

**HEAT AND MASS TRANSFER IN
MEAT PROCESSING**

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ABSTRACT

Over time, cooking of food has become a complex industry, with opportunities for improvement. The growing importance of the industrial and institutional food processing the processing equipment has become more and sophisticated. Modern smokehouses and ovens are equipped with instruments for monitoring and controlling air temperature, humidity and velocity. While offering a potential for incorporating energy efficient heat processing, faster adaptation to new products like low-fat products and more, control requires knowledge of the heat and mass transfer mechanisms for the food product during the cooking process. In addition, knowledge of the parameters important for the consumer acceptability and for the storage of the product is needed.

The goal of this research was the development of a mechanistic model for a food product commonly referred to as frankfurter, bolonga or hot dog. The model had to account for the heat and mass transfer within the product and for the interactions of the food product with its environment. Different models were developed using different assumptions for the building blocks of the model. The cooking process was simulated and the results compared to data to determine the importance and effects the parameters of the models have on temperature history and moisture loss of the product.

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NOMENCLATURE

ROMAN SYMBOLS

Symbol	Description
A	adsorption potential in Dubinin-Polanyi isotherm
a_w	water activity
B	mass transfer driving force
Bi	Biot number for heat transfer
C	specific heat (kJ/kg K)
c_p	specific heat of air at constant pressure (kJ/kg K)
c_w	equilibrium constant in linear isotherm
D	moisture diffusivity within the food product (m^2/s)
D_w	mass diffusivity of water in air
\dot{E}	energy flux (W)
E_0	characteristic energy of adsorption in Dubinin-Polanyi isotherm
erfc	complementary error function ($erfc(x)=1-erf(x)$)
f	fugacities
g	mass transfer conductance (kg/m^2s)
g'''	energy generation rate per unit volume (W/m^3)
$G_{diff,w}$	diffusional mass flux due to concentration differences
h	convective heat transfer coefficient (W/m^2K)

 ROMAN SYMBOLS cont.

Symbol	Description
i	enthalpies (J/kg)
$i_{fg,0}$	standard heat of evaporation at temperature T_0
J_0	Bessel function of first kind of order zero
J_1	Bessel function of first kind of order one
k	thermal conductivity (W/mK), in 1.2.2 (cal/sec.cm K)
K_{ij}	phenomenological coefficients in irreversibel thermodynamics
Le	Lewis number
M_w	molecular weight of water (18 g/gmole)
\dot{M}	mass flux (kg/s)
m_w	mass concentration of water (kg water/kg humid air)
P	absolute pressure (bar)
$P_{sat}(T)$	saturation pressure of water at temperature T (bar)
P_w	partial pressure of water (bar)
P_w^0	vapor pressure in 1.2.2 (atm)
q	energy flux
r	radius (m)
R	outer radius (m)
R	gas constant in 1.2.2 (1.98 cal/gmole K)
R_v	gas constant in 1.2.2 (82.1 atm cm ³ /gmole K)
t	time (s)
T	Temperature (°C)

 ROMAN SYMBOLS cont.

Symbol	Description
u	velocity component in x-direction of boundary layer
v	velocity component in y-direction of boundary layer
w	water concentration in the food product (kg water/kg food product)
X	Moisture content per dry basis (kg water/kg dry matter)
X_0	total microporous volume in Dubinin-Polanyi isotherm
X_{\min}	minimum moisture content of the food product
z	distance from the surface

 GREEK SYMBOLS

Symbol	Description
α	thermal diffusivity (m^2/s)
ΔH_s	molar heat of sorption in 1.2.2 (cal/gmole)
ϕ	relative humidity
λ_n	eigenvalues
μ	dynamic viscosity (kg/m s)
ω_s	humidity ratio of air (kg water/kg dry air)
ρ	mass density (kg/m^3)
ρ_a	mass density of humid air (kg humid air / m^3)
ρ_s	density of solid with no moisture in 1.2.2 (g/cm^3)
ρ_d	density per dry matter (kg dry matter/ m^3)

SUPERSCRIPTS

Symbol	Description
''	per unit area
'''	per unit volume
p	indicates the number of the time step

SUBSCRIPTS

Symbol	Description
a	air
e	equilibrium
fpr	food product
gen	amount of energy generated in a control volume
in	amount of mass or energy transferred into a control volume
L	L-state in derivation of mass and energy flux within the boundary layer
n	indicates the number of the node
out	amount of mass or energy transferred out of a control volume
s	surface
store	mass or energy storage in a control volume
T	T-state in derivation of mass and energy flux within the boundary layer
w	water
0	initial conditions
∞	free stream conditions

CHAPTER
ONE

INTRODUCTION

Cooking of food products is an everyday occupation in which about everybody is involved. In recent times the cooking process has become more an industrial process of food. With the growing importance of the industrial and institutional food processing, the equipment has become more sophisticated. Modern smokehouses and ovens are equipped with instruments for programming and controlling of air temperature, humidity and velocity, offering a potential for incorporating energy efficient heat processing, for faster adaptation to new products such as low-fat products and more. All this requires the knowledge of the heat and mass transfer mechanisms for the food product during the cooking process as well as the knowledge of the parameters important for the consumer acceptability and for the storage of the product. The focus of this work is processing of a food commonly referred to as frankfurter, bologna or hot dog. The goal of this research was the development of a model for this product to make simulations of the cooking process possible. Therefore the model had to account for the heat and mass transfer within the product, the interactions of the food product with its environment, and the

determination of temperature and the moisture content within the product. These latter two are important parameters for the storage of the food product.

1.1 BACKGROUND

The task of acquiring an understanding of the heat and mass transfer mechanisms for food stuff is more difficult than for other industrial processes due to the complexity of the products involved. In the cooking process of food products many phenomena can occur, such as the melting of fats, the denaturation of proteins or the evaporation of water from the product. All these effects influence the temperature and the moisture history of the product, which themselves are the main variables for the quality of the food product, where preservation, storage and consumer acceptance are concerned. What adds to the complexity of the task are the material properties of the foodstuffs. They are usually a function of temperature and composition, both of which may change to some extent during the cooking process. There has been much research on the processing of food and its material properties. Most of the research has been done on drying applications. The processes of drying and cooking are related, and in both cases the heat and mass transfer for the product must be modeled. Thus the same fundamental equations for heat and mass transfer apply to both processes. To use these equations simplifying assumptions must be made. In the following section three approaches for the modeling of food processing and their different assumptions will be introduced.

1.2 REVIEW OF LITERATURE

In this section three approaches in modeling the heat and mass transfer for the processing of food stuff and the main assumptions that apply will be discussed briefly. The first approach discussed is that of irreversible thermodynamics that is usually associated with Luikov's work. The second approach is the so called shrinking-core model developed by King. The third approach is a model introduced by Spielbauer, which accounts for heat and mass transfer between product and environment, but does not account for mass transfer within the product.

1.2.1 Irreversible Thermodynamics Approach

The major reference for this section are relations developed by Luikov, (see [3] and [9]). The application of the irreversible thermodynamics modeling on the cooking process is almost ideally suitable due to the existence of two driving forces. This model was first developed by Luikov to understand the transport phenomena of wicks in heat pipes, but was soon applied to other problems such as drying of capillary-porous bodies in general. The interrelation of the driving forces is due to the fact that an enthalpy transport is associated with every mass transport and that liquids are transferred not only due to concentration gradients but also due to temperature gradients (Soret effect). Luikov assumed that moisture is transferred both as liquid and as vapor, where the fraction of vapor to be transferred is assumed to be 10^{-5} the fraction of the liquid mass transferred. Luikov developed a model that even accounted for possible pressure gradients within the material. The following equations apply then:

$$\frac{\partial T}{\partial t} = K_{11}\nabla^2 T + K_{12}\nabla^2 X + \{K_{13}\nabla^2 P\} \quad (1.2.1)$$

$$\frac{\partial X}{\partial t} = K_{21}\nabla^2 T + K_{22}\nabla^2 X + \{K_{23}\nabla^2 P\} \quad (1.2.2)$$

$$\left\{ \frac{\partial X}{\partial t} = K_{31}\nabla^2 T + K_{32}\nabla^2 X + K_{33}\nabla^2 P \right\} \quad (1.2.3)$$

The coefficients K_{ij} in these equations are called phenomenological coefficients. Since they are rather complex expressions they will not be used in this work and will not be rewritten explicitly here. The terms and equations in brackets vanish if pressure gradients are not taken into account. For the most simple case reported in [3] the phenomenological coefficients K_{11} and K_{22} are the straight coefficients for heat and mass transfer respectively. Thus K_{11} is the same as α , the thermal diffusivity, and K_{22} is the mass diffusion coefficient D . K_{21} can be expressed as $K_{21} = \gamma D$, where γ has dimensions of inverse unit temperature and is called thermo gradient coefficient. The other coupling coefficient K_{12} can be written as $K_{12} = \tau D$, where τ is a characteristic temperature defined as i_{fg}/C . For the limit transition, where the coupling coefficients K_{21} and K_{12} turn into zero, the classic heat conduction and diffusion equations are obtained.

Despite the fact that the irreversible thermodynamics approach is very suitable to the modeling of cooking and drying processes it does not enjoy much popularity among food engineers due to the problems of obtaining values for the phenomenological coefficients.

1.2.2 Shrinking-core Model

King presents an analysis of the ways heat and mass transfer interact to govern rates of sorption and desorption [8]. The analysis was developed for dried foodstuffs. King observed the following anomalies for reported diffusivity data: The pressure effects and

the diffusivities suggest vapor phase diffusion. However, the values of the diffusivities are many orders or magnitude smaller than typical gas phase diffusion. The temperature dependence at high pressures is greater than that for gas phase diffusion. To resolve these anomalies King makes the following set of assumptions: That the foodstuffs can be considered porous materials, that mass transfer within the solid medium occurs by vapor phase diffusion only and that the diffusivity within the medium D' is the regular vapor phase diffusion D times a constant that accounts for the geometry of the solid, that the vapor space is in local equilibrium with the sorbed moisture content everywhere and at any time, that the heat of sorption is constant with respect to temperature and moisture content, that the heat effects of sorption or desorption are large in comparison to sensible heat changes of the medium. Applying these assumptions King arrived at an effective diffusivity shown in eq. 1.2.4, (see nomenclature at the beginning of this work).

$$D_{\text{eff}} = \frac{M_w D' P_w^0}{\rho_s R_v T} \frac{P}{P - P_w} \left(\frac{\partial \phi}{\partial X} \right)_T \left(\frac{\alpha}{1 - \alpha} \right) \quad (1.2.4)$$

where :

$$\alpha = \frac{k R R_v T^3}{\Delta H_s^2 D' \phi P_w^0} \quad (1.2.5)$$

α determines whether the diffusional transport is controlled by mass transfer or by heat transfer. It is not obvious how the heat transfer can control the diffusional transport. In this case the heat transfer can limit the amount of mass transferred because energy is needed inside the medium to evaporate the water that is then transferred to the surface. Thus the amount of water transferred depends on the amount of energy transferred into the solid. The term $\frac{M_w P_w^0}{\rho_s R_v T} \left(\frac{\partial \phi}{\partial X} \right)_T$ represents the ratio of moisture density in the vapor phase to the moisture density in the adsorbed phase. By reexamining previous experiments the validity of the model for drying applications was shown. However, there is no evidence that this model and its assumptions are valid for cooking where

higher moisture contents are present. The assumption that moisture is transferred in the vapor phase is especially questionable, because for high moisture contents the pores of the medium are likely to be filled with water.

1.2.3 MS-Thesis by James L. Spielbauer

In his thesis [12] Spielbauer has written a simulation program for the entire cooking process of frankfurters in convection ovens. In this program a model for the food product was used that assumed that moisture transfer within the product occurs in liquid phase only, and the moisture evaporates only at the surface of the product. The partial differential equation for the heat conduction problem was approximated using finite differences. Internal mass transfer was not taken into account. Thus the boundary condition for the evaporation of moisture was restricted to two cases. One where the surface is assumed to be totally wet and moisture evaporates from the product; this assumes an infinite moisture diffusivity inside the product. The other where no moisture evaporates and the behavior of the product is that of a dry body; this assumes zero moisture diffusivity.

Experiments showed that for the boundary condition of a wet surface the simulation results agreed with the data at the beginning of the cooking process where the product still has a high moisture content. The experiments showed further that for the boundary condition of dry surface the simulation results agreed with the data later in the cooking process where the product is dried out to some extent in the surface regions. However, the model was not able to predict either the behavior for the transition from wet to dry surface or the time at which this transition occurs. Thus there was a need to develop a model that was able to account for the effects of the transition too.

That the model was able to represent the beginning of the process implies that the assumption that moisture is transferred in the liquid phase is valid. Thus the present work tried to modify the model in a way that it additionally accounts for the moisture transport within the product and that it represents the equilibrium between the moisture content of the food product and the relative humidity of the surrounding air. The predicted temperature histories of the model should show the following characteristic behavior that is shown in Fig. 1.1 and is taken from [12]:

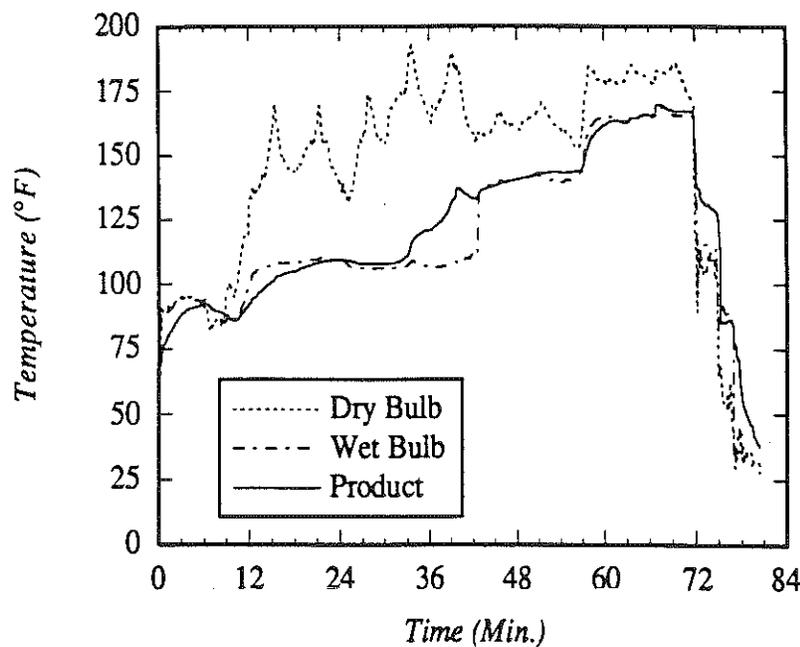


Figure 1.1 Temperature data from Spielbauer [12]

In this test of an actual product, the temperature of the product reaches the wet-bulb temperature quite fast, remains at approximately that temperature for some time and then approaches the dry-bulb temperature. This happens in Fig. 1.1 in the time range from 12 to 40 min. After 40 min. a new cooking zone is entered which changes the thermal response of the product.

1.3 ORGANIZATION

The research in this report is divided into four additional chapters.

In chapter 2 the development of the product model is presented in four parts. Differential equations for heat and mass transfer within the product are formulated. The transfer within the boundary layer is modeled and the equilibrium between product and surrounding air is represented with different isotherms. These parts are brought together in a finite difference method to yield a complete model of the food product during the cooking process. Chapter 3 discusses the problem of property estimations for the foodstuff in general and for processed meat products in particular. Chapter 4 describes the results for the different versions of the simulation program and compares them to the expected results. In chapter 5 the conclusions to the results of this work and recommendations for future work are presented.

DEVELOPMENT OF THE SIMULATION MODEL

The development of the model for the food product is broken in four parts. The first part accounts for heat and mass transfer within the food product. The second accounts for the convective transfer of heat and moisture through the boundary layer. In the third part these will be linked by introducing representations of the equilibrium at the surface of the food product. The fourth part shows how the governing differential equations can be represented using a finite difference method.

2.1 PRODUCT MODEL

The equations for conductive heat and diffusional moisture transfer within the food product are presented separately. Beginning with the general partial differential equations, various assumptions are made to yield a transient radial conduction and diffusion formulation. Analytical solutions are presented for both the conduction and the diffusion problems.

2.1.1 Mathematical Description of the Product

The first step in modeling the conduction and diffusion situation is to select an appropriate coordinate system to the geometry of the problem. The food product is most easily represented in cylindrical coordinates as shown in Fig. 2.1. The energy and mass balances are performed on a control volume shown in Fig. 2.1 and 2.2 :

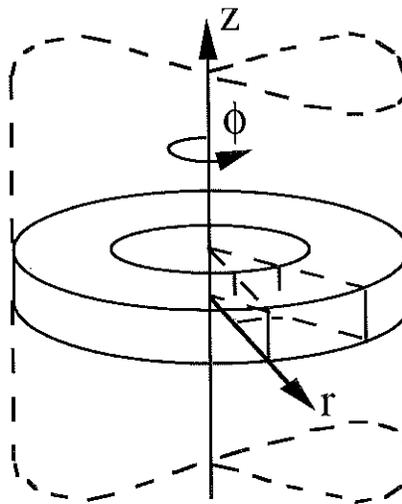


Figure 2.1 Cylindrical coordinate representation and control volume for the food product

Performing the energy and mass balances yields equations 2.1.1 and 2.1.2 respectively.

$$\dot{E}_{\text{store}} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_{\text{gen}} \quad (2.1.1)$$

$$\dot{M}_{\text{store}} = \dot{M}_{\text{in}} - \dot{M}_{\text{out}} \quad (2.1.2)$$

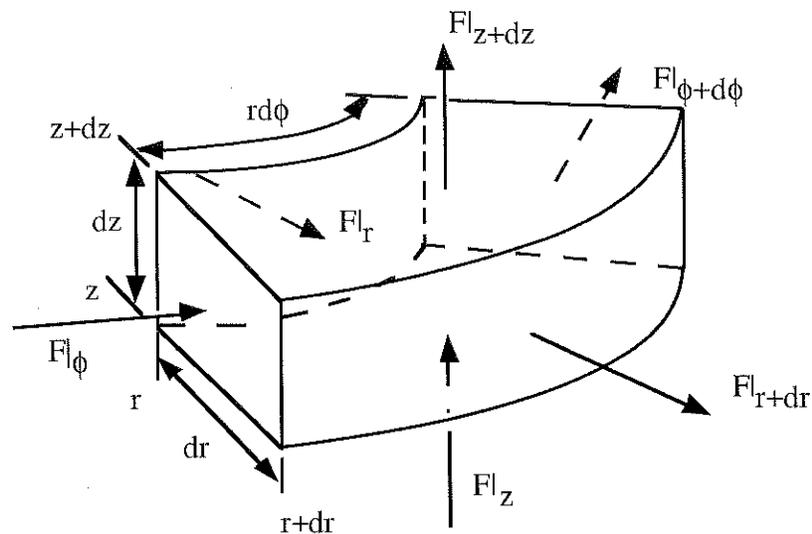


Figure 2.2 Control volume where F is either energy or mass flux

2.1.2 Governing Differential Equations

The energy balance of equation. 2.1.1 becomes the governing partial differential equation for transient conduction within a cylinder equation. 2.1.3 substituting the appropriate rate expressions and taking the limit as Δz , Δr and $\Delta \phi$ go to zero.[1]

$$\rho C \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(k r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + g''' \quad (2.1.3)$$

The same is done for the mass balance and equation. 2.1.2 yields the governing differential equation for transient diffusion within a cylinder:

$$\frac{\partial X}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(D r \frac{\partial X}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left(D \frac{\partial X}{\partial \phi} \right) + \frac{\partial}{\partial z} \left(D \frac{\partial X}{\partial z} \right) \quad (2.1.4)$$

Equations 2.1.3 and 2.1.4 can be simplified by two assumptions. The first assumption is that the radial gradients of temperature and moisture are much greater than the axial and circumferential gradients. This assumption implies that end effects are negligible and that the surface temperature and moisture content are uniform around the product surface. The material properties (ρ , D , k , C) are assumed constants.

Applying these assumptions to equations 2.1.3 and 2.1.4 are reduced them to equations 2.1.5 and 2.1.6 respectively:

$$\rho C \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial r^2} + \frac{k}{r} \frac{\partial T}{\partial r} \quad (2.1.5)$$

$$\frac{\partial X}{\partial t} = D \frac{\partial^2 X}{\partial r^2} + \frac{D}{r} \frac{\partial X}{\partial r} \quad (2.1.6)$$

2.1.3 Analytical Solutions

An analytical solution exists for both equations (2.1.5 and 2.1.6). The following boundary and initial conditions are applied to equation 2.1.5:

$$\text{B.C.:} \quad \left(\frac{\partial T}{\partial r} \right)_{r=R} = \frac{h}{k} (T_{\infty} - T(R,t)) \quad (2.1.7)$$

$$T(0,t) < \infty \quad (2.1.8)$$

$$\text{I.C.:} \quad T(r,0) = T_0 \quad (2.1.9)$$

The solution for the heat conduction problem is reported in [12]. The solution can be represented by equations 2.1.10 through 2.1.13:

$$T(r,0) = \frac{T(r,t) - T_\infty}{T_0 - T_\infty} = 2\text{Bi} \sum \frac{J_0(\lambda_n r) \text{Exp}(-\alpha \lambda_n^2 t)}{[(\lambda_n P)^2 + \text{Bi}^2] J_0(\lambda_n R)} \quad (2.1.10)$$

$$\lambda_n R = \frac{\text{Bi} J_0(\lambda_n R)}{J_1(\lambda_n R)} \quad (2.1.11)$$

$$\text{Bi} = \frac{hR}{k} \quad (2.1.12)$$

$$\alpha = \frac{k}{\rho C} \quad (2.1.13)$$

To evaluate equation 2.1.10 involves solving the eigenfunction in equation 2.1.11 to find the eigenvalues λ_n . The analytical solution does not account for changes in the free stream temperature, the heat loss due to evaporation, or property changes. However the analytical solution provides a base to show that the finite difference method meets the results of the analytical solution [12].

For the diffusion problem the analytical solution was developed making the following three assumptions. There are no temperature variations within the food product. The moisture loss penetrates only a thin layer, thus the food product can be treated as a semi-infinite slab. The surface is at equilibrium with the free stream humidity. This is a reasonable assumption because the mass transfer Biot number is on the order of magnitude of 100. For sake of simplicity it was then further assumed that the equilibrium can be represented with a linear relationship like eq. 2.1.14:

$$\phi_{s,a} = c_w X_{s,\text{fpr}} \quad (2.1.14)$$

The problem then reduces to the following set of equations (2.1.15 through 2.1.18)

$$\frac{\partial X}{\partial t} = D \frac{\partial^2 X}{\partial z^2} \quad (2.1.15)$$

$$\text{I.C.:} \quad X(t=0, z) = X_0 \quad (2.1.16)$$

$$\text{B.C.:} \quad X(t, z=0) = X_s \quad (2.1.17)$$

$$X(t, \infty) = X_0 \quad (2.1.18)$$

The solution is shown in equation 2.1.19:

$$\frac{X - X_0}{X_s - X_0} = \text{erfc} \left(\frac{z}{\sqrt{4Dt}} \right) \quad (2.1.19)$$

This solution is of little practical use, because for the cooking process the surface temperature varies over a wide range and the assumption of constant boundary condition is not valid. However the analytical solution provides a rough estimate for the penetration depth and can thus be compared to the results of the finite differences solution, which is developed in chapter 2.4.

2.2 CONVECTIVE HEAT AND MASS TRANSFER WITHIN THE BOUNDARY LAYER

This section presents a method for calculating the mass and energy flux for the combined heat and mass transfer between product and surrounding fluid. This will be done in rather detailed form because of the complexity of the matter. For a more detailed description of this problem see Kays [7]. Fluid flows around objects are very complicated problems, but can be greatly simplified using the Boundary Layer Theory. Prandtl discovered that for most applications the influence of viscosity is confined to an

extremely thin region at the body's surface. This region is called boundary layer. The fact that the boundary layer is relatively thin compared to the dimensions of the body allows approximating flows over curved surfaces as a flow over a flat plate using a cartesian coordinate system like shown in Figure 2.3.

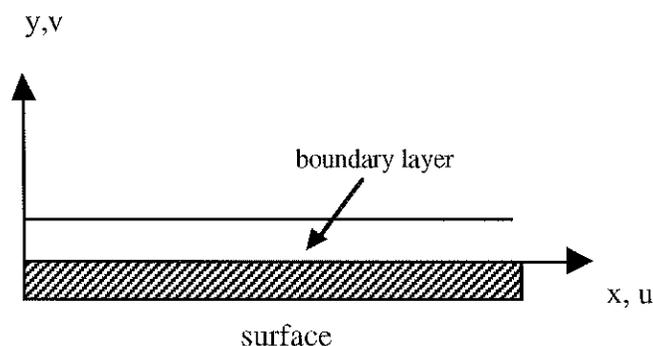


Figure 2.3 Coordinate System for the Boundary Layer

A fundamental assumption of Boundary Layer Theory is that the fluid immediately adjacent to the body is at rest relative to the body and that its temperature is equal to the surface temperature of the body. There are different kinds of boundary layers: The momentum, the temperature and the concentration boundary layer. Where the region, where the fluid conditions change from the values at the body's surface to the free-stream conditions is referred to as the boundary layer. The different boundary layers vary in thickness.

Applying the usual assumptions the differential equations for mass diffusion within the boundary layer can be represented in the following system of differential equations for water vapor in air:

continuity:
$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \quad (2.2.1)$$

momentum:
$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial v}{\partial y} - \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) + \frac{\partial p}{\partial x} = 0 \quad (2.2.2)$$

mass diffusion:
$$\rho u \frac{\partial m_w}{\partial x} + \rho v \frac{\partial m_w}{\partial y} - \frac{\partial}{\partial y} \left(\rho D_w \frac{\partial m_w}{\partial y} \right) = 0 \quad (2.2.3)$$

energy equation:
$$\rho u \frac{\partial i}{\partial x} + \rho v \frac{\partial i}{\partial y} - \frac{\partial}{\partial y} \sum_j \rho D_{wj} \frac{\partial m_j}{\partial y} - \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) = 0 \quad (2.2.4)$$

2.2.1 Mass Flux

To determine the mass flux from the product the control volume in Fig. 2.4 is considered.

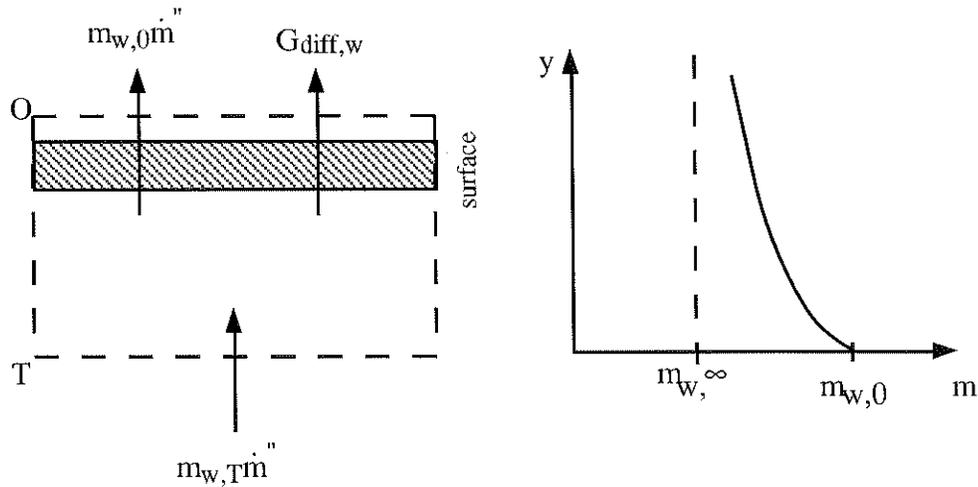


Figure 2.4 Control volume for the mass flux and typical concentration profile

The O-state describes a surface at infinitesimal distance in the fluid phase, while the T-state is a reference state that describes a surface of finite distance within the product. Its principal characteristic is that there are no gradients either of concentration or

temperature. This means that all mass and energy transferred at the T-state is due to fluid flow. The reference state T may be totally fictitious as pointed out in [7]. $m_{w,T}\dot{m}''$ is the mass flux of water vapor from the product. $G_{\text{diff},w}$ is the mass flux across the liquid-vapor interface due to the concentration gradient within the boundary layer. A typical concentration profile is pictured in Fig. 2.4. Performing a mass balance for water on the control volume in Fig. 2.4 results in:

$$m_{w,T}\dot{m}'' = m_{w,0}\dot{m}'' + G_{\text{diff},w} \quad (2.2.5)$$

The water concentration m_w at the reference state T is equal to one, because water is the only substance transferred. The rate of diffusion due to a concentration gradient can be described by Fick's law given by eq. 2.2.6:

$$G_{\text{diff},w} = -\rho_a D_w \frac{\partial m_w}{\partial y} \quad (2.2.6)$$

Substituting eq. 2.2.6 into 2.2.5 yields:

$$\dot{m}'' = \frac{[-\rho_a D_w \frac{\partial m_w}{\partial y}]_0}{m_{w,0} - 1} \quad (2.2.7)$$

To evaluate eq. 2.2.7 the concentration gradient at the product surface must be known. It could be determined by solving eq. 2.2.1 through 2.2.3 .

2.2.2 Energy Flux

A similar procedure like in section 2.2.1 can be performed for the energy flux at the surface. Therefore the control volume in Fig. 2.5 is considered.

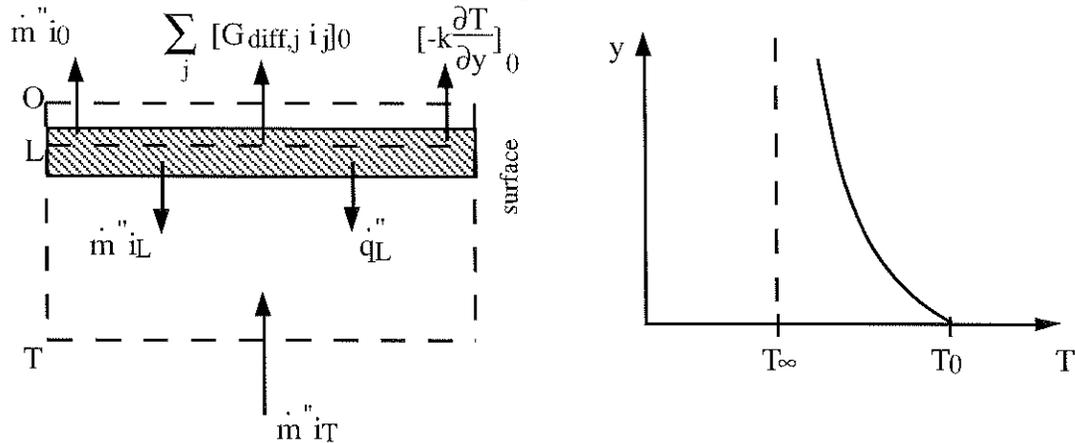


Figure 2.5 Control volume for energy balance and typical temperature profile

The L-state indicates a surface infinitesimal inside the solid phase, where all water is assumed to be liquid. \dot{q}''_L is assumed to be due to conduction, while $\dot{m}'' i_L$ and $\dot{m}'' i_T$ are due to convective energy transport. There is no conduction over the T surface, since all gradients are zero. For the control volume bounded by L and T the energy balance results in:

$$\dot{m}'' i_L - \dot{q}''_L - \dot{m}'' i_T = 0$$

$$\text{or} \quad \dot{q}''_L = \dot{m}'' (i_L - i_T) \quad (2.2.8)$$

At the O-surface it is now necessary to consider the convective transport with the flow

$\dot{m}'' i_0$, the air and water flows due to diffusion $\sum_j [G_{\text{diff},j} i_j]_0$ as well as the energy flux due to conduction $[-k \frac{\partial T}{\partial y}]_0$. Performing the energy balance over the O-T control volume, substituting Fick's law and solving for the mass flux again yields eq. 2.2.9 .

$$\dot{m}'' = \frac{[-k \frac{\partial T}{\partial y}]_0 + \sum_j [-\rho_a D_j i_j \frac{\partial m_j}{\partial y}]_0}{i_0 - i_T} \quad (2.2.9)$$

Again the concentration and temperature gradient at the surface are needed to calculate the mass flux. This could be done by solving the system of differential equations for the boundary layer, now including the energy equation 2.2.4 .

2.2.3 Lewis Number unity Assumption

First the definition of the Lewis number is introduced in eq. 2.2.10, it is the ratio of heat and mass transfer in a certain medium. $Le = \frac{\rho D_j}{\frac{k}{c_p}}$ (2.2.10)

To apply the Lewis number unity assumption the first term in eq. 2.2.9 can be rewritten for ideal gases , using $di_j = c_{pj} dT$ and $\sum_j m_j c_{pj} = m c_p$. The assumption of ideal gas behavior is valid for the binary mixture of water vapor and air

$$[-k \frac{\partial T}{\partial y}] = \frac{k}{c_p} [c_p \frac{\partial T}{\partial y}] = \frac{k}{c_p} [\sum_j c_{pj} m_j \frac{\partial T}{\partial y}] = \frac{k}{c_p} [\sum_j m_j \frac{\partial i_j}{\partial y}] \quad (2.2.11)$$

where:

c_p = specific heat of the mixture at constant pressure

c_{pj} = specific heat of component j at constant pressure

Substituting eq. 2.2.11 into eq. 2.2.9 :

$$\dot{m}'' = \frac{\frac{k}{c_p} \left[\sum_j m_j \frac{\partial i_j}{\partial y} \right]_0 + \sum_j [\rho D_j i_j \frac{\partial m_j}{\partial y}]_0}{i_0 - i_T} \quad (2.2.12)$$

Applying now the Lewis number unity assumption, which is represented by eq. 2.2.13 eq. 2.2.12 becomes 2.2.14

$$Le = \frac{\rho D_j}{\frac{k}{c_p}} = 1 \quad \text{or} \quad \frac{k}{c_p} = \rho D_j \quad (2.2.13)$$

$$\dot{m}'' = \frac{\frac{k}{c_p} \sum_j \left([m_j \frac{\partial i_j}{\partial y}]_0 + [i_j \frac{\partial m_j}{\partial y}]_0 \right)}{i_0 - i_T} \quad (2.2.14)$$

The numerator of eq. 2.2.14 can be rewritten as:

$$\frac{k}{c_p} \sum_j \left([m_j \frac{\partial i_j}{\partial y}]_0 + [i_j \frac{\partial m_j}{\partial y}]_0 \right) = \frac{k}{c_p} \sum_j \frac{\partial}{\partial y} (m_j i_j) = \frac{k}{c_p} \frac{\partial i}{\partial y} \quad (2.2.15)$$

Thus eq. 2.2.14 becomes:

$$\dot{m}'' = \frac{[\frac{k}{c_p} \frac{\partial i}{\partial y}]_0}{i_0 - i_T} \quad (2.2.16)$$

Although eq. 2.2.9 is greatly simplified it is still not possible to calculate the mass flux from eq. 2.2.6 or 2.2.16 because the gradients of concentration or enthalpy are unknown. By comparing eq. 2.2.6 and 2.2.16 one may note that they have the same form and the concept of the conserved property P is introduced and the equations are rewritten in the form of eq. 2.2.17:

$$\dot{m}'' = \frac{[\lambda \frac{\partial P}{\partial y}]_0}{P_0 - P_T} \quad (2.2.17)$$

Where P is either the water concentration or the enthalpy. The problem is now to solve eq. 2.2.17 for various boundary conditions of P . The general profile for P in the boundary layer was shown in Fig. 2.4 and 2.5. Examination of eq. 2.2.17 indicates, that the equation is almost linear in P . Non-linearities may arise due to λ depending on P or temperature influence on the transport properties. Assuming that the gradient at the surface is proportional to the difference of the values of P at the surface and for free stream conditions yields:

$$\lambda \left[\frac{\partial P}{\partial y} \right]_0 = g(P_\infty - P_0) \quad \text{or} \quad g = \frac{\lambda \left[\frac{\partial P}{\partial y} \right]_0}{(P_\infty - P_0)} \quad (2.2.18)$$

Substituting eq. 2.2.18 into 2.2.17 results in:

$$\dot{m}'' = g \frac{P_\infty - P_0}{P_0 - P_T} = gB \quad (2.2.19)$$

where:

$$g = \text{mass transfer conductance (kg/m}^2\text{s)}$$

$$B = \frac{P_\infty - P_0}{P_0 - P_T} = \text{mass transfer driving force (-)} \quad (2.2.20)$$

For the energy flow one obtains:

$$\dot{m}'' = g \frac{i_\infty - i_0}{i_0 - i_T} \quad (2.2.21)$$

Since i_T is unknown eq. 2.2.21 can still not be solved. Because of the similarity of mass and heat transfer for Lewis number of unity the mass transfer conductance and the heat transfer coefficient are related in the following manner:

$$g = \frac{h}{c_p} \quad (2.2.22)$$

For large rates of mass transfer the following correction should be made [7]:

$$g = g^* \left[\frac{\ln(1 + B)}{B} \right], \text{ where } g^* = \frac{h}{c_p} \quad (2.2.23)$$

Where h is the heat transfer coefficient for no mass flux at the surface.

For the mass flux:

$$\dot{m}'' = g \frac{m_{w,\infty} - m_{w,0}}{m_{w,0} - m_{w,T}} \quad (2.2.24)$$

Eq. 2.2.24 can be solved because $m_{w,T}$ is known to be unity. Now that the mass flux from the product can be calculated it is also possible to calculate the heat flux to the product. Therefore eq. 2.2.8: $\dot{q}_L'' = \dot{m}''(i_L - i_T)$ is recalled. To eliminate i_T , eq. 2.2.20 is written in terms of enthalpies and solved for i_T :

$$i_T = i_0 - \frac{i_\infty - i_0}{B} \quad (2.2.25)$$

Then eq. 2.2.8 becomes:

$$\dot{q}_L'' = \dot{m}'' \left(i_L - i_0 - \frac{i_\infty - i_0}{B} \right) \quad (2.2.26)$$

From eq. 2.2.19:

$$\frac{\dot{m}''}{B} = g = \frac{h}{c_p} \quad (2.2.27)$$

Substituting eq. 2.2.27 into 2.2.26 results in eq. 2.2.28:

$$\dot{q}_L'' = \dot{m}''(i_L - i_0) - \frac{h}{c_p}(i_\infty - i_0) \quad (2.2.28)$$

Where the enthalpy of water can be calculated using eq. 2.2.29 [7].

$$i_w = c_p(T - T_{ref}) + i_{fg} \quad (2.2.29)$$

The reference state is chosen such that the enthalpy of liquid water is zero. Then let $i_{w,0} = 0$ at $T_{ref} = T_0$, then $i_L = i_{fg,0}$ and $i_\infty = c_p(T_\infty - T_0)$. Substituting these definitions into eq. 2.2.28 yields:

$$\dot{q}_L'' = h(T_\infty - T_0) - \dot{m}'' i_{fg,0} \quad (2.2.30)$$

2.3 REPRESENTATION OF THE EQUILIBRIUM AT THE PRODUCT SURFACE

Both heat and mass transfer are modeled within the product and within the boundary layer. Thus the missing part in the model is to account for the phase transition of the moisture from within the product to the air. A graph or a mathematical expression relating the moisture content of a medium to the relative humidity is referred to as an isotherm. To keep the model simple the first attempt was to describe the relation between moisture content of the product and the relative humidity of the surrounding air in a linear fashion. Since this does not agree with the data reported in [12] a non-linear isotherm was used in the model.

2.3.1 Approximation of the Saturation Pressure of Water

As mentioned isotherms express the relation between moisture content of a medium and relative humidity where the relative humidity of water in air is defined as:

$$\phi_{s,a} = \frac{P_w}{P_{sat}(T_s)} \quad (2.3.1)$$

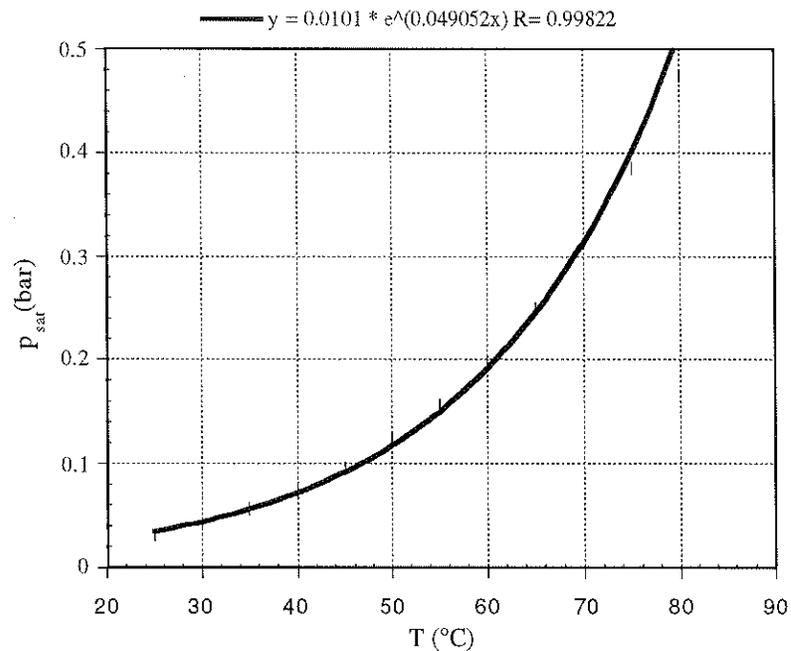


Figure 2.6 Saturation pressure of pure water versus temperature

Since the saturation pressure changes tremendously with temperature the simulation program must take this into account. Therefore data from [13] were approximated by an exponential function shown in eq. 2.3.2. Figure 2.6 shows that the curve represents the data very well for the temperature range in question.

$$P_{sat} = 0.101 * \exp(0.049052 * T) \quad (2.3.2)$$

2.3.2 Using a Linear Isotherm

For sake of simplicity it was assumed that the relation between moisture content of the food product and the relative humidity of air can be represented using a linear isotherm as shown in eq 2.1.14 :

$$\phi_{s,a} = c_w X_{s,hd} \quad (2.1.14)$$

Where c_w is the equilibrium constant between moisture content and relative humidity. Knowing the partial pressure of water for the product temperature from eq. 2.3.2 the humidity ratio of air can be calculated from

eq. 2.3.3:

$$\omega_s = 0.622 \frac{P_w}{P - P_w} \quad (2.3.3)$$

The water concentration is then given by eq. 2.3.4:

$$m_s = m_{w,0} = \frac{\omega_s}{1 + \omega_s} \quad (2.3.4)$$

Now knowing $m_s = m_{w,0}$ the mass flux from the product can be calculated using eq. 2.2.23.

A review of literature as well as comparing the simulation data to the experimental data in [12] show that the linear isotherm does not hold.

2.3.3 Food Product Isotherm

In [6] Iglesias and Chirife report isotherms for all kinds of foodstuffs, including meat . In [5] Igbeka and Blaisdell report isotherms for processed meat products, namely bologna, at different temperatures. All these data show that the isotherms for the food product in question are sigmoidal in shape.

The isotherms for foodstuff are usually reported in terms of moisture content versus water activity. The water activity accounts for the deviations of the partial pressure of water over the foodstuff compared to that over pure water. Thus the water activity can be represented by the ratio of the fugacities for the product and for pure water at a given temperature:

$$a_w = \frac{f}{f_0} = C \frac{P}{P_0} = C \phi \quad (2.3.6)$$

The deviation between f/f_0 and p/p_0 is less than two per thousand [4]. Thus the relative humidity was used to model the product equilibrium. However the water activity has its own significance. It is a good indicator for the stability of foodstuffs because it measures the availability of water for reactions. As a reference for the measurement of the moisture content a dry basis is chosen, which is reasonable since the overall mass changes tremendously during the cooking process due to the moisture loss. The problem of modeling the surface equilibrium with an isotherm is that isotherms are dependent on temperature and the surface temperature of the food product varies over a wide range in the cooking process. Most of the research on isotherms is done with regard to drying applications and there are some relations to account for temperature changes for the most commonly used types of isotherms. These relations are not applicable to the cooking process due to the higher moisture content of the food product. Igbeka and Blaisdell use a Henderson isotherm to describe the isotherms of bologna. The Henderson isotherm

uses two parameters that vary with temperature and moisture content. This variation makes it cumbersome to use in a simulation program.

An isotherm that can account for temperature changes continuously is the Dubinin-Polanyi isotherm [2]. It uses the adsorption potential theory proposed by Polanyi. He arrived at a theoretical relationship between average potential energy of adsorbed molecules and adsorbed amount using kinetic gas theory. The adsorption potential is defined as:

$$A = -RT \ln \left(\frac{P}{P_0} \right) = \text{const. for } X = \text{const.} \quad (2.3.7)$$

Using the differential heat of adsorption given by the Clausius-Clapeyron equation:

$$q_{\text{ad}} = -RT^2 \frac{d \ln p}{dT} \Big|_X \quad (2.3.8)$$

it can be shown that:

$$q_{\text{ad}} = \Delta i_{\text{fg}} + A \quad (2.3.9)$$

where Δi_{fg} is the latent heat of condensation of the pure liquid. Thus the adsorption potential is the difference in free energy between adsorbed phase and saturated liquid phase of the adsorptive. The expression for the amount adsorbed as a function of A is called characteristic curve:

$$X = f(A) \quad (2.3.10)$$

Dubinin showed that the characteristic curve can often be expressed by:

$$X = X_0 \exp\left(-\left(\frac{A}{E_0}\right)^n\right) \quad (2.3.11)$$

where:

E_0 = characteristic energy of adsorption

For heterogeneous pore distributions there may be different volumes and characteristic energies of adsorption and eq. 2.3.11 becomes:

$$X = X_{0,1} \exp\left(-\left(\frac{A}{E_{0,1}}\right)^n\right) + X_{0,2} \exp\left(-\left(\frac{A}{E_{0,2}}\right)^n\right) \quad (2.3.12)$$

For the application in modeling the equilibrium of the food product in the cooking process two assumptions were made. First the exponent n was assumed to be unity. To account for the fact that most foodstuff is hygroscopic the second term was assumed to be equal to the minimum moisture content X_{\min} at equilibrium with dry air and thus the characteristic energy is a large finite number, such that eq. 2.3.12 can be written as:

$$X = X_{0,1} \exp\left(-\frac{A}{E_{0,1}}\right) + X_{\min} \quad (2.3.13)$$

Using this expression it is possible to generate isotherms similar in shape to those reported in [5]. An example for such isotherms are shown in Fig. 2.7, where Dubinin-Polanyi isotherms are plotted for different characteristic energies at different temperatures.

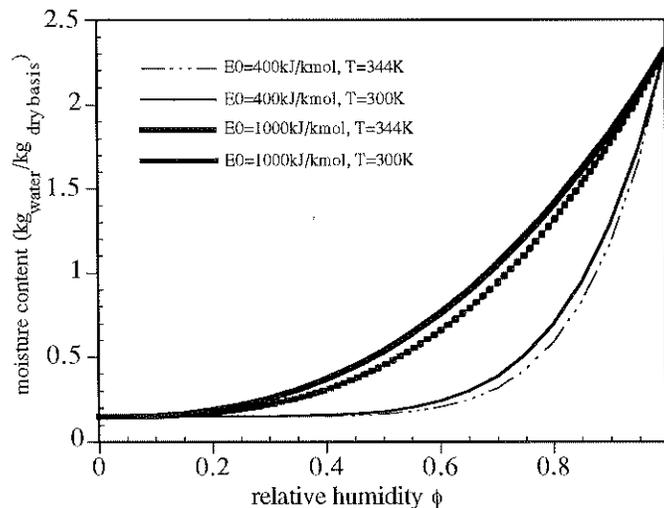


Figure 2.7 Examples of Dubinin-Polanyi isotherms

The validity of the Dubinin-Polanyi isotherm for foodstuff needs to be examined in future work and in case of failure be replaced, e.g. by the Henderson isotherm.

Rewriting eq. 2.3.12 and 2.3.6, the partial pressure of water vapor can be expressed in terms of the moisture content of the food product:

$$p_w = p_{\text{sat}} \exp\left(\frac{E_{0,1}}{RT} \left(\frac{X_s - X_{\text{min}}}{X_{0,1}}\right)\right) \quad (2.3.14)$$

where p_{sat} is determined using eq. 2.3.2 and $X_{0,1}$ is equal to the initial moisture content minus the minimum moisture content of the product X_{min} . The water concentration of the air can then be calculated using eq. 2.3.5 and 2.3.6.

2.4 FINITE DIFFERENCE METHOD

The equations derived in the previous sections are now brought together to provide a complete heat and mass transfer model of the food product. Since there are no analytical solutions for the combined conduction and diffusion problem, the derivatives in the partial differential equations are approximated by finite differences. Therefore the food product was broken in a number of nodes as shown in Fig. 2.8:

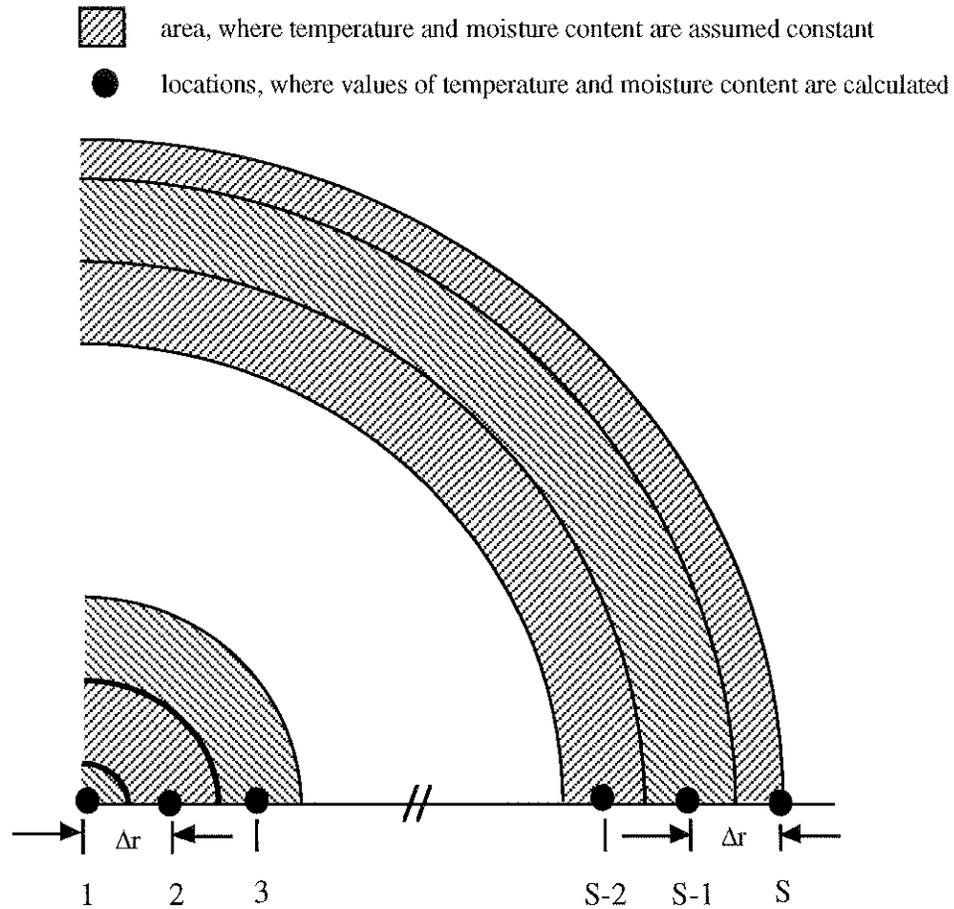


Figure 2.8 Finite difference representation of the product

In this work the number of values of temperature and moisture content were chosen to be equal to 40 both for temperature and moisture content.

2.4.1 Approximating the Derivatives

There are different ways of approximating derivatives. Approximations for the spatial derivatives with second order accuracy are used. This is done by performing central differences, as shown in eq. 2.4.1 and 2.4.2:

$$\frac{\partial y}{\partial r} = \frac{y_{n+1}^p - y_{n-1}^p}{2\Delta r} + O(\Delta r^2) \quad (2.4.1)$$

$$\frac{\partial^2 y}{\partial r^2} = \frac{y_{n+1}^p - 2y_n^p + y_{n-1}^p}{\Delta r^2} + O(\Delta r^2) \quad (2.4.2)$$

where:

y = either temperature T or moisture content X

The approximation of the time derivative is performed as a forward difference, as expressed in eq. 2.4.3 .

$$\frac{\partial y}{\partial t} = \frac{y_n^{p+1} - y_n^p}{\Delta t} + O(\Delta t) \quad (2.4.3)$$

Doing this one obtains explicit finite difference equations, where only known values of temperature and moisture content are used for the calculation of y_n^{p+1} . Thus no iterations are needed and the calculations are straight forward. However the time steps for an explicit calculation are smaller than that for an implicit version, where the value of y_n^{p+1} is determined by iterations. This reduces the savings of calculation time for the explicit version to some extent. For the inner nodes replacing the derivatives with differences is straight forward, while for the center node and the surface node energy and water balances must be performed to account for the boundary conditions.

2.4.2 Inner Nodes

For the inner nodes eq. 2.4.1 through 2.4.3 are substituted into the differential equations 2.1.5 and 2.1.6 and solved for the unknown values of temperature and moisture content, as shown in eq. 2.4.4 and 2.4.5 respectively:

$$T_n^{p+1} = \frac{\Delta t k}{2 \rho_d (1 + X_n^p) C r_n \Delta r} (T_{n+1}^p - T_{n-1}^p) + \frac{\Delta t k}{\rho_d (1 + X_n^p) C \Delta r^2} (T_{n+1}^p - 2T_n^p + T_{n-1}^p) + T_n^p \quad (2.4.4)$$

$$X_n^{p+1} = \frac{\Delta t D}{2 r_n \Delta r} (X_{n+1}^p - X_{n-1}^p) + \frac{\Delta t D}{\Delta r^2} (X_{n+1}^p - 2X_n^p + X_{n-1}^p) + X_n^p \quad (2.4.5)$$

These equations are valid only for constant k , D , C and ρ . Mittal et al. report in [10] that the diffusivity changes over some orders of magnitude for changing temperature and especially for changing moisture content. It is thus desirable to account for this effect and modify eq. 2.1.6 and 2.4.5 accordingly. The partial differential equation for the diffusion of moisture can be expressed in terms of variable diffusivity by eq. 2.4.6:

$$\frac{\partial X}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} (r D \frac{\partial X}{\partial r}) \quad (2.4.6)$$

Approximating the derivatives and solving for the X_n^{p+1} yields:

$$X_n^{p+1} = \frac{\Delta t}{r_n \Delta r} \left(r_{n+\frac{1}{2}} D^{(n+\frac{1}{2})} \frac{X_{n+1}^p - X_n^p}{\Delta r} - r_{n-\frac{1}{2}} D^{(n-\frac{1}{2})} \frac{X_n^p - X_{n-1}^p}{\Delta r} \right) + X_n^p \quad (2.4.7)$$

2.4.3 Center Node

For the center node a energy and moisture balance must be performed to account for the

boundary conditions that $\frac{\partial T}{\partial r} \Big|_{r=0} = 0$ and $\frac{\partial X}{\partial r} \Big|_{r=0} = 0$ respectively. The control volume for

the balances is sketched in Fig. 2.9:

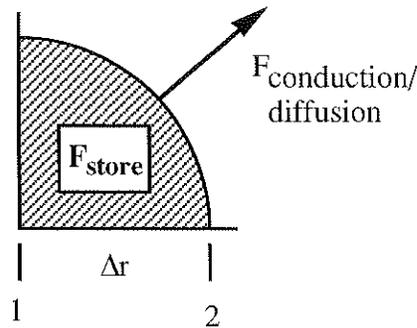


Figure 2.9 Representation of the center node, where F is either an energy flux or mass flux

The balances yield the following equations:

$$\dot{E}_{\text{store}} = - \dot{E}_{\text{conduction}} \quad (2.4.8)$$

and

$$\dot{m}_{\text{store}} = - \dot{m}_{\text{diffusion}} \quad (2.4.9)$$

Substituting an expression for the energy storage within the node and Fourier's law for the conduction term into eq. 2.4.8. Since the moisture content in the center remains fairly constant changes in the energy storage due to varying moisture content were not taken into account.

$$\rho_d C (1 + X) V_{12} \frac{\partial T}{\partial t} = k A_2 \left. \frac{\partial T}{\partial r} \right|_2 \quad (2.4.10)$$

Where the spatial derivative is approximated by the central difference for the second node, which is:

$$\left. \frac{\partial T}{\partial r} \right|_2 = \frac{T_3^p - T_1^p}{2\Delta r} \quad (2.4.11)$$

Substituting eq 2.4.11 into 2.4.10 and solving for T_1^{p+1} :

$$T_1^{p+1} = \frac{\Delta t k A_2}{2 \rho_d (1 + X_1^p) C V_{12} \Delta r} (T_3^p - T_1^p) + T_1^p \quad (2.4.12)$$

Substituting Fick's law and an expression for the moisture storage into the moisture balance eq. 2.4.9 can be written as:

$$\rho_d V_{12} \frac{\partial X}{\partial t} = \rho_d D A_2 \left. \frac{\partial X}{\partial r} \right|_2 \quad (2.4.13)$$

Replacing the derivatives according to eq. 2.4.3 and the equivalent of eq. 2.4.11 for moisture content, the unknown moisture content X_1^{p+1} can be expressed by:

$$X_1^{p+1} = \frac{\Delta t D A_2}{2 V_{12} \Delta r} (X_3^p - X_1^p) + X_1^p \quad (2.4.14)$$

For variable diffusivities the values of D are calculated for the conditions at $r = \frac{1}{2} \Delta r$.

2.4.4 Surface Node

Similar to the center node an energy and moisture balances must be performed for the surface node to account for the boundary conditions. Therefore the control volumes shown in Fig. 2.10 were considered.

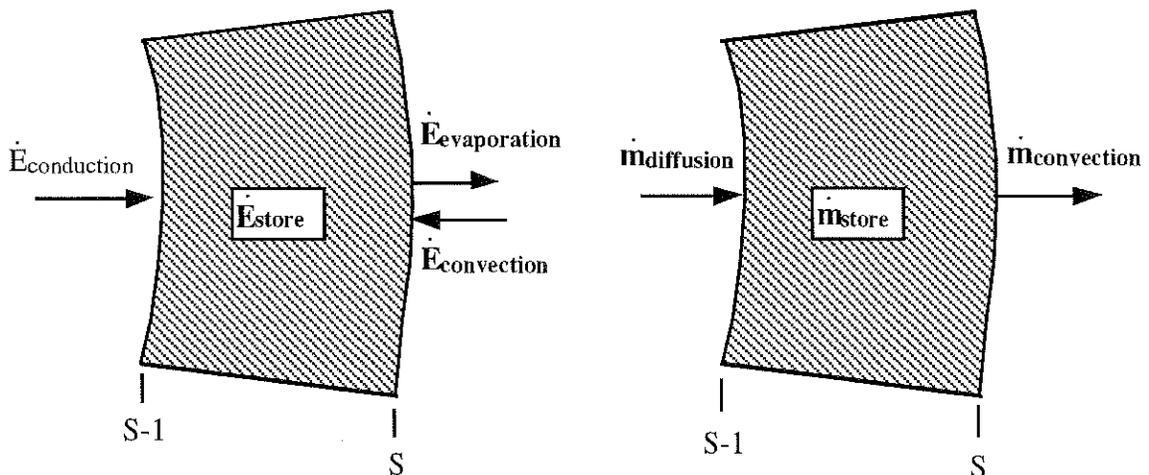


Figure 2.10 Representation of the surface node for energy and mass balances.

The balances result in the following equations:

$$\dot{E}_{\text{store}} = \dot{E}_{\text{conduction}} + \dot{E}_{\text{convection}} - \dot{E}_{\text{evaporation}} \quad (2.4.15)$$

$$\dot{m}_{\text{store}} = \dot{m}_{\text{diffusion}} - \dot{m}_{\text{convection}} \quad (2.4.16)$$

where $\dot{E}_{\text{evaporation}}$ is the amount of energy needed to evaporate the water that is transferred from the food product. For the surface node the expression for the energy storage has to account for changes of the moisture content. This is illustrated by eq. 2.4.17:

$$\frac{\partial}{\partial t} (\rho_d (1 + X) C V_S T) = \rho_d C V_S \left(T \frac{\partial X}{\partial t} + (1 + X) \frac{\partial T}{\partial t} \right) \quad (2.4.17)$$

where the term $\rho_d (1 + X)$ represents the whole mass of the surface node including water.

Substituting this expression and appropriate expressions for the other terms eq. 2.4.15 turns into:

$$\rho_d C V_S \left(T \frac{\partial X}{\partial t} + (1 + X) \frac{\partial T}{\partial t} \right) = -k A_{S-1} \frac{\partial T}{\partial r} \Big|_{S-1} + A_S h (T_\infty - T_S) - \dot{m}_{\text{convection}} \Delta i_{fg} \quad (2.4.18)$$

where $\dot{m}_{\text{convection}}$ can be calculated using eq. 2.2.24, then one obtains:

$$\dot{m}_{\text{convection}} = A_S \dot{m}'' = A_S g \frac{m_{w,\infty} - m_{w,0}}{m_{w,0} - m_{w,T}} \quad (2.4.19)$$

The spatial derivative is again approximated using a central difference now at the S-1 node. This is:

$$\frac{\partial T}{\partial r} \Big|_{S-1} = \frac{T_S^p - T_{S-2}^p}{2\Delta r} \quad (2.4.20)$$

Using 2.4.3 and 2.4.20 and solved for the unknown surface temperature T_S^{p+1} eq. 2.4.18 becomes:

$$T_S^{p+1} = \frac{\Delta t k A_{S-1}}{2 \rho_d (1 + X_S^p) C V_S \Delta r} (T_{S-2}^p - T_S^p) + \frac{\Delta t h A_S}{\rho_d (1 + X_S^p) C V_S} (T_\infty - T_S^p) + \frac{\Delta t \dot{m}_{\text{convection}} \Delta i_{fg}}{\rho_d (1 + X_S^p) C V_S} - \frac{T(X_S^{p+1} - X_S^p)}{(1 + X_S^p)} + T_S^p \quad (2.4.21)$$

The simulation results showed a final temperature difference of one °C between the version where the moisture loss in the surface node was taken into account and the version where the moisture content in the surface node was assumed to be constant. Thus the results where moisture loss in the surface node is accounted for are shown in the remainder of this work. No attempt was made to account for the effects of moisture migration on the energy balance within the product because the moisture differences are much smaller there and the effect at the surface node was not that strong.

Substituting the appropriate terms into eq. 2.4.16 yields:

$$\rho_d V_S \frac{\partial X}{\partial t} = -\rho_d D A_{S-1} \left. \frac{\partial X}{\partial r} \right|_{S-1} - \dot{m}_{\text{convection}} \quad (2.4.22)$$

Replacing the derivatives by the differences given by eq. 2.4.3 and the equivalent of eq. 2.4.19 for moisture content and solving for the X_S^{p+1} eq. 2.4.21 becomes:

$$X_S^{p+1} = \frac{\Delta t D A_{S-1}}{2 V_S \Delta r} (X_{S-2}^p - X_S^p) + \frac{\Delta t g A_S}{\rho_d V_S} \frac{m_{w,\infty} - m_{w,0}}{m_{w,0} - m_{w,T}} \quad (2.4.23)$$

The above equations were used to write a FORTRAN program to simulate the behavior of the food product for constant ambient conditions. The program can be found in the appendix.

CHAPTER
THREE

ESTIMATION OF PROPERTIES AND
INITIAL CONDITIONS

The estimation of the transport and thermal properties of the foodstuff is a complex task. This is because of the heterogeneous mixture of the food product and the changes of temperature and composition that occur during the cooking process. The changes of density, specific heat, thermal conductivity and diffusivity due to the changes in composition and temperature are of different magnitudes. For density, specific heat and thermal conductivity these changes are relative small and their changes do not have the same impact on the thermal behavior of the product. Thus these properties are treated as constants, as is done in the first part of this chapter. In the second part the estimation of the diffusivity and a correlation for variable diffusivities as a function of temperature, fat-protein ratio and moisture content are described. The third part deals with the estimation of the initial conditions of the food product.

3.1 ESTIMATION OF CONSTANT PROPERTIES

During the cooking process a high percentage of the moisture in the food product is evaporated, especially in the surface region. This change in composition also changes the properties of the food product. Since water is the only substance transferred the density can be treated as a constant if it is expressed in terms of kg dry matter per unit volume. This is also convenient because the moisture content is expressed in kg water per kg dry matter. Because the temperature of the food product is mainly determined by the amount of energy needed for the latent heat of evaporation than by the amount needed for the sensible temperature changes of the food product itself. Thus deviations in the thermal properties cause relatively small errors in the simulation of the product temperature and the assumption that the thermal properties, specific heat and thermal conductivity remain constant is made. To estimate the constant properties two initial composition of the food product were assumed:

	composition 1	composition 2
Moisture content:	70%	60%
lipids (fats):	15%	15%
protein:	15%	25%

Table 3.1 Initial composition of the food product

Gekas reports in [3] characteristic values of density, specific heat and thermal conductivity for these components as shown in Table 3.2:

	ρ (kg/m ³)	C (kJ/kgK)	k (W/mK)
water	1000	4.182	0.6
lipids (fats)	1380	1.55	0.2
protein	930	1.67	0.18

Table 3.2 Properties of food components

Using these values and summing over the components according to their concentration the following values for density and specific heat are obtained for the different compositions. The summation over the components is not possible for the thermal conductivity, which value is used depends on the model that is assumed for the conductive transport of energy. Because of the high concentration of water in the food product the model shown in Fig. 3.1 is assumed and the thermal conductivity of water can be used for the entire product because water is present everywhere.

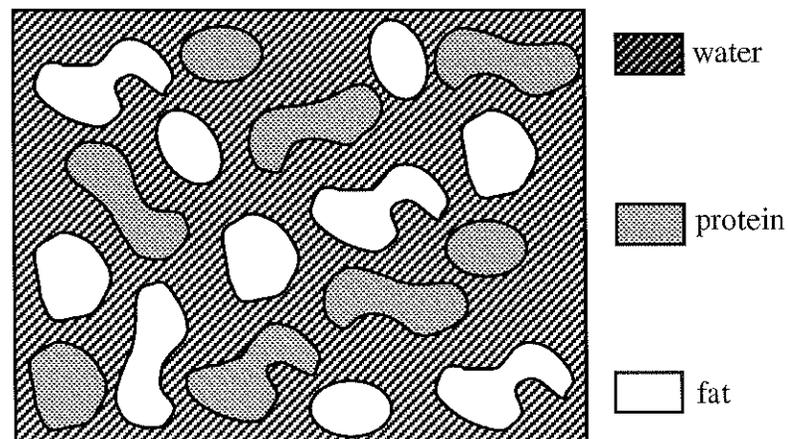


Figure 3.1 Model for the thermal conductivity

	composition 1	composition 2
density	$\rho = 1046 \text{ kg/m}^3$	$\rho = 1020 \text{ kg/m}^3$
specific heat	$C = 3.41 \text{ kJ/kgK}$	$C = 3.16 \text{ kJ/kgK}$
thermal conductivity	$k = 0.6 \text{ W/mK}$	$k = 0.6 \text{ W/mK}$

Table 3.3 Properties for different compositions of the food product

To transform the density into a density in terms of dry matter the concentrations of the dry components have to be used. Fat and protein are referred to as dry components, thus for the first and the second composition 30% and 40% of the weight respectively is assumed to be dry. Then the density in terms of dry matter ρ_d is equal to 30% and 40% respectively of the density ρ . If the density is expressed in terms of dry matter the overall mass (including water) of a volume element is given by the following equation:

$$m = \rho_d (1 + X) V \quad (3.1.1)$$

For some of the earlier versions of the simulation program the diffusivity was assumed to be constant. The values for the constant diffusivities were taken from Gekas [3] and are values that were measured for meat at moisture contents close to the initial moisture content of the food product. The values reported in [3] for beef lie in the range from 1×10^{-11} to 5×10^{-10} . Thus values of 1×10^{-11} and 1×10^{-10} were used in the program.

Thus the following constant properties are used for the different compositions in the simulation program:

	composition 1	composition 2
density:	$\rho_d = 314 \text{ kg}_d/\text{m}^3$	$\rho_d = 434 \text{ kg}_d/\text{m}^3$
specific heat:	$C = 3.41 \text{ kJ/kgK}$	$C = 3.16 \text{ kJ/kgK}$
thermal conductivity:	$k = 0.6 \text{ W/mK}$	$k = 0.6 \text{ W/mK}$
diffusivity:	$D = 1 \times 10^{-11} - 1 \times 10^{-10} \text{ m}^2/\text{s}$	$D = 1 \times 10^{-11} - 1 \times 10^{-10} \text{ m}^2/\text{s}$

Table 3.4 Properties of the food product used in the simulations

3.2 ESTIMATION OF VARIABLE PROPERTIES

As stated previously, the amount of water evaporated mainly influences the temperature of the food product. The amount of water evaporated depends mainly on the diffusivity of moisture within the food product. This is because the mass transfer Biot number for the food product is on the order of 100 and thus the transport within the product is the main resistance. Thus it is important to account for the changes of the diffusivity with moisture content. This is done by using an expression reported by Mittal et al. in [10] shown in eq. 3.2.1:

$$D = 0.0029 \exp(-0.4419 \text{ FP} - \frac{4892.7}{T} + 11.51 \text{ C}) \quad (3.2.1)$$

where:

FP = Fat-protein ratio (-)

T = Temperature (K)

C = dimensionless moisture concentration defined as:

$$C = \frac{X - X_e}{X_0 - X_e} \quad (3.2.2)$$

where: X_e = equilibrium moisture content

The above equation was obtained through measurements using a slab configuration for moisture losses up to 6% and has a coefficient of determination of 0.88. It is further reported in [10] that the moisture transport at least till 58°C is by Fickian diffusion. A plot of the diffusivity for different temperatures and moisture content is shown in Fig. 3.2:

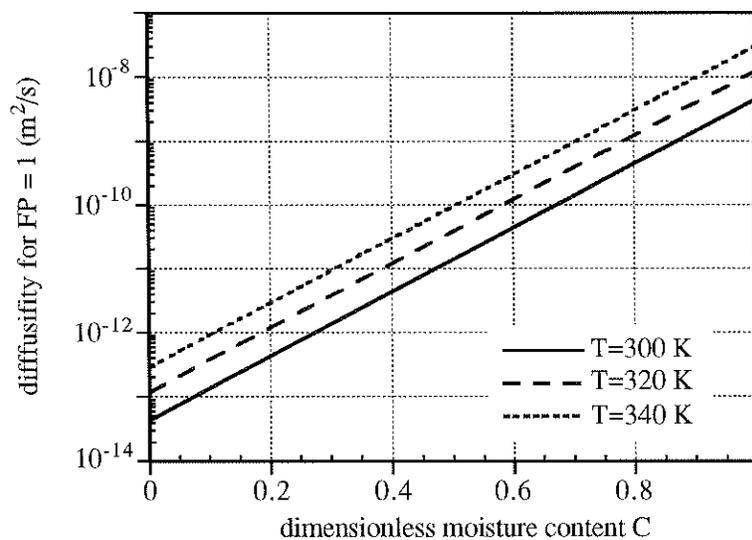


Figure 3.2 Moisture diffusivities as a function of temperature T and moisture content C for a fat-protein ratio of one

Expressions for the diffusivity in processed meat products for a slab and a cylindrical configuration as a function of fat-protein ratio and temperature only are given in [11]. Since the diffusivity changes for different moisture content over some orders of magnitude, as shown in Fig. 3.2, the expression of eq. 3.2.1 was used in the simulation program.

3.3 INITIAL CONDITIONS

The initial temperature of the food product was chosen to be slightly above room temperature, namely 27°C. The initial moisture content was given by the assumed initial composition. Using eq. 3.3.1 the initial moisture content in kg water per kg dry matter was computed from eq. 3.3.1 for the two initial compositions assumed above:

$$X = \frac{W}{1 - w} \quad (3.3.1)$$

For the first composition one obtains a value of $X_0 = 2.33$ kg water / kg dry matter and for the second $X_0 = 1.5$ kg water / kg dry matter

CHAPTER
FOUR

RESULTS OF THE SIMULATIONS

In this chapter the results for the different versions of the simulation program are presented to show the development of the model. The most significant measure for the validation of the model is the surface temperature, and the results will be presented in plots of the surface temperature versus time, in the plots the wet and dry bulb temperatures will be shown as bold lines. First the erf-solution from section 2.1.3 is compared to the finite difference solution for the moisture content. Then the surface temperature for the version where the equilibrium at the surface is represented by an equilibrium constant is compared to results where no water evaporation was taken into account. In the next section the results for the version where a Dubinin-Polanyi isotherm was used to represent the surface equilibrium are compared to those for the equilibrium constant. Finally the model which accounts for the variable diffusivity and uses the isotherm is compared to the that using constant diffusivity and the isotherm.

In the simulation program constant ambient conditions were used, but it can be modified for variable ambient conditions quiet easily. From [12] the ambient temperature was

assumed to be 71°C with a wet-bulb temperature of 39°C , which is equivalent to a humidity ratio of $\omega = 0.032$. The humidity ratio can be transformed into a water concentration using eq. 2.3.4 , which is $m_{w,\text{inf}} = 0.031$. In [12] the heat transfer coefficient h was determined and this work uses an average value that is $17.5 \text{ W/m}^2\text{K}$.

4.1 VALIDATION OF FINITE DIFFERENCES APPROXIMATION

To compare the analytical solution to the finite difference approximation the same assumptions as for the erf-solution were made. These are isothermal conditions of the food product, representation of the equilibrium by a constant. However for the finite difference solution the convective transport within the boundary layer was taken into account. Thus this comparison shows the influence of the boundary layer on the amount of water transferred. The results for the analytical solution are shown in Fig. 4.1, those for the finite difference solution in Fig 4.2:

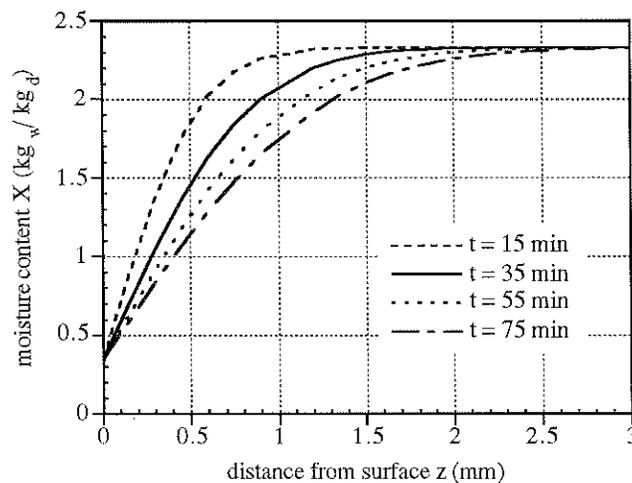


Figure 4.1 Erf-solution for the moisture content of the food product

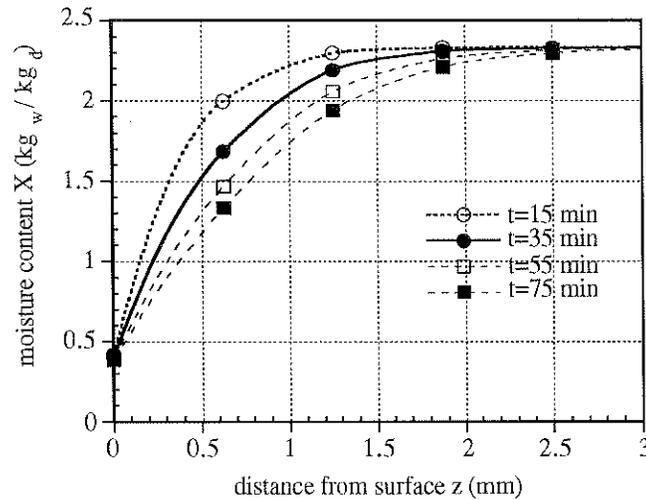


Figure 4.2 Finite difference solution for the moisture content of the food product

The results show that the penetration depth is about equal for both solutions and that the values at certain times are quite similar. The similarity between the solutions implies that the finite difference approximation represents the moisture content history in the food product accurately. Since the penetration depth is relatively small compared to the diameter of the food product the assumptions made to develop the erf-solution are justified.

4.2 LINEAR ISOTHERM MODEL

The first version of the program used a linear isotherm to account for the equilibrium at the surface. In Fig 4.3 results from this version are compared to results where no water evaporation was taken into account.

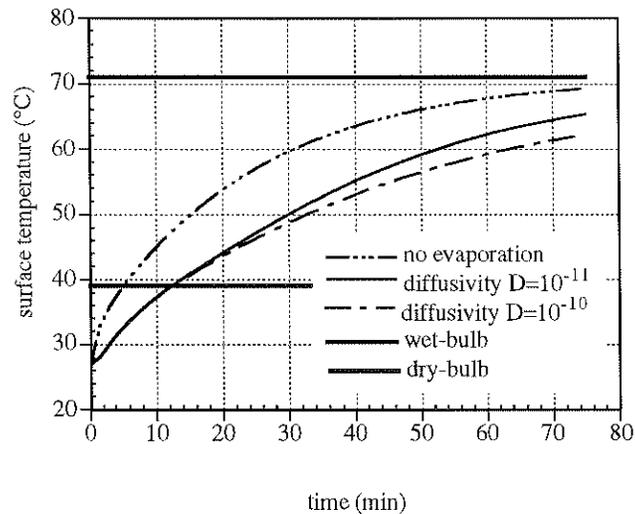


Figure 4.3 Surface temperature history with evaporation

(at diffusivities of $D = 10^{-10} \text{ m}^2/\text{s}$ and $D = 10^{-11} \text{ m}^2/\text{s}$) and without evaporation

Fig 4.3 shows that the water evaporation significantly influences the temperature of the food product. Thus it is important to account for the amount of water evaporated accurately. Fig 4.3 shows too that the version using an equilibrium constant does not account for the water transport accurately, because the surface temperature approaches directly the dry-bulb temperature and does not show the behavior reported in [12]. This behavior shows the following pattern as shown in section 1.2.3: The surface temperature reaches the wet-bulb temperature rather quickly and remains there for some time and then approaches the dry-bulb temperature. Since the version using an equilibrium constant only slows down the increase of temperature it is not able to meet the actual data and a new representation of the equilibrium had to be used.

4.3 DUBININ-POLANYI ISOTHERM MODEL

The first version using an equilibrium constant could not represent the data correctly. Thus in the next version the surface equilibrium was represented by an Dubinin-Polanyi isotherm. Using this isotherm the model has three parameters, the diffusivity of moisture in the food product and the characteristic energy of adsorption and the minimum moisture content for the isotherm. Fig. 4.4 shows plots for different values of these parameters.

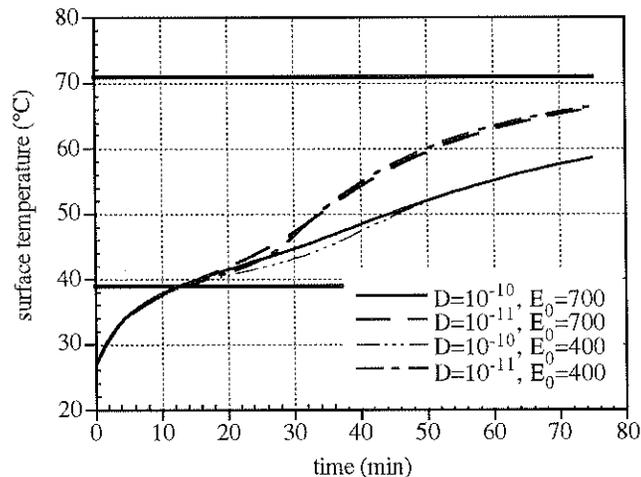


Figure 4.4 Surface temperature using an isotherm with a minimum moisture content of $0.05 \text{ kg}_w / \text{kg}_d$ and constant diffusivities

Fig. 4.4 shows that for this version the typical temperature history is obtained. This is especially true for lower values of the characteristic energy of adsorption. The temperature crosses the wet-bulb temperature in a slightly sloped plateau. As shown in Fig. 4.5 the slope of the plateau decreases with decreasing values of the minimum moisture content and increasing diffusivity.

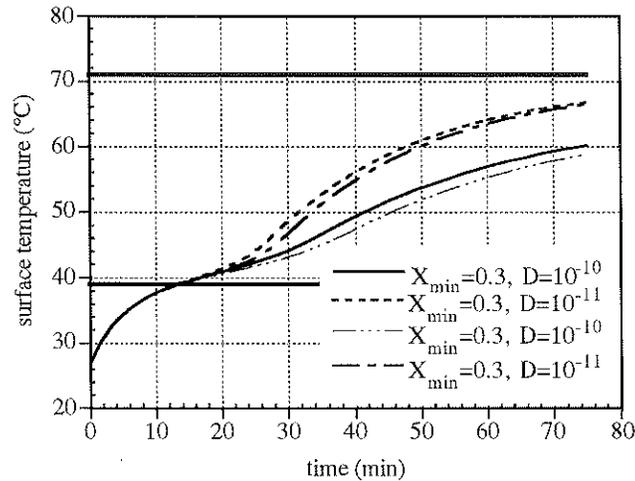


Figure 4.5 Surface temperature using an isotherm with an characteristic energy of adsorption of 400 kJ/kmole

This means that the isotherm represents the surface equilibrium well enough that the temperature shows the typical behavior. However the approach towards the dry-bulb temperature is rather slow. This could be due to the fact that the moisture diffusivity decreases with decreasing moisture content and does not remain constant as it is assumed for this version of the model. There are two questions that should be addressed in future work. The first is to proof that the Dubinin-Polanyi isotherm is able to represent sorption data for the food product. Second the influence of the casing that surrounds the food product during cooking should be examined. This could be done in combination with the first point.

4.4 DUBININ-POLANYI ISOTHERM AND VARIABLE DIFFUSIVITY MODEL

In section 4.3, it was found that the approach towards the dry-bulb temperature is not as fast as the data in [12] imply when a constant diffusivity is used. Thus the relation reported by Mittal et al. and introduced in section 3.2 was built in the model to account for the changes of diffusivity with changing temperature and moisture content. The number of parameters is reduced to two because the diffusivity is now determined by other quantities. The parameters remaining are those of the isotherm. Fig. 4.6 and 4.7 show different surface temperature histories for different values of these parameters. In Fig 4.6 the characteristic energy of adsorption is kept constant and the minimum moisture content is changed:

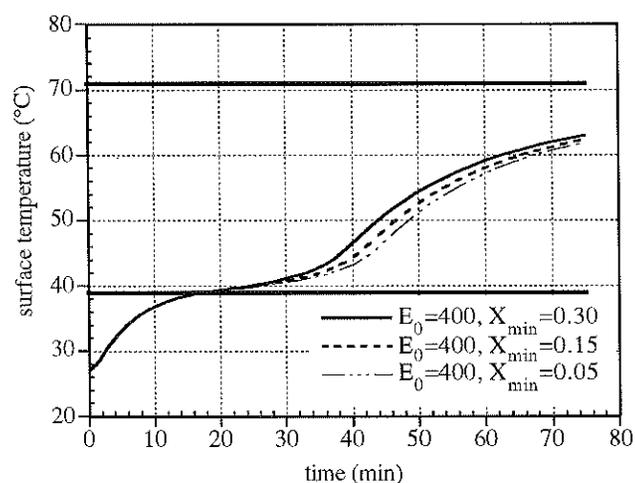


Figure 4.6 Surface temperature for different minimum moisture contents

From the sorption data reported in [5] a minimum moisture content of 0.05 kg water / kg dry matter is the most reasonable for the temperature range in question. For this value the temperature history shows the flattest plateau at the wet-bulb temperature, however the approach toward the dry-bulb temperature still may be too slow. This can only be determined by experimental data and should be done in future work. In Fig. 4.7 the temperature history for different values of the characteristic energy of adsorption at constant minimum moisture content of 0.05 kg water / kg dry matter is plotted:

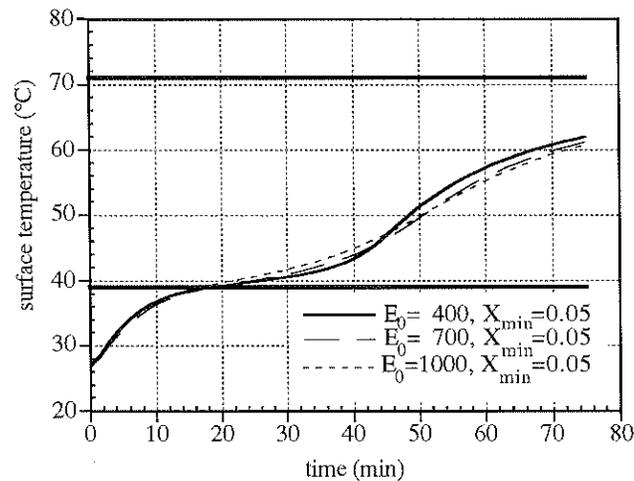


Figure 4.7 Surface temperature for different characteristic energies of adsorption

Fig. 4.7 indicates that the slope of the plateau crossing the wet-bulb temperature decreases somewhat for decreasing characteristic energy of adsorption. It shows further that the approach toward the dry-bulb temperature is somewhat faster for decreasing characteristic energy of adsorption. Thus the value of 400 kJ/kmole or even smaller values are the most realistic and future work on the Dubinin-Polanyi isotherm can answer this question. It is also interesting to look at the changes of the diffusivity during the cooking process. This is done in Fig. 4.8 for the diffusivity of the surface node at

different values of the characteristic energy of adsorption at a constant minimum moisture content of 0.05 kg water / kg dry matter :

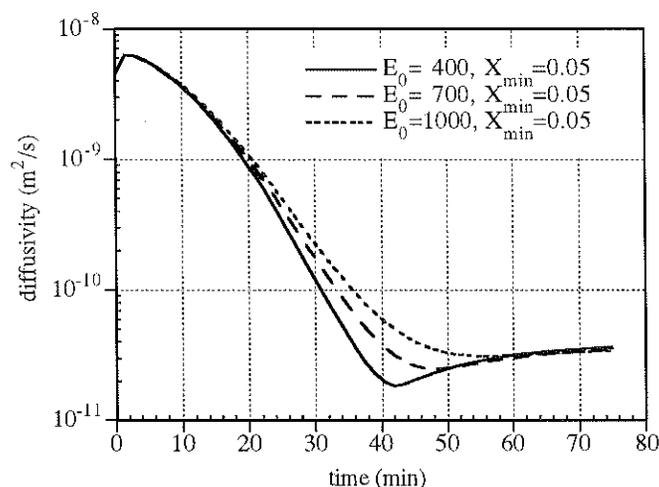


Figure 4.8 Moisture diffusivity of the surface node for different characteristic energies of adsorption

Fig. 4.8 demonstrates why the surface temperature increases faster for lower values of the characteristic energy of adsorption. For these cases the moisture diffusivity decreases faster and thus there is less moisture transferred to the surface node and less water evaporated so that the temperature rises faster. Fig. 4.8 shows further that the diffusivity gradually increases due to the increasing temperature when the minimum moisture content is reached.

Finally some other results of the simulation are also presented. The simulation program calculates the accumulated moisture loss for the food product during cooking, which is relatively easy to determine in experiments. The results for the total moisture loss are presented in Fig 4.9 for a constant minimum moisture content of 0.05 kg water / kg dry matter and different characteristic energies of adsorption.

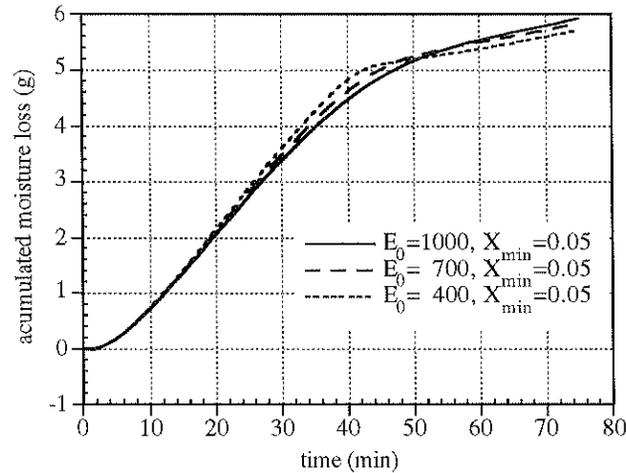


Figure 4.9 Accumulated moisture loss for the food product

Fig. 4.9 shows two amazing facts. One is that for the lower characteristic energies of adsorption the moisture loss is smaller even though the relative humidity at the surface for low values is always higher than that for high characteristic energies of adsorption. This is due to the fact that for the higher characteristic energies of adsorption the moisture is transferred faster in the beginning so that the diffusivity drops faster and less moisture can be transferred from the interior to the surface and evaporated from the food product. The second fact is that for these values there is a moisture gain of the food product as Fig. 4.10 shows:

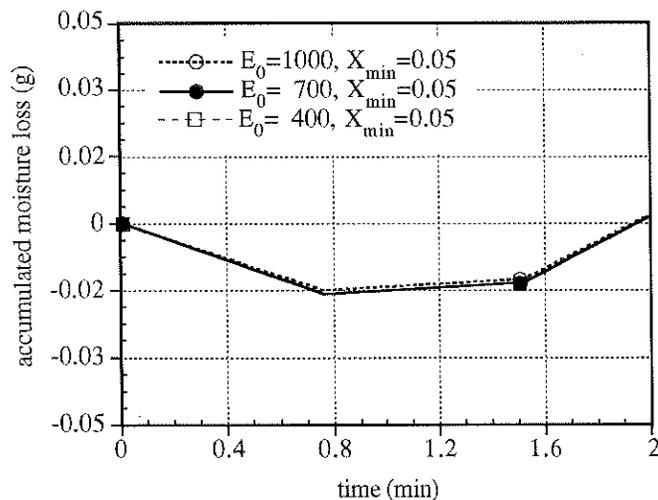


Figure 4.10 Moisture gains for the food product in the first minutes of cooking

This gain of moisture is due to the fact that the temperature of the food product entering the "oven" is below the dew point temperature of the surrounding humid air. The condensation of water vapor on the food product speeds up the temperature increase by releasing the energy of condensation.

Fig. 4.9 shows another evidence for the validity of this model. For times greater than 40 min. there is hardly any more moisture evaporated from the product. Thus the food product would exhibit the behavior of a dry body. This is what was found for the temperature data of the product. Fig. 4.11 shows a figure from [12] where the actual temperature of the product and the predicted temperatures from the simulation for either dry or wet surface are shown. The product temperature lies in between the predicted temperatures for dry and wet surfaces. For the following cooking zones the product temperature approaches the predicted temperature for dry surfaces.

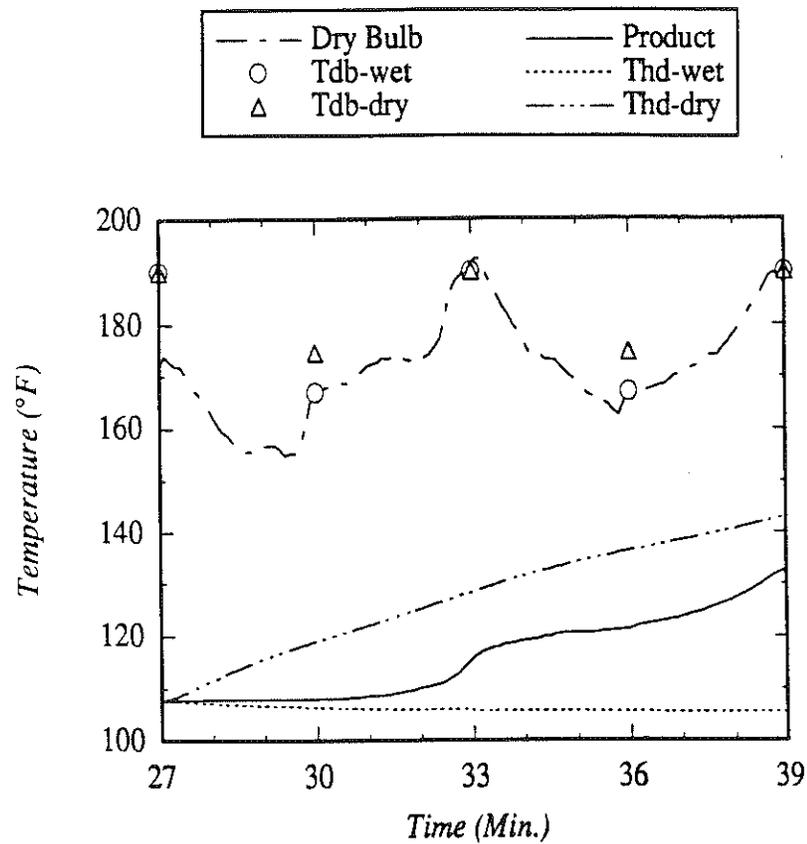


Figure 4.11 Temperature data from [12]

Spielbauer reports in [12] that the thermal response of the food product is strongly dependent on the initial moisture content. It was found that products with lower initial moisture content increase above the wet-bulb temperature much faster than those with higher moisture content. To find out if the model developed is able to account for this effect a simulation with an initial composition of lower moisture content introduced in section 3.1 as composition 2 was performed. Fig. 4.12 indicates that the model

represents the findings correctly. The plateau at the wet-bulb temperature for the low moisture content is somewhat sloped and short.

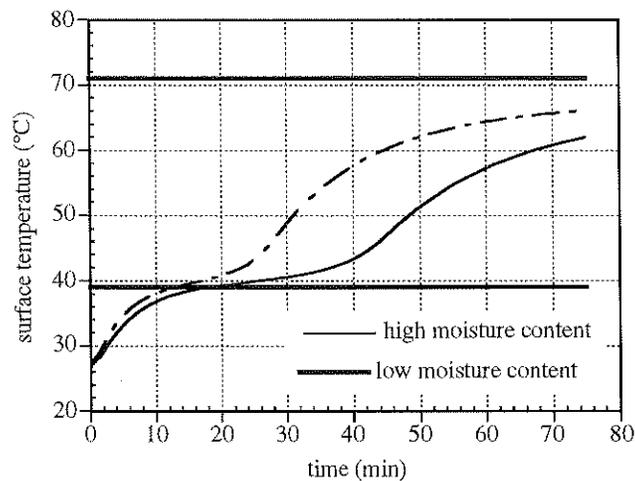


Figure 4.12 Surface temperature for the different initial conditions of the food product

Overall the model is able to account for the main characteristics of the temperature history of the food product, however future work should verify if it is also able to represent the cooking process quantitatively correct.

CONCLUSIONS AND RECOMMENDATIONS

The goal of this research project was the development of a model of the combined heat and mass transfer during the cooking of processed meat products. Chapter two and three describe the development of the model and the estimation of the properties used for the simulations. In Chapter four the results from the different models were discussed by comparing them to actual data reported in [12]. It was found that the model can predict the characteristic behavior of the temperature history of the food product.

5.1 CONCLUSIONS

The simulation program uses a finite difference method to model the temperature and moisture distribution within the product. By comparing the results from the simulation program to analytical solutions it was verified that the product predicts the temperature and the moisture distribution accurately.

The simulation program is also able to predict the different thermal responses of the food product for different initial moisture contents as it was reported in [12]. This implies that

the assumptions that moisture is transferred internally in the liquid phase and that moisture only evaporates at the surface are valid. It is possible to predict the thermal response of the food product accurately using these assumptions.

The results of the simulation program showed further that the modeling of the equilibrium between the moisture content of the food product and the relative humidity of the surrounding air and the moisture diffusivity within the product are important and have the biggest influence on the thermal response of the product.

5.2 RECOMMENDATIONS

1. Since it was beyond the scope of this work to acquire actual data the first recommendation for future work is to do experiments to verify the validity of the results predicted by the simulation program. Measurement of the overall moisture loss of the product for the entire process or the accumulated moisture loss over time are recommended. These quantities are relatively easy to measure and are also computed by the simulation program. Data for the accumulated moisture loss over time could provide some insight in the validity of the model and the mechanisms of the heat and mass transfer during cooking.
2. The most important question that should be addressed by future work is the modeling of the equilibrium between product and surrounding air. This work used an isotherm model with a Dubinin-Polanyi isotherm. It is desirable to verify whether it is possible to model the food product equilibrium using this type of isotherm, which was developed to model adsorption in micro porous materials. Although food products have already been modeled as micro porous materials it might not be possible for this application because the food product is surrounded by a casing during the cooking

process that is likely to change the equilibrium. The possibility that the casing requires time to reach equilibrium and that measurements at temperatures are needed where the food product might change its overall properties due to melting of fats, chemical reactions or degradation when temperature and moisture content are high needs to be considered.

3. It should be verified whether the way the properties were estimated is valid. The assumption of constant thermal conductivity and heat capacity should be examined. It is assumed that the thermal conductivity is that of water because it is assumed that there is always enough moisture present to transfer the heat with that conductivity. This is a reasonable assumption for the most parts of the food product and even for the surface at the beginning of the process, but might not be true for the surface region which is dried out shortly after the beginning of the cooking process. The heat capacity of water is four times as high as the heat capacities of fat and protein. Thus the heat capacity will decrease with decreasing moisture content and it should be examined if it is necessary to account for this change at least for the surface region, where the effect is strongest, due to the highest moisture loss.
4. The relation that is used to account for the changes of the moisture diffusivity due to changes in temperature and moisture content is based on experimental data and can be trusted, but it should be verified if it is applicable to this problem.

 APPENDIX A

FORTTRAN code of the simulation program:

```

*****
**      Simulation program that computes temperature and moisture distributions   *
*      in hot dogs using a finite difference method to approximate the         *
*      derivatives in the partial differential equations for heat conduction    *
*      and diffusion. A Dubinin-Polanyi-isotherm is used to represent the     *
*      equilibrium at the surface of the product. The simulation accounts for  *
*      changes of moisture diffusivity due to changes of temperature and      *
*      moisture content of the food product.                                  *
*****
      program evapiso
*****
*      Declaration of Variables                                               *
*                                                                              *
*      a1, a, b, as, bs, cs    parameters in finite differences for temperature *
*      alm, am, bm, asm    parameters in finite differences for               *
*                          moisture content                                  *
*      conc    dimensionless moisture content of food product                *
*      cp      heat capacity of the food product                            *
*      d,d2    moisture diffusivities for different nodes in food product    *
*      df      driving force for convection of watervapor                   *
*      dr      finite radius                                                  *
*      dtime   finite time interval                                          *
*      e0      characteristic energy of adsorption in isotherm              *
*      filename1-3 name of output files                                     *
*      fp      fat to protein ratio of food product                         *

```

*	h	heat convection to food product	*
*	hm	mass conductance to food product	*
*	k	heat conductivity within the food product	*
*	mdot	massflux for finite time	*
*	mevap	total waterloss of food product	*
*	minf	mass water per mass humid air for free stream conditions	*
*	ms	mass water per mass humid air at the surface	*
*	new	label for timesteps in finite difference method	*
*	old	label for timesteps in finite difference method	*
*	pa	partial pressure of water vapor at the surface	*
*	psat	saturation pressure	*
*	pi		*
*	r	outer radius of food product	*
*	rho	density of food product	*
*	rn	radiae of the finite difference nodes	*
*	t	temperature distribution over the food product	*
*	time	counter over timesteps	*
*	tinf	temperature of airstream	*
*	t0	initial temperature of food product	*
*	tstep	number of timesteps computed	*
*	ur	universal gas constant (8.314kJ/kg/kmol) used in isotherm	*
*	wa	humidity ratio at surface	*
*	winf	humidity ratio of airstream	*
*	x	distribution of moisture content over the food product	*
*	xe	equilibrium moisture content of food product (per dry matter)	*
*	xeq	approximation of equilibrium moisture content	*
*	xiso	constant in isotherm	*
*	xmin	minimum moisture content for food product (per dry matter)	*
*	x0	initial moisture content of food product (per dry matter)	*
*	xs	moisture content of food product (per dry matter) at the surface	*

real dtime, dr, cp, k, d, d2, rho, h, hm, tinf, winf, t0, x0, pi

```

real a1, a1m, a, am, b, bm, as, asm, bs, bsm, cs, csm, r, ur
real mdot, mevap, ms, minf, df, xmin, xiso, e0, conc, fp, xeq
double precision xe, psat, xs, pa, wa
integer nodes, tstep, new, old, time
real t, x, rn
dimension t(40,2),x(40,2),rn(40)
character*20 filename1, filename2, filename3

```

```

*** define variables*****

```

```

pi=acos(-1.)
rho=314.
mdot=0.
mevap=0.
h=17.5
cp=3410.
tinf=71.
winf=.032
minf=winf/(1+winf)
ur=8.314
nodes=39
k=0.6
r=0.0125

```

```

*** reading data from input file inpu tevap.dat***

```

```

open(23,file='inpu tevap.dat',status='old')
rewind(23)
print*,'length of the finite time step dtime?'
read(23,*) dtime
write(6,*) dtime
print*,'number of timesteps to calculate?'
read(23,*)tsteps

```

```
write(6,*)tsteps
print*, 'initial temperature of food product?'
read(23,*) t0
write(6,*) t0
print*, 'initial moisture content of food product?'
read(23,*) x0
write(6,*) x0
print*, 'fat to protein ratio of the food product ?'
read(23,*) fp
write(6,*) fp
print*, 'characteristic heat of adsorption for isotherm?'
read(23,*) e0
write(6,*) e0
print*, 'minimum moisture content of food product?'
read(23,*) xmin
write(6,*) xmin
print*, 'name of output file for temperature distribution data?'
read(23,501) filename1
write(6,*) filename1
print*, 'name of output file for moisture distribution data?'
read(23,501) filename2
write(6,*) filename2
print*, 'name of output file for surface temperature data?'
read(23,501) filename3
write(6,*) filename3
501 format (a)
close(23)
hm=0.017
xiso=x0-xmin
dr=r/nodes
new=1
old=2
time=tsteps/50-1
```

```

***   define initial values for all nodes*****
      do 10 i=1,40
        t(i,old)=t0
        x(i,old)=x0
        rn(i)=(i-1)*dr*1000
      10 continue

***   opening output files *****
      open(20,file=filename1, status='new')
      open(21,file=filename2,status='new')
      open(22,file=filename3,status='new')

***   calculation of the values of t, x for all nodes and timesteps*****

      do 20 j=2,tsteps
        time=time+1
        do 21 i=1,40

          xeq=xiso*exp(ur*(tinf-4+273)
            *log(winf/((0.0101*exp(0.049052*(tinf-4))
            *(winf+0.622)))/e0) +xmin

***   innermost node*****
          if (i .eq. 1) then
            a1=dtime*k/(dr*dr*rho*cp*(1+x(1,old)))
            t(1,new)=t(1,old)*(1-a1)+a1*t(3,old)
            d=diff(fp,t(1,old),t(2,old),x(1,old),x(2,old),x0,xeq,i)
            a1m=dtime*d/(dr*dr)
            x(1,new)=x(1,old)*(1-a1m)+a1m*x(3,old)

          else if (i .eq. 40) then

***   surfacenode*****

***   calculation of the humidity of the surrounding air *****

```

```

*** saturation pressure at surface temp ***
    psat=0.0101*exp(0.049052*t(40,old))

*** partial pressure pa from isotherm ***
    xs=x(40,old)
    if (xs .lt. 0.4) then
        xe=xiso*exp(ur*(t(40,old)+273.)
            *log(winf/(psat*(winf+0.622)))/e0) +xmin
        if (xs .lt. xe) then
            xs=xe
        endif
    endif
    pa=psat*exp(e0*log((xs-xmin)/xiso)/(ur*(t(40,old)+273.)))

*** air humidity from partial pressure***
    wa=0.622*pa/(1-pa)
    ms=wa/(1+wa)

*** definition of driving force for mass transfer ***
    df=(ms-minf)/(1-ms)
    if (df .gt. 0.01) then
        df = log(1+df)
    endif

*** calculation of moisture content ***
    d=diff(fp,t(40,old),t(39,old),x(40,old),x(39,old),x0,xeq)
    asm=dtime*d*(r-dr)/((r*r-(r-dr)**2)*dr)
    x(40,new)=asm*x(38,old)+(1-asm)*x(40,old)
        -2.*r*dtime*hm*df/(rho*(r*r-(r-dr)**2))

    mdot=2.*pi*r*.15*hm*df
    ievap=(2501.4-2.319*t(40,old))*1000

```



```

***      writing values of t, x to output files*****
          write(20,990) (t(i,new), i=1,20)
          write(20,990) (t(i,new), i=21,40)
          write(21,992) (x(i,new), i=1,20)
          write(21,990) (x(i,new), i=21,40)
          write(22,998) (j*dtime/60), t(20,new), df, d, mevap*1000
990      format(20(1x,f5.2))
992      format(20(1x,f5.3))
998      format(f8.3,3x,f7.3,3x,f8.5,3x,e15.6,3x,f8.3)
          endif

***      changing of old and new values in t, x *****
          if (new .eq. 2) then
              new=1
              old=2
          else
              new=2
              old=1
          endif

20      continue

***      printing of overall waterloss to screen and to file for surface temp***
          write(6,*) 'overall waterloss in [g]'
          write(6,999) mevap*1000
999      format(f9.6)

          end

          function diff (fp, t1, t2, x1, x2, x0, xe)
***      this function computes the diffusivity as a function of temperature
***      and moisture content *****
***      xav  average moisture content in node *****
***      tav  average temperature in node *****

```

```

real xav, tav

xav = (x1+x2)/2.
tav = (t1+t2)/2.

conc=(xav-xe)/(x0-xe)
diff = (0.0029*exp(-0.4419*fp-4892.7/(tav+273))+11.55*conc))/3600 return
end

```

Input file of the simulation program:

0.125	length of finite timestep
36000	number of timesteps
27	initial temperature of hotdog
2.33	initial moisture content of hotdog (per dry basis)
1	fat to protein ratio of hot dog
1000	characteristic energy in isotherm
0.05	minimum moisture content of hotdog
temp5.dat	output file for temperature distribution data
water5.dat	output file for moisture distribution data
temp20e.dat	output file for surface temperature and driving force data

 APPENDIX B

Data for Figures 4.4 and 4.5:

Surface temperature for different values of the characteristic energy of adsorption, the minimum moisture content, and the diffusivity versus time.

time	D	10 ⁻¹⁰	10 ⁻¹⁰	10 ⁻¹⁰	10 ⁻¹¹	10 ⁻¹¹	10 ⁻¹¹
in	E ₀	400	400	700	400	400	700
min	X _{min}	0.05	0.3	0.05	0.05	0.3	0.05
0.0000	27.000	27.000	27.000	27.000	27.000	27.000	27.000
1.5000	30.070	30.069	30.066	30.066	30.069	30.069	29.910
3.0000	32.344	32.344	32.340	32.340	32.344	32.343	32.170
4.5000	33.990	33.991	33.999	33.999	33.990	33.992	33.786
6.0000	35.273	35.279	35.310	35.310	35.276	35.283	35.064
7.5000	36.298	36.313	36.381	36.381	36.308	36.323	36.126
9.0000	37.132	37.158	37.276	37.276	37.152	37.180	37.033
10.500	37.823	37.864	38.039	38.039	37.859	37.903	37.830
12.000	38.407	38.468	38.707	38.707	38.468	38.535	38.546
13.500	38.910	38.995	39.310	39.310	39.004	39.100	39.209
15.000	39.355	39.468	39.862	39.862	39.494	39.627	39.838
16.500	39.758	39.906	40.380	40.380	39.958	40.137	40.451
18.000	40.135	40.323	40.875	40.875	40.415	40.653	41.063
19.500	40.496	40.732	41.356	41.356	40.882	41.195	41.686
21.000	40.850	41.142	41.827	41.827	41.377	41.791	42.332
22.500	41.205	41.564	42.298	42.298	41.917	42.480	43.013
24.000	41.568	42.005	42.773	42.773	42.537	43.313	43.738
25.500	41.945	42.473	43.255	43.255	43.278	44.371	44.518

27.000	42.342	42.984	43.745	44.203	45.716	45.363
28.500	42.762	43.545	44.246	45.393	47.194	46.279
30.000	43.217	44.166	44.757	46.815	48.604	47.265
31.500	43.713	44.853	45.280	48.240	49.926	48.314
33.000	44.258	45.603	45.813	49.579	51.167	49.403
34.500	44.855	46.399	46.357	50.834	52.332	50.501
36.000	45.504	47.212	46.910	52.013	53.426	51.581
37.500	46.200	48.016	47.471	53.121	54.453	52.623
39.000	46.926	48.800	48.036	54.160	55.416	53.619
40.500	47.660	49.556	48.603	55.134	56.319	54.567
42.000	48.386	50.283	49.170	56.048	57.165	55.464
43.500	49.094	50.980	49.734	56.905	57.959	56.313
45.000	49.780	51.648	50.291	57.708	58.703	57.115
46.500	50.442	52.286	50.839	58.460	59.400	57.871
48.000	51.078	52.896	51.377	59.165	60.053	58.584
49.500	51.689	53.478	51.903	59.826	60.666	59.255
51.000	52.275	54.034	52.414	60.445	61.239	59.887
52.500	52.836	54.564	52.911	61.025	61.777	60.481
54.000	53.373	55.070	53.393	61.569	62.281	61.040
55.500	53.886	55.552	53.860	62.078	62.753	61.565
57.000	54.377	56.012	54.310	62.555	63.196	62.059
58.500	54.847	56.451	54.745	63.002	63.611	62.523
60.000	55.295	56.869	55.164	63.421	63.999	62.959
61.500	55.724	57.268	55.568	63.814	64.364	63.368
63.000	56.134	57.648	55.956	64.182	64.705	63.753
64.500	56.525	58.011	56.330	64.526	65.025	64.114
66.000	56.899	58.357	56.690	64.849	65.325	64.453
67.500	57.257	58.687	57.035	65.151	65.605	64.772
69.000	57.598	59.002	57.367	65.435	65.869	65.071
70.500	57.925	59.302	57.686	65.700	66.115	65.352
72.000	58.237	59.589	57.992	65.949	66.347	65.615
73.500	58.536	59.863	58.286	66.182	66.563	65.863
75.000	58.820	60.124	58.567	66.400	66.766	66.095

 APPENDIX C

Data for Figures 4.6, 4.7, and 4.11:

Surface temperature for different values of the characteristic energy of adsorption and the minimum moisture content versus time where the frame indicates the initial composition with lower moisture content:

time	E_0	400	400	400	400	700	1000
(min)	X_{\min}	0.05	0.05	0.15	0.3	0.05	0.05
0.0000	27.000	27.000	27.000	27.000	27.000	27.000	27.000
1.5040	28.468	29.252	28.468	28.468	28.179	28.466	
3.0040	30.747	32.060	30.747	30.747	30.285	30.745	
4.5040	32.654	34.170	32.654	32.655	32.120	32.660	
6.0040	34.170	35.705	34.171	34.172	33.624	34.193	
7.5040	35.361	36.822	35.363	35.366	34.840	35.409	
9.0040	36.293	37.647	36.296	36.302	35.821	36.374	
10.504	37.026	38.266	37.032	37.041	36.617	37.146	
12.004	37.604	38.750	37.613	37.627	37.265	37.771	
13.504	38.063	39.151	38.075	38.095	37.798	38.283	
15.004	38.430	39.512	38.447	38.476	38.240	38.709	
16.504	38.730	39.866	38.752	38.792	38.613	39.074	
18.004	38.980	40.239	39.010	39.064	38.935	39.394	
19.504	39.196	40.658	39.235	39.308	39.221	39.686	
21.004	39.389	41.152	39.440	39.536	39.482	39.962	
22.504	39.570	41.757	39.636	39.760	39.730	40.231	
24.004	39.747	42.534	39.831	39.989	39.973	40.503	
25.504	39.927	43.605	40.032	40.234	40.219	40.785	

27.004	40.116	45.147	40.249	40.502	40.476	41.083
28.504	40.321	47.026	40.486	40.803	40.750	41.403
30.004	40.548	48.860	40.751	41.146	41.046	41.750
31.504	40.802	50.549	41.052	41.546	41.369	42.126
33.004	41.091	52.087	41.399	42.019	41.726	42.534
34.504	41.423	53.480	41.805	42.595	42.120	42.976
36.004	41.809	54.741	42.285	43.323	42.554	43.458
37.504	42.265	55.881	42.874	44.276	43.038	43.980
39.004	42.817	56.912	43.619	45.514	43.578	44.543
40.504	43.509	57.843	44.595	46.913	44.179	45.147
42.004	44.405	58.684	45.848	48.297	44.850	45.794
43.504	45.573	59.445	47.238	49.606	45.597	46.480
45.004	46.938	60.132	48.603	50.829	46.422	47.206
46.504	48.312	60.755	49.891	51.966	47.324	47.967
48.004	49.616	61.319	51.092	53.021	48.293	48.758
49.504	50.836	61.829	52.208	53.999	49.306	49.573
51.004	51.969	62.293	53.242	54.905	50.335	50.404
52.504	53.019	62.714	54.199	55.743	51.352	51.242
54.004	53.990	63.097	55.085	56.519	52.334	52.077
55.504	54.889	63.446	55.904	57.237	53.271	52.900
57.004	55.719	63.764	56.661	57.901	54.156	53.704
58.504	56.486	64.055	57.361	58.516	54.989	54.481
60.004	57.195	64.321	58.008	59.086	55.769	55.227
61.504	57.849	64.564	58.606	59.613	56.498	55.939
63.004	58.454	64.788	59.160	60.102	57.179	56.616
64.504	59.013	64.994	59.672	60.555	57.813	57.256
66.004	59.530	65.184	60.146	60.975	58.404	57.861
67.504	60.008	65.359	60.585	61.365	58.953	58.430
69.004	60.451	65.521	60.992	61.728	59.465	58.964
70.504	60.860	65.671	61.369	62.064	59.940	59.466
72.004	61.240	65.811	61.719	62.377	60.383	59.937
73.504	61.591	65.941	62.044	62.669	60.794	60.377
75.000	61.917	66.062	62.345	62.940	61.175	60.789

 APPENDIX D

Data for Figures 4.9 and 4.10:

The accumulated moisture loss of the product for different values of the characteristic energy of adsorption and the minimum moisture content versus time :

time	E_0	400	700	1000
(min)	X_{\min}	0.05	0.05	0.05
0.0000	-0.0072200		-0.0072200	-0.0072200
1.5040	0.0014300		0.0020000	0.0014400
3.0040	0.0043600		0.0048000	0.0043000
4.5040	0.0064000		0.0066300	0.0062700
6.0040	0.0079700		0.0080000	0.0077800
7.5040	0.0092000		0.0090700	0.0089400
9.0040	0.010120		0.0099100	0.0098300
10.504	0.010860		0.010540	0.010470
12.004	0.011420		0.011040	0.010970
13.504	0.011840		0.011420	0.011330
15.004	0.012150		0.011700	0.011570
16.504	0.012370		0.011890	0.011720
18.004	0.012510		0.012010	0.011790
19.504	0.012600		0.012070	0.011790
21.004	0.012630		0.012070	0.011740
22.504	0.012620		0.012010	0.011630
24.004	0.012560		0.011910	0.011470
25.504	0.012460		0.011760	0.011270
27.004	0.012310		0.011560	0.011020
28.504	0.012120		0.011320	0.010740

30.004	0.011870	0.011020	0.010420
31.504	0.011560	0.010680	0.010060
33.004	0.011170	0.010290	0.0096900
34.504	0.010680	0.0098700	0.0092500
36.004	0.010050	0.0093400	0.0087900
37.504	0.0092400	0.0087500	0.0083000
39.004	0.0080700	0.0080900	0.0077900
40.504	0.0063900	0.0073500	0.0072600
42.004	0.0040100	0.0065200	0.0067100
43.504	0.0019600	0.0056400	0.0061500
45.004	0.0014500	0.0047300	0.0056000
46.504	0.0013900	0.0038600	0.0050600
48.004	0.0014000	0.0031200	0.0045500
49.504	0.0014400	0.0025800	0.0040800
51.004	0.0015000	0.0022400	0.0036600
52.504	0.0015500	0.0020500	0.0033100
54.004	0.0016100	0.0019400	0.0030100
55.504	0.0016600	0.0018900	0.0027800
57.004	0.0017100	0.0018600	0.0025900
58.504	0.0017600	0.0018500	0.0024500
60.004	0.0018100	0.0018500	0.0023500
61.504	0.0018400	0.0018600	0.0022600
63.004	0.0018800	0.0018700	0.0022000
64.504	0.0019100	0.0018800	0.0021500
66.004	0.0019400	0.0019000	0.0021200
67.504	0.0019600	0.0019100	0.0020900
69.004	0.0019800	0.0019300	0.0020700
70.504	0.0020000	0.0019400	0.0020500
72.004	0.0020200	0.0019500	0.0020400
73.504	0.0020300	0.0019600	0.0020300
75.000	0.0020400	0.0019700	0.0020200

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