

**A METHOD FOR ESTIMATING  
THE LONG-TERM PERFORMANCE OF  
DIRECT-COUPLED PHOTOVOLTAIC SYSTEMS**

by

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## **Abstract**

This thesis introduces a new method to estimate the long-term performance of direct-coupled and maximum power-tracked photovoltaic (PV) systems without battery storage. A number of models exist for estimating maximum power-tracked system performance, but the maximum power-tracking feature is included here as a benchmark for comparison to direct-coupled system performance, the true object of this research. While the output from either type of PV system is dependent on weather and PV array characteristics, the output from direct-coupled PV systems is dependent on the applied load as well. As a result, estimating the performance of direct-coupled systems is more complex than for maximum power-tracked systems.

The method developed here is computationally simple. A reduced set of hourly weather data is generated from widely available long-term monthly-average global solar and ambient temperature data. Correlations are used to estimate hourly weather variations within a day and daily variations within a month. A small number of "typical day" groups are used to approximate the long-term distribution of daily weather in each month. The "typical days" within each group are assumed to be identical which reduces the number of computations yet, subject to the accuracy of the correlations, retains the accuracy of long-term simulations.

To assess the validity of the new method, a program titled DCPVSIMP (for Direct-Coupled PV model, simplified version) was written. Five potential I-V curve sub-

models for this program were evaluated; the one which was selected corresponded well with a large sampling of experimental I-V curve data from outside sources, and the information required to use it is commonly available from PV manufacturers. A detailed version of this program, identical in all respects except that it uses hourly TMY data, titled DCPVDET, was also written to provide a basis for evaluating the weather generation component.

Both the DCPVSIMP and DCPVDET versions were compared to two established models for maximum power-tracking systems, PVFORM and PV f-Chart. Monthly and annual estimates were within  $\pm 1\%$  relative to PVFORM and  $+5$  to  $6\%$  relative to PV f-Chart. PV f-Chart includes a variable correction factor which decreases estimates of the absorbed radiation at off-normal incidence angles. For performance estimates at moderate northern latitudes the overall effect of this term is about a  $5\%$  decrease in annual output. The difference between the DCPVDET model and a "5 typical day" version of the DCPVSIMP model was found to be less than  $1\%$  for maximum power-tracking systems.

Monthly and annual performance for over 800 cases using the DCPVDET model, based on three locations and a large variety of direct-coupled resistive, fixed voltage, and DC motor loads, was compared to 3, 5, 10, and 20 "typical day" versions of the DCPVSIMP model. The overall annual  $\%$  root mean square (RMS) difference between the two models ranged from  $3.8\%$  for the "3 typical day" version to  $3.2\%$  for the "20 typical day" version, with most of the reduction in the  $\%$  RMS difference occurring between the 3 and 5 "typical day" versions ( $3.4\%$  RMS for the 5 day version). For all

versions the % mean bias difference (MBD) was less than 1%. The worst monthly results ranged between 5 to 6% RMS among the four versions tested, with a % MBD of less than 1 %.

The body of the report includes a derivation of the new direct-coupled performance estimating method and a description of the DCPVSIMP and DCPVDET models, statistical evaluations of the models and their components, and a set of graphs illustrating typical applications of the DCPVSIMP model for direct-coupled system design.

## Acknowledgments

4:30 AM in the Solar Lab, *naturally*. But it feels great; after 16 months of stumbling around deep in research hell' and 3 1/2 more spent noodling with an endless stream of plots, equations, and dangling modifiers, if Mr. Goodprinter cooperates, I can download this baby onto an unsuspecting planet by daybreak. Despite the all-nighters, the steady diet of cardboard pizza, battling the equipment, and risking it all in a building with a Swiss cheese roof and a hurricane ventilator that swirls gases of unexplained origin ("it smerrs bad, but it woorn't hoolt you"), it doesn't seem so bad now. Hey, where do you sign up for the Ph.D?

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\* from Minnerly, B.V., UW-Madison Solar Energy Laboratory, 1988]



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## Nomenclature

A	Completion factor
a,b	Constants in Liu and Jordan correlation
AC	Alternating Current
C <sub>1</sub> ,C <sub>2</sub>	Constants in MIT model
C <sub>STAT</sub>	Static torque loss constant, N-m
C <sub>VISC</sub>	Viscous torque loss constant, N-m-s
D	Diode diffusion factor
DC	Direct Current
EMF	Electromotive Force, Volts
F	Cumulative fraction of days per month
F	Objective function
F.F.	Fill Factor, dimensionless
G <sub>SC</sub>	Solar constant, 1353 W/m <sup>2</sup>
H	Total Dynamic Pressure Head, meters
H	Total horizontal radiation, KJ/m <sup>2</sup> -day
H <sub>D</sub>	Daily diffuse radiation, KJ/m <sup>2</sup> -day
H <sub>O</sub>	Daily extraterrestrial horiz. radiation, KJ/m <sup>2</sup> -day fIR Hour
HRX	Dimensionless hour I Current, Amps
I <sub>D</sub>	Diode current, Amps
I <sub>L</sub>	Light-generated current, Amps
I <sub>O</sub>	Reverse saturation current, Amps
k	Boltzmann constant, 1.381E-23 joules/Kelvin

$k$	Motor dimensionless flux coefficient
$K_{MAX}$	Maximum clearness index
$K_{MIN}$	Minimum clearness index
$K_T$	Clearness index
$K_T$	Monthly-average clearness index
kWh	Kilowatt-hour
L3P	Lumped, 3 Parameter I-V Model
L4P	Lumped, 4 Parameter I-V Model
2M5P	Lumped 2 Mechanism, 5 Parameter I-V Model
2M6P	Lumped 2 Mechanism, 6 Parameter I-V Model
$L_A$	Armature Inductance, Henrys
$L_F$	Field Inductance, Henrys
LINEAR	Simple linear maximum power only model
$M_{AF}$	Mutual Armature-Field Inductance, Henrys
MBD	Mean Bias Difference
MIT	Mass. Inst. of Tech. I-V Model
MPT	Maximum Power Tracker
$n$	number of mechanisms
$N$	Day of year
$n,p$	negative, positive charges
NCS	Number of cells in series
NMSEI	New Mexico Solar Energy Institute

NP	Number in Parallel
NS	Number in Series
P	Power, Watts
PCU	Power Conditioning Unit
PV	Photovoltaic
Q	Volumetric flowrate, m <sup>3</sup> /sec.
q	Electron Charge constant, 1.602E-19 Coulombs
R <sub>A</sub>	Armature Resistance, Ohms
R <sub>B</sub>	Ratio of tilt/horiz. beam radiation
r <sub>D</sub>	Hourly fraction of daily diffuse radiation
R <sub>F</sub>	Field Resistance, Ohms
R <sub>L</sub>	Load Impedance or Resistance, Ohms
RMS	Root mean square
R <sub>S</sub>	Apparent series resistance, Ohms
R <sub>SH</sub>	Apparent shunt resistance, Ohms
r <sub>T</sub>	Hourly fraction of total daily radiation
SW	Specific weight of water, 9807 N/m <sup>3</sup>
T	Temperature
T <sub>C</sub>	Cell Temperature
T <sub>LOAD</sub>	Load torque, N.m
T <sub>LOSS</sub>	Loss torque, N.m
TMY	Typical Meteorological Year
U <sub>L</sub>	Overall loss coefficient, W/m <sup>2</sup> .K

V	Voltage, Volts
Z	Consolidating term for algebraic simplification

### Greek Symbols

$\beta$	Temperature coefficient of maximum power, 1/°C
$\delta$	Declination, degrees
$\varepsilon_A$	Armature EMF, Volts
$\varepsilon_G$	Bandgap energy, Volts
$\eta$	electric conversion efficiency
$\gamma$	Dimensionless curve fit factor
$\lambda$	Bendt correlation variable
$\Lambda$	Consolidating term for algebraic simplification
$\mu_{ISC}$	Temperature coefficient of short circuit current, Amps
$\mu_{VOC}$	Temperature coefficient of open circuit voltage, Volts
$\omega$	Hour angle, degrees
$\omega$	speed, radians/sec.
$\omega_S$	Sunset angle, degrees
$\varphi$	Permanent magnet flux, Webers
$\varphi$	Latitude, degrees
$\Phi$	Irradiance, W/m <sup>2</sup>
$(\overline{\tau\alpha})$	Transmittance-Absorptance product
$\zeta$	Bendt correlation variable

### Subscripts

A, AMB	ambient
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DET	detailed
KNOWN	given condition
MP	maximum power
NEW	new condition
NOCT	Nominal Operating Cell Temperature
OC	open circuit
OPT	optimum
P	pump
REF	reference condition
SC	short circuit
SIMP	simplified