Second Law Analysis of Solid Desiccant Rotary Dehumidifiers[1]

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This paper presents a second law analysis of solid desiccant rotary dehumidifiers. The equations for entropy generation for adiabatic flow of humid air over a solid desiccant are developed. The generation of entropy during operation of a rotary dehumidifier with infinite transfer coefficients is investigated and the various sources of irreversibility are identified and quantified. As they pass through the dehumidifier, both the process and regeneration air streams acquire nonuniform outlet states, and mixing both of these air streams to deliver homogeneous outlet streams is irreversible. Transfer of mass and energy between the regeneration air stream and the desiccant matrix occurs across finite differences in vapor pressure and temperature and these transfer processes generate entropy. The second law efficiency of the dehumidifier is given as a function of operating conditions and the effect of finite transfer coefficients for an actual dehumidifier is discussed. It is shown that operating the rotary dehumidifier at conditions that minimize regeneration energy also yields a local maximum for the second law efficiency.

Introduction

Desiccant air conditioning and industrial drying systems using a rotary dehumidfier have been proposed as an alternative to conventional vapor compression units. In these systems, air is dried by passing it over the desiccant and the heat of sorption is removed by sensible cooling. The air may then be further cooled by adiabatic humidification to yield cool and dry air. The energy consumption and system performance in commercial and residential applications have been studied [1–3]. The efficiency of a desiccant air conditioning system is given by the thermal COP, i.e., the ratio of cooling capacity to required regeneration energy. Experimental and calculated COP's of desiccant cooling systems have been reported in the range of 0.5 to 1.0 [1, 4].

Jurinak et al. [1] evaluated the performance of residential air conditioners which use adiabatic silica gel dehumidifiers in gas-fired, solar-fired, and solar-assisted ventilation and recirculation type systems. For each configuration, they computed the COP of three different air conditioners with individual component performance ranging from state of the art to effectiveness approaching 1. Their study gives maximum values of calculated COP's ranging from 0.5 to 1. In a second law analysis of open cycle desiccant cooling systems, Lavan et al. [5] showed that the COP of these systems has a maximum value that depends on ambient conditions. Their model for reversible desiccant air conditioners gives a maximum COP of 4.66 for ARI standard ambient and room conditions. However, in his discussion on high performance open cycle adiabatic desiccant cooling systems, Maclaine-cross [13] showed a cycle with reversible components which has infinite COP

for ARI conditions. He also proposed a reversible cycle with high sensible heat performance which has a COP ranging from 2.7 to 5.8 for ARI conditions depending on the latent load ratio.

Lavan et al. [5] presented a second-law overall system-analysis of desiccant air conditioners. A second law analysis of the individual components in these systems can give additional understanding of the feasible performance of desiccant cooling systems. The performance of power and refrigeration cycles is decreased by the thermodynamic irreversibilities that occur in the system. The sources of irreversibility in desiccant cooling systems are the heat exchanger, the evaporative coolers, and the dehumidifier. Maclaine-cross et al. [6] proposed a wet surface heat exchanger that may operate as a reversible regenerative evaporative cooler for infinite transfer coefficients for heat and mass. This paper reports the results of a second law analysis of adiabatic, silica gel rotary dehumidifiers.

In a first part, the entropy generation for adiabatic flow of moist air through a solid desiccant matrix and during operation of a rotary dehumidifier with infinite transfer coefficients is investigated. The various causes of irreversibility during dehumidifier operation are identified and quantified. The major sources are mixing of the air stream, which leaves the dehumidifier to produce a uniform outlet state (process and regeneration air streams), the transfer of energy and mass across finite temperature, and vapor pressure differences (shocks) between the desiccant matrix and the regeneration air stream. It is shown that the adiabatic flow of the process air stream over the desiccant is reversible with respect to the exchange of mass and energy.

The required regeneration energy for a given cooling load of desiccant cooling systems may be minimized by choosing optimum values of dehumidifier-rotational speed, regeneration

air mass flow rate, and regeneration temperature [2]. In a second part of the analysis, the second law efficiency of solid desiccant rotary dehumidifiers is defined and determined as a function of these operating conditions. It is shown that operating the rotary dehumidifier at conditions for minimum regeneration energy also yields a local maximum for the second law efficiency. The second law efficiency of an actual dehumidifier with finite transfer coefficients is given as a function of the number of transfer units.

Rotary Dehumidifier Model

The model for performance prediction of solid-desiccant rotary dehumidifiers used in this analysis is the effectiveness-NTU model that has been developed by Van den Bulck et al. [7]. A discussion of this model and comparison with other models is given in [2]. The concepts of this model that are needed for this analysis are briefly outlined below.

The nomenclature and coordinate system for the rotary dehumidifier are illustrated in Fig. 1. The adsorbent matrix is arranged as a rotating cylindrical wheel of length L and has a total mass of dry desiccant M_d. Two air streams are passed simultaneously in counterflow through two separated sections of the regenerator. The process air stream has a low temperature and high relative humidity while the regeneration air stream has a high temperature and low relative humidity. For each period, the axial coordinate, z, is defined as positive in the fluid flow direction, while the rotary position is indicated by the time coordinate θ.

The model which describes the exchange of heat and mass between the moist air and the desiccant matrix is based on the following conventional assumptions for this situation [8]:

1. The matrix is modeled as being of parallel passage form, consisting of a homogeneous solid with constant matrix characteristics and porosity, through which an air-water vapor mixture flows with constant velocity. Pressure drop effects through the bed are small with respect to absolute pressure [4] and are neglected.
2. The state properties of the air streams are spatially uniform at the inlet of each period. There is no mixing or carry over of process and regeneration air streams.
3. A transient one-dimensional approach is applied. There is no radial variation of fluid or matrix states, and dif-
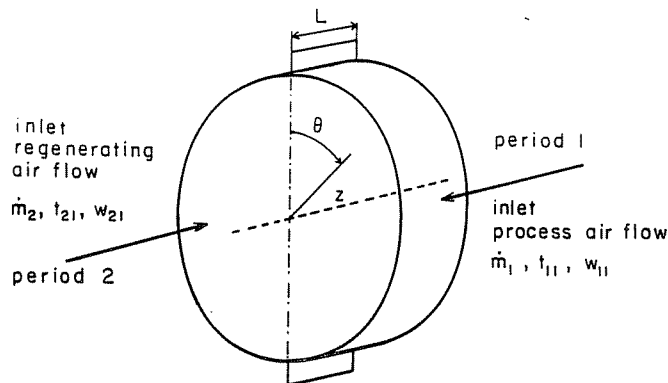

Fig. 1 Nomenclature and coordinate system for the rotary dehumidifier

fusion fluxes of heat and mass due to tangential gradients of matrix and air state properties are neglected.
4. The axial heat conduction and water vapor diffusion flux are negligible in both the matrix and the air streams.
5. The thermal and moisture capacities of the air entrained in the matrix is negligible compared to the matrix capacities.
6. Transport of water vapor through the air occurs only through ordinary diffusion and transport of heat occurs only through ordinary heat conduction. Flux coupling is neglected.
7. The heat and mass transfer processes between the desiccant matrix and the air stream can be described by lumped transfer coefficients.
8. The periodic steady state performance of the dehumidifier is considered.

The capacitance rate parameters, Γ_1 and Γ_2, are defined as the ratio of matrix to fluid mass capacity rate:

$$\Gamma_j = \frac{M_{dj}\tau_{dj}}{\theta_j M_{fj}} = \frac{M_d}{T\dot{m}_j}; \quad j = 1,2 \qquad (1)$$

and the following dimensionless coordinates are introduced:

Nomenclature

a = availability of humid air (Table 1) (J/kg dry air)

A_j = total heat transfer area of the dehumidifier matrix in period j (m²)

COP = coefficient of performance (dimensionless)

c_p = moist air specific heat (J/kg dry air K)

h = heat transfer coefficient (W/m²K)

h_w = mass transfer coefficient (kg dry air/m²s)

i = moist air enthalpy (J/kg dry air)

i_s = heat of sorption (J/kg)

i_v = water vapor enthalpy (J/kg)

i_{vap} = heat of vaporization (J/kg)

I = desiccant matrix enthalpy (J/kg dry desiccant)

I_w = enthalpy of adsorbed water (J/kg)

L = axial flow length through the matrix (m)

Le = NTU_t/NTU_w, overall Lewis number (dimensionless)

$\dot{m}$ = mass flow rate (kg dry air/s)

M_d = mass of desiccant in the dehumidifier matrix (kg dry desiccant)

M_f = mass of air in the dehumidifier matrix (kg dry air)

NTU_t = $hA/\dot{m}c_p$, overall number of transfer units for heat transfer (dimensionless)

NTU_w = $h_wA/\dot{m}$, overall number of transfer units for mass transfer (dimensionless)

P = pressure (Pa)

r = relative vapor pressure (dimensionless)

s = moist air entropy (J/kg dry air K)

s_v = water vapor entropy (J/kg K)

S = desiccant entropy (J/kg dry desiccant K)

S_{gen} = generated entropy (J/kg K)

S_w = entropy of adsorbed water (J/kg K)

$V_{s,i}$ = ith shock wave speed (dimensionless)

t = temperature (°C)

T = time required for a complete rotation of the matrix (s) or absolute temperature (K)

w = moist air humidity ratio (kg/kg dry air)

$$x = \frac{z}{L}; \quad 0 \leq x \leq 1$$

$$\tau = \frac{\theta}{\tau_{dj}} \frac{M_{fj}}{M_{dj}} = \frac{\theta}{\theta_j} \frac{1}{\Gamma_j}; \quad 0 \leq \tau \leq \frac{1}{\Gamma_j} \qquad (2)$$

Under the assumptions 1 to 8, the conservation and transfer rate equations for period j of the heat and mass regenerator have been written as [8]:

$$\frac{\partial w_f}{\partial x} + \frac{\partial W_m}{\partial \tau} = 0$$

$$\frac{\partial w_f}{\partial x} = NTU_{w,j}(w_m - w_f)$$

$$\frac{\partial i_f}{\partial x} + \frac{\partial I_m}{\partial \tau} = 0 \qquad (3)$$

$$\frac{\partial i_f}{\partial x} = NTU_{t,j} \frac{\partial i_f}{\partial t_f}(t_m - t_f) + i_v NTU_{w,j}(w_m - w_f)$$

In these equations, $NTU_{t,j}$ and $NTU_{w,j}$ are the overall Number of Transfer Units for heat and mass, respectively, and are defined as conventional. The Lewis number, Le, is the ratio of overall resistance for mass transfer to the overall resistance for heat transfer, $NTU_{t,j}/NTU_{w,j}$, and assumed to be the same for both periods. Equations (3) are coupled through the thermodynamic property relationships for the desiccant-air-water vapor mixture, which may be expressed in the following functional forms:

$$w_m = w_m(W_m, t_m)$$

$$i_f = i_f(w_f, t_f) \qquad (4)$$

$$I_m = I_m(W_m, t_m)$$

The initial conditions for this system of equations are:

for $0 \leq \tau \leq \frac{1}{\Gamma_j}$; $j = 1,2$: $\begin{cases} w_f(x=0,\tau) = w_{j1} \\ i_f(x=0,\tau) = i_{j1} \end{cases} \qquad (5)$

The periodic equilibrium boundary conditions for the matrix state properties are

for $0 \leq x \leq 1$:

$$\lim_{\tau_1 \to (1/\Gamma_1)} - W_m(x,\tau_1) = \lim_{\tau_2 \to 0} + W_m(1-x,\tau_2)$$

$$\lim_{\tau_1 \to (1/\Gamma_1)} - I_m(x,\tau_1) = \lim_{\tau_2 \to 0} + I_m(1-x,\tau_2) \qquad (6)$$

$$\lim_{\tau_1 \to 0} + W_m(x,\tau_1) = \lim_{\tau_2 \to (1/\Gamma_2)} - W_m(1-x,\tau_2)$$

$$\lim_{\tau_1 \to 0} + I_m(x,\tau_1) = \lim_{\tau_2 \to (1/\Gamma_2)} + I_m(1-x,\tau_2)$$

Two state properties of the moist air are required to fully characterize the process outlet state of a dehumidifier. These properties may be obtained by using a conventional effectiveness approach, in which the dehumidifier is compared to a corresponding dehumidifier with infinite transfer coefficients. Two effectiveness factors for the process air outlet state are needed. The effectiveness for humidity ratio is defined as:

$$\epsilon_w = \frac{w_{11} - \bar{w}_{12}}{w_{11} - (\bar{w}_{12})_{id}}, \qquad (7a)$$

and for enthalpy:

$$\epsilon_h = \frac{\bar{i}_{12} - i_{11}}{(\bar{i}_{12})_{id} - i_{11}}, \qquad (7b)$$

where the subscript id indicates the outlet state of a dehumidifier operating at the same inlet conditions, the same Γ_j parameters, and with infinite overall transfer coefficients for heat and mass. Equations (3), (5), and (6) show that the effectiveness are functions of inlet temperature and humidity, capacitance rate parameters Γ_j, transfer parameters $NTU_{t,j}$, and the Lewis number Le. Effectiveness expressions for silica gel rotary dehumidifiers are presented in [7]. These expressions are obtained by combining the solutions for an ideal dehumidifier with values from a numerical analysis of a dehumidifier with finite transfer coefficients [8].

Nomenclature (cont.)

W = matrix water content (kg/kg dry desiccant)

x = axial coordinate measured from period entrance (dimensionless)

y = mole fraction (dimensionless)

z = axial displacement through matrix measured from period entrance (m)

Γ_j = jth operating parameter of the rotary dehumidifier defined in equation (1) (dimensionless)

ϵ_h = enthalpy effectiveness of the rotary dehumidifier (dimensionless)

ϵ_w = humidity ratio effectiveness (dimensionless)

θ = time (s)

θ_j = duration of period j (s)

λ_i = ith characteristic wave speed (dimensionless)

η_{II} = second law efficiency (dimensionless)

τ = time coordinate (dimensionless)

τ_{dj} = dwell time of a fluid particle in period j (s)

Subscripts

a = air

d = desiccant

f = evaluated at fluid state

id = ideal outlet state

int = evaluated at the intersection point state

j = period index

m = evaluated at, or in equilibrium with, the matrix state

o = evaluated at the standard state

R = evaluated on the regeneration line

s = evaluated at the shock state

t = heat transfer or temperature

v = water vapor

w = mass transfer or moisture

11 = process air inlet state of the dehumidifier

12 = process air outlet state of the dehumidifier

21 = regeneration air inlet state of the dehumidifier

22 = regeneration air outlet state of the dehumidifier

Superscripts

$\cdot$ = rate

$-$ = average value for a period

The Ideal Rotary Dehumidifier. In the ideal dehumidifier, the overall heat and mass transfer coefficients are infinite. Thus at all times, each differential desiccant-moist air subsystem is in complete thermodynamic equilibrium (i.e., thermal and vapor pressure equilibrium).

The conservation equations (3) may then be expressed as:

$$\frac{\partial w_m}{\partial x} + \frac{\partial W_m}{\partial \tau} = 0$$

$$\frac{\partial i_m}{\partial x} + \frac{\partial I_m}{\partial \tau} = 0$$

(8)

Equations (8) combined with the property relationships (4) and the initial and boundary conditions (5), (6), form a system of two coupled conservation laws. Each is a hyperbolic partial differential equation and is nonlinear because of the nonlinear property relationships. Solutions may be obtained by the method of characteristics and the shock wave method [9].

A detailed description of the method of characteristics and the theory of shock waves as applied to equations (8) is given in [7]. The flow of process air over a desiccant that is initially in equilibrium with regeneration air is characterized by the propagation of two separated expanding rarefaction waves, each carrying one of the two Riemann invariants of the system of equations (8). These Riemann invariants are called the F_i potentials [8], and the speed at which they propagate is λ_i. The air state before the first F_1 wave is the initial, regeneration air state. The air state behind the second F_2 wave is the inlet process air state. The air state between the F_1 and F_2 wave is the intersection point state (t_{int}, w_{int}). For this case, the flowing exchange process is continuous and described by equations (8). For the case of regeneration air passed over a desiccant that is initially in equilibrium with process air, the flow is characterized by the propagation of two separated shock waves. The speed of the ith shock is $V_{s,i}$. The air state before the first shock is the initial, process air state. The air state behind the second shock is the inlet regeneration air state. The air state between the two shocks is the shock state (t_s, w_s). For this case, the flow-exchange process is discontinuous and described by integral conservation conditions, rather than partial differential equations [7]. The method of characteristics and the shock wave theory provide a set of analytical equations that allow prediction of the performance of an ideal dehumidifier for the entire range of operating parameters, Γ_j, and for any inlet conditions. The functional form of these equations is presented in [7].

The characteristic behavior of a rotary dehumidifier may be demonstrated by its operating chart. In general, the operating chart [7] shows the fundamental modes of operation of regenerative heat and mass exchangers on a capacitance rate diagram. An operating chart for a silica gel rotary dehumidifier with infinite transfer coefficients is illustrated in Fig. 2 for specified inlet conditions. In this figure, three regions of operation are shown on a $\Gamma_1 - \Gamma_2$ diagram. The solid line A-B-D represents the dehumidifier regeneration line. The area to the right of the regeneration line in the chart is the region of complete regeneration. If the dehumidifier were to be operated at an operating point (Γ_1, Γ_2) within the region of complete regeneration, all of the desiccant particles would be in complete equilibrium with the regeneration air inlet state before they enter period 1 (i.e., the second shock would move completely through the matrix during the regeneration period). Similarly, the solid line E-B-G in Fig. 2 represents the dehumidifier processing line. The area to the left of the processing line is the region of complete processing. If the dehumidifier were to be operated at an operating point (Γ_1, Γ_2) within the region of complete processing, all of the desiccant particles would reach equilibrium with the process air during
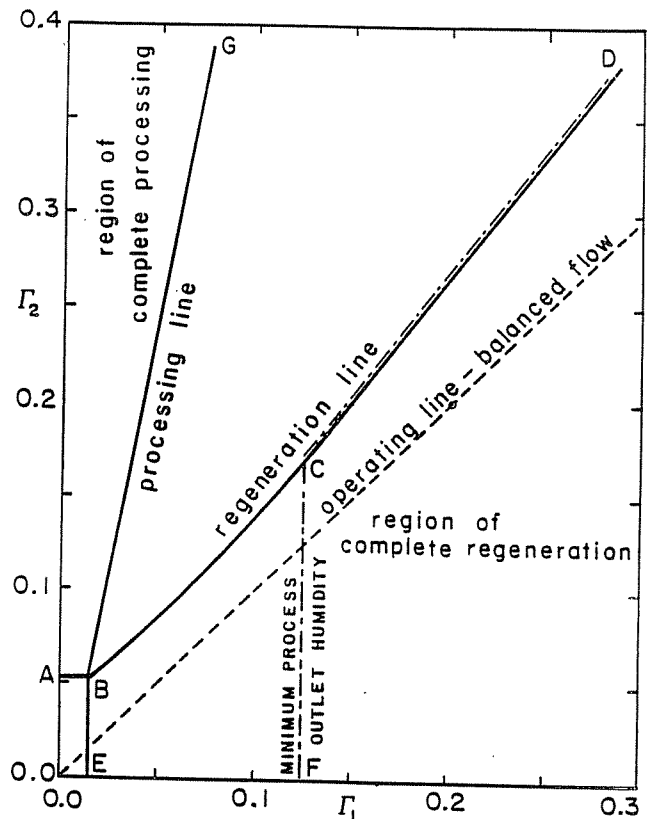

Fig. 2 Operating chart for a regenerative silica gel dehumidifier with infinite transfer coefficients [7]; $t_{11} = 25°C$, $w_{11} = 0.015$ kg/kg, $t_{21} = 85°C$, $w_{21} = 0.015$ kg/kg

the processing period before they enter period 2 (i.e., the trailing edge of the F_2 wave would move completely through the matrix during the processing period). The period-average humidity ratio of the process air outlet stream is a local minimum along the dashed line F-C-D.

The analysis presented in this paper is illustrated for rotary dehumidifiers, and is also valid for periodic switching of process and regeneration air in counterflow through fixed desiccant beds.

Properties of Moist Air and Silica Gel

Following Wepfer et al. [10], water vapor at low pressures ($p < 2$ psia) may be treated as an ideal gas. For humid air at atmospheric pressure, this vapor pressure corresponds to a humidity ratio of 0.1 kg/kg. In dehumidifier systems, the maximum humidity is that of the regeneration air outlet stream which is typically 0.035 kg/kg or less. For this analysis, moist air may therefore be treated as an ideal gas mixture. The available energy of the mixture is determined according to Wepfer et al. [10]. Conventional state property correlations which are used in this analysis are listed in Table 1.

The following accepted assumptions are made with respect to the properties of the wet desiccant [11]:
1. The gaseous adsorptive is an ideal gas mixture of air and water vapor.
2. The solid adsorbent is completely inert. All its thermodynamic properties are the same in both the presence and in and absence of the adsorbate (adsorbed phase),
3. Surface area, structure and specific heat of the adsorbent are independent of temperature and pressure.

With these assumptions the thermodynamic functions of the adsorbate may be discussed alone, e.g., the heat of adsorption is then the difference in enthalpy of the adsorptive and adsorbate. The properties of regular density silica gel which are used for this analysis are listed in Table 2.

Table 1 Thermodynamic properties of moist air

$$T_o = 273.15 \text{ K}$$
$$P_o = 1 \text{ atm}$$
$$T_f = t_f + 273.15$$

Composition

$$y_v = \frac{w_f}{w_f + 0.62195}$$

$$y_a = 1. - y_v$$

Enthalpy

$$i_a(t_f) = 1004.8 \ t_f$$
$$i_v(t_f) = 2502680. + 1800.3 \ t_f$$
$$i_f = i_a(t_f) + w_f i_v(t_f)$$
$$i_{f,o} = i_a(t_{f,o}) + w_f i_v(t_{f,o})$$

Entropy

$$s_a(t_f, w_f) = 1004.8 \ \ln(T_f/T_o) - 287.0 \ \ln(y_a)$$
$$s_v(t_f, w_f) = 1800.3 \ \ln(T_f/T_o) - 461.5 \ \ln(y_v)$$
$$s_f = s_a(t_f, w_f) + w_f s_v(t_f, w_f)$$
$$s_{f,o} = s_a(t_{f,o}, w_{f,o}) + w_f s_v(t_{f,o}, w_{f,o})$$

Availability

$$a = (i_f - i_{f,o}) - T_{f,o}(s_f - s_{f,o})$$

Table 2 Thermodynamic properties of regular density silica gel and water vapor (from [12])

Water Vapor

saturation pressure $\log_{10}\left(\dfrac{P_{ws}}{218.167}\right)$

$$= \frac{-z}{t_f + 273.15} \ \frac{a + bz + cz^3}{1 + dz}$$

$$z = 374.12 - t_m$$
$$a = 3.2437814$$
$$b = 5.86826 \ 10^{-3}$$
$$c = 1.1702379 \ 10^{-8}$$
$$d = 2.1878462 \ 10^{-3}$$

relative vapor pressure $r = y_v P_o / P_{ws}$
heat of vaporization $i_{vap} = 2502680. - 2386.5 \ t_m$

Heat of Sorption

$$h^* = 1.0 + 0.2843 \ \exp(-10.28 \ W_m)$$
$$i_a = h^* \ i_{vap}$$

Equilibrium Isotherm

$$r = (2.112 \ W_m)^{h^*} (29.91 \ P_{ws})^{h^* - 1}$$

Enthalpy

$$I_m = 921.096 \ t_m + \int_0^{W_m} (i_v - i_s) dW_m$$

Entropy

$$S_m = 921.096 \ \ln\left(\frac{T_m}{T_o}\right) + \int_0^{W_m} \left(s_v - \frac{i_s}{T_m}\right) dW_m$$

Second Law Analysis of Solid Desiccant Dehumidifiers

The conservation of water vapor and energy for adiabatic flow over a desiccant for infinite transfer coefficients is expressed by equations (8). Neglecting the storage in air, the entropy generated during the flow-exchange process may be written as:

$$\frac{\partial s_m}{\partial x} + \frac{\partial S_m}{\partial \tau} = \dot{S}_{gen} \tag{9}$$

where $\dot{S}_{gen}$ is the entropy generated per unit desiccant mass, per unit dimensionless time, τ, and displacement, x. Under the assumption of constant total pressure, it is shown in Appendix A that the entropy equation may be expressed in terms of enthalpy and humidity as:

$$\frac{\partial s_m}{\partial x} + \frac{\partial S_m}{\partial \tau} = \frac{1}{T_m}\left(\frac{\partial i_m}{\partial x} + \frac{\partial I_m}{\partial \tau}\right)$$

$$+ \left(s_v - \frac{i_v}{T_m}\right)\left(\frac{\partial w_m}{\partial x} + \frac{\partial W_m}{\partial \tau}\right) \tag{10}$$

This equation holds also for thermodynamically nonideal behavior of adsorptive and adsorbate-adsorbent. In this case, s_v and i_v are the partial molal entropy and enthalpy of the water vapor in the air. The only requirement for equation (10) is constant pressure throughout the matrix.

Equation (10) demonstrates that if the exchange of mass and energy is described by equations (8), the flow-exchange process is thermodynamically reversible. The ideal dehumidifier theory [7] shows that equations (8) describe the flow of process air past a regenerated desiccant. No entropy is generated during the dehumidification of moist air by passing it over a desiccant for infinite transfer coefficients.

During the regeneration of the desiccant by hot air with infinite transfer coefficients, the exchange process is characterized by two shocks propagating through the desiccant. Energy and water vapor are transferred across a finite difference in temperature and vapor pressure and entropy is generated. For complete regeneration, the entropy generated by the two shocks is calculated in Appendix B and given by:

$$\dot{S}_{gen,1} = (S_s - S_{11}) - \frac{(s_s - s_{11})}{V_{s,1}}$$

$$\dot{S}_{gen,2} = (S_{21} - S_s) - \frac{(s_{21} - s_s)}{V_{s,2}} \tag{11}$$

where $\dot{S}_{gen,i}$ is the entropy generated by the ith shock, moving at speed $V_{s,i}$, per unit dry desiccant mass. Both entropy generation terms are positive provided that the correct solution of the integral conservation equations [7] is chosen for the shock state ("generalized entropy condition," [9]).

For periodic operation of desiccant dehumidifiers, two additional sources of irreversibilities may be identified. The outlet state of a rotary dehumidifier is nonhomogeneous with respect to angular position at the exit face because of the existence of the two waves. Mixing of the air stream leaving the dehumidifier to make it uniform is irreversible and generates entropy. The total entropy generation during operation of rotary dehumidifiers is thus the sum of the entropy generated by the two shocks and the mixing of both process and regeneration air streams at the outlet. The entropy generation may be presented as a function of the two capacitance rate parameters, Γ_i, for specified inlet conditions. In graphical form, this leads to entropy-generation contours in the dehumidifier operating chart.

Figure 3 illustrates lines of constant entropy generation per unit process air mass flow rate in the dehumidifier operating chart for a given pair of inlet states. On the Γ_2 axis, the entropy generated per unit process mass flow rate is then zero. Some contours have elliptical shape with large eccentricity and major axis coinciding with the regeneration line. For most operating conditions it is not possible to distinguish between the various sources of irreversibilities (shocks and mixing) in Fig. 3. The entropy generation terms may be separately calculated for the regime where the regions of complete processing and regeneration overlap in the operating chart of the dehumidifier, area OABE in Fig. 2. As an example, Table 3 lists the entropy generated by the alternate irreversibility
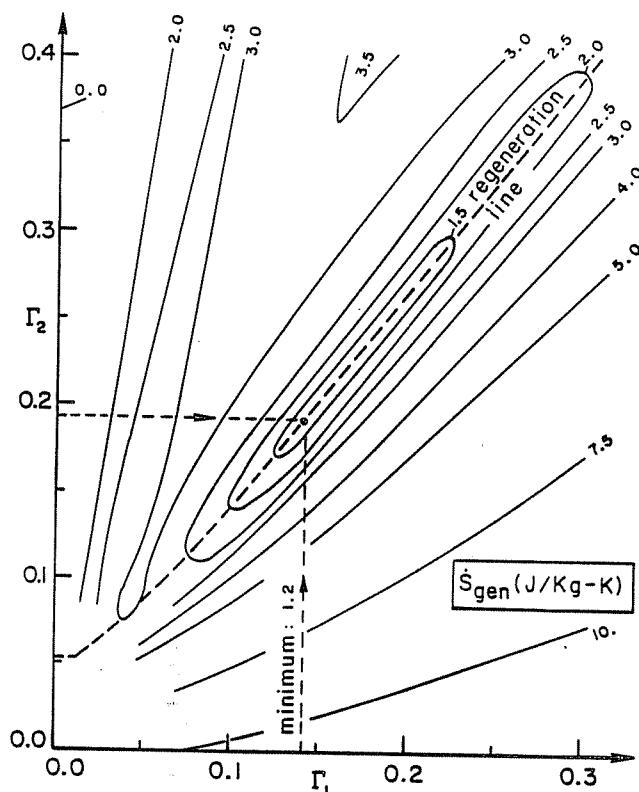
Fig. 3 Contours of entropy generation per unit process air mass flow rate in the dehumidifier operating chart from Fig. 2

Table 3 Entropy generation* during dehumidifier operation at point B (Fig. 2). Inlet states are (25.0°C, 0.015 kg/kg) and (85.0°C, 0.015 kg/kg).

Mixing process air stream:		
from first wave	0.0482	2.0
from second wave	0.9643	39.2
total	1.0125	41.2
Mixing regeneration stream:	0.0265	1.1
Shock waves:		
first shock	0.0019	0.1
second shock	1.4171	57.6
total	1.4190	57.7
Total:	2.4580 J/kg K	100.0%

*Per unit process-air mass flow rate

sources for operating point B in Fig. 2. At this point the second shock has just reached the exit face of the wheel during regeneration, and similarly, the trailing edge of the second expansion wave has moved completely through the matrix. As shown in Table 3, 97 percent of the total entropy generated is produced by the propagation of the second shock through the desiccant (58 percent) and the nonuniformity of the process outlet flow caused by the second wave reaching the exit face of the dehumidifier wheel (39 percent).

Minimum entropy generation during dehumidifier operation may be achieved by operating the dehumidifier at values for the capacitance parameters given by:

$$\Gamma_1 = \lambda_{2,\text{int}} \tag{12a}$$

$$\Gamma_2 = \Gamma_{2,R}(\Gamma_1 = \lambda_{2,\text{int}}) \tag{12b}$$

At the value for Γ_1 given by equation (12a), the leading edge

of the second expansion wave has just reached the process-period exit face of the desiccant wheel. The uniformity of the process outlet stream is then maximized. Also, at that value for Γ_1, the second wave has maximally expanded during the processing period within the desiccant matrix. The formation of the second shock during the regeneration period is then maximally delayed. The value for Γ_2 given by equation (12b) assures maximum uniformity of the regeneration outlet stream. The lower focus of the elliptical contours in Fig. 3 is the operating point given by equation (12). The total entropy generation has a local minimum at that point. The averaged process outlet humidity is minimum at a point on the regeneration line slightly to the left of the point given by equations (12a) and (12b) in Fig. 3.

Second Law Efficiency of Desiccant Dehumidifiers

Dehumidifier performance has been studied from a first law point of view [2], in which the required regeneration heat supply is compared to the system cooling load. The dehumidifier may also be studied from a second law point of view. In this approach availability, rather than energy, is used to determine component performance. The second law efficiency of the rotary dehumidifier is defined as:

$$\eta_{\text{II}}^{(1)} = \frac{\dot{m}_1(a_{12} - a_{11})}{\dot{m}_2(a_{21} - a_{22})} \tag{13}$$

In the analysis, the dead state is the process air inlet state (state 11), i.e., $a_{11} = 0$. The decrease in available energy of the regeneration air stream is used to increase the availability of the process air stream.

For a given pair of inlet states, the efficiency may be calculated as a function of the capacitance parameters. Figure 4 illustrates lines of constant second law efficiency in the operating chart of the dehumidifier for infinite transfer coefficients. The contours in Fig. 4 have the form of skewed parabolas. For specified Γ_1, the efficiency is maximal for operating points on the regeneration line. For these operating points, the outlet state of the regenerating air stream is maximally uniform. The second law efficiency is zero along the ordinate axes. For $\eta_{\text{II}}^{(1)} > 0.85$ the contours have elliptical shape (not shown in Fig. 4) that are centered around a common focus. The major axes of the ellipses coincide with the regeneration line. The ordinates of the common focus are given by equations (12a) and (12b), at which the second law efficiency is maximum.

The analysis of open cycle, recirculation and ventilation-type desiccant cooling systems has shown that such a system can meet a given cooling load with the least supply of regeneration energy, if the dehumidifier is operated such that the process outlet humidity is minimum for given inlet states [2]. For infinite transfer coefficients and inlet states as specified in Figs. 2–4, the averaged humidity ratio of the process air outlet stream is minimum for the operating point $(\Gamma_1 = 0.127, \Gamma_2 = 0.172)$. At this point, the second law efficiency is 0.883, whereas the maximum value occurs at (0.146, 0.197) and is 0.895. This indicates that values for the capacitance parameters that optimize desiccant system performance from a first law point of view [2] also guarantee nearly optimal performance based upon a second law formulation.

The second law efficiency of dehumidifiers in open cycle cooling and drying systems may decrease if the regeneration air outlet stream is exhausted in the surroundings (e.g., ventilation and recirculation systems). In this case, the second law efficiency may be defined more appropriately by:

$$\eta_{\text{II}}^{(2)} = \frac{\dot{m}_1 a_{12}}{\dot{m}_2 a_{21}} \tag{14}$$

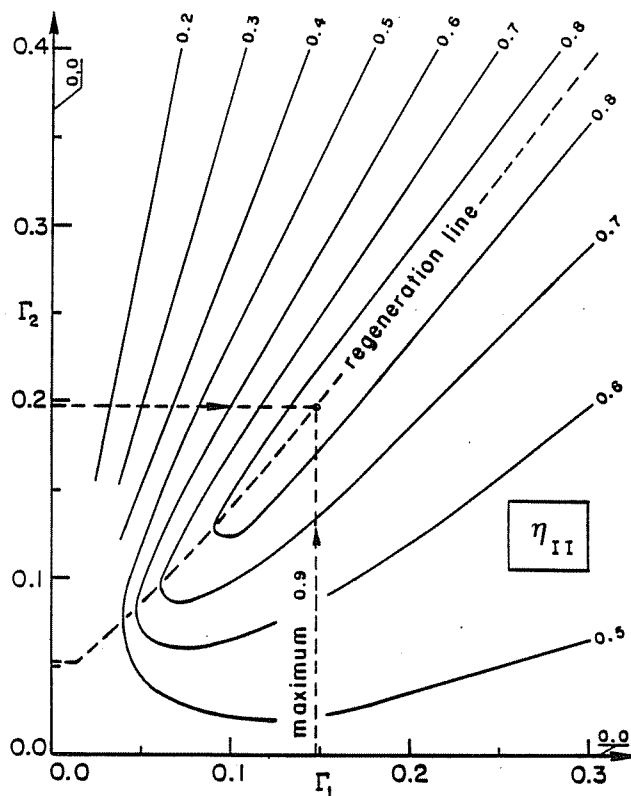This efficiency has similar contours in the operating chart as

Fig. 4 Second law efficiency contours in the dehumidifier operating chart from Fig. 2

Fig. 5 Effect of the overall number of transfer units on the second law efficiency of the rotary dehumidifier: $t_{11} = 25°C$, $w_{11} = 0.015$ kg/kg, $t_{21} = 85°C$, $w_{21} = 0.015$ kg/kg, $\Gamma_1 = 0.15$, $\Gamma_2 = 0.18$, $C^* = 0.87$, Le = 1

$\eta_{II}^{(1)}$ in Fig. 4. The operating points for maximum $\eta_{II}^{(1)}$ and $\eta_{II}^{(2)}$ coincide. At this operating point, the maximum second law efficiency of the ideal dehumidifier decreases from 0.895 to 0.723 by exhausting the regeneration-air outlet stream.

Finite transfer coefficients cause transfer of energy and water vapor across finite temperature and vapor pressure differences. This further decreases the second law efficiency of desiccant dehumidifiers. For well-designed dehumidifiers, the operating point that minimizes the averaged humidity of the process-air outlet stream is (0.15, 0.18), for the inlet conditions as specified in Figs. 2–4 [2]. Figure 5 shows the computed second law efficiency $\eta_{II}^{(1)}$ of an actual dehumidifier as a function of the overall number of transfer units, NTU_o, for this operating point. NTU_o is defined by [7]:

$$\frac{1}{NTU_o} = \frac{1}{NTU_{t,1}} + \frac{C^*}{NTU_{t,2}} \quad (15)$$

In Fig. 5, $C^* = 0.87$, $NTU_{t,2} = (\Gamma_2/\Gamma_1)NTU_{t,1}$ and Le = 1. This figure shows that the second law efficiency of actual regenerative desiccant dehumidifiers increases with the overall number of transfer units. Whereas the maximum is 0.82 for infinite transfer coefficients, actual values for $\eta_{II}^{(1)}$ range from 0.60 to 0.70 for the range of NTU_o of interest.

Conclusion

Entropy generation for adiabatic flow of humid air over solid desiccants with infinite transfer coefficients is investigated. Reversible flow is achieved during dehumidification of the air stream by the desiccant. Regeneration of the adsorbent is irreversible due to the occurrence of shock waves that accompany the transfer processes. The entropy production during operation of ideal regenerative dehumidifiers is discussed. The various sources of irreversibilities are identified as mixing of the dehumidifier outlet streams to provide uniform outlet states for both air streams and the transfer of energy and mass by shock waves. Values for the capacitance
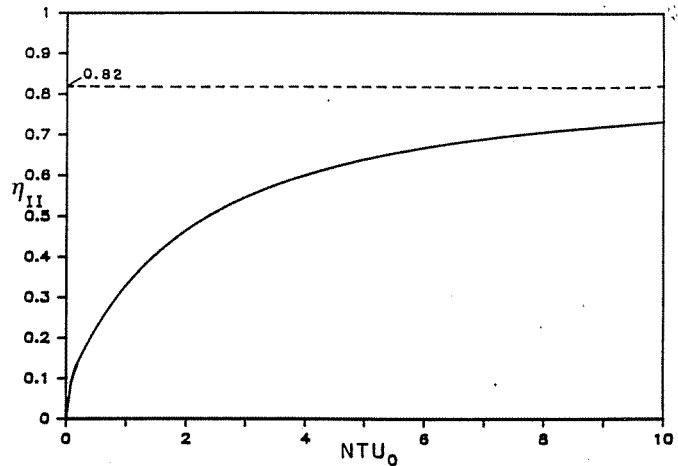
rate parameters that minimize entropy generation are arrived at theoretically.

The efficiency of solid desiccant regenerative dehumidifiers based upon the second law is calculated and presented as a function of the capacitance-rate operating conditions. Values for these operating parameters that maximize second law efficiency also minimize required regeneration energy for a given dehumidifier system cooling load.

The maximum second law efficiency of ideal regenerative desiccant dehumidifiers is typically about 0.85. For actual dehumidifiers with finite transfer coefficients, this efficiency ranges from 0.60 to 0.70.

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A P P E N D I X   A

Rewrite the conservation equation for energy:

$$\frac{\partial i_m}{\partial x} + \frac{\partial I_m}{\partial \tau} = 0 \tag{A.1}$$

by expressing:

$$\frac{\partial i_m}{\partial x} = \frac{\partial i}{\partial T}\bigg)_w \frac{\partial T_m}{\partial x} + \frac{\partial i}{\partial w}\bigg)_T \frac{\partial w_m}{\partial x} \tag{A.2}$$

and

$$\frac{\partial I_m}{\partial \tau} = \frac{\partial I}{\partial T}\bigg)_w \frac{\partial T_m}{\partial \tau} + \frac{\partial I}{\partial W}\bigg)_T \frac{\partial W_m}{\partial \tau} \tag{A.3}$$

If the pressure is constant, terms in equations (A.2) and (A.3) may be rewritten as:

$$\frac{\partial i}{\partial w}\bigg)_T = i_v \tag{A.4}$$

and

$$\frac{\partial I}{\partial W}\bigg)_T = I_w \tag{A.5}$$

i_v and I_w are the partial molal enthalpies of the water vapor in the air and the adsorbed water in the desiccant, respectively. At equilibrium, these enthalpies are related through the heat of adsorption by:

$$I_w = i_v - i_s \tag{A.6}$$

Substituting equations (A.2)–(A.6) into equation (A.1) yields:

$$\frac{\partial i_m}{\partial x} + \frac{\partial I_m}{\partial \tau} = \frac{\partial i}{\partial T}\bigg)_w \frac{\partial T_m}{\partial x}$$
$$+ \frac{\partial I}{\partial T}\bigg)_w \frac{\partial T_m}{\partial \tau} - i_s \frac{\partial W_m}{\partial \tau} + i_v \left[\frac{\partial w_m}{\partial x} + \frac{\partial W_m}{\partial \tau} \right] \tag{A.7}$$

The entropy equation (9) is rewritten as follows:

$$\frac{\partial s_m}{\partial x} = \frac{\partial s}{\partial T}\bigg)_w \frac{\partial T_m}{\partial x} + \frac{\partial s}{\partial w}\bigg)_T \frac{\partial w_m}{\partial x} \tag{A.8}$$

and

$$\frac{\partial S_M}{\partial \tau} = \frac{\partial S}{\partial T}\bigg)_w \frac{\partial T_M}{\partial \tau} + \frac{\partial S}{\partial W}\bigg)_T \frac{\partial W_m}{\partial \tau} \tag{A.9}$$

For constant pressure, terms in equations (A.8) and (A.9) may be expressed as:

$$\frac{\partial s}{\partial T}\bigg)_w = \frac{1}{T_m} \frac{\partial i}{\partial T}\bigg)_w \tag{A.10}$$

$$\frac{\partial s}{\partial w}\bigg)_T = s_v \tag{A.11}$$

and similarly

$$\frac{\partial S}{\partial T}\bigg)_w = \frac{1}{T_m} \frac{\partial I}{\partial T}\bigg)_w \tag{A.12}$$

$$\frac{\partial S}{\partial W}\bigg)_T = S_w \tag{A.13}$$

At equilibrium, the partial molal entropies s_v and S_w of the adsorptive and adsorbate are related by:

$$S_w = s_v - \frac{i_s}{T_m} \tag{A.14}$$

Substituting equations (A.8)–(A.14) into equation (9) yields:

$$\frac{\partial s_m}{\partial x} + \frac{\partial S_m}{\partial \tau} = \frac{1}{T_m} \left[\frac{\partial i}{\partial T}\bigg)_w \frac{\partial T_m}{\partial x} \right.$$
$$\left. + \frac{\partial I}{\partial T}\bigg)_w \frac{\partial T_m}{\partial \tau} - i_s \frac{\partial W_m}{\partial \tau} \right] + s_v \left[\frac{\partial w_m}{\partial x} + \frac{\partial W_m}{\partial \tau} \right] \tag{A.15}$$

Inserting equation (A.7) into (A.15) gives:

$$\frac{\partial s_m}{\partial x} + \frac{\partial S_m}{\partial \tau} = \frac{1}{T_m} \left(\frac{\partial i_m}{\partial x} + \frac{\partial I_m}{\partial \tau} \right)$$
$$+ \left(s_v - \frac{i_v}{T_m} \right) \left(\frac{\partial w_m}{\partial x} + \frac{\partial W_m}{\partial \tau} \right) \tag{A.16}$$

which is also equation (10).

A P P E N D I X   B

The entropy generated by the propagation of the first shock through the desiccant matrix is calculated. The procedure is similar for the second shock.

The control volume considered is the system between the location of the first shock at time θ, z and at time $\theta + d\theta$, $z + dz$. At time θ, the equilibrium state of air and desiccant is state 11. After passage of the shock, this state is the shock state s. The entropy equation for this system may be written as:

$$(S_s - S_{11})dM_d = (s_s - s_{11})\dot{m}d\theta + d\dot{S}'_{gen,1} \tag{B.1}$$

$d\dot{S}'_{gen,1}$ is the entropy generated during the time $d\theta$ in the volume dz. The mass of the desiccant in the system is expressed in terms of the shock speed using equation (2) by:

$$dM_d = \frac{M_d}{L}dz = \left(\frac{dx}{d\tau} \right)\dot{m}d\theta$$
$$= V_{s,1}\dot{m}d\theta \tag{B.2}$$

Inserting equation (B.2) into (B.1) and simplifying gives:

$$(S_s - S_{11})V_{s,1} = (s_s - s_{11}) = \dot{S}'_{gen,1} \tag{B.3}$$

$\dot{S}'_{gen,1}$ is the entropy generated per unit process-air mass flow rate and per unit dimensionless time, τ.

The total time required for the first shock to pass from the inlet to the exit face of the desiccant is $1/V_{s,1}$. Multiplying equation (B.3) by this total time and rearranging gives the total entropy generated during passage of the first shock:

$$\dot{S}_{gen,1} = (S_s - S_{11}) - \left(\frac{s_s - s_{11}}{V_{s,1}} \right)$$

which is also the first part of equation (11).