CHAPTER THREE

WIND TURBINE CLUSTER MODELING

Wind, she brews a heady kettle. Human beings love it--love it.
Gods above are not above it.

--Robert Frost

Most of today's utility wind projects are composed of wind turbines installed in a group, or *cluster*. Much of the literature uses the term \array", since the layout of a wind project often follows a repeated pattern, but because of the discussion of data arrays in this thesis, the term cluster is used to avoid confusion.

The performance of a wind turbine cluster is more complex to model than the performance of a single wind turbine because of wake interactions among the turbines in the cluster. Each turbine in the cluster operates autonomously, but the wind reaching a turbine may be influenced by the other turbines in the cluster. This influence is a function of wind speed, direction and relative position. Analysis of wind clusters employs single turbine performance models, but also including wind direction impacts and wake interaction effects. For this reason, analyses of wind turbine clusters are most concerned with wake models and cluster geometries. Results of cluster analyses include statistics of merit for both individual turbines and the cluster as a whole.

3.1 Wake Models

Wind turbine cluster models ultimately rely on wake models for prediction of the profile of the wind speed deficit behind the rotor. Wake models have traditionally been based on wind tunnel and wind cluster field data for validation of theoretical models. These models have common attributes:

- A Gaussian profile of the wake
- Characterization of the wind velocity in cylindrical coordinates (x, distance downwind; r, radial distance outward from wake centerline)
- Wake radius defined in a manner similar to boundary layer thickness
- Wakes divided into regions: a near region, transition region, and far-wake region.

Figure 3.1 is a graphical depiction of the wake regions utilized in the various models. A number of wake models have been put forth, but the best known are those employed in the cluster models developed by P.B.S. Lissaman (1982) and Veenhuizen and Lin (1989).

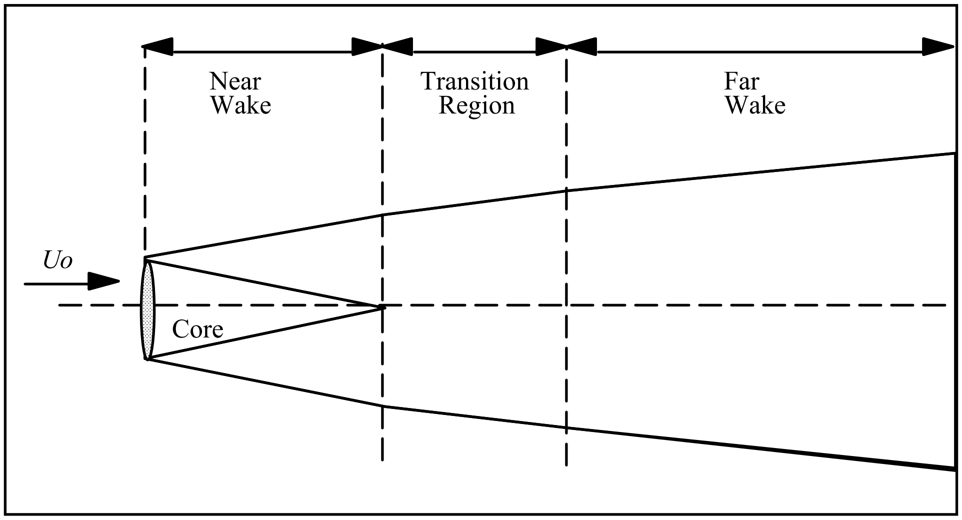

Figure 3.1 Graphical description of wake regions.

3.1.1 *Early Wake Research*

The far wake is of primary interest; the near wake and transition wake regions are 0 to 5 *x* downwind from the rotor, (*x* is the distance downwind, measured in upwind-turbine rotor radii) and wind turbine spacings at these close distances are rare. The original Lissaman wake model for the far wake was based on a relation described by Abromovitch (1963) for the profile of a \co-flowing" jet. Lissaman later updated his model based on early field data. In Europe, several other wake models (Crespo, et al., 1988), (Van Oort, 1988) received some early attention. Instead of fluid mechanical models, the European models employed statistical (Gaussian) profiles of the wake in the transverse direction.

3.1.2 *The UIC Wake Model*

Veenhuizen and Lin (1985) of United Industries Corporation (UIC) formulated a set of relations, based on empirical data, to calculate the wake deficit at location *x* and *r*, where x is the number of (upstream turbine) rotor radii downwind, and r is the number of (downstream) rotor radii crosswind. Based on a review of literature available on the validation of the various models, the Veenhuizen and Lin model has received the best validation results and was chosen for the wake deficit model incorporated in the cluster model developed in this thesis. The wake does not have a boundary *per se*, but a boundary is taken to be the locus of points where the wind speed is 99.9% of the free-stream wind speed. Figure 3.2 shows the typical profiles of the wake deficit as a function of distance behind the rotor.

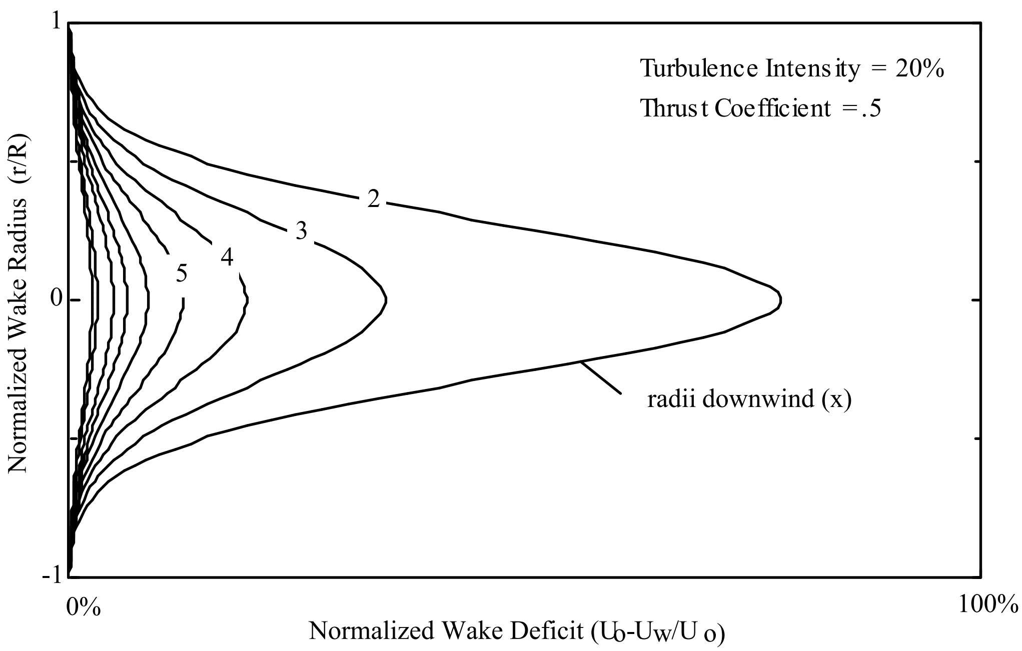

Figure 3.2 Typical profiles of the wake deficit as a function of distance behind the rotor.

The UIC far-wake model used is appropriate after a critical distance downwind. This critical distance is calculated by the equation:

$$x_{critical} = \sqrt{C_T / \sigma} \tag{3.1}$$

where $x_{critical}$ is defined as the minimum distance downwind where the far wake model is appropriate, C_T is the turbine thrust coefficient (equation 2.28), and σ is the free-stream turbulence intensity in the transverse, radial direction.

The transverse turbulence intensity, σ, is approximated by the relation

$$\sigma = 50 \Big/ \sqrt{U_0} \tag{3.2}$$

for U_0 measured in m/s. The wake deficit, ΔU, at x (upwind turbine) radii downwind and r (upwind turbine) radii crosswind is determined using the relation

$$\Delta U = \left(C_T / 4\,\sigma^2 x^2\right) \exp\left(-r^2 / 2\,\sigma^2 x^2\right) \tag{3.3}$$

Although the wake radius is not required for energy calculations, it is useful for graphical representation of superimposed wakes. Solving equation 3.3 for ΔU = 99.9% results in the following equation for the wake radius,

$$R_{wake} = \left[2\,\sigma^2 x^2 \ln\left(4\,\sigma^2 \frac{x^2}{1000 \cdot C_T} \right) \right]^{1/2} \tag{3.4}$$

Using relations 3.2 and 3.3, it is possible to estimate the wind speed at any point location in the wake.

3.2 Cluster Performance Modeling

The overall process of wind turbine cluster modeling is a superset of the single turbine modeling process. A cluster modeling process is utilized to estimate the rotor wind speed of each turbine in the cluster prior to applying the single turbine performance algorithms. The process utilized follows the flow diagram presented in Figure 3.3.

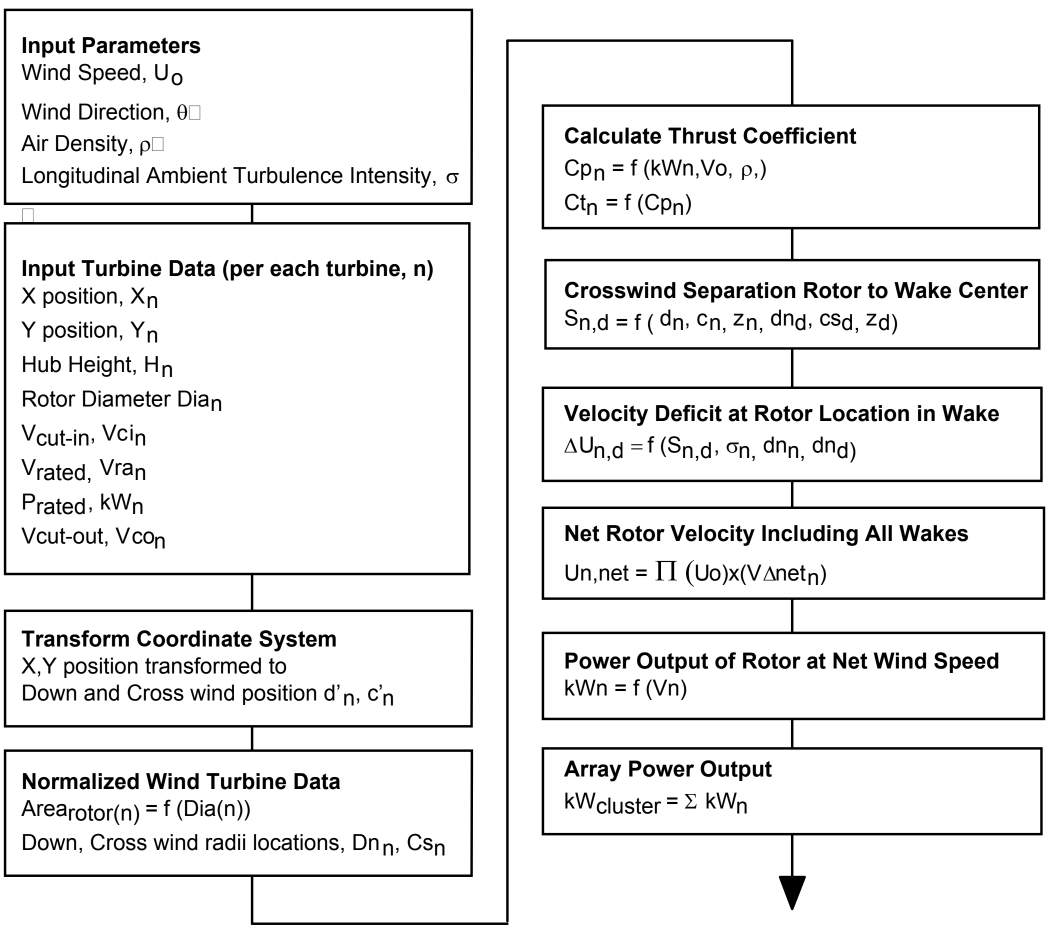

Figure 3.3 Wind cluster modeling algorithm.

3.2.1 *Coordinate Transformations*

In order to begin modeling clusters, three sets of coordinate systems are defined. The first is the wind vector coordinate system. Following convention taken from the meteorological community, Northerly winds mean winds from the North, defined as originating from the zero (360) degree direction and increasing in the clockwise direction. For example, Westerly winds are winds coming from the 270 degree direction. Note that this convention is not the same as a 270 degree vector, since mathematically vectors are defined by where they point *toward* not *from*.

The second convention is the originating wind turbine plan view coordinate system. This follows a traditional positive eastward (e) and positive northward (n) direction. The third convention is the coordinate system transformed orthogonal to the wind direction: downwind (d) and crosswind (c).

Coordinate transformations are performed so that the e and n plan view coordinates are transformed into d and c coordinates, as defined by the wind direction vector for each compass position. For each position, new d, c coordinates are created relative to a downwind d axis and a crosswind c axis. Transformations of coordinates per compass direction allows the wake deficit equations to be applied within a consistent d, c coordinate system.

The coordinate transformations are carried out in two steps: *rotation* and *translation*. First, the coordinates are rotated into downwind and crosswind directions using the following trigonometric formulations (Beyer, 1991):

$$d = e\cos(\theta) - n\sin(\theta) \quad \text{and} \quad c = e\sin(\theta) + n\cos(\theta) \tag{3.5}$$

After the rotation is performed, it is possible that some coordinate values will be negative. In order to eliminate these negative values, and to "push" the cluster into the first (positive, positive) quadrant, the rotated values are *translated* so that the nearest turbines are moved to the zero line, and all turbine crosswind positions are shifted so that the lowest position lies on the zero line (Beyer, 1991). The translation is performed by adding a fixed distance, equal to the most negative coordinate value, to all the coordinate values associated with each axis

$$c_{\min} = \min(c_1, c_2, c_3, \ldots c_N) \quad c_{offset} = 0 - c_{\min} \quad c'_n = c_n + c_{offset} \tag{3.6}$$

$$d_{\min} = \min(d_1, d_2, d_3, \ldots d_N) \quad d_{offset} = 0 - d_{\min} \quad d'_n = d_n + d_{offset} \tag{3.7}$$

The variables c'_n and d'_n are the final, rotated and translated coordinates for each turbine n. The coordinate transformation operation is carried out for all angles in which cluster calculations are performed. Figure 3.4 illustrates the transformation process for an example cluster of four wind turbines labeled A, B, C, and D, in winds out of the East-Northeast at 22°. The first plot shows the cluster in North and East coordinates. The second plot shows the cluster rotated to be orthogonal to the wind in d, c coordinates. The third cluster is in the same orientation, but shifted over out of the negative region.

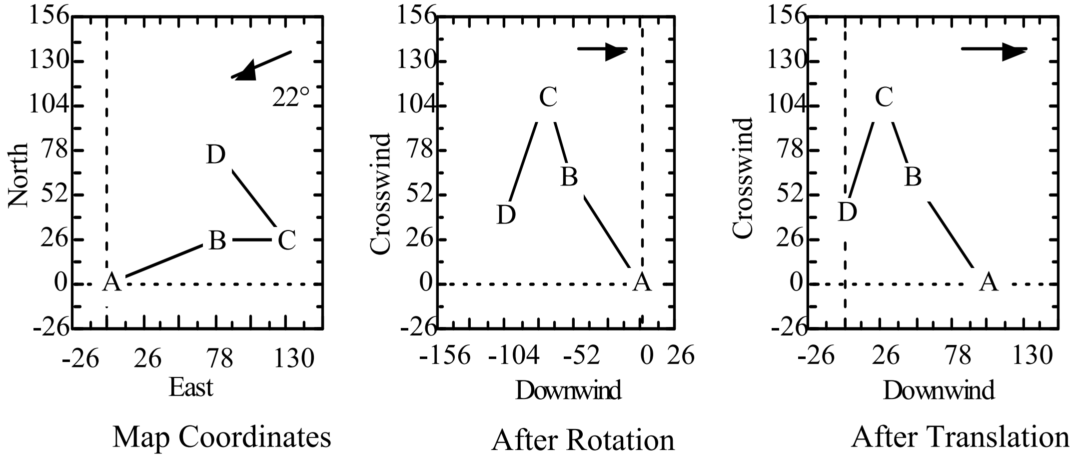

Figure 3.4 Effect of the coordinate transformation.

3.2.2 *Wake Superposition Methodology*

The position of the centerline of a wake is described by its crosswind position in the *d, c* coordinate system. In order to determine the separation distance between a turbine and the centerline of the wake from an upwind turbine, the difference in rotor heights must be taken into account. For turbines *i* and *j,* the separation distance, $S_{i,j}$, is the hypotenuse of the triangle with the lengths equal to their relative distance on the ground and their relative height difference, or

$$S_{i,j} = \sqrt{\left(c_i - c_j\right)^2 + \left(z_i - z_j\right)^2} \quad (3.8)$$

The UIC wake deficit model, equation 3.3, is then used to determine the wake deficit at the *d, c* position of the hub center. The UIC model furnishes a wake deficit value given the separation distance, $S_{i,j}$; the relative downwind position $\left(d_i - d_j\right)$; the C_T of the upwind rotor; and the transverse turbulence intensity, σ.

An addition to the UIC model used in this work is the inclusion of additional transverse turbulence created by the upwind turbine, equation 2.33. The free-stream and turbine-induced turbulence are combined at this point in the calculation process because the relative downwind distance of the upwind and downwind turbines is calculated at this point, and the resultant turbulence value is used in the wake deficit calculation.

Although the wind turbine average rotor speed is the average of the wind speeds integrated over the area of the downwind rotor, this value is similar to the value of the point at the rotor center. Use of the rotor center to represent the integrated average of a property applied to the entire rotor area is analogous to the convention used for assigning the average wind speed for a rotor in the atmospheric boundary layer: the wind speed at the rotor center may not equal the average for the rotor in a highly sheared flow; it may be underestimated, but the difference is rarely significant from a power output modeling perspective. Finally, use of the wake deficit equation at a representative point is computationally much faster than an integration process, especially considering the eventuality of applying the process repeatedly to a large number of turbines.

3.2.3 *In-wake Rotor Wind Speeds*

Wake superposition is defined as the convention where the combined wake effect at a turbine location results from the superposition of the deficits at that position from all upwind turbines. Computationally, wake superposition is the multiplication of wake deficit values from all upwind turbines to the downwind turbine, to the free stream wind speed. The net wind speed reaching a particular downwind turbine is therefore the free stream wind speed multiplied by the assigned deficit factors, as described by the relation:

$$U_{turbine} = U_0 \cdot \prod_{1}^{N} (1 - \Delta U_n) \tag{3.9}$$

This is a geometric model; the free-stream wind speed can never be reduced to a negative value.

3.2.4 *In-wake Turbine Power Output*

The in-wake turbine power output for each turbine in a cluster is calculated exactly the same as for single turbines, except that the input wind speed is not the free-stream rotor wind speed, but the in-wake rotor wind speed, $U_{WT,n}$. The *cluster efficiency* of a wind turbine in the cluster is defined as

$$\eta_{WT,n} = \frac{P\big|_{U_{WT,n}}}{P\big|_{U_0}}$$

(3.10)

Cluster efficiencies of individual turbines can range from 30% to 100%.

3.2.5 *Cluster Performance*

Cluster performance is defined as the performance of the entire cluster, with wake effects, compared to performance without wake effects. Cluster performance and cluster efficiency can be mapped as a function of wind direction, similar to the example shown in Figure 3.5. The cluster efficiencies portrayed in the figure are for four AWT 250 kW turbines located along a north/south line, spaced very closely at three rotor diameters apart. Figure 3.6 shows the total power output of the cluster.

Wind speed reductions within a cluster may not have an effect on reducing output power if the free-stream wind speeds are high enough so that the downwind turbines remain operating above their rated wind speed; there is no sensitivity to wind speed decreases at high wind speed. Similarly, if wind speeds are low, it is possible for a wind turbine to be shut-off by an upwind turbine which has reduced wind speeds to below the cut-in value. The sensitivity of cluster efficiency to wind speed is illustrated in Figure 3.5. Figure 3.6 illustrates the impact of increasing wind speed on cluster power output. At free-stream wind speeds just above cut-in, 6 m/s, the last turbine in the row has been shut-down because the wind speed at its rotor is below its cut-in wind speed. At wind speeds just above rated, 18 m/s, the turbines are producing maximum rated output for most directions, except when the wind direction is parallel with the row, when the last turbines in the row are in winds below rated and the total output is reduced. At wind speeds far above the rated wind speed, 22 m/s, the decrease in wind speed occurs, but all turbines continue to produce maximum power.

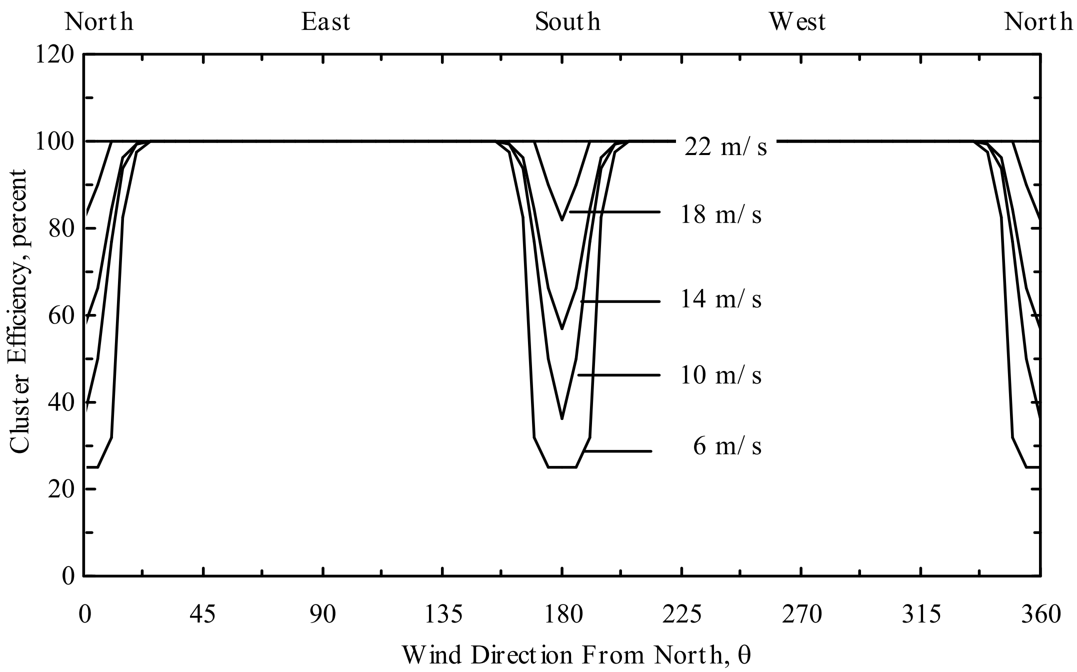

Figure 3.5 Cluster efficiency as a function of wind speed and direction.

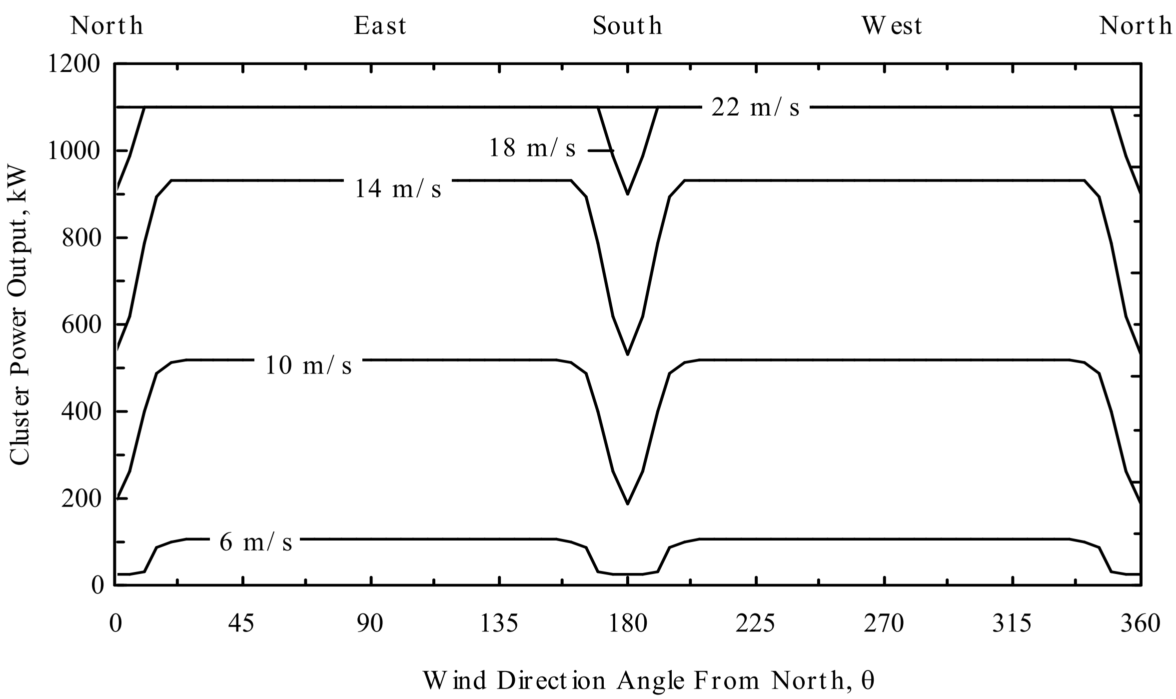

Figure 3.6 Cluster power output as a function of wind speed and direction.

3.3 TRNSYS Type 88: Wind Turbine Cluster

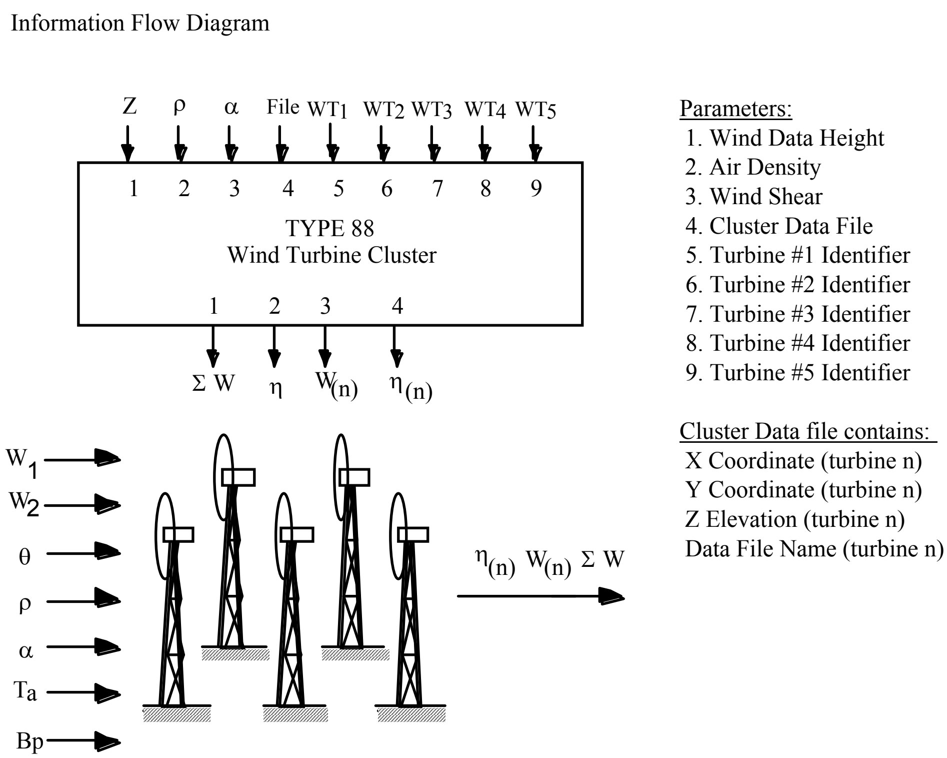

Figure 3.7 TRNSYS Type 88: Wind Turbine Cluster

TRNSYS Type 88 models the output of a wind turbine cluster, employing the wake deficit model, equation 3.3, developed and verified by the U.S. Department of Energy (Veenhuizen, 1988). In the first section of Type 88, the wind turbine model, location and hub height data are read in. Then an array is filled with the operating characteristics of each of the identified turbines based on its model description. The model description files used by Type 88 are identical to the data files read by Type 85.

Next, individual turbine power output is calculated for each wind speed (per m/s increment) through 25 m/s, and for each ten degrees of compass direction.

The cluster model proceeds along the following process to calculate performance:

1. Input turbine x, y, z location and power curve data.
2. For each wind direction, from 0 to 350, in ten degree increments:

   A. Transform coordinates to new a downwind, crosswind system.

   B. For each wind speed, from cut-in to cut-out:

   - Calculate wake deficit per turbine location due to all turbines.

     (Calculate rotor averaged wind speed at its location in wake.)

   - Apply all deficits to wind speed to obtain a turbine net local wind speed.

   - Calculate power output of the turbine at the net local wind speed.

   - Sum wind turbine power outputs to determine cluster totals at that speed.

The net result is a FORTRAN array dimensioned by wind turbine, wind speed, and wind direction. Type 88 then interpolates the array per turbine, per timestep. This pre-processing of cluster performance is performed prior to time-series analysis. The preprocessing of power output is more computationally efficient for time-series analyses. During time-series operation, Type 88 interpolates the two-dimensional array for each wind turbine according to the wind speed and direction in the time step.

The maximum number of turbines that can be simulated by Type 88 is determined by the size of the data array size parameter. Larger clusters can be handled by resetting the array size parameter in the FORTRAN source code and recompiling. It is anticipated, however, that the current setting of 99 will probably not be a constraint for the near future.

Type 88 outputs the per-turbine energy production in each time step for up to five selected turbines in the cluster, as well as the cluster total energy and cluster efficiency. The TRNSYS wind turbine cluster component is the first implementation of a cluster array loss model in a time series context. The cluster model is listed in Appendix A.