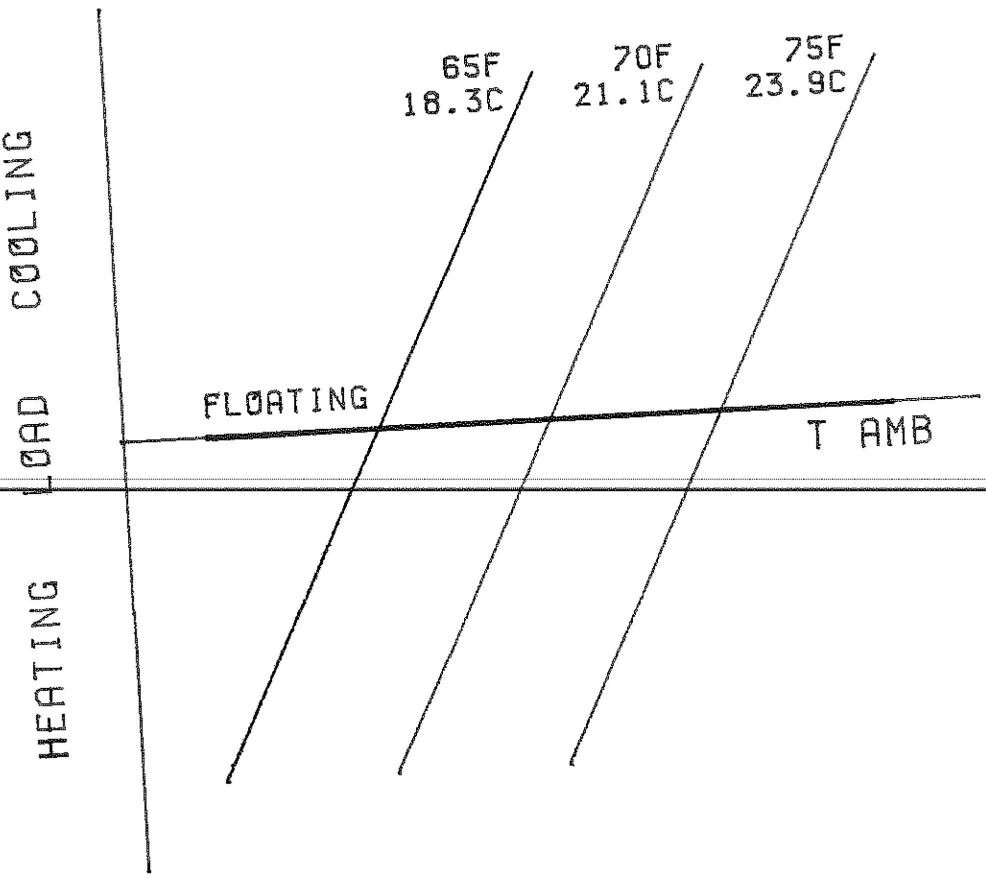


Figure 4.4.2 Typical Load Versus Ambient Temperature Curves for Different Room Temperature Setpoints and for Floating Room Temperature.

Two different types of simulation methods were used to investigate the CRTF, the model outlined in Chapter 3 and extended in Appendix C, and a TRNSYS standard component model based in part on the ASHRAE transfer functions.

4.5 Transfer Functions from the 3-Element Room Model

The 3-element room model outlined in Chapter 3 provides a simulation tool which accounts for conduction, solar and/or lighting gains, long wave radiant exchange, the capacitance of the mass of the building, and a dead band thermostat. It has sufficient detail, therefore, to test the CRTF method for a simulation model.

A transfer function of the form:

$$\dot{Q}_\tau = \sum_{i=o}^{n} (a_i T_{s-a,\tau-i\Delta} + b_i T_{r,\tau-i\Delta} + c_i q_{solar,\tau-i\Delta}) + \sum_{j=1}^{n} f_j \dot{Q}_{\tau-j\Delta} \qquad (4.5.1)$$

was assumed. The model was run for a particular construction for various days and thermostat settings. The driving weather data was Madison TMY data. The heating/cooling load was then correlated with the previous loads, the current and previous sol-air temperatures, room temperatures, and solar heat gains.

Both approaches described in the last section were used for creating data to include in the linear regression. First, a few spring days with the potential of requiring both heating and cooling were chosen. The loads were simulated for single setpoints (no deadband) of 65F, 70F, and 75F. Then a few days scattered about the year were simulated to obtain the data for the floating temperature mode. Again, the goal is to have representative coverage of the operating range of room temperature. Second, several days of simulated loads and temperatures were obtained for a specific deadband. Days in winter, spring, and summer were used in the regression.

A linear regression was then done for different cases using MINITAB, a statistical package from Penn State University [13]. The cases used combinations of number and type of days, and number of previous time steps. Table 3.4.1 shows results for four typical cases. Case I used six days of data, three days at setpoints of 65F, 70F, and 75F (18.3C, 21.1C, and 23.9C), and three days of floating temperatures. In Case II, 5 additional days of data were added to Case I to fill in gaps. In both I and II, the current and five previous hours of data were used in the regression. Case III used the same

Table 3.4.1 Comparison of Errors in Load Prediction by CRTF's for Combinations of Regression Data and Inputs

Case	Number of Days in Regression	Number of Previous Timesteps	Average Weekly % RMS Error*	Largest Weekly % RMS Error*
I	6	5	2.0%	8.3%
II	11	5	0.4%	1.8%
III	11	3	0.4%	2.2%
IV	21	3	0.3%	1.6%

*% RMS error based on RMS difference between loads by Model and CRTF for all hours of heating or cooling in a week, divided by the mean heating or cooking load for these hours as calculated by the model.

data as II, except only the current and 3 previous hours of data were used in the regression. Case IV used a week each of winter, spring, and summer data with a specific deadband of 67F to 73F (19.4C to 22.8C), and used the current and 3 previous hours of data in the regression. The percentage RMS (root mean square) errors and the mean bias errors between the model and the transfer functions were calculated for all hours in a week over the whole year to determine whether there was any seasonal dependence in the fits. The third column of Figure 3.4.1 shows the mean of the 52 weekly RMS errors. The last column shows the highest weekly RMS error for the whole year.

Comparison of Cases I and II shows the importance of getting a good distribution of data over the operating conditions. Comparison between Cases II and III shows that reduction of the number of previous hours of data included in the transfer function may not significantly increase the errors. The number of terms required depends on the construction. For these cases, the number of calculations for each timestep can be reduced by over 25% with little loss of accuracy. The regression analysis has to be studied to determine whether the number of terms can probably be reduced. One indicator is the ratio of each coefficient to its standard deviation, the T-ratio. A T-ratio which is significantly smaller than the others probably indicates the term is not very significant and can be removed from the regression. The current and three previous hours of inputs seems to be a good place to begin.

Case IV shows that using data from a simulation for the same deadband for which the CRTF is to be used gives the best fit, but only by a small margin. However, the resulting function is very specific, and cannot be applied as accurately for a different deadband, whereas Case III, for example, can be used for different deadbands, or even time of day thermostat setbacks with little sacrifice in accuracy.

All of the above cases were done using an exterior frame wall with R=7 hr-ft^2-F/Btu (1.23 m^2-K/W) insulation and 20% double glazing. The interior partition-floor-ceiling was 4 in (0.1 m) poured concrete. While this is not a conventional construction, it mixes light and heavy building materials, and therefore introduces different "time constants" into the system, thereby testing the method more severely.

Figure 4.5.1 shows a comparison between the heating loads calculated by the room model and the room transfer

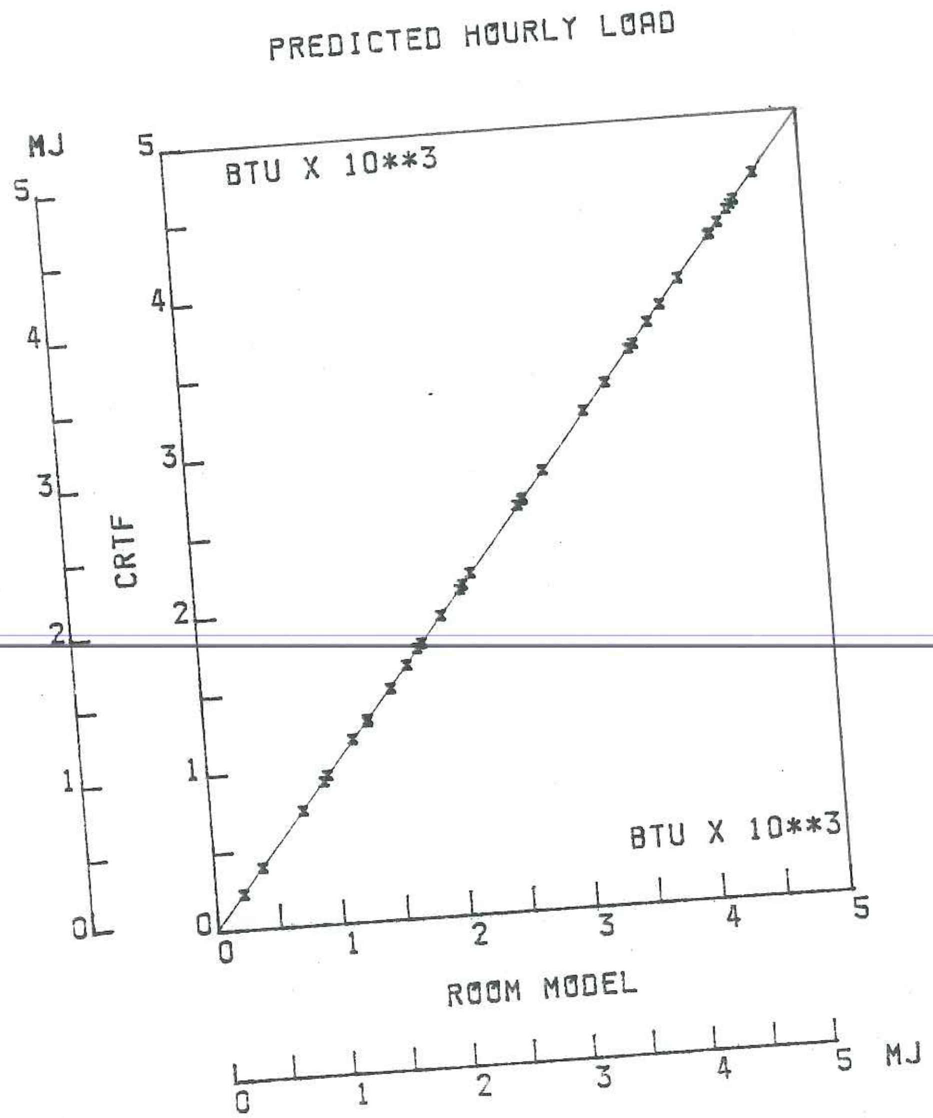


Figure 4.5.1 Hourly Load Comparisons Between the Room Model and the CRTF for March 8-12 in Madison.

function for Case IV for March 8-12. This was a week with a percentage RMS error of 0.93%, three times the average for the year. Figure 4.5.2 shows the similar comparison for the room temperature during this period when the heat was off and the room temperature was floating in the deadband.

The CRTF is very successful in predicting loads and temperatures for the 3-element room. The accuracy obtained by using a specific deadband in the regression data is slightly better, but the transfer function loses generality.

4.6 Transfer Functions from a TRNSYS Building Model

TRNSYS, the transient simulation program from the Solar Energy Laboratory of the University of Wisconsin [11], contains separate components which can be coupled together to model a multi-zone building. The main components in such a model are the Type 17 Wall or Flat Roof, and the Type 19 Room. Each perimeter zone consists of one or more Type 17 exterior components which calculate conduction and transmitted solar gains coupled along with other inputs, such as lights and occupancy, with the Type 19 Room. Type 19 uses the ASHRAE room transfer functions for constant room temperature (Sec-

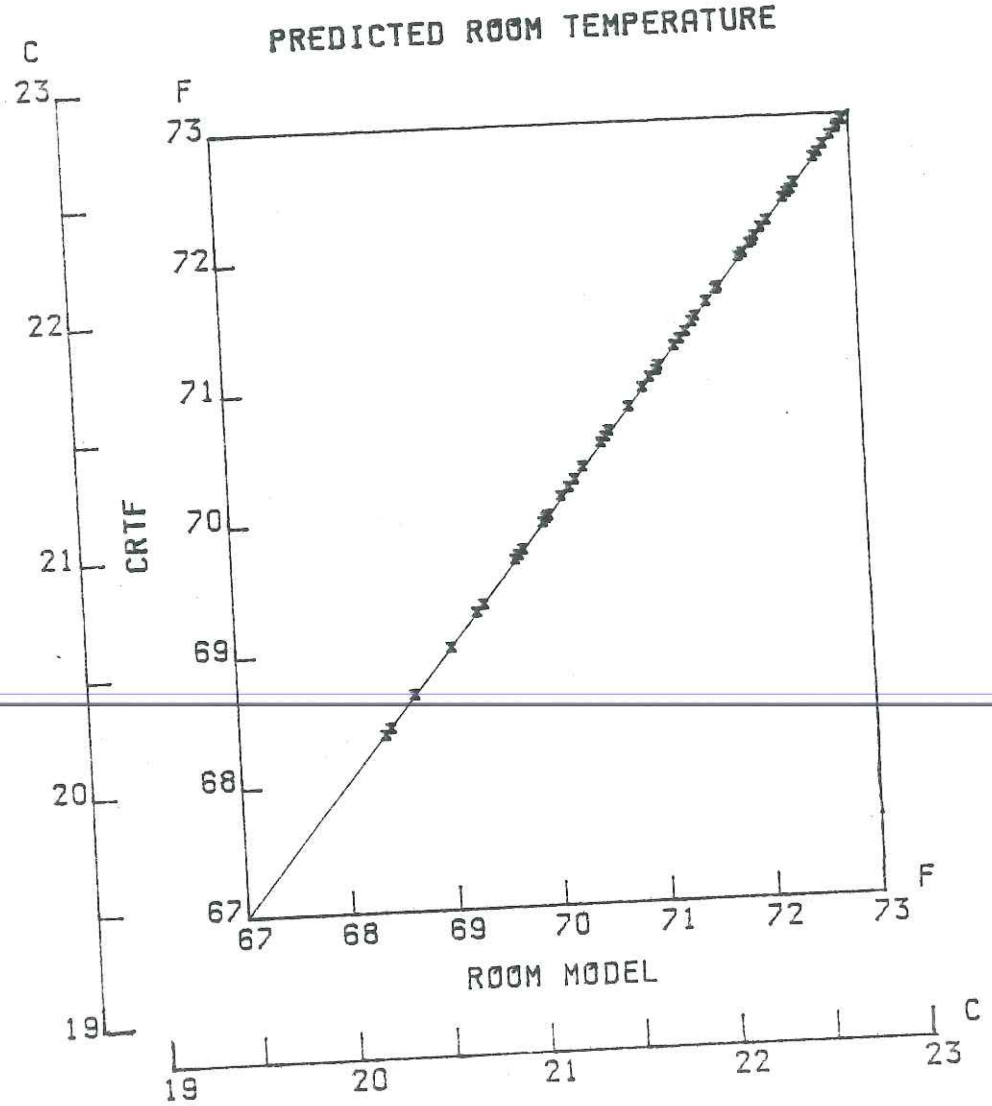


Figure 4.5.2 Room Temperature Comparisons Between the Room Model and the CRTF for March 8-12 in Madison.

tion 4.2) to distribute the gains in time in order to compute the current load. In the Type 19 energy rate control mode, the room temperature floats within a dead-band between setpoints during periods when no heating or cooling is required. The room temperature is determined by the solution of the differential equation for a single lumped room capacitance, a user-supplied parameter. If the mode changes between heating or cooling and floating temperature during a time step, Type 19 splits the time step into two parts, makes the appropriate calculations for each, and outputs the load and average room temperature for the timestep. During time steps when the mode changes, a nonlinearity is introduced by the two part solution. It is not clear what a single node room capacitance should encompass. It is used to determine the room air temperature, but if all the mass of the building is included in the capacitance, then the mass is also at the same temperature as the room air, while in a real building the mass temperature will usually lag or lead the room temperature. The question arises whether the exterior wall and roof mass should be included in the room mass at all. The wall transfer function includes the effect of storage of energy conducted between room air and the outside, but the walls can also store energy radiated from lights and occupants, or solar radiation reflected back to the exterior wall. The TRNSYS components were used in a building model to test the CRTF method.

A six-zone office building model, originally modeled with TRNSYS by Ottenstein [12], was used. It consists of four perimeter zones, a roof zone, and a core zone. The three story building has 2700 m^2 (29,000 ft^2) of floor area with brick and block exterior walls with 20% double glazing, and a poured concrete roof. Madison TMY data was used for the simulations.

Two modifications of the standard TRNSYS components were used. The original Type 17 Wall uses transfer functions for constant room temperature, while Type 19 allows the room temperature to vary. Type 17 uses the current room temperature as the "constant" room temperature in the transfer function. Horn [8] has shown that this introduces a sharp nonlinearity and can result in significant errors. The standard Type 17 was initially used in the building model; however, when Horn's modified Type 17 using wall transfer for variation room temperature was used instead, there was a significantly better correlation of the data, and the errors in predictions of loads and temperatures by the resulting CRTF were significantly reduced. Therefore, it was used in all subsequent simulations. This requires obtaining the c coefficients directly from the Mitalas-Arsenault program. Second, Type 19 uses the ASHRAE room transfer function coefficients from 1972. These were changed in 1977, and the new coefficients were used in this work. Also, the room temperature output by Type 19 is an average over the timestep, while the final temperature would be more appropriate for use in transfer functions since ramps are assumed between time steps. Therefore, the final temperature was output from Type 17.

Since transfer functions assume linear systems driven by ramp inputs, there may be difficulties using TRNSYS to test the comprehensive room transfer function method. As discussed above, TRNSYS Type 19 introduces nonlinearities at time steps in which the mode changes. The transfer function must assume the system has been in the same mode for the entire time step, and that the inputs and responses are following ramps. Therefore, tests of the CRTF using TRNSYS-generated data may not be an appropriate test of the method. It may, however, indicate how flexible the transfer function approach is when used in situations for which it is not entirely appropriate, and may give some indications of whether the CRTF method will work for real buildings where there is noise in the data, and the inputs will not be smooth ramps.

There seem to be two possible applications of transfer functions for a multi-zone building. If the whole building is being held at a constant temperature, or if only knowledge of the total load of the building on the plant is required, rather than knowledge of the loads and temperatures in individual zones, then a single transfer function for the whole building could perhaps be found. This would necessitate use of some kind of average sol-air and indoor temperature. Such an approach might be useful for use in detailed simulation of the plant. Then the building could be treated as a black box with a transfer function giving loads for input conditions. The second possible application would be to obtain a CRTF for each zone as was done for the 3-element model in the last section. This would be useful for simulations of air handling unit--zone interactions, heat pumping between zones, etc.

The simulated total hourly load for all six zones during July was used to test the use of the building transfer function. All six zones were maintained at a temperature of 25.5C (77.9F). An area-weighted sol-air temperature was used as the representative outside temperature. The current building cooling load was correlated with the three previous hourly cooling loads, the current gains, previous three solar and lighting gains, and sol-air temperatures, and the current occupancy load. The first seven days of July were used in the regression.

Figure 4.6.1 compares TRNSYS with the transfer function for three typical days in mid-July. The agreement is quite good. The RMS error between the transfer function and the TRNSYS simulation for all hours in July is 1.8% of the mean hourly cooling load. This seems to indicate that an area-weighted sol-air temperature can be used in the transfer function, rather than the sum of all the computed conduction loads. However, more research should be done before generalizing.

Next, the temperature and loads in the south zone were used to test the comprehensive room transfer function approach. Hourly TRNSYS simulations were made for the whole year in Madison for a deadband of 20C to 25.5C (68F to 77.9F) for a room capacitance of 40 $KJ/C\text{-}m^2$ (2.0 $Btu/F\text{-}ft^2$). Some limited simulations and tests were also done for a room capacitance of 200 $KJ/C\text{-}m^2$ (9.8 $Btu/F\text{-}ft^2$). The TRNSYS default convergence tolerance

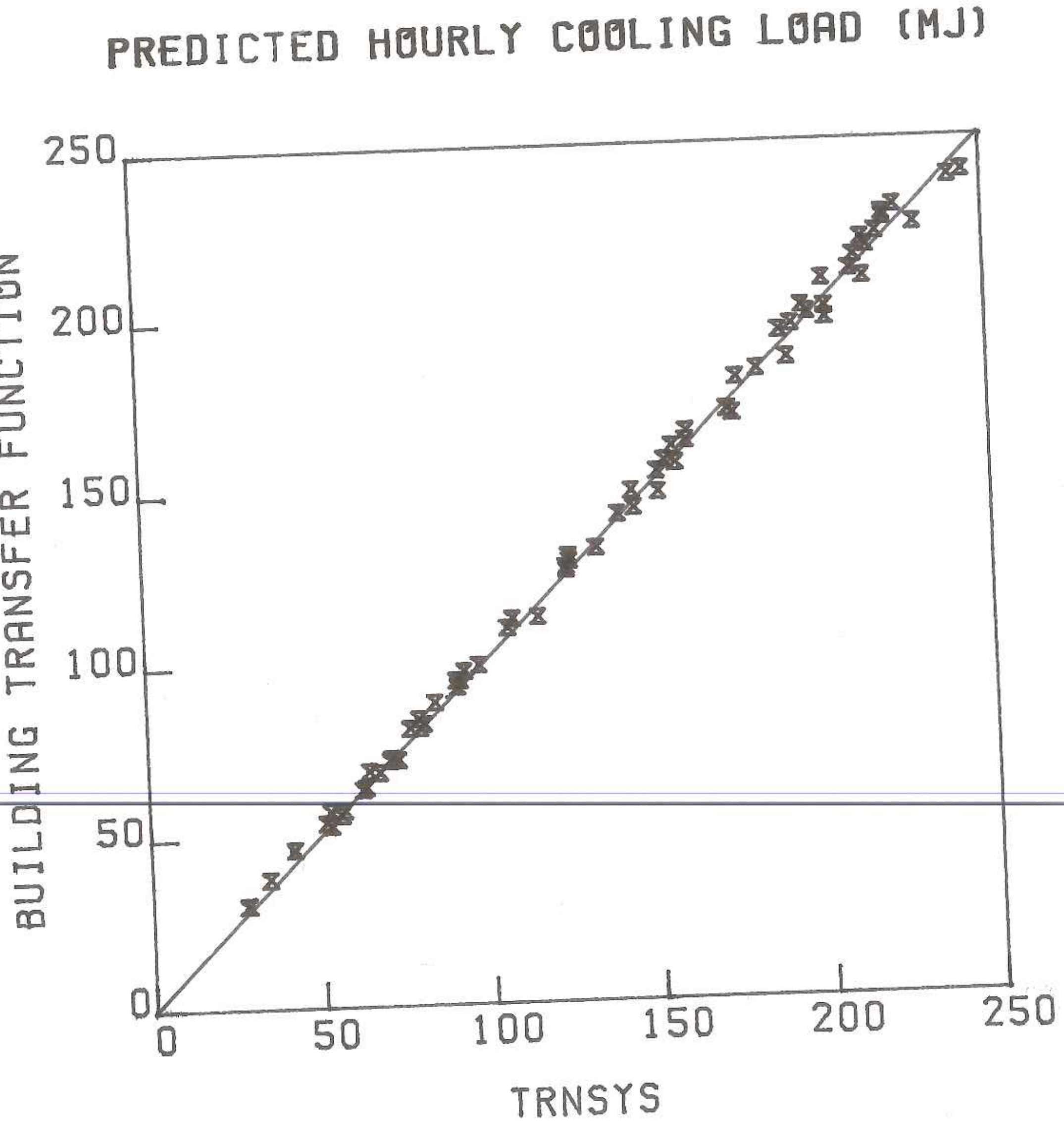


Figure 4.6.1 Comparison of Hourly Building Cooling Loads Between TRNSYS and the Transfer Function for 3 Typical July Days.

of 1% was reduced to 0.1% after it was discovered that significant numerical errors were sometimes occurring in the spring and fall when the zone was alternating between heating, floating, and cooling modes.

Five days each of winter, spring, and summer hourly simulation results were used in the regressions. The TRNSYS simulation loads and room temperatures were then compared with those predicted by the transfer function for every hour of the year. The RMS and bias errors were calculated over all hours in each week to determine whether there was a seasonal error.

The results for the lower room capacitance case were investigated in more detail than those for the higher capacitance. For the CRTF using sol-air temperatures, the results were somewhat disappointing. While the total annual heating load was underpredicted by only 2.6%, the mean of all of the weekly RMS error percentages weighted by the number of hours of heating during the week was 7.2%. For about 48 hours during the year, the errors were over 50%. The largest errors were during spring and fall when the room was switching from heating to deadband to cooling and back again, often in a single day. The average RMS error weighted by the weekly loads, rather than by number of heating hours in the week, would be much smaller since the large errors occurred at times of small loads. For the cooling season, the total annual load was underpredicted by 1.9%, with an hour-weighted average of the cooling weeks RMS error of 5.0%. The fact that the average weekly RMS error percentages are large compared to the errors in the annual totals is because the large errors occurred during periods of low loads.

The loads and temperatures during one of the weeks with the largest errors in predictions is shown in Figure 4.6.2. The RMS errors in the heating load for the week is 35%, while the cooling load error is 24%. Though the errors are large, this week represents only 0.4% of the annual heating load and 0.5% of the annual cooling load. Figure 4.6.3 shows similar comparisons for a spring week with an RMS error in the cooling load of 4 %, near the average for the year, and represents 3.4% of the total annual load. The majority of cooling occurs during weeks when the room temperature rarely leaves the setpoint, and the typical RMS errors for these weeks is on the order of 2.5%.

The regression was done again for the lower capacitance case using the computed conduction loads instead of the sol-air temperatures. For this case, the annual

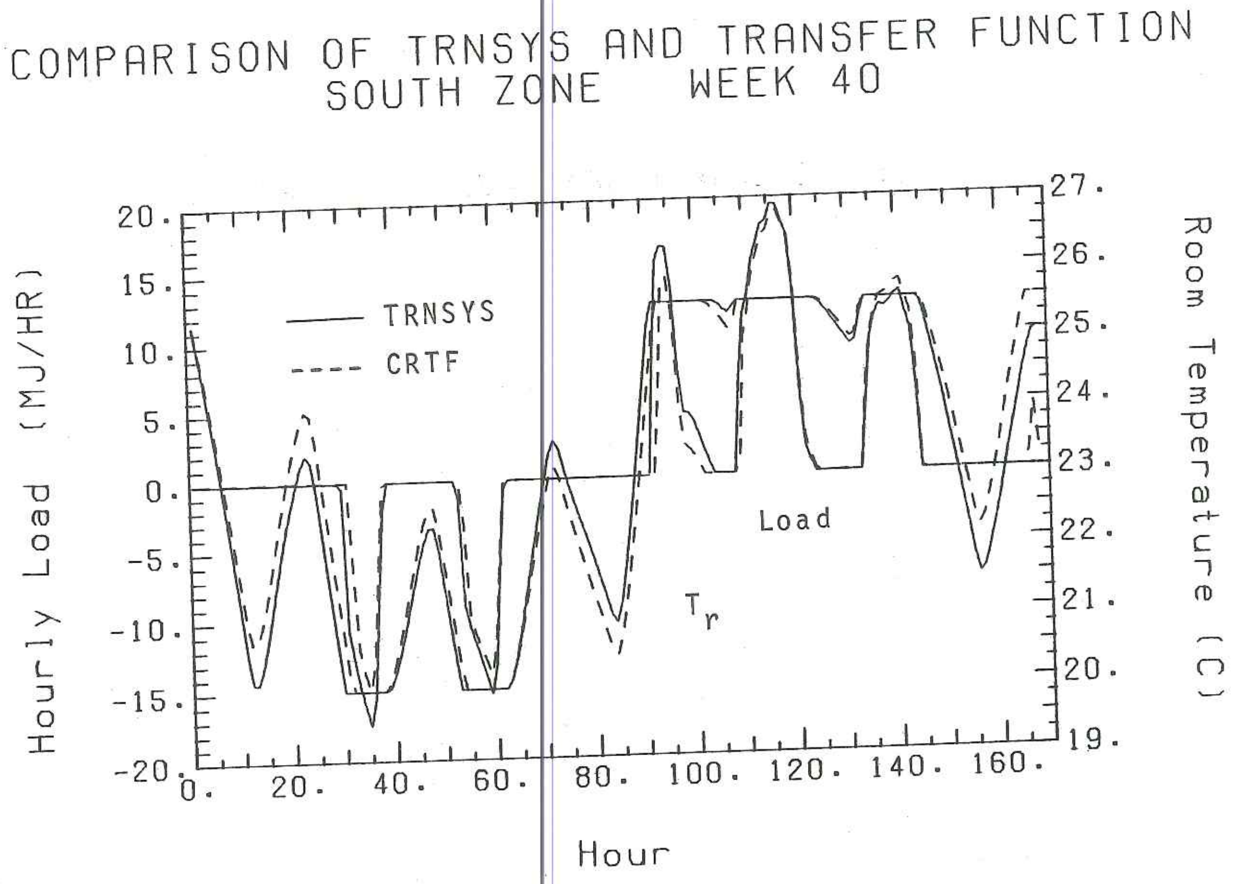


Figure 4.6.2 Comparison of Hourly Loads and Temperatures Between TRNSYS and the CRTF for Week 40.

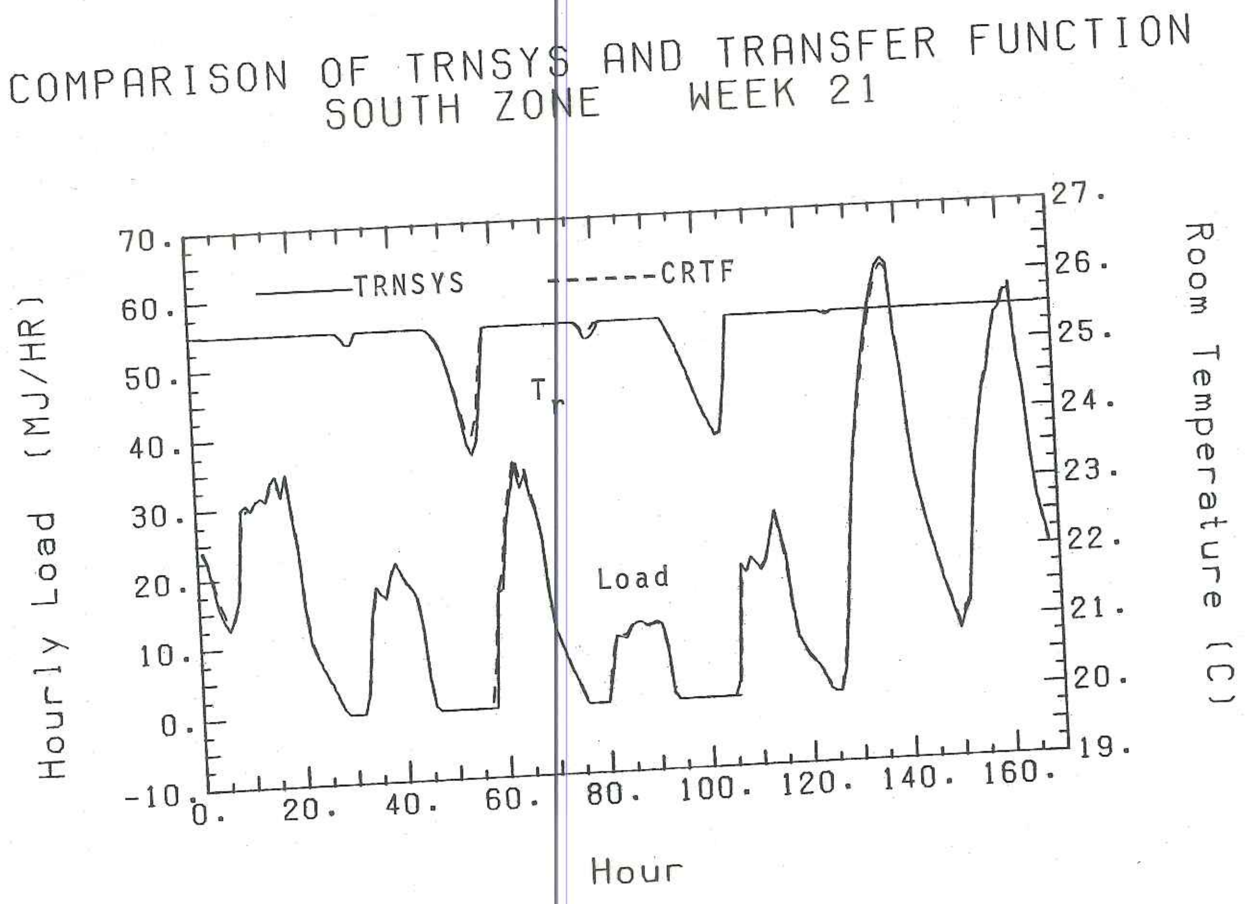


Figure 4.6.3 Comparison of Hourly Loads and Temperatures Between TRNSYS and the CRTF for Week 21.

heating load was off by only 0.4% with an average percent RMS error of 2.1%. There were only two weeks, totaling 10 hours of heating, with errors over 10%. The annual cooling load was off by 0.2% with an average weekly percent RMS error of 1.3%. There were six weeks, totaling 53 hours of cooling, with weekly errors over 10%. The standard deviation about the regression line was reduced by 55% over that of the regression using sol-air temperatures.

The differences in accuracy of these two cases are difficult to explain. It would seem that since conduction gains are calculated using wall transfer functions, the regression should "include" the wall transfer function in some way within the comprehensive room transfer function. One possibility investigated was to include the current ambient temperature in the first regression to perhaps get a better handle on the differences between conduction through the walls and that through the glass. Hopefully, the part of the conduction through glass would be better correlated by the ambient term. However, the standard deviation about the regression line was only reduced by 15%. Another possibility is that the use of the room capacitance with the ASHRAE room transfer function may have a secondary effect on the conduction calculations. Also, Type 17 uses the average room temperature computed by Type 19 in the conduction calculation. This could give rise to problems since it is the final room temperature which is used in the regression. Each of these problems would tend to introduce problems at changes of mode, which are the times when the comprehensive room transfer functions are at their worst for the TRNSYS zone. More research needs to be done on this difference between the regressions using sol-air temperatures and conduction gains.

For the case of very high effective capacitance, an interesting problem arose. Since the changes in room temperatures are much smaller than they are for low capacitance, the regression routine eliminated previous room temperatures from the fit, as they were so close in magnitude that they were insignificant. Since it is the changes in temperature, rather than the absolute temperature that are critical in getting an accurate transfer function, it was decided to subtract off a bias equal to the midpoint of the deadband, from the room temperatures used in the regression. Then differences on the order of tenths of a degree become more significant when the maximum temperature is on the order of degrees rather than tens of degrees. Using the biased room temperature improved standard deviation about the regression line by nearly an order of magnitude.

The detailed analysis of errors done for the low capacitance case was not repeated for the high capacitance case. On the basis of the analysis of the regression, the errors in the fit may be on the order of twice those for the lower capacitance case. However, this capacitance is high, and assumes most of the mass in a heavy building is active in the storage of energy, which is probably not true for the diurnal storage cycles present in spring and fall days when the errors in the fit are large. As discussed above, the question of what is included in the room capacitance is uncertain. While the errors for the comprehensive room transfer function for the high capacitance case are probably significant, real buildings will probably have a much lower room capacitance.

4.7 Modified Room Transfer Function Approaches

Three variations on the comprehensive room transfer function were tried in the course of finding a better fit for the TRNSYS zone model. These methods were tried early in the research efforts before the modifications to the standard TRNSYS components were made, so it is unknown how well they might work with these modifications. They are briefly outlined here as methods which may be looked at again if the comprehensive room transfer function does not work well for actual buildings.

First, the changes in room temperature, rather than the absolute temperatures can be used in the regression. This did not work as well as hoped, probably since there were no absolute temperatures, the predicted values were not "pegged," and bias errors never get corrected until the room temperature reaches a setpoint again. Perhaps including the current room temperature in the regression could help prevent errors from accumulating. One of the problems with transfer functions is that because they use previous values of outputs, future outputs are affected by a poor prediction in a recent hour.

Second, separate regressions can be done for the loads and for the room temperatures, using only data from heating or cooling modes for the first, and only from the floating temperature mode for the second. For all regressions done in this way, the individual regression fits were excellent. The problem arose when changing over from one correlation to the other at changes of mode during simulation of a zone with a deadband. The question is whether it is appropriate to use previous outputs predicted by one correlation as inputs to the other. Doing so led to errors at the mode changes which then were carried on through the simulation for several hours. Apparently, a discontinuity exists between the two transfer functions.

Finally, a method was used which worked very well for the TRNSYS zone, however, it is probably not a fair test with TRNSYS since its method is quite similar to that of TRNSYS Type 19. It may be useful for a real building if the comprehensive room transfer method fails. The heating or cooling load is exactly equal to the energy transferred to the room air in energy rate control. If a transfer function is found for the load at a setpoint, then this transfer function also gives the amount of energy transferred to the room air, or to the room capacitance, during the floating temperature mode. Knowing this energy input and the change in room temperature for each time step, a linear least squares fit can be done to relate the energy transfer with the change in room temperature. The ratio relating these two is then a kind of average linear room capacitance. To use the method, always compute the net energy input to the room using the transfer function, regardless whether the room is in a heating or cooling mode, in which case the energy transferred to the air is the load on the equipment, or in the floating mode, in which case there is no load and the energy is transferred to the linear capacitance. If the room is initially being heated or cooled and the sign of the energy input changes from one time step to the next, then the zone will enter the deadband and the load will be zero. The average linear capacitance is then used with the energy input to the room calculated by the transfer function to give the change in room temperature over the previous step. If the room temperature exceeds a setpoint during a time step, then the mode changes to heating or cooling again, and loads are calculated. This method could perhaps be used in a real building. During periods of heating or cooling when the load equals the net energy transfer to the air, data could be collected to use in a regression to find the transfer function. During times when the temperature is floating, the changes in temperature and the other inputs could be collected. The transfer function derived from the loads then provides an estimate for the energy transfer to the room, and together with the temperature differences, the average linear capacitance could be obtained. This assumes that there really is a single internal capacitance for a real building, which is not apparent at all.

4.8 Conclusions and Recommendations

The comprehensive room transfer function approach worked very successfully in predicting the loads and room temperatures calculated by the 3-element room model developed in Chapter 3. The method also worked well in predicting the total building cooling load for constant space temperature as calculated by a TRNSYS model of a multizone building. The results were not as good in predicting the loads and temperatures calculated by TRNSYS for a single zone with a deadband, however the errors were acceptable for most applications. It is not clear whether the errors are due to some inherent flaw in the comprehensive transfer function, or whether there is a problem arising from the way TRNSYS does certain calculations. The fact that the method works so well for the 3-element room model, which is based directly on the equations governing the heat transfer within the room, seems to support the approach. Measured data from existing buildings should be used to see whether the uncertainties in the data prevent finding an accurate CRTF.

5.0 CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The transfer function method for calculating the transient conduction in walls and roofs is convenient and economical for use in modeling building heating and cooling loads. The limitation of a single surface resistance is easily overcome by use of equivalent air temperatures. The equivalent air temperature idea also allows the radiation and convection heat transfer to be separated, rather than combining them by the assumption of surroundings at air temperature, and therefore improves the accuracy of the model. The limitation of constant surface heat transfer coefficients can also be overcome easily by reformulating the transfer function so that time-varying coefficients can be incorporated; however, since convection coefficients for a particular building are not well known, this improvement requires further justification for inclusion in a model. The simultaneous solution of the equations governing the heat transfer at each surface along with the equation representing the room air provides an exact solution for a particular time step, and is easy to compute. It results in rapid, economical hourly simulations of equipment loads and room temperatures.

The comprehensive transfer function is more convenient to use for a particular building or room, and requires fewer calculations than the present room transfer function methods of ASHRAE. They are not, however, generalized, and data, either from a simulation or an existing building, must first be obtained. The result is a customized function for each building or room, and presumably results in better results than a general transfer function from ASHRAE. The comprehensive room transfer function combines the possibility of heating or cooling load calculations with a thermostatic control which can include a deadband and night setback. It also is more convenient in that the sol-air temperatures can be used directly, rather than requiring separate conduction gains calculations.

The comprehensive room transfer function is very successful in predicting the loads and temperatures calculated by the room model developed in this thesis, and for those calculated by a TRNSYS model of a six zone building operated at a constant building temperature. There are some difficulties in the prediction of loads and temperatures for a TRNSYS zone model with a deadband during weeks when there are frequent changes of mode between heating, floating temperature, and cooling. However, using the conduction loads in the function rather than sol-air temperatures gives very good predictions. The reasons for the difference is not obvious, but they probably arise from the mixing in TRNSYS of the ASHRAE room transfer function methods with a single node room thermal capacitance, and with the ability of TRNSYS to divide up timesteps if a change of mode occurs during the timestep, whereas the transfer function must assume the mode has been the same for the whole timestep and that the inputs are ramps. Even with the errors in the TRNSYS tests, the method seems promising for application to real buildings.

5.2 Recommendations

The following recommendations are made regarding future research and use of TRNSYS room models and the comprehensive room transfer function method:

1. TRNSYS Type 19 should include the option of using the ASHRAE room air transfer functions rather than the single room capacitance. This will enable a user to calculate loads and temperature using purely ASHRAE methods.
2. TRNSYS Type 19 should be updated to use the

1977 room transfer functions.

3. TRNSYS Type 17 should include the option of using wall transfer functions for variable room temperature. This would require providing users with a table of transfer function coefficients which include the individual c coefficients, since ASHRAE does not.

4. A new TRNSYS zone component should be developed which solves all equations for heat transfer within a single component along the lines of Appendix C. This would require extensive parameter lists for a single component and a matrix inversion routine within TRNSYS, however, the possibility exists in such a room model to model the actual physics much more accurately.

5. The simulations done to provide data for fitting the comprehensive room transfer function should perhaps be driven with artificial weather and input schedules, rather than using real weather data and typical operating schedules. It is possible that the different gains, e.g., solar lights, and occupancy, have such similar distributions that the regression cannot completely

identify them separately.

6. The simulation data used in fitting a transfer function should be randomly perturbed to see how this affects the quality of the fit. This could represent the noise in measurements from real buildings to determine if the regression fits are too sensitive to be used with data from real buildings.
7. Data from an existing building, or preferably several buildings, should be used to test the comprehensive room transfer function method.
8. If the comprehensive room transfer function method does not work for real buildings, the modified methods of Section 4.7 should be tried again.
9. Since most buildings use proportional temperature level control rather than energy rate control, ways of including this in the transfer function to calculate the actual load on the room at a given temperature, determine how much of that load will be extracted, and then calculate the new room temperature as usually done except that the excess energy

not removed by the system should be added in with the instantaneous convection gain term. Of course, one could also go back to a separate transfer function relating energy extraction, zone gains, and the zone temperature similar to what is currently done by ASHRAE.

Appendix A Sample Input and Output for the Mitalas-Arsenault Program

The computer program written by Mitalas and Arsenault computes the coefficients for multi-layer slab transfer functions. The default case computes the coefficients giving the heat fluxes for temperature inputs. The input card images are:

1	Time Step
2	2 line label for the element
3	Format (80 A1/80 A1)
4	Properties and label for each slab
.	Format (5 F10., 30 A1)
.	thickness, conductivity, density, specific
.	heat, resistance
n	(resistance is only specified for surfaces or spaces)
n+1	blank card ends layers
n+2	blank card default case
n+3	blank card ends input or begin new wall with Card 1

The program returns 4 sets of coefficients, usually denoted a, b, c, and d in order in each row.

$$q''_{\substack{\text{inside}\\ \text{surface}}} = \sum_{l=o} b_l T_o^{\,l} - \sum_{l=o} c_l T_i^{\,l} - \sum_{l=1} d_l q''^{\,l}_{\text{inside}}$$

$$q''_{\substack{\text{outside}\\ \text{surface}}} = \sum_{l=o} a_l \, T_o^{\,l} - \sum_{l=o} b_l T_i^{\,l} - \sum_{l=1} d_o \, q''^{\,l}_{\text{outside}}$$

```
        1.0
        ASHRAE EXTERIOR WALL 37  H=6
        HO=6  HI=.33 CONVECTION ONLY
                                                    .167
        .0833     .4      116.      .20                     A1 1 IN. STUCCO

        .275      .025    2.0       0.2                     B4 3 IN. INSUL
        .0624     .42     100.      .20                     E1 3/8 IN. GYP
                                                    3.0
```

```
@XQT J*M.TRANS
@ADD J*M.A37PP

                                   ASHRAE EXTERIOR WALL 37  H=6
                                   HO=6  HI=.33 CONVECTION ONLY

LAYER THICKNESS    CONDUCTIVITY          DENSITY         SP HEAT
          RESISTANCE

                                        DESCRIPTION OF LAYER

  1       .0000          .0000                .0000          .0000
               .1670
  2       .0833          .4000             116.0000          .2000
               .0000                    A1 1 IN. STUCCO
  3       .2750          .0250               2.0000          .2000
               .0000                    B4 3 IN. INSUL
  4       .0624          .4200             100.0000          .2000
               .0000                    E1 3/8 IN. GYP
  5       .0000          .0000                .0000          .0000
             3.0000


                                      THERMAL CONDUCTANCE,    U=  .069
                                              SAMPLING TIME INTERVAL,
 DT=     1.000

                                                  COEFFICIENTS FOR RA
MP INPUT

             J                     D/B                     1/B
              A/B                     D(Z)
             0         .17493918932161+001      .20394355871254-002
 .28991641968647+000 .10000000000000+001
             1        -.29442794591089+001      .11159020781100-001
-.30483383521528+000-.84035448028044+000
             2         .12157852948605+001      .35675660544314-002
 .31741074658767-001 .84472758713692-001
             3        -.40918512992034-002      .40647271626414-004
-.16989939440322-004-.21195172978776-004
             4         .79297678246275-006      .95095713130720-009
 .14547288976999-008 .67438313995961-017
             5         .15465498127883-016      .81418508814194-019
 .16785148989926-017
```

Appendix B Derivation of Wall Transfer Functions for Variable Surface Heat Trasnfer Coefficients

In order to use variable surface heat transfer coefficients, the wall transfer functions are calculated without surface resistances by the Mitalas-Arsenault program. The program returns four sets of coefficients--a, b, c, and d--which are used in different combinations to find heat fluxes at either the outside or inside surface.

The driving functions are the outside surface temperature, T_o, and the inside surface temperature, T_i. But these are not known directly. Only the sol-air temperature, T_{sa}, and the room temperature, T_r, are known. The objective is to combine the transfer functions with the surface heat transfer equations to give an expression for the heat flux at the inside surface.

$$q_i = f\,(T_{sa},\ T_r,\ h_o,\ h_i)$$

The flux at the outside surface expressed as a transfer function is:

$$q_o^0 = a_0\ T_o^0 - b_0\ T_i^0 + \left(\sum_{l=1} a_l\ T_o^l - \sum_{l=1} b_l\ T_i^l\right) - \sum_{l=1} d_l\ q_o^l \qquad \text{(A.1)}$$

Let the contribution from previous time steps, the terms in parentheses be called P. Solving for T_o^0,

$$T_o^0 = \frac{q_o^0 + b_0 T_i^0 - P}{a_0} \tag{A.2}$$

but,

$$q_o^0 = h_o^0 (T_{sa}^0 - T_o^0) \tag{A.3}$$

Therefore

$$T_o^0 = \frac{h_o^0 T_{sa}^0 + b_0 T_i^0 - P}{a_0 + h_o^0} \tag{A.4}$$

For the inside surface

$$q_i^0 = b_0 T_o^0 - c_0 T_i^0 + \left(\sum_{l=1} b_l T_o^l - \sum_{l=1} c_l T_i^l - \sum_{l=1} d_l q_i^l \right) \tag{A.5}$$

Let these previous contribution terms in parentheses be called R. Then substituting Equation A.4 into A.5

$$q_i^0 = b_0 \left(\frac{h_o^0 T_{sa}^0 + b_o T_i - P}{a_0 + h_o^0} \right) - c_0 T_i^0 + R \tag{A.6}$$

or

$$q_i^0 = \left(b_0 \cdot \frac{b_0}{a_0 + h_o^0} - c_0 \right) T_i^0 + \frac{b_0\, h_o^0\, T_{sa}^0 - b_o P}{a_0 + h_o^0} \tag{A.7}$$

$$+ R$$

Let

$$x_0 = \frac{b_0}{a_0 + h_o^0}$$

then,

$$q_i^0 = (b_0 x_0 - c_0)\, T_i^0 + h_0 x_0\, T_{sa}^0 - x_0 P + R \tag{A.8}$$

But,

$$T_i^0 = \frac{q_i^0}{h_i^0} + T_r^0$$

Therefore

$$q_i^0 = (b_0 x_0 - c_0) \left(\frac{q_i^0}{h_i^0} + T_r^0 \right) + h_o^0\, x_0\, T_{sa}^0 \tag{A.9}$$

$$- x_0 P + R$$

Let

$$y_0 = \frac{b_0\, x_0 - c_0}{h_i^0}$$

Then, solving for q_i^0

$$q_i^0 = \frac{y_0\, h_i^0\, T_r^0 + x_0 h_o^0\, T_{sa}^0 - x_0 P + R}{1 - y_0} \tag{A.10}$$

Equation A.10 gives the heat flux at the interior surface as a function of the current heat transfer coefficients and the current and previous sol-air and room temperatures.

Appendix C Equations Governing a Multi-Element Zone Model

This is a brief development of the system of equations governing the n surface temperatures and the air temperature in a zone. The terms are combined and rearranged to form a concise matrix equation. Once the temperatures are known, the total energy transferred to the room can be calculated by summing the convection transfers at the n surfaces. If the air temperature is to be held constant by the HVAC equipment, then this total energy is the load. If the room temperature is allowed to float, then the net energy transfer to the air must be zero, since the air is assumed to have negligible thermal capacitance.

The building elements are modeled with transfer functions. However, the function has been modified to give the interior surface temperatures as the output, rather than the inside surface heat flux. An energy balance at the inside surface shows that for $l = 0,1,2,...$

$$\hat{b}_l = \frac{b_l}{h_c}$$

$$\hat{c}_l = \frac{c_l}{h_c} - d_l$$

$$\hat{d}_l = d_l$$

where $d_0 = 1$ by definition.

The objective is to set up the matrix equation

$$\left[Z_{j,k} \right] \left(T_{s,j} \right) = \left(X_j \right)$$

so that

$$\left(T_{s,j} \right) = \left[Z_{j,k} \right]^{-1} \left(X_j \right)$$

where

$$T_{s,n+1} = T_r$$

There are two Z matrices and solutions depending on whether T_r is constant or floating.

In the following derivatives, j and k are used as surface indices, l as a time step index, and i means inside, o outside.

<u>Exterior Elements</u>

For each exterior element

$$T^0_{s,j} = - \hat{c}^0_j \, T^0_{e,i,j} - \sum_{l=1} \hat{c}^l_j \, T^l_{e,i,j} + \sum_{l=0} \hat{b}^l_j \, T_{e,o,j}$$

$$- \sum_{l=1} \hat{d}^l_j \, T^l_{s,j}$$

where,

$$T_{e,o} = T_a + \alpha^* I_t/h_o$$

$$T_{e,i} = T_r + \frac{S''_j \sum_{k=1}^{n} h_{r,j-k} (T_{s,k} - T_{s,j})}{h_{c,i}}$$

$$h_{r,j-k} = 4\,\sigma_{fjk}\,\overline{T}^3$$

S''_j is the absorbed solar/lighting flux and can be any fraction of the total gain to the room divided by the area of surface j.

Then, $T^o_{s,j} =$

$$\left[\left(\sum_{1=0} \hat{b}^1_j T^1_{e,o,j} - \sum_{1=1} \hat{c}^1_j T^1_{e,i,j} - \sum_{1=1} \hat{d}^1_j T^1_{s,j}\right) - \hat{c}^o_j \left(\frac{S^0_j + \sum_{k=1}^{n} h_{r,j-k} T^0_k}{h_{c,j}} + T^0_r\right)\right] \Bigg/ \left(1 - \frac{\hat{c}^o_j}{h_{c,j}} \sum_{k=1}^{n} h_{r,j=k}\right)$$

$$\text{Let } \beta_j = 1 - \frac{\hat{c}^0_j}{h_{c,j}} \sum_{k=1}^{n} h_{r,j-k}$$

Then

$$\sum_{k=1}^{n+1} Z_{jk} T_{s,k} = X_j$$

where $$Z_{j,k} = - \frac{\hat{c}_j^0 \; h_{r,j-k}}{\beta_j \; h^0_{c,y}} \qquad j \neq k$$

$$Z_{j,j} = - \frac{\hat{c}_j^0 \; h_{r,j-k}}{\beta_j \; h_{c,j}} - 1$$

$$Z_{j,n+1} = \hat{c}_j^o / \beta_j$$

(n+1 is the index of T_r)

$$X_j = - \frac{\frac{\hat{c}_j^0 \; s''^0_j}{h_{c,j}} + \sum_{l=1} \hat{c}_j^l \; T^l_{e,i,j} + \sum_{l=1} \hat{d}_j^l \; T^l_{s,j} - \sum_{l=o} \hat{b}_j^l \; T^l_{e,o,j}}{\beta_j}$$

Glazing

Treat as exterior walls with

$$\hat{b}^0 = \frac{U_g}{h_{c,i}} \quad \text{where} \quad U_g = \frac{1}{R_g + 1/h_{c,i}}$$

$$\hat{c}^0 = \frac{U_g}{h_{c,i}} - 1 = \hat{b}^0 - 1$$

$$\hat{d}^o = 1$$

$$\hat{b}^1 = \hat{c}^1 = \hat{d}^1 = o \text{ for all } 1 > o$$

Interior Elements

If there is no heat transfer by conduction through interior elements, then

$$T_{e,i} = T_{e,o}$$

and,

$$\beta_j = 1 + \frac{\hat{b}_j^0 - \hat{c}_j^0}{h_{c,j}} \sum_{k=1}^{n} h_{r,j-k}$$

$$Z_{j,k} = -\frac{\hat{b}_j^0 - \hat{c}_j^0 \quad h_{r,j-k}}{\beta_j \; h_{c,i,j}} \qquad j \neq k$$

$$Z_{j,j} = -\frac{\hat{b}_j^0 - \hat{c}_j^0 \quad h_{r,j-k}}{\beta_j \; h_{c,i,j}} - 1$$

$$Z_{j,n+1} = -\frac{\hat{b}_j^0 - \hat{c}_j^0}{\beta_j}$$

$$X_j = -\frac{\dfrac{\hat{b}_j^0 - \hat{c}_j^0}{h_{c,j}} \quad S''^0_j + \sum_{1=1} \hat{b}_j^1 - \hat{c}_j^1 \quad T_{e,i,j} - \sum_{1=1} \hat{d}_j^1 \, T_{s,j}^1}{-\beta_j}$$

Room Air

There are two possible solutions. For fixed room temperature,

$$Z_{n+1,k} = 0 \qquad k \neq n+1$$

$$Z_{n+1,n+1} = 1$$

$$X_{m+1} = T_{r,set}$$

For floating T_r, the net energy transfer by con-vertion to the air is zero, giving

$$Z_{n+1,k} = h_{c,j} A_j \quad \text{for} \quad 1 \leq j \leq n$$

$$Z_{n+1,n+1} = -\sum_{k=1}^{n} h_{c,kAk} - \rho c_p \dot{V}_{infil}$$

$$X_{n+1} = \rho c_p \dot{V}_{infil} T_a - \dot{Q}_{inst} - \dot{Q}_{aux}$$

which includes the possibility of additional convective gains to the air from people, lights and equipment, and the possibility of auxiliary energy input, e.g., in a temperature level control where the room temperature is not fixed.

The system of n+1 equations is then solved simultan-eously at each time step for the n surface temperatures and the room temperature. If the heat transfer proper-ties in the Z terms are constant, the two matrices for the fixed and floating solution need to be inverted just once.

Appendix D Program Listings

```
C PROGRAM MODEL SIMULATES A ROOM CONSISTING OF EXTERIOR WALL,
C GLAZING, AND INTERIOR PARTITION-FLOOR-CEILING.  INTERIOR
C CONVECTION AND LONG-WAVE RADIATION EXCHANGE ARE TREATED
C SEPARATELY.  SOLAR GAIN IS ALLOWED THROUGH GLAZING AND IS
C CONSIDERED DISTRIBUTED EVENLY ON THE P-F-C.  THE WALL AND
C P-F-C ARE REPRESENTED BY TRANSFER FUNCTIONS DRIVEN BY
C SOL-AIR OR EQUIVALENT TEMPERATURES ON BOTH SIDES.
C ENERGY RATE CONTROL IS USED WITH A THERMOSTAT WITH A
C DEADBAND BETWEEN HIGH AND LOW SETPOINTS.  "FREE COOLING"
C IS USED IF OUTSIDE AIR ALLOWS, AND IT IS LIMITED BY VENTILATION
C RATE.
C TMY DATA IS USED FOR THE WEATHER DATA.
C SUBROUTINE RADVER COMPUTES THE SOLAR IRRADIATION ON A VERTICAL
C SURFACE AND SUBROUTINE SIMUL INVERTS THE MATRICES.
C
      DIMENSION AO(2),BO(2),CO(2),DO(2),A(6,2),B(6,2),C(6,2),D(6,2)
      DIMENSION TSA(6),TWE(6),TPE(6),NN(2),QW(6),QP(6)
      DIMENSION ITEXT(2,20),DUMMY(4)
      EQUIVALENCE (TR,T(1)),(TP,T(2)),(TW,T(3)),(TG,T(4))
      DIMENSION SET(4,4),FLT(4,4),EF(4,4),T(4),Z(4)
      DIMENSION TROOM(6),RADWAL(6),SOLAR(6),QLD(6),TAMBNT(6),TSAWAL(6)
      DIMENSION MODAY(12)
      DATA MODAY/31,28,31,30,31,30,31,31,30,31,30,31/
      DATA HMAX,CMAX,CDD,HDD,HL,CL/6*0.0/
C INPUT HEAT TRANSFER COEFFICIENTS AND GLASS RESISTACE
C ABBREVIATIONS: G,GLASS  W,WALL  P,PARTITION  C,CONVECTION
C             R,RADIATION  O,OUTSIDE  T,TEMP  Q,FLUX
C  IF ISI=1, UNITS ARE SI METRIC
      IPRT=-1
      WRITE(*,115)
115   FORMAT('1')
C INPUT CONVECTION COEFS.: OUTSIDE,INSIDE WALL, GLAZING, P-C-F
C      RADIATION COEFS. (4*SIGMA*TBAR**3) WALL-PCF, GLAZING-PCF
C      RG IS GLAZING CONDUCTION RES. ONLY, NO SURFACE RES.
C      SOLAR ABSORBTIVITY:OUTSIDE WALL, GLAZING   GLAZING TRANSMIT
C      LATITUDE, AZIMUTH, SI UNITS FLAG (SI=1) USED IN TMY CONVERSION
      READ(*,*) HO,HCW,HCG,HCP,HRW,HRG,RG
      READ(*,*) ALFW,ALFG,TAU,PHI,GAM,ISI
C  INPUT THERMOSTAT SETPOINTS: LOW,HIGH
C  INPUT FRACTION GLAZING AND PCF AS RATIO TO TOTAL EXT. AREA
C    E.G. FG=.2 IS 20% GLAZING;FP=5 IS 5:1 RATIO PCF TO TOTAL EXT AREA
C  INPUT AIR DENSITY AND CP, AND MAX VENT FLOWRATE
C      PER UNIT TOTAL EXT. AREA; ENTER ZEROES TO SKIP OPTION
      READ(*,*) TSETLO,TSETHI
      READ(*,*) FG,PAREA
      READ(*,*) RHO,CPAIR,VDOT
      VENT=RHO*CPAIR*VDOT
C READ THE STARTING MONTH AND DAY TO BEGIN HOURLY  PRINTING,
C  AND THE NUMBER OF HOURS TO BE PRINTED.
      READ(*,*) IMO1,IDAY1,NHOURS
```

```
C INPUT TRANSFER FUNCTION COEFFICIENTS
C  EXTERIOR WALL 1ST, THEN PCF
C  A,B,C,D ARE COEFS FROM MITALAS ARSENAULT PROGRAM
C  CURRENT TIMESTEP FIRST (0), THEN THE NN PREVIOUS
      DO 5 J=1,2
      READ(*,105) (ITEXT(J,K),K=1,20)
105   FORMAT(20A4)
      READ(*,*) NN(J),A0(J),B0(J),C0(J),D0(J)
      DO 5 I=1,NN(J)
      READ(*,*) A(I,J),B(I,J),C(I,J),D(I,J)
5     CONTINUE
      WRITE(*,104) (ITEXT(1,K),K=1,20),(ITEXT(2,K),K=1,20)
104   FORMAT(' ',20A4)
      WRITE(*,106) HO,HCW,HCG,HCP,HRW,HRG,RG
106   FORMAT(/ 10X,'HEAT TRANSFER COEFFICIENTS'/' HO=',F5.3,
     1   ' HCW=',F5.3,' HCG=',F5.3,' HCP=',F5.3,' HRW=',
     2   F5.3,' HRG=',F5.3/' GLAZING RESISTANCE ',F5.3/)
      WRITE(*,107) ALFW,ALFG,TAU,PHI,GAM
107   FORMAT(' WALL ALPHA=',F5.3,/,' GLASS ALPHA=',F5.3,
     1 5X,'TAU=',F5.3,/,' LATITUDE=',F6.2,5X,'AZIMUTH=',F6.2)
      IF(ISI.NE.1) WRITE(*,*) 'UNITS: BTU,HR,FT,LBM,DEG F'
      IF(ISI.EQ.1) WRITE(*,*) 'UNITS: KJ,HR,M,KG,DEG C'
      WRITE(*,109) FG,PAREA,TSETLO,TSETHI
109   FORMAT(' FRACTION GLAZING=',F5.3,'  INTERIOR AREA RATIO=',F5.3,
     1 /' THERMOSTAT SETPOINTS....  LOW=',F7.2,
     2' HIGH='F7.2)
      WRITE(*,102) VDOT
102   FORMAT(' VENTILATION VOLUMETRIC RATE:',F10.0,' PER HOUR')
      WRITE(*,*) 'PRINTOUT FOR',IMO1,IDAY1,NHOURS
C INITIALIZE VECTORS AND VARIABLES
      FW=1.-FG
      UG=1./(RG+1./HO)
      QPOLD=0.
      TAV=(TSETLO+TSETHI)/2.
      DO 10 I=1,6
      TSA(I)=TAV
      QW(I)=0.
      QP(I)=0.
      TWE(I)=TAV
      TPE(I)=TAV
10    CONTINUE
      BC=B0(2)-C0(2)
      X1=FG*HRG/PAREA
      X2=FW*HRW/PAREA
      BCH=BC/HCP
      XX=X1+X2
      EF(1,1)=-FW*HCW-PAREA*HCP-FG*HCG
      EF(1,2)=PAREA*HCP
      EF(1,3)=FW*HCW
      EF(1,4)=FG*HCG
      EF(2,1)=BC+HCP
```

```
      EF(2,2)=-HCP-XX*(1.+BCH)
      EF(2,3)=X2*(1.+BCH)
      EF(2,4)=X1*(1.+BCH)
      EF(3,1)=HCW-CO(1)
      EF(3,2)=HRW*(1.-CO(1)/HCW)
      EF(3,3)=HRW*(CO(1)/HCW-1.)-HCW
      EF(3,4)=0.0
      EF(4,1)=HCG
      EF(4,2)=HRG
      EF(4,3)=0.0
      EF(4,4)=-(UG+HCG+HRG)
C  COPY MATRIX INTO FLT AND INVERT
      DO 20 I=1,4
      DO 20 J=1,4
20    FLT(I,J)=EF(I,J)
      DET=SIMUL(4,FLT,DUMMY,1.E-6,-1,4)
C SET UP SET AND INVERT
      DO 22 I=2,4
      DO 22 J=1,4
22    SET(I,J)=EF(I,J)
      SET(1,1)=1.0
      SET(1,2)=0.0
      SET(1,3)=0.0
      SET(1,4)=0.0
      DET=SIMUL(4,SET,DUMMY,1.E-6,-1,4)
C  BEGINNING OF HOURLY CALCULATION LOOP
15    CONTINUE
C  READ TMY DATA FOR HOUR
      READ(*,112,END=999)IMO,IDAY,IHR,RADBN,RAD,TAMB,ISNOW
      IF(IMO.GT.12) STOP
      IF((IDAY.EQ.IDAY1).AND.(IMO.EQ.IMO1).AND.(IHR.EQ.1))
     1  IPRT=NHOURS
116   CONTINUE
112   FORMAT(10X,I2,1X,I2,1X,I2,1X,F4.0,1X,F4.0,
     1 1X,F4.1,36X,I1)
      IF(ISI.EQ.1) GO TO 29
      TAMB=TAMB*1.8+32.
      RADBN=RADBN*.088076
      RAD=RAD*.088076
29    CONTINUE
      RADT=0.0
      IF(RAD.EQ.0.) GO TO 17
      CALL RADVER(RADT,IMO,IDAY,IHR,RADBN,RAD,ISNOW,GAM,PHI)
17    TSAW=TAMB+ALFW*RADT/HO
      TSAG=TAMB+ALFG*RADT/HO
      IF(TAMB.GT.TSETHI) CDD=CDD+(TAMB-TSETHI)
      IF(TAMB.LT.TSETLO) HDD=HDD+(TSETLO-TAMB)
C  CALCULATE HISTORIES PP & PW
      PP=0.0
      DO 30 I=1,NN(2)
30    PP=PP+(B(I,2)-C(I,2))*TPE(I)-D(1,2)*QP(I)
```

```
      PW=0.0
      DO 32 I=1,NN(1)
32    PW=PW+B(I,1)*TSA(I)-C(I,1)*TWE(I)-D(I,1)*QW(I)
C  SET UP Z VECTOR
      Z(1)=0.
      Z(2)=-PP-(1.+BCH)*FG/PAREA*TAU*RADT
      Z(3)=-BO(1)*TSAW-PW
      Z(4)=-UG*TSAG
C  FIND TR
      TR=0.
      DO 50 J=1,4
50    TR=TR+FLT(1,J)*Z(J)
      INDIC=0
      IF(TR.GT.TSETHI) INDIC=1
      IF(TR.LT.TSETLO) INDIC=-1
      IF (INDIC.EQ.0) GO TO 60
      IF (INDIC.EQ.1) T(1)=TSETHI
      IF (INDIC.EQ.-1) T(1)=TSETLO
      Z(1)=T(1)
C  FIND OTHER TEMPS
      DO 54 I=2,4
54    T(I)=0.0
      DO 55 I=2,4
      DO 55 J=1,4
55    T(I)=T(I)+SET(I,J)*Z(J)
      GO TO 70
60    DO 58 I=2,4
58    T(I)=0.0
      DO 59 I=2,4
      DO 59 J=1,4
59    T(I)=T(I)+FLT(I,J)*Z(J)
      GO TO 70
70    CONTINUE
C  MOVE HISTORY STACKS DOWN
      DO 72 I=2,NN(1)
      L=NN(1)+1-I
      LP1=L+1
      TSA(LP1)=TSA(L)
      TWE(LP1)=TWE(L)
72    QW(LP1)=QW(L)
      DO 74 I=2,NN(2)
      L=NN(2)+1-I
      LP1=L+1
      TPE(LP1)=TPE(L)
74    QP(LP1)=QP(L)
C EVALUATE EQUIVALENT TEMPERATURES
      TSA(1)=TSAW
      TPE(1)=TR-(X1*(TP-TG)+X2*(TP-TW))/HCP+TAU*RADT*FG/(PAREA*HCP)
      TWE(1)=TR-HRW*(TW-TP)/HCW
C CALCULATE LOADS
      QW(1)=BO(1)*TSAW-CO(1)*TWE(1)+PW
```

```
      QP(1)=BC*TPE(1)+PP
      QWFW=FW*HCW*(TW-TR)
      QPAR=PAREA*HCP*(TP-TR)
      QGFG=FG*HCG*(TG-TR)
      QLOAD=QGFG+QPAR+QWFW
      IF (ABS(QLOAD).LT.0.01) GO TO 89
      IF(QLOAD.LT.0.) GO TO 85
C UPDATE COOLING LOAD AND CHECK FOR FREE COOL
      IF(TAMB.GE.TSETHI) GO TO 82
      QV=VENT*(TSETHI-TAMB)
      FAN=QLOAD/QV
      IF(FAN.GT.1.) FAN=1.
      CFAN=CFAN+FAN
      QV=AMIN1(QV,QLOAD)
      QLOAD=QLOAD-QV
      CDUMP=CDUMP+QV
82    CL=CL+QLOAD
      IF(QLOAD.GT.CMAX) CMAX=QLOAD
      GO TO 89
C UPDATE HEATING LOAD AND CHECK FOR FREE HEAT
85    IF(TAMB.LE.TSETLO) GO TO 87
      QV=VENT*(TSETLO-TAMB)
      FAN=QLOAD/QV
      IF(FAN.GT.1.) FAN =1.
      HFAN=HFAN+FAN
      QV=AMAX1(QV,QLOAD)
      QLOAD=QLOAD-QV
      HDUMP=HDUMP-QV
87    HL=HL-QLOAD
      IF(QLOAD.LT.HMAX) HMAX=QLOAD
89    CONTINUE
C MOVE TEMPERATURE, SOLAR AND LOAD STACKS DOWN
      DO 78 I=2,6
      L=7-I
      LP1=L+1
      TSAWAL(LP1)=TSAWAL(L)
      TROOM(LP1)=TROOM(L)
      RADWAL(LP1)=RADWAL(L)
      SOLAR(LP1)=SOLAR(L)
      TAMBNT(LP1)=TAMBNT(L)
      QLD(LP1)=QLD(L)
78    CONTINUE
      TSAWAL(1)=TSAW
      TROOM(1)=TR
      SOLAR(1)=RADT*TAU*FG
      RADWAL(1)=RADT
      TAMBNT(1)=TAMB
      QLD(1)=QLOAD
C OUTPUT HOURLY INFO IF WITHIN RANGE OF PRINTING OPTION
      IF(IPRT.LT.1) GO TO 79
       IPRT=IPRT-1
```

```
      WRITE(*,200) QLD(1),TROOM(1),TAMBNT(1),TSAWAL(1),SOLAR(1)
200   FORMAT(18F7.2)
79    CONTINUE
C OUTPUT MONTHLY SUMMARY AND RESET
      IF(IHR.LT.23) GO TO 15
      IF(IDAY.NE.MODAY(IMO)) GO TO 15
      HDD=HDD/24.
      CDD=CDD/24.
      HMAX=-HMAX
      WRITE(*,114) IMO,HL,HDD,HMAX,HDUMP,HFAN,
     1 CL,CDD,CMAX,CDUMP,CFAN
114   FORMAT (' SUMMARY FOR MONTH ',I2,/
     1 ,' HEATING LOAD:',F10.0,3X,'DEGREE DAYS:',F6.0,
     2 3X,' HOURLY MAX:',F8.2,'   GAIN:',F10.0,3X,'FAN HRS:',F5.0/
     3 ,' COOLING LOAD:',F10.0,3X,'DEGREE DAYS:',F6.0
     4 ,3X,' HOURLY MAX:',F8.2,'   DUMP:',F10.0,3X,'FAN HRS:',F5.0/)
      HDD=0.
      CDD=0.0
      HL=0.
      CL=0.
      HMAX=0.
      CMAX=0.
      CFAN=0.
      HFAN=0.
      CDUMP=0.
      HDUMP=0.
      GO TO 15
999   STOP
      END
```

```
      FUNCTION SIMUL(N,A,X,EPS,INDIC,NRC)
C  THIS FUNCTION USES GAUSS-JORDAN ELIMINATION TO COMPUTE
C THE INVERSE IN PLACE. N IS THE NUMBER OF ROWS, A IS THE MATRIX
C X IS THE SOLUTION VECTOR, EPS IS THE TOLERANCE FOR A SINGULAR
C MATRIX, INDIC: NEGATIVE, COMPUTE INVERSE IN PLACE
C ZERO,SOLVE SYSTEM WITH AUGMENTED MATRIX OF COEFS IN N BY N+1
C A MATRIX.  IF POSITIVE,  SOLVE SYSTEM BUT DON'T INVERT
C NRC IS THE DIMENSION OF A
C THE FUNCTION RETURNS THE DETERMINANT
C**************
C THIS PROGRAM IS FROM APPLIED NUMERICAL METHODS BY
C CARNAHAN, LUTHER, AND WILKES, WILEY, NEW YORK (1969)
C**************
      DIMENSION IROW(50),JCOL(50),JORD(50),Y(50),A(NRC,NRC),X(N)
      MAX=N
      IF(INDIC.GE.0) MAX=N+1
      IF(N.LE.50) GO TO 5
      WRITE(*,200)
      SIMUL=0.
      RETURN
5     DETER=1.
      DO 18 K=1,N
      KM1=K-1
      PIVOT=0.
      DO 11 I=1,N
      DO 11 J=1,N
      IF(K.EQ.1) GO TO 9
      DO 8 ISCAN=1,KM1
      DO 8 JSCAN=1,KM1
      IF(I.EQ.IROW(ISCAN) ) GO TO 11
      IF(J.EQ.JCOL(JSCAN) ) GO TO 11
8     CONTINUE
9     IF (ABS(A(I,J)).LE.ABS(PIVOT) ) GO TO 11
      PIVOT=A(I,J)
      IROW(K)=I
      JCOL(K)=J
11    CONTINUE
      IF(ABS(PIVOT).GT.EPS) GO TO 13
      SIMUL=0.
      RETURN
13    IROWK=IROW(K)
      JCOLK=JCOL(K)
      DETER=DETER*PIVOT
      DO 14 J=1,MAX
14    A(IROWK,J)=A(IROWK,J)/PIVOT
      A(IROWK,JCOLK)=1./PIVOT
      DO 18 I=1,N
      AIJCK=A(I,JCOLK)
      IF(I.EQ.IROWK) GO TO 18
      A(I,JCOLK)= -AIJCK/PIVOT
```

```
      DO 17 J=1,MAX
17    IF(J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
18    CONTINUE
      DO 20 I=1,N
      IROWI=IROW(I)
      JCOLI=JCOL(I)
      JORD(IROWI)=JCOLI
20    IF(INDIC.GE.0) X(JCOLI)=A(IROWI,MAX)
      INTCH=0
      NM1=N-1
      DO 22 I=1,NM1
      IP1=I+1
      DO 22 J=IP1,N
      IF(JORD(J).GE.JORD(I)) GO TO 22
      JTEMP=JORD(J)
      JORD(J)=JORD(I)
      JORD(I)=JTEMP
      INTCH=INTCH+1
22    CONTINUE
      IF (INTCH/2*2.NE.INTCH) DETER=-DETER
      IF(INDIC.LE.0) GO TO 26
      SIMUL=DETER
      RETURN
26    DO 28 J=1,N
      DO 27 I=1,N
      IROWI=IROW(I)
      JCOLI=JCOL(I)
27    Y(JCOLI)=A(IROWI,J)
      DO 28 I=1,N
28    A(I,J)=Y(I)
      DO 30 I=1,N
      DO 29 J=1,N
      IROWJ=IROW(J)
      JCOLJ=JCOL(J)
29    Y(IROWJ)=A(I,JCOLJ)
      DO 30 J=1,N
30    A(I,J)=Y(J)
      SIMUL=DETER
      RETURN
200   FORMAT('N TOO BIG')
      END
```

```
      SUBROUTINE RADVER(RADT,IMO,IDAY,IHR,RADBN,RAD,ISNOW,GAM,PHI)
C  THIS SUBROUTINE COMPUTES THE SOLAR IRRADIATION ON
C     A VERTICAL SURFACE, RADT, GIVEN THE MONTH,DAY,HR
C     NORMAL BEAM RAD, TOTAL HORIZ RAD, ISNOW FLAG
C     AZIMUTH,  LATITUDE.
      DIMENSION ND(12),RHO(2)
      DATA ND/0,31,59,90,120,151,181,212,243,273,304,334/
      DATA RHO/.2,.7/
      DATA IENTER,C/0,.0174532/
      IR=ISNOW+1
      IF(IENTER.NE.0) GO TO 1
      CPHI=COS(C*PHI)
      SPHI=SIN(C*PHI)
      CGAM=COS(C*GAM)
      SGAM=SIN(C*GAM)
      IENTER=1
 1    CONTINUE
      IF(IDAY.EQ.IDAYO) GO TO 2
      DAY=FLOAT(ND(IMO)+IDAY)
      DEL=23.45*SIN(.0172142*(284.+DAY))
      SDEL=SIN(C*DEL)
      CDEL=COS(C*DEL)
      IDAYO=IDAY
 2    CONTINUE
      OM=.2618*(FLOAT(IHR)-12.5)
      COM=COS(OM)
      SOM=SIN(OM)
      CTH=-SDEL*CPHI*CGAM+CDEL*SPHI*CGAM*COM+CDEL*SGAM*SOM
      CTHZ=CDEL*CPHI*COM+SDEL*SPHI
      RB=CTH/CTHZ
      RADB=RADBN*CTHZ
      IF(RADB.LT.0.) RADB=0.
      RADD=RAD-RADB
      IF((CTH.LT.0.).OR.(CTHZ.LT.0.)) RB=0.
      RADT=RADB*RB+.5*(RADD+RAD*RHO(IR))
      RETURN
      END
```

```
C THIS PROGRAM USES THE COEFFS. OF A COMPREHENSIVE ROOM TRANSFER
C FUNCTION OBTAINED BY REGRESSION OF DATA GENERATED BY THE 3-SURFACE
C MODEL TO CALCULATE HOURLY HEATING AND COOLING LOADS AND ROOM TEMPS.
C THE LOADS ARE PER UNIT AREA TOTAL EXTERIOR AREA.
C IT IS SET UP TO USE 5 PREVIOUS LOAD COEFFS. AND 6 EACH OF
C SOL-AIR TEMPS, ROOM TEMPS, AND SOLAR HEAT GAIN (PER UNIT EXT AREA)
C IT COMPARES THE HOURLY SIMULATION FROM THE MODEL WITH THE
C VALUES CALCULATED BY THE CRTF, AND DOES SOME ERROR ANALYSIS
C FOR EACH WEEKLY PERIOD (168 HRS)
C THE SIMULATION AND INPUT DATA IS READ IN THE SAME FORMAT AS IT WAS
C PROBABLY INPUT TO THE REGRESION ROUTINE.
C  HERE IT IS ASSUMED THE CURRENT AND 5 PREVIOUS LOADS, TSA, TR AND
C SOLAR GAINS ARE INPUT.
C TSETLO AND TSETHI ARE THE THERMOSTAT POINTS. NW, # WEEKS TO
C BE ANALYZED
C IFPRIN =1 IS A FLAG TO PRINT HOURLY VALUES FOR IPRHR CONSECUTIVE
C HOURS.  HOURLY VALUES ARE PRINTED ONLY IF FLAGGED
      DIMENSION CQ(6),CT(6),CR(6),CS(6)
200   FORMAT(18F7.2/6F7.2)
      F(Q1,Q2,Q3,Q4,Q5,T0,T1,T2,T3,T4,T5,
     1R0,R1,R2,R3,R4,R5,S0,S1,S2,S3,S4,S5)
     1=CQ(2)*Q1+CQ(3)*Q2+CQ(4)*Q3+CQ(5)*Q4+CQ(6)*Q5
     1+CT(1)*T0+CT(2)*T1+CT(3)*T2+CT(4)*T3+CT(5)*T4+CT(6)*T5
     1+CR(1)*R0+CR(2)*R1+CR(3)*R2+CR(4)*R3+CR(5)*R4+CR(6)*R5
     1+CS(1)*S0+CS(2)*S1+CS(3)*S2+CS(4)*S3+CS(5)*S4+CS(6)*S5
      IHOUR=0
      IPRHR=-1
      INIT=0
       READ(*,*) (CQ(J),J=2,6),CT,CR,CS
      READ(*,*) TSETLO,TSETHI,NW
      READ(*,*) IFPRIN
      IF(IFPRIN.NE.1)GO TO 10
      READ(*,*) IPRHR
10    CONTINUE
      WRITE(*,*)'TSETLO',TSETLO,'TSETHI',TSETHI
      WRITE(*,*) ' WEEKS',NW,' HOURLY PRINT',IFPRIN,IPRHR
      WRITE(*,*) 'REGRESSION COEFFICIENTS:'
      WRITE(*,*) (CQ(J),J=2,6),CT,CR,CS
      WRITE(*,*) '  '
      DO 500 IWK=1,NW
      NC=0
      NH=0
      NT=0
      SSQH=0.
      CBIAS=0.
      HBIAS=0.
      TBIAS=0.
      SSQC=0.
      SQCS=0.
      SQHS=0.
```

```
      STS=0.
      STC=0.
      SQHC=0.
      SQCC=0.
      SST=0.
      DO 499 IHR=1,168
      READ(8,200)QS,D1,D2,D3,D4,D5,T0,T1,T2,T3,T4,T5,
     1 RS,D6,D7,D8,D9,D10,S0,S1,S2,S3,S4,S5
      IF(INIT.NE.0) GO TO 60
      INIT=1
      Q0=QS
      R0=RS
      R1=D6
      R2=D7
      R3=D8
      R4=D9
      R5=D10
      Q1=D1
      Q2=D2
      Q3=D3
      Q4=D4
      Q5=D5
      MODE=SIGN(1.,Q0)+2.
60    GO TO (70,100,80),MODE
C     MODE 1 HEATING
70    R0=TSETLO
      Q0=F(Q1,Q2,Q3,Q4,Q5,T0,T1,T2,T3,T4,T5,
     1 R0,R1,R2,R3,R4,R5,S0,S1,S2,S3,S4,S5)
      IF(Q0.LE.0.) GO TO 150
      MODE=2
      GO TO  100
C     MODE 3  COOLING
80    R0=TSETHI
      Q0=F(Q1,Q2,Q3,Q4,Q5,T0,T1,T2,T3,T4,T5,
     1 R0,R1,R2,R3,R4,R5,S0,S1,S2,S3,S4,S5)
      IF(Q0.GE.0.) GO TO 150
      MODE=2
      GO TO 100
C     MODE 2  FLOATING
100   CONTINUE
      Q0=0.
      R0=CQ(2)*Q1+CQ(3)*Q2+CQ(4)*Q3+CQ(5)*Q4+CQ(6)*Q5
     1+CT(1)*T0+CT(2)*T1+CT(3)*T2+CT(4)*T3+CT(5)*T4+CT(6)*T5
     1+CR(2)*R1+CR(3)*R2+CR(4)*R3+CR(5)*R4+CR(6)*R5
     1+CS(1)*S0+CS(2)*S1+CS(3)*S2+CS(4)*S3+CS(5)*S4+CS(6)*S5
      R0=R0/(-CR(1))
      IF(R0.LE.TSETHI) GO TO 110
      MODE=3
      GO TO 80
110   IF(R0.GE.TSETLO) GO TO 150
      MODE=1
```

```
      GO TO 70
150   CONTINUE
      IF(IPRHR.LE.0) GO TO 160
      IHOUR=IHOUR+1
      IPRHR=IPRHR-1
      WRITE(*,159) IHOUR,QS,Q0,RS,R0,T0,S0
159   FORMAT(I5,3X,2F7.2,3X,2F7.2,3X,2F7.2)
160   CONTINUE
      Q5=Q4
      Q4=Q3
      Q3=Q2
      Q2=Q1
      Q1=Q0
      R5=R4
      R4=R3
      R3=R2
      R2=R1
      R1=R0
      IF(QS.EQ.0.) GO TO 450
      IF(QS.LT.0.) GO TO 440
      NC=NC+1
      DC=Q0-QS
      CBIAS=CBIAS+DC
      SSQC=SSQC+DC*DC
      SQCS=SQCS+QS
      GO TO 450
440   NH=NH+1
      DH=Q0-QS
      HBIAS=HBIAS+DH
      SSQH=SSQH+DH*DH
      SQHS=SQHS+QS
450   CONTINUE
      NT=NT+1
      STS=STS+RS
      STC=STC+R0
      DR=R0-RS
      SST=SST+DR**2
      TBIAS=TBIAS+DR
      IF(Q0.LT.0.) SQHC=SQHC+Q0
      IF(Q0.GT.0.) SQCC=SQCC+Q0
499   CONTINUE
      TSBAR=STS/NT
      TCBAR=STC/NT
      QHSBAR=SQHS/NH
      QHCBAR=SQHC/NH
      QCSBAR=SQCS/NC
      QCCBAR=SQCC/NC
      TSIG=SQRT(SST/NT)
      QHSIG=SQRT(SSQH/NH)
      QCSIG=SQRT(SSQC/NC)
      HBIAS=HBIAS/NH
```

```
      CBIAS=CBIAS/NC
      TBIAS=TBIAS/NT
      WRITE(*,401) QHSBAR,QHCBAR,QHSIG,HBIAS,NH,QCSBAR,QCCBAR,
     1 QCSIG,CBIAS,NC
     1 ,TSBAR,TCBAR,TSIG,TBIAS,NT
401    FORMAT(4F9.4,I4,4F9.4,I4,4F9.4,I4)
      WRITE(9,401) QHSBAR,QHCBAR,QHSIG,HBIAS,NH,QCSBAR,QCCBAR,
     1 QCSIG,CBIAS,NC
     1 ,TSBAR,TCBAR,TSIG,TBIAS,NT
500   CONTINUE
      STOP
      END
```

REFERENCES

(1) Stephenson, D.G. and Mitalas, G.P., "Cooling Load Calculations by Thermal Response Factor Method," ASHRAE Transactions, 73, III.1.1-III.1.7 (1967).

(2) Stephenson, D.G. and Mitalas, G.P., "Calculation of Heat Conduction Transfer Functions for Multi-Layer Slabs," ASHRAE Transactions, 78, 117-126 (1972).

(3) Mitalas, G.P. and Arsenault, J.G., "Fortran IV Program to Calculate Z-Transfer Functions for the Calculation of Transient Heat Transfer through Walls and Roofs," (Presented at the 1st Symposium on Use of Computers for Environmental Engineering Related to Buildings held at the National Bureau of Standards, Washington, D.C., 1970).

(4) Ceylan, H.T. and Meyers, G.E., "Long-time Solutions to Heat Conduction Transients with Time-Dependent Inputs," Journal of Heat Transfer, 102, 115-120 (1980).

(5) ASHRAE Handbook of Fundamentals, New York: American Society of Heating, Refrigerating, and Air Conditioning Engineers, Inc. (1977).

(6) Pawelski, M.J., "Development of Transfer Function Load Models and Their Use in Modeling the C.S.U. Solar House I," M.S. Thesis in Mechanical Engineering, University of Wisconsin, Madison, Wisconsin (1976).

(7) Kimura, Ken-Ichi, Scientific Basis of Air Conditioning, London: Applied Science Publishers (1977).

(8) Horn, J.C., "Investigation of Various Thermal Capacitance Models," M.S. Thesis in Mechanical Engineering, University of Wisconsin-Madison (1982).

(9) Chen, Z. and Mitchell, J.W., "Response Coefficients of Room Thermal Processes," Proceedings of the Sixth National Passive Solar Conference, Portland, Oregon (1981).

(10) SOLMET Typical Meteorological Year, Tape Deck 9734, National Oceanic and Atmospheric Administration, Environmental Data Service, National Climatic Center, Asheville, North Carolina.

(11) Klein, S.A. et al., "TRNSYS--A Transient Simulation Program," University of Wisconsin-Madison, Engineering Experiment Station Report 38-10, Version 11-1 (1981).

(12) Ottenstein, J., "Application of Solar Energy in Multizone Buildings," M.S. Thesis in Mechanical Engineering, University of Wisconsin-Madison (1979).

(13) MINITAB, Version 81.1, Penn State University-Statistics Dept., University Park, Pennsylvania, Copyright 1981.