

---

## *CHAPTER TWO*

---

### WIND TURBINE PERFORMANCE MODELS

*"There is something in the wind."*

| Shakespeare, Comedy of Errors

#### **2.1 Fundamentals of Wind Energy Systems**

Wind turbines transform the kinetic energy of moving air into useful work. In order to understand this process, a control-volume (*c.v.*) is constructed as shown in Figure 2.1, representing a three-dimensional streamtube of air. The rotor is represented by an *actuator disk* interspersed in the flow.

Although the process is described for horizontal axis wind turbines, the energy analysis is generally applicable for all wind turbines, and is appropriate for both horizontal (propeller-type) and vertical (Darrieus) wind turbines.

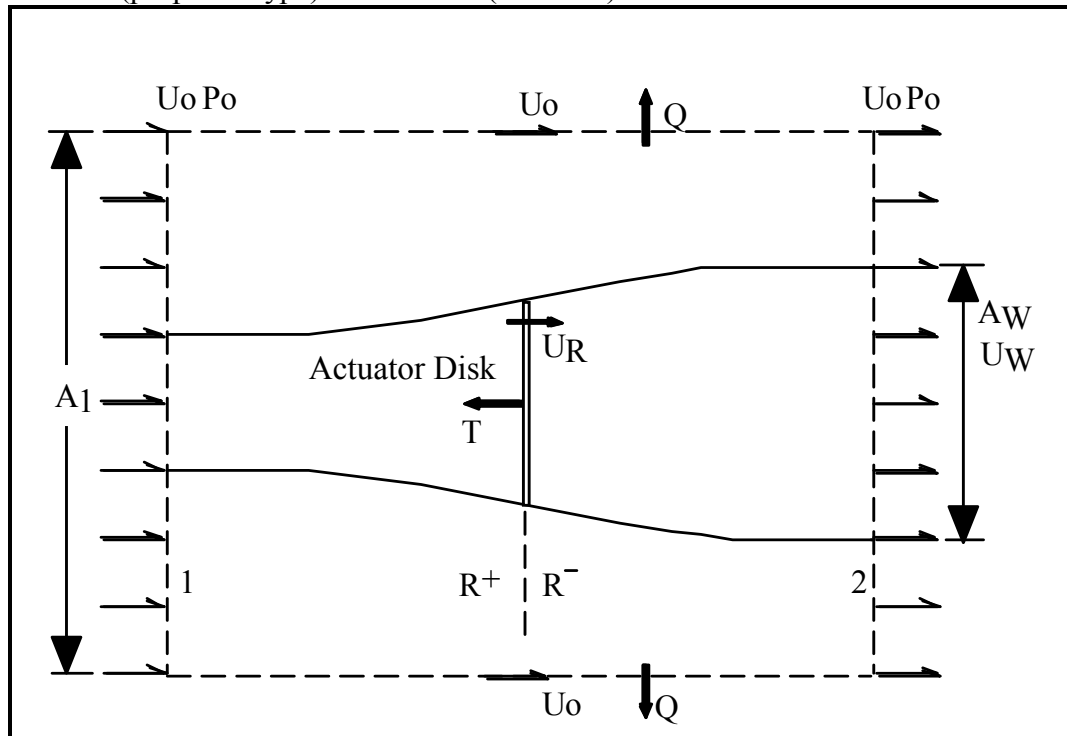


Figure 2.1 Streamtube control volume with actuator disk wind turbine model.

The control volume method applied to wind turbine fluid dynamics is the *actuator disk model*, which was originally developed by Rankine [Spera, 1989] to model marine propellers. For wind turbines, the rotor is a homogeneous disk which removes (rather than furnishes) energy to the moving fluid. Although insufficient for analysis of rotor geometry, the model is appropriate for analysis of axial mass, momentum, and energy balances. The following physical assumptions are employed:

- Constant, incompressible, irrotational, flow at constant temperature
- No mass flow across the streamtube boundary

- Point 1 is far upstream; point R is at the rotor, and point 2 is far downstream.

The position of point 2 is at the hypothetical point where the streamtube boundary is parallel to the horizontal c.v. boundary. At this point downstream the static pressure is constant and equals the free stream static pressure,  $P_o$ .

The actuator disk approach to the momentum theory analysis of wind turbines does not include turbulent mixing between the air in the streamtube and the air in the balance of the c.v. Thus, the placement of the downstream boundary of the c.v. is arbitrary once the streamtube lines become parallel. When the condition that the streamtube lines are parallel is met, then the mass transfer  $Q$  ( and the momentum associated with  $Q$ ) leaving the c.v. due to the existence of the rotor is completed. The following variables are defined:

$U_o$  : Velocity in the free stream

$P_o$  : Pressure in the free stream

$A_1$  : Area of the c.v. far upstream of the rotor.

$U_R$  : Velocity through the rotor

$P_R^+$  : Pressure just upstream of the rotor

$P_R^-$  : Pressure just downstream of the rotor

$U_w$  : Velocity far downstream in the rotor wake

$A_w$  : Area far downstream in the rotor wake

$Q$  : Mass flux out the sides of the c.v.

### 2.1.1 *Momentum Theory*

If it is assumed that the density of the air does not change, then mass continuity through the streamtube requires

$$A_1 U_0 = A_R U_R = A_W U_W \quad (2.1)$$

Since  $U_0 > U_R > U_W$  then it follows that  $A_1 < A_R < A_W$  and the streamtube expands. The mass balance for the c.v. is

$$A_1 U_0 - A_W U_W - (A_1 - A_W) U_0 - Q = 0 \quad (2.2)$$

Rearranging equation 2.2, and solving for Q yields an expression for the mass flow rate out of the c.v.

$$Q = A_W (U_0 - U_W) \quad (2.3)$$

Mass flow rate can be expressed as  $\rho A U$ . Conservation of momentum in the horizontal direction results in

$$\rho A_1 U_0^2 - \rho A_W U_W^2 - \rho (A_1 - A_2) U_0^2 - \rho U_0 Q - D = 0 \quad (2.4)$$

or, after rearranging,

$$D = \rho A_W U_0^2 - \rho A_W U_W^2 - \rho U_0 Q \quad (2.5)$$

Substituting for Q by using equation 2.3 results in

$$D = \rho A_W U_0^2 - \rho A_W U_W^2 - \rho U_0 [A_W (U_0 - U_W)] \quad (2.6)$$

which, after rearranging, becomes

$$D = \rho A_w U_w (U_0 - U_w) \quad (2.7)$$

Figure 2.2 shows the wind speed, plus static, dynamic and total pressure across the rotor. Bernoulli's equation is next used to describe the pressure difference across the rotor. Upstream of the rotor:

$$p_0 + \frac{1}{2} \rho U_0^2 = p_R^+ + \frac{1}{2} \rho U_R^2 \quad (2.8)$$

and downstream of the rotor:

$$p_R^- + \frac{1}{2} \rho U_R^2 = p_0 + \frac{1}{2} \rho U_0^2 \quad (2.9)$$

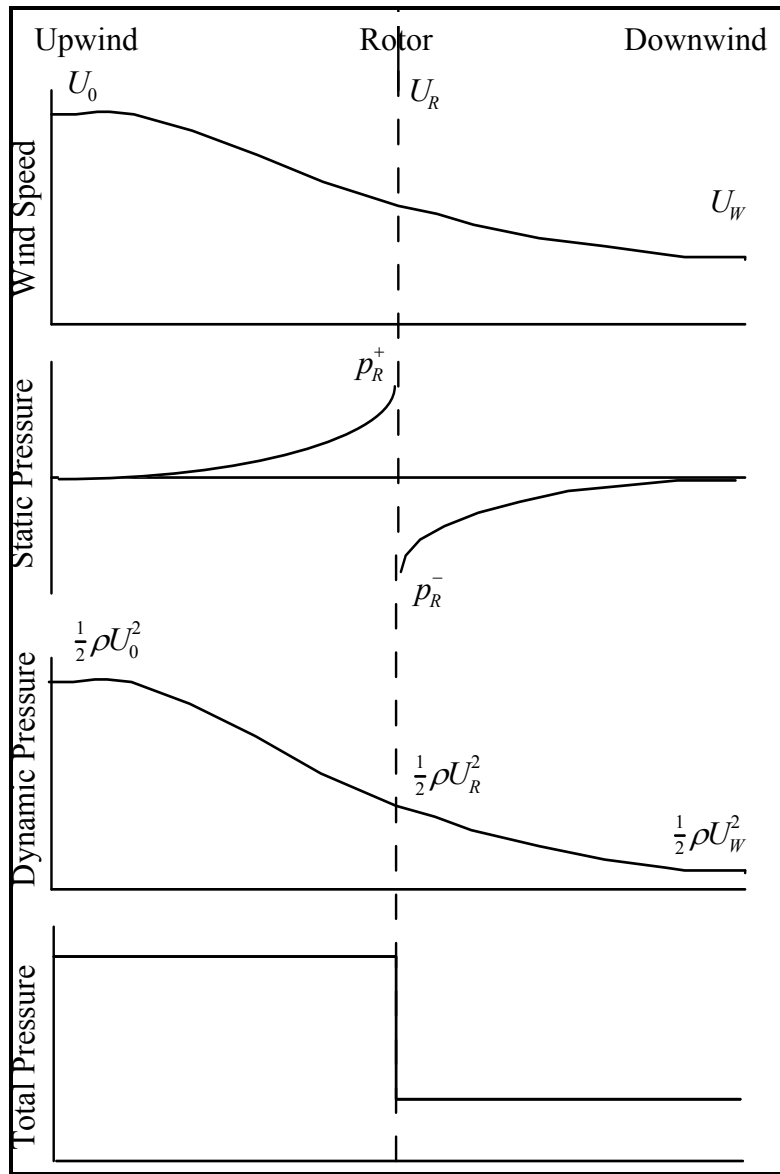


Figure 2.2 Wind speed, plus static, dynamic and total pressure across the rotor.

The pressure difference across the rotor is then equivalent to the difference between equation 2.8 and 2.9, or

$$p_R^+ - p_R^- = \frac{1}{2}\rho(U_0^2 - U_W^2)$$

(2.10)

The thrust force,  $D$  can be expressed as the pressure difference applied to the rotor area (hence, *actuator disk*). The expression in this case is

$$D = A_R (p_R^+ - p_R^-)$$

(2.11)

It is then possible to combine equations 2.10 and 2.11 to create an expression for the thrust:

$$D = \frac{1}{2} \rho A_R (U_0^2 - U_W^2)$$

(2.12)

Two expressions for thrust have been developed. Combining them leads to the following expression

$$\rho A_W U_W (U_0 - U_W) = \frac{1}{2} \rho A_R (U_0^2 - U_W^2)$$

(2.13)

Recall that  $A_W U_W = A_R U_R$ , so that equation 2.13 becomes

$$\rho A_R U_R (U_0 - U_W) = \frac{1}{2} \rho A_R (U_0^2 - U_W^2) \tag{2.14}$$

Canceling out like terms and simplifying, results in

$$U_R (U_0 - U_W) = \frac{1}{2} (U_0^2 - U_W^2)$$

(2.15)

Recalling that  $(U_0^2 - U_w^2) = (U_0 - U_w)(U_0 + U_w)$  then equation 2.15 reduces to an expression for the wind velocity at the rotor,

$$U_R = \frac{U_0 + U_w}{2} \quad (2.16)$$

which means that the wind speed at the rotor is the average of the upstream and downstream wind speeds. The term  $a$  is defined as the *axial induction factor* (or the *retardation factor*) and is a measure of the influence of the rotor on the wind, such that

$$U_R = U_0(1 - a) \quad (2.17)$$

Equations 2.16 and 2.17 can then be combined to yield an expression for the downstream wind speed in terms of the free stream wind speed, or

$$U_w = U_0(1 - 2a) \quad (2.18)$$

The power output of a wind turbine can then be written as the product of the thrust times velocity, or

$$P = DU_R \quad (2.19)$$

Equation 2.12 can be substituted into equation 2.19 to create an expression for the power output



$$P = \left[ \frac{1}{2} \rho A_R (U_0^2 - U_w^2) \right] U_R$$

(2.20)

Equation 2.17 can be used to replace  $U_R$ , and equation 2.18 can be used to replace  $U_w$  in equation 2.20, resulting in

$$P = \frac{1}{2} \rho A_R U_0 (1-a) \left( U_0^2 - [U_0(1-2a)]^2 \right)$$

(2.21)

Simplifying equation 2.21 yields

$$P = \frac{1}{2} \rho A_R U_0^3 4a(1-a)^2 \tag{2.22}$$

The power coefficient for a wind turbine,  $C_p$ , is defined as the power of the turbine divided by the power in the wind, where the power in the wind is

$$P = \frac{1}{2} \rho A_R U_0^3$$

(2.23)

Dividing equation 2.22 by 2.23 yields an expression for the power coefficient as a function of the axial induction factor

$$C_p = 4a(1-a)^2$$

(2.24)

The maximum power coefficient,  $C_{p_{max}}$ , is found by finding  $dP/da = 0$  using equation 2.22 and solving for  $a$ , where the solutions are  $a = 1$  and  $a = 1/3$ . The solution

for  $a = 1$  results in the minimum value of  $C_p$ , 0, while  $a = 1/3$  results in the maximum value of  $C_p$ , where

$$C_{p_{\max}} = 4 \left( \frac{1}{3} \right) \left[ 1 - \frac{1}{3} \right]^2 = \frac{16}{27} = 59.3\% \quad (2.25)$$

The value of 59.3% as a maximum power coefficient was first derived by Betz in 1919, and has since been called Betz's limit. The value of the coefficient is that, when multiplied by the area of the rotor and power in the wind, it describes the power output of the wind turbine, or

$$P = C_p \rho A U_0^3 \quad (2.26)$$

Figure 2.3 shows  $C_p$  as a function of the axial induction factor. The value of  $C_p$  for a wind turbine is determined by its tip-speed-ratio (the ratio of blade tip speed in the plane of the rotor to the free stream wind speed  $U_0$ ). For variable pitch or variable speed wind turbines, tip-speed-ratio is selected by the turbine for maximum  $C_p$  up to the rated power output, then for operation at wind speeds above the rated wind speed,  $C_p$  falls in order to maintain constant rated power. For stall-regulated turbines, rotational speed is fixed, so tip-speed-ratio is a measure of the ratio of free-stream wind velocity to the fixed rotor speed. Figure 2.4 shows  $C_p$  as a function of wind speed for a typical stall-regulated wind turbine and power-regulated turbine.

The  $C_p$  values applicable for most wind turbine applications are those associated with axial induction factors between 0 and 0.5; values less than 0 are associated with propeller operation, and values above 1.0 are associated with propeller brakes. For wind turbines, values between 0.5 and 1.0 are not encountered in practice because in this region stall regulated turbines have lower tip-speed-ratios, and pitch-regulated wind

turbine feathering to a lower, rather than a higher thrust coefficient. Pitching to the lower thrust coefficient achieves lower structural loads than pitching to the higher thrust coefficient.

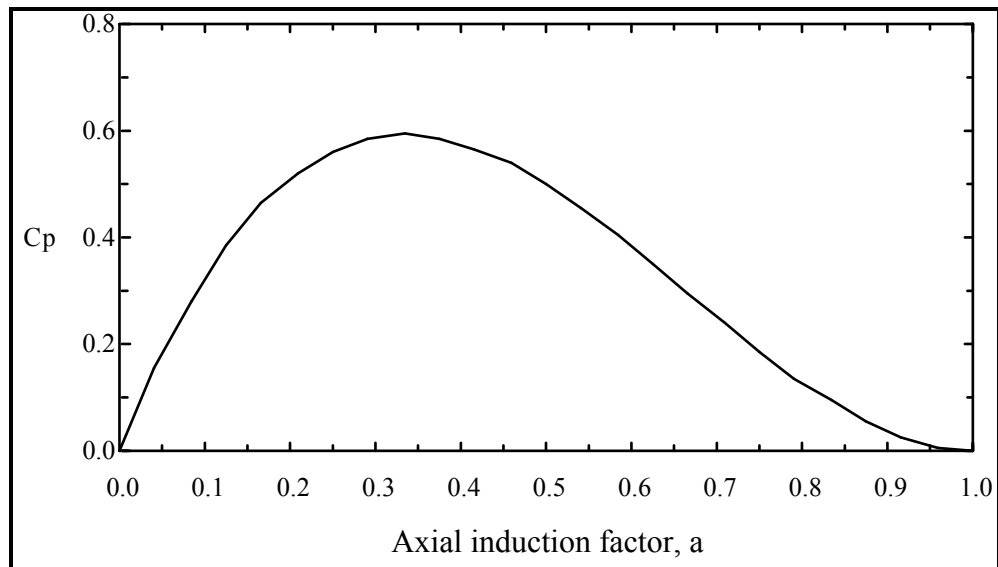


Figure 2.3  $C_p$  as a function of axial induction factor

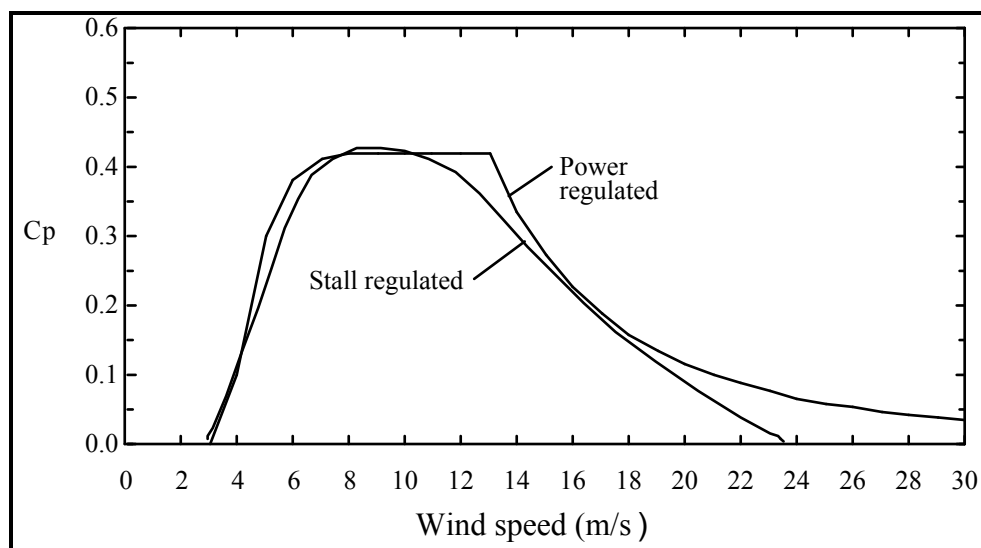


Figure 2.4 Typical  $C_p$  versus wind speed curves for stall-regulated and power regulated turbines.

### 2.1.2 Thrust Coefficient

The *thrust coefficient*,  $C_T$ , for a wind turbine is defined as the ratio of the wind turbine drag force divided by the force of the wind over an equivalent swept area. Determination of  $C_T$  is required for turbine wake and cluster calculations. The term "thrust" is commonly used instead of "drag" because turbine design shares a significant amount of its nomenclature and theoretical development with propeller theory. Most manufacturers do not explicitly publish thrust coefficient data. However, it is possible to derive the thrust coefficient at a given wind speed if the  $C_p$  or turbine power output is known, based on momentum theory.

Wilson and Lissaman (1974) developed a method for relating  $C_T$  to the axial induction factor. Recall from equation 2.19 that  $P = DU_R$ , so  $D = P/U_R$ . Substituting for  $P$  using equation 2.17, and canceling like terms results in the expression

$$D = \frac{1}{2} \rho A_R U_0^2 [4a(1-a)] \quad (2.27)$$

The thrust coefficient is the turbine thrust divided by the wind force applied to the rotor swept area, or

$$C_T = \frac{D}{\frac{1}{2} \rho A_R U_0^2} \quad (2.28)$$

Eliminating  $T$  between equations 2.28 and 2.27 results in an expression for  $C_T$  as a function of  $a$ ,

$$C_T = 4a(a - 1) \quad (2.29)$$

Recalling from equation 2.24 that  $C_p$  can also be expressed as a function of the axial induction factor,  $C_p = 4a(1 - a)^2$ , then it is possible to relate the thrust coefficient to the power coefficient through the axial induction factor. Figure 2.5 shows  $C_T$  and  $C_p$  as a function of the axial induction factor in the  $a = 0$  to 0.5 region. Figure 2.6 shows the curve of  $C_T$  as a function of  $C_p$ , for  $C_p$  between 0 and 0.593 and  $C_T$  between 0 and 1.0. The region  $C_T = 0$  to 1.0 is appropriate for practical wind turbines because  $C_p$  cannot exceed Betz's limit and because commercial turbines, with low rotor solidities, do not commonly exhibit thrust coefficients greater than 1.0.

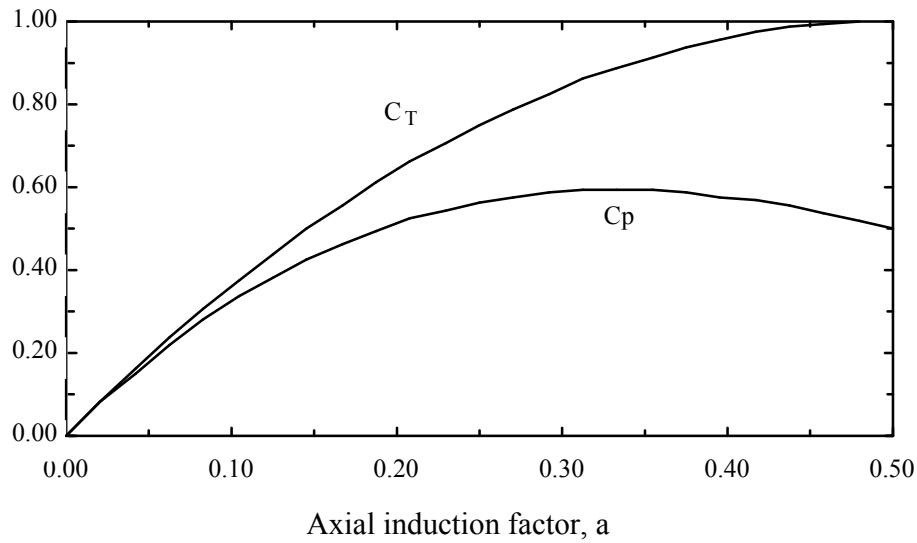


Figure 2.5  $C_T$  and  $C_p$  as a function of the axial induction factor.

Because equation 2.16 is implicit, it is not possible to explicitly characterize  $C_p$  as a function of the axial induction factor. However, it is possible to determine a value

for  $C_T$  by using the axial induction factor as a numeric solution parameter, as shown in Figure 2.6. An EES listing is included in Appendix A.

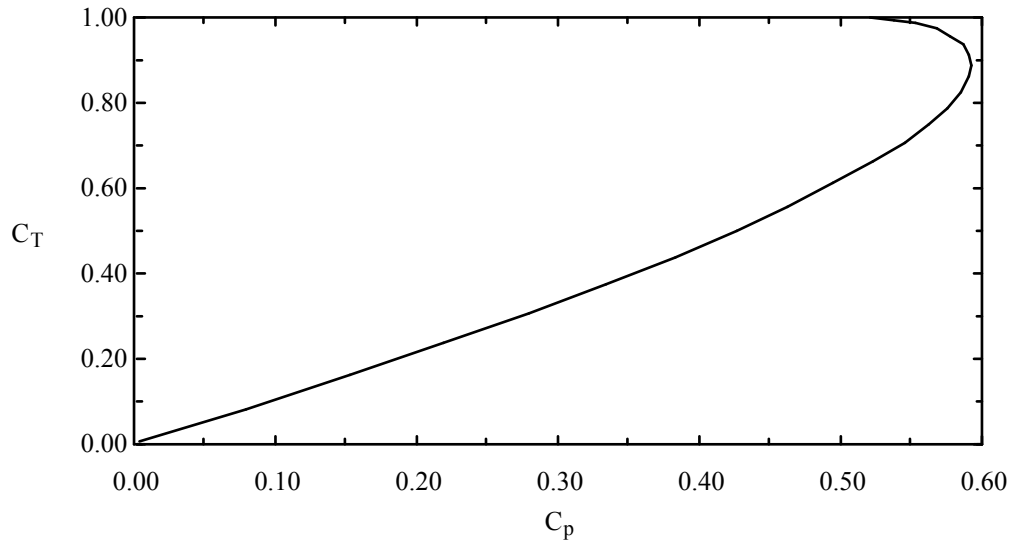


Figure 2.6  $C_T$  as a function of  $C_p$ .

## 2.2 Factors Influencing Wind System Performance

It has been shown earlier in this chapter that the energy extracted by a wind turbine results from a change in momentum of the air moving through the rotor plane. The mass flow rate of air is affected by its density, which is a function of temperature and pressure, and velocity. Estimated wind velocity at the turbine rotor is based on information concerning flow obstructions and assumptions made concerning speed-up with height above ground. In addition, the power conversion efficiency of a wind turbine is influenced by variance in the wind speed (turbulence), orthogonality to the flow (yaw error) and aerodynamic effects (blade cleanliness).

### 2.2.1 Air Density

The energy extracted by a wind turbine results from the change in momentum of the air moving through the rotor. The mass flow rate of air is a function of its density; therefore, the change in momentum is a function of the density of the air passing through the rotor. As described by the ideal gas law, the density of air is a function of its temperature and pressure. Air density is also dependent on humidity ratio, although Rohatgi and Nelson (1994) have shown that this influence is negligible for wind energy applications.

In the atmosphere, the air density at a particular altitude is a function of the temperature and pressure of the air at the time and at that location. Both air pressure and temperature fall as a function of altitude. Hydrostatic pressure is commonly described as

$$\frac{dp}{dz} = -\rho g \quad (2.30)$$

So the pressure difference from one altitude to the next is

$$\Delta P = -\int_1^2 \rho g dz \quad (2.31)$$

Introducing the ideal gas law  $p = \rho RT$ , substituting for  $\rho$  in equation 2.30, separating the variables, and integrating between two points yields

$$\int_1^2 \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R} \int_1^2 \frac{dz}{T} \quad (2.32)$$

Assuming constant temperature, integrating equation 2.32 over  $z$  yields an expression for the pressure difference from point 1 to point 2 in elevation

$$p_2 = p_1 \exp \left[ -\frac{g(z_2 - z_1)}{RT} \right] \quad (2.33)$$

For elevations where wind energy applications apply, temperature decreases linearly with altitude. The temperature "lapse rate" (White, 1994) is

$$T \approx T_0 - Bz \quad \text{where } B = 6.5 \text{ K/km of altitude.} \quad (2.34)$$

Equation 2.34 can be inserted into equation 2.32 and integrated, resulting in an expression for pressure decrease with altitude taking into the temperature lapse rate

$$p_2 = p_1 \left[ 1 - \frac{Bz}{T_0} \right]^{\frac{g}{RB}} \quad (2.35)$$

where the dimensionless exponent  $\frac{g}{RB} = 5.26$  for air, and  $T_0 = 288K$ .

The air density at an elevation is a function of the combined effects of pressure and temperature, according to the ideal gas law. The air density at an elevation, in kg/m<sup>3</sup>, is therefore

$$\rho_{elev} = \frac{p_{elev}}{RT_{elev}} \quad (2.36)$$



The resulting variation in air density is shown in Figure 2.7. The left axis shows the percent of sea-level density, while the right axis shows the numeric value. At an altitude of 3,000 meters, the density has fallen to approximately 80 percent of sea-level density. An EES program listing of the air density calculation model is contained in Appendix A.

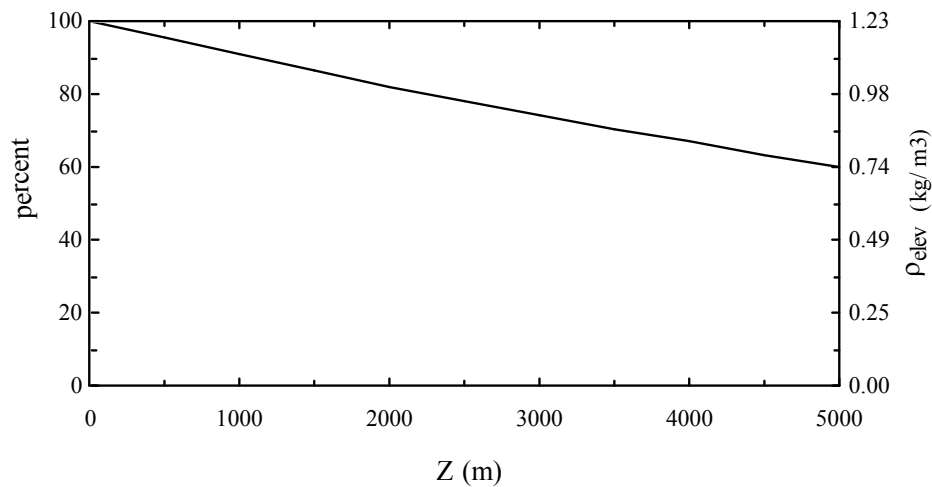


Figure 2.7. Air density as a function of elevation.

In most cases, wind turbine power output at a given wind speed is a linear function of air density. Wind turbines are usually not considered practical at high elevations, despite the stronger wind speeds. Since temperature and pressure also vary with weather, air density varies as well over time. Figure 2.8 shows percent of standard air density for weather conditions which are typical over the course of a year.

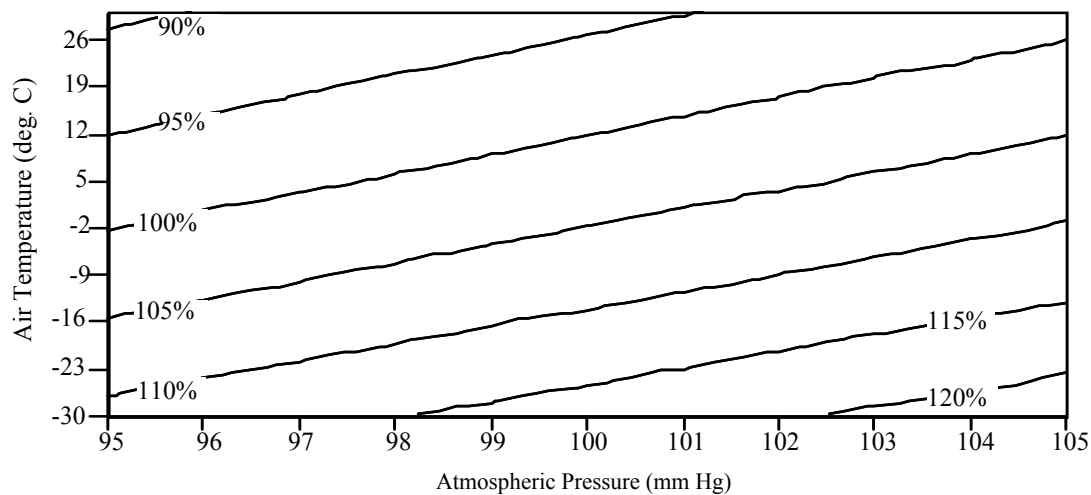


Figure 2.8 Percent of standard air density for weather conditions which are typical over the course of a year.

Wind turbine power output also varies slightly due to variations in air density because the Reynolds number, upon which airfoil performance depends, also depends on  $\rho$ . However, the influence on power output, at the variations in density occurring in the atmosphere are small, so that Reynolds effects are not usually considered in power output modeling. Different types of turbines respond differently to air density changes depending on the method of power regulation employed. The output of fixed pitch wind turbines (stall-regulated and Darrieus vertical axis wind turbines) varies linearly with air density ratio. In this case, the vector addition of free-stream and rotor wind velocities results in an apparent airfoil angle of attack which moves into the "stalled" region of the airfoil lift-drag curve. Fixed pitch is problematic for commercial operators of wind turbines because the pitch of the turbine, as installed, is fixed and therefore based on a mean assumption for site air density. Since air density varies constantly, the turbine is rarely operating at an optimum. Fortunately, the range of "near-optimum" is broad. However, it is common practice to set conservative pitch angles on stall-regulated turbines to avoid overloading the turbine generator at times of high air density and high wind (typically, winter conditions).

Turbines with variable speed rotors and pitch-controlled blades are of a "power-regulation" class. For these turbines, output is linearly proportional to air density up to the maximum power rating of the turbine. Maximum power is achievable, albeit at a higher wind speed in low density conditions. The wind speed at which the rotor cuts-in and can reach its rated output occurs at a *wind power density* equivalent to the wind power density at standard conditions. Wind power density ( $\text{W/m}^2$ ) is a cubic function of wind speed, therefore

$$U_{rated,\rho} = U_{rated} (\rho_0/\rho)^{1/3} \quad (2.37)$$

Figure 2.9 shows power curve variations for a power-regulated turbine as a function of air density. Note that the turbine reaches a fixed maximum, but that the point at which it reaches the maximum depends on air density.

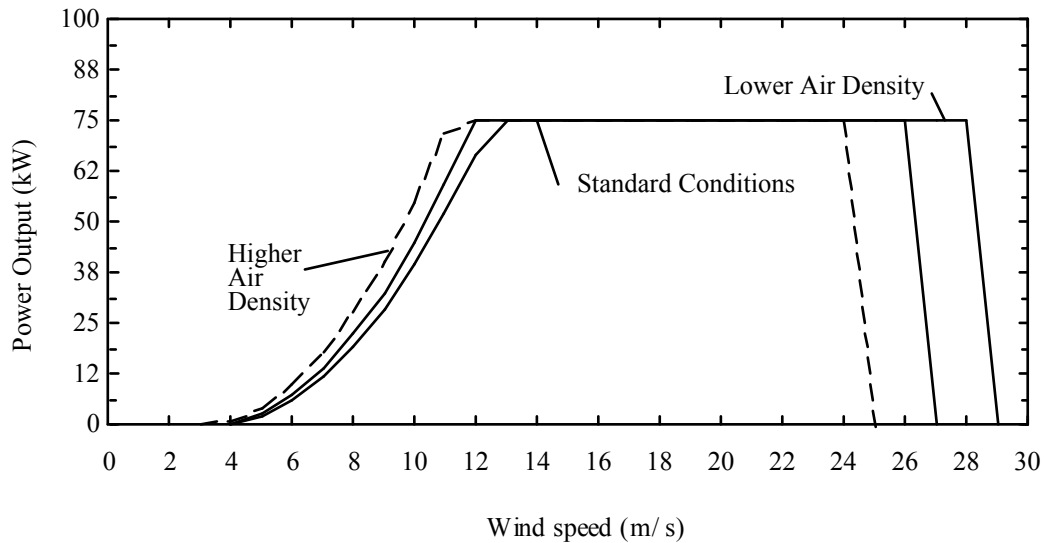


Figure 2.9 Output of a power-regulated wind turbine as a function of air density.

### 2.2.2 Vertical Wind Shear

The modeling of vertical wind shear (the change in wind speed per change in height above the ground), is historically based on boundary layer theory applied to the atmosphere. Two vertical wind shear models are used in wind energy applications.

The first is the "one seventh power- law" model, based on the theoretical work of Von Karman (Koepl, 1982):

$$\left(\frac{U_1}{U_2}\right) = \left(\frac{Z_1}{Z_2}\right)^\alpha$$

(2.38)

A single parameter,  $\alpha$ , determines the rate of wind speed increase as a function of height. Under ideal boundary layer conditions, the value of alpha is taken to be 1/7 (0.14). However, under actual conditions, the value of alpha constantly varies, and depends on a variety of factors influencing vertical turbulence intensity, including:

- Local upwind surface roughness, determined by whether the air is moving over water, prairie, bushes or trees.
- Large scale surface characteristics in the "fetch", or upwind area, such as mountainous far upwind, or buildings and other structures nearby upwind.
- Atmospheric stability, as defined by the temperature gradient with height.
- Other wind turbines. Turbines increase turbulence due to vortex shedding.

Efforts to include these effects into more detailed, multiple-parameter shear models for wind energy applications have produced a variety of alternatives. The logarithmic profile

$$U_2 = U_1 \ln(Z_2/Z_0) / \ln(Z_1/Z_0)$$

(2.39)

is incorporated in the WaSP computer model used in Europe. The value of  $Z_0$  is the surface roughness length. In some circumstances, a value for the surface roughness is not known, or is a source of subjectivity. For this reason, the power law model continues to be most often used by meteorologists, especially where multi-height wind data is obtainable from a site. In this case, the shear exponent, alpha, implicitly incorporates influences due to surface roughness. For this reason, many wind data collection sites monitor wind velocity at two or more heights, allowing the meteorologist to infer a vertical wind shear exponent between heights. Combined with the capability to disaggregate time series data into directional components, then it is possible to map the shear exponent by time and direction.

In time series modeling of wind turbine performance, vertical wind shear data is important in two cases. In the first case, the wind turbine power curve may have been collected at a height which was not the turbine hub height. For example, wind data may have been collected at ten meters for a wind turbine which was operating at 25 meter height above the ground. Though not a common situation, power curve data have been published with this height mismatch, since it can misleadingly represent a more powerful turbine.

In the second case, it is more common that site wind data has been collected at a different height from the turbine hub height. In this case, a hierarchy of approaches is appropriate. The most rigorous approach is to utilize a time series of vertical wind shear values calculated from two wind speed data sets, each from a different height above ground bracketing the height of the wind turbine. The equation for determining the alpha value from heights 1 and 2 is obtained by solving the power law equation for  $\alpha$ :

$$\alpha = \frac{\ln U_2 - \ln U_1}{\ln Z_2 - \ln Z_1}$$

(2.40)

A less rigorous approach is to calculate wind speeds based on wind data where both heights were below the turbine hub height: in this case the very same model is employed, but for extrapolation, rather than interpolation.

The least rigorous approach is to apply an alpha estimate which does not vary with time. Unfortunately, this is often the case with typical availability of historic wind data, where annual average alpha exponents for sites are often provided with average wind speed data and wind roses.

The sensitivity of hub height wind speed on the value of the vertical wind shear exponent is very strong. Figure 2.10 was prepared to present an "envelope" of possible variations in wind speed estimates, based on vertical wind shear assumptions, where the value of 1/7 was used as a baseline. The figure shows that an error of 10% in wind power output can occur with a five point error in shear exponent. Considering the fact that wind power is a cubic function of wind speed, the importance of vertical wind shear data cannot be overstated.

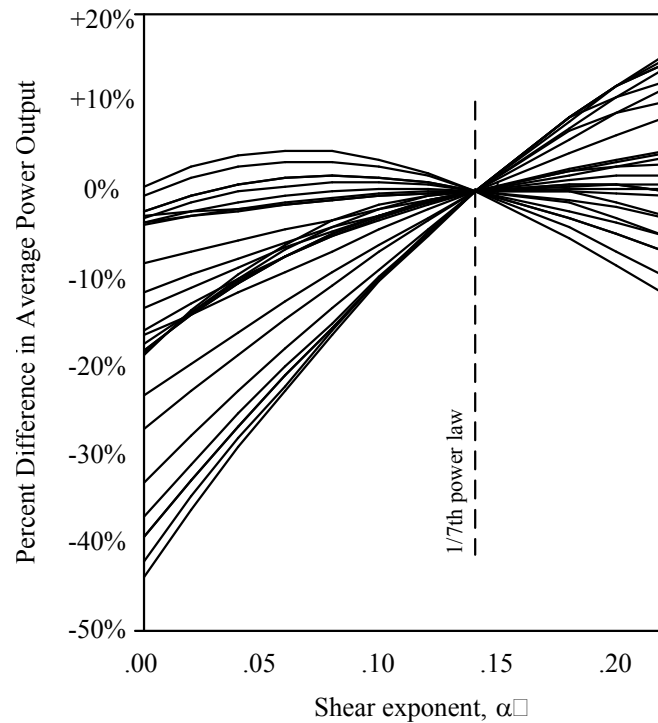


Figure 2.10 Example variation in wind turbine power output as a function of shear exponent. Lines represent various typical data height and turbine height values across a probable range.

### 2.2.3 Turbulence

Turbulence has a variety of impacts in wind energy applications. As discussed earlier in this chapter, turbulence is a major factor in contributing to the fatigue of turbine components. Turbulence is also important from an energy perspective. It contributes to errors in the preparation of power curves. Higher turbulence fosters mixing, reducing wake effects in clusters. It also causes power fluctuations, since pitch controlled blades may not be able to adjust their blade pitch sufficiently quickly to follow the rapidly varying wind speeds, which can result in potentially unstable power output events. For stall-regulated wind turbines, turbulence has a similar effect, due to a hysteresis

phenomenon, termed *dynamic stall*. Data from wind tunnel tests have confirmed the existence of dynamic stall; if the angle of attack of an airfoil is changed rapidly, stall is delayed. The same effect occurs as angle of attack is returned from the stall condition; stall is maintained for a brief amount of time before settling back. When a turbine airfoil encounters turbulence, the effect is a change in apparent angle of attack. This effect has been one of the areas NREL has attempted to mitigate in its development of airfoils designed specifically for wind turbines.

The  $U$  component is downwind, the  $V$  component is vertical, and the  $W$  component is the crosswind component of the wind velocity vector. Turbulence is described as the variation in wind velocity, where the associated turbulence intensities are defined as  $U'$ ,  $V'$  and  $W'$ . The only component which is actually measured in most wind energy site assessments is  $U'$ , based on the variation in the wind speed measurements from a cup anemometer or propeller anemometer mounted on the front of a wind vane.  $U'$  is important from an energy perspective because of its role in airfoil aerodynamics, such as dynamic stall.  $W'$  is important because it influences the horizontal component of the rotor wake structure, and  $V'$  influences vertical wind shear and the vertical component of the rotor wake structure.

The time base for turbulence data is usually over an hour, with measurements stored at 1 Hz. A turbulence intensity of 10% is equivalent to a standard deviation of 1m/s in an hour with a mean value of 10 m/s.

Connell (1986, 1988) has investigated turbulence in the atmosphere, as it relates to wind power applications. His research revealed that turbine blades pass through a combination of turbulence components as a rotation is completed. This mixing effect was simulated by computer re-sampling of turbulence data from a ring of anemometers,



using a rotational approach. Veers (1994) has made important contributions in the characterization of the turbulent flow field, especially as it applies to fatigue damage, to the extent of creating algorithms based on Connell's findings for synthesizing turbulent flow fields with properties similar to observed data.

Typically, for energy calculations, turbulence data is most important for cluster effects calculations. Simplifying assumptions are made of the crosswind structure. First, it is assumed that  $V'$  and  $W'$  are similar. This is important, since otherwise wakes would be anisotropic in the radial direction. The modeling of ovoid wakes would then be required, resulting in increased computational complexity with negligible improvement to the accuracy of cluster energy calculations.

The second simplification is the assumption that crosswind radial turbulence (using a cylindrical coordinate system) is linearly proportional to longitudinal turbulence, about 60 percent of the longitudinal turbulence intensity. This assumption has not been challenged since it was presented by (Lissaman 1982), principally because of the unavailability of field research in the area of radial turbulence characteristics.

Turbulence also becomes important in cluster analyses because upwind turbines impart additional turbulence to the flow-field. Builtjes and Vermeulen (1992) described this effect, showing that the turbulence from upwind turbines is superimposed on the in-flow turbulence. Figure 2.11 shows this effect. The additional turbulence is given by

$$\frac{U'}{U} \cdot \frac{1}{C_T} = \frac{1}{7} \left[ 1 - \frac{2}{5} \ln(2x) \right] \quad (2.41)$$

where the expression on the left side is a dimensionless number expressing turbulence divided by the mean speed and thrust coefficient of the turbine, and the right side is a dimensionless number based on  $x$ , defined as rotor radii downwind.

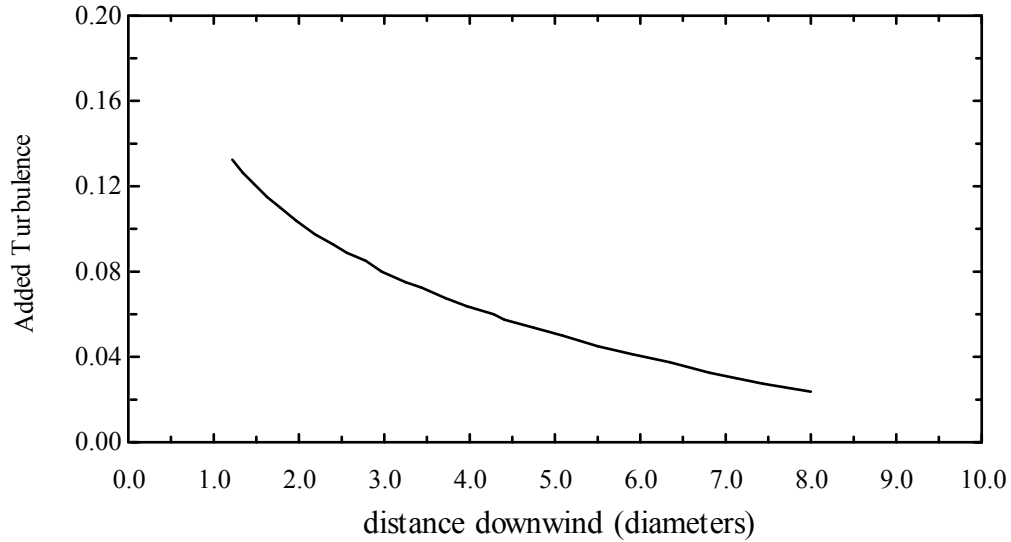


Figure 2.11 Turbulence added to free stream turbulence, as a function of diameters downwind.

When turbulence data from multiple sources are combined, the resulting net turbulence is the r.m.s of the constituent components. So, when adding free-stream and wake-induced turbulence, the net result is calculated as

$$U'_{net} = \sqrt{U'^2_{free-stream} + U'^2_{induced}} \quad (2.42)$$

Applying the assumption of 60% tangential turbulence intensity to equation 2.42 results in

$$\sigma_{nat} = 0.6 \cdot \sqrt{U_{free-stream}^2 + U_{induced}^2} \quad (2.43)$$

where the term  $\sigma_{nat}$  refers to the tangential turbulence intensity (parallel to the plane of the rotor).

#### 2.2.4 Wakes

When installed in multiple-unit arrays or clusters, wind turbines have the potential to interact with each other when a downwind turbine is in the *wake* of an upwind wind turbine. The wake of the upwind turbine can be visualized as a plume of reduced flow having a generally Gaussian shape and boundary-layer characteristics in the radial crosswind direction. The extent of the wake interaction is dependent on the general wind direction (placement), turbulence intensity in the radial crosswind direction (mixing), and distance between turbines (strength), the number of upwind turbines (superposition), and whether wind speeds are reduced below the turbines rated speed (sensitivity). Figure 2.12 shows a typical wake interaction situation for a hypothetical wind cluster.

In general, the impact of wake effects on a wind turbine cluster is a reduction in power output for certain wind speeds and directions. Knowledge of the wind speed by direction distribution at a site results in a determination of an efficiency factor associated with the layout of the wind cluster. Thus, it is possible to perform scenario analyses of possible cluster layouts in order to minimize wake effects. In real life projects, the easiest approach is to increase the distances between turbines. This can increase the cost of a project, however. The costs per land area are then introduced in the analysis to seek an optimum.

In the case of wind turbines arranged along ridge lines, some wind project operators have shut off wind turbines along the rows. This has had two effects: the first influences the apparent turbine distances. The second (and most important to the operator) is the reduced in-flow turbulence (and resulting increase in service-life) experienced by the operating wind turbines. The detailed modeling of wind turbine clusters is presented in Chapter 3.

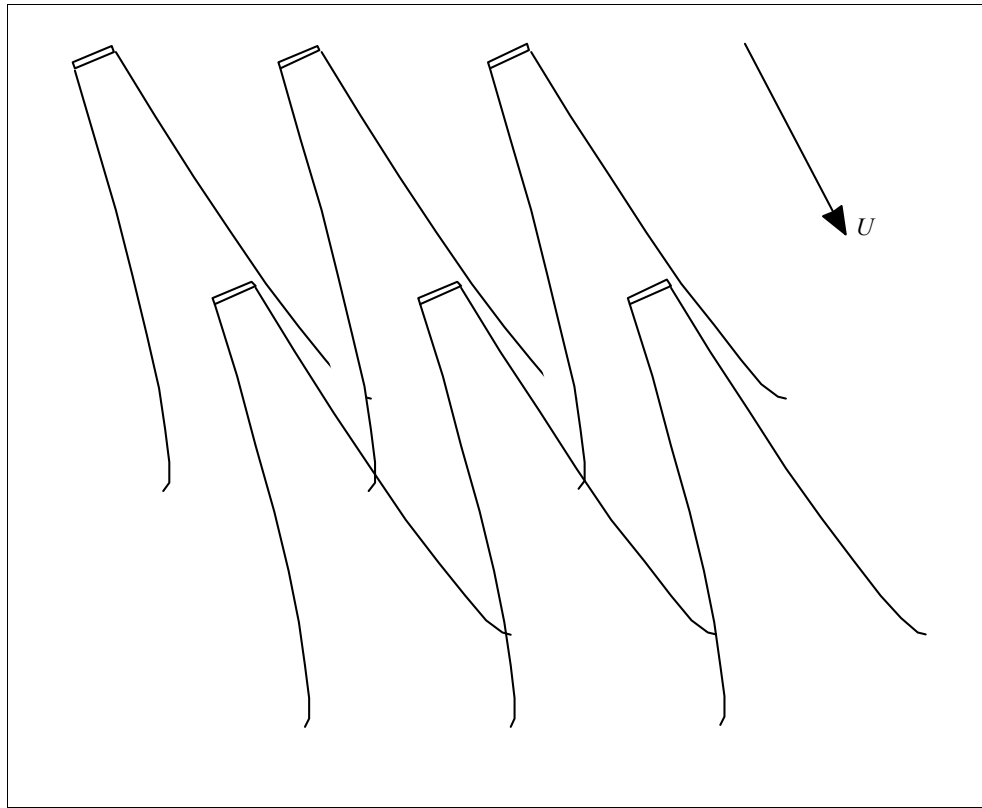


Figure 2.12 Wake interactions in a hypothetical wind cluster.

### 2.2.5 *Other Factors*

A variety of other factors influence wind turbine power output. These can be divided into two categories: aerodynamic and operational. Aerodynamic impacts are those influences on the aerodynamic performance of the rotor airfoils. The most important aerodynamic impacts are the sources of increased surface roughness of the blades, and the prime cause is dead insects built up on the leading edges of the blades. Also recognized are ultra-violet light degradation of the surfaces of the blades, and air pollution. Installations near highways have reported that turbine blades had been soiled by aerosols from the exhaust of diesel engines powering trucks on the highway.

Operational influences are either external- or control-system sources of sub-optimal operation. External causes include power outages or inadvertent shut-down. Control-system causes include sub-optimal cut-in of the rotor due to anemometer or software error. A common cause of sub-optimal operation of horizontal axis turbines is yaw-error: the misalignment of the rotor to the wind which can happen when variation in wind direction occurs faster than the response rate of the yaw drive.

Operational factors are commonly lumped into a percent-loss factor or efficiency factor.

## 2.3 **TRNSYS Type 85: Wind Turbine**

The algorithms discussed earlier in this chapter were formalized into a TRNSYS module, Type 85. The single wind turbine component simulates the energy output of a wind turbine based on input characteristics. After reading in a file containing turbine performance parameters and power curve data, Type 85 determines shear corrections,

air density, and resulting air density effect on the power curve each timestep. Vertical wind shear corrections are based on a power-law model, with exponent values derived from a time-series, calculated from available data, or input by the user. Air density is calculated using the ideal gas law and the temperature lapse rate model. Air density corrections for empirical power curves are applied using the AWEA / IEA methodologies (AWEA, 1988) (IEA, 1982).

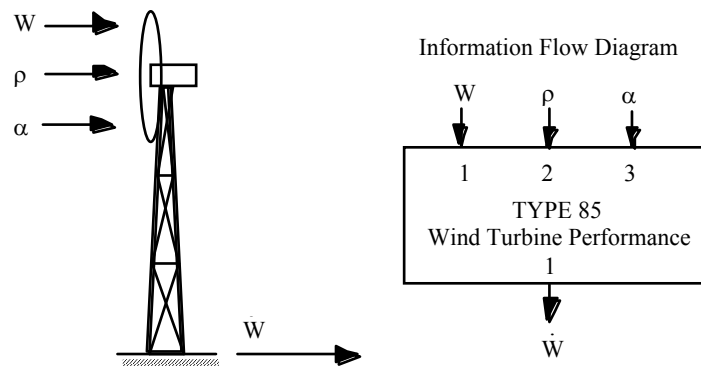


Figure 2.13 TRNSYS Type 85: Wind Turbine.

The simulation of the wind turbine produces outputs of power output, coefficient of performance, on/off status, etc. In the first section of the code, the variables are dimensioned, the parameters are identified, and the inputs are listed.

Because multiple units of the type may be used, Type 85 sets up a data array, where each column of the array is the data for each unit of Type 85. Since power curve data are two-dimensional (Power versus wind speed) the data array contains:

- A unit (turbine) number: up to 60

- Variable names and wind speed values: up to 99 values of wind data
- Values for variables and power output (kW) corresponding to wind speeds

After the turbine data files are read in, and the parameters are read in, Type 85 determines what kind of data are available for air density calculations. If time series data are available, then they are used. Otherwise, if temperature and pressure data are available, they are used to calculate air density over time. If temperature and pressure data are not available, then a fixed value is used as an input.

The next operation corrects the power curve wind data to hub height when the power curve data collection height was lower than the hub height. This routine adjusts the wind speed values in the power curve data file to hub height values using the power law and an exponent. Next, the power curve terms are interpolated to estimate a power output value. This is performed in all cases by determining the power coefficient at the bracketing wind speeds, then applying the interpolated power coefficient and timestep air density. This variable-density cubic approach allows for the use of standard tabular lists of power curve data, but also handles sparse data tables more accurately than simple linear interpolation: the interpolation in the power regulation region of the power curve is more realistic, and corresponds with the mechanistic model of rotor performance.

Type 85 calculates maximum power differently based on the type of power control. If the rotor is stall-regulated, maximum power is determined by the ratio of hourly air density over power-curve air density. If the turbine is power regulated, the maximum power is not affected by air density. The speed at which the wind turbine reaches rated power output is altered, however. The new rated wind speed is calculated using equation 2.28.

The cut-out wind speed (if there is one) is the wind speed at which the force of the wind is at a maximum. Recalling that thrust forces are a function of the square of the velocity, the equivalent force determines the new cut-out wind speed, or

$$U_{cut-out,\rho} = U_{cut-out} (\rho_0/\rho)^{1/2}$$

(2.34)

A "turbine hours" value is calculated as the time step hourly fraction, and is valuable in the event a user wishes to know the number of total run hours per year, for example. The outputs of the Type 85 are:

- Energy per time step
- Power Coefficient
- Turbine hours.

A listing of Type 85 is given in Appendix B. A demonstration TRNSED deck is listed in Appendix C.