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SOL-AIR HEATING AND COOLING DEGREE-DAYS

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Abstract—A set of relationships is developed which allow the estimation of sol-air heating and cooling degree-days at any base temperature. The method makes use of existing relationships for hourly utilizability and degree-days, and the only input required is the monthly-average solar radiation and ambient temperature. Solar radiation which is absorbed on outside surfaces and solar radiation which is transmitted through glazings are accounted for in the analysis. Estimated sol-air degree-days are compared with sol-air degree-days determined using long-term hourly data, and good agreement is observed. Sample calculations for a simple structure illustrate the use of sol-air degree days.

1. INTRODUCTION

Solar radiation and ambient temperature control the net energy exchange between a building and its environment. The combined effect of these variables on building energy use must be considered. One way of accomplishing this is to use weather statistics to estimate monthly heating and cooling loads. Heating and cooling degree-days provide a simple means of estimating the monthly building load, but the effect of solar radiation is not accounted for. Ambient temperature bin data, while being more versatile, also fail to include solar effects.

The increasing popularity of passively heated solar homes has led to design methods which include the effects of both ambient temperature and solar radiation. The solar load ratio method [1] presents a set of empirical relationships based on simulations of passive solar buildings. Another direct gain design method is that developed by Monsen *et al.* [2]. Both methods rely on heating degree-days to account for ambient temperature effects. While solar effects are treated somewhat differently, the monthly-average incident solar radiation is an input to each method. Any location for which these statistics are available can be considered. However, the methods only directly consider solar radiation which enters through south facing apertures, and there are no provisions for the estimation of cooling loads. In addition, the interaction of ambient temperature and solar radiation, which may affect the building load, is only considered indirectly.

The estimation of the solar contribution to building cooling loads has received recent attention. The modified bin method developed by ASHRAE [3] and the modified building load and temperature bin calculation method (MBLTBM) [4] are based on ambient temperature bin data. The ASHRAE method treats solar radiation as a linear function of ambient temperature. The MBLTBM uses the seasonal average incident solar radiation for gains due to transmission, while solar radiation absorbed by walls is a linear function of ambient temperature.

The approach to combining solar radiation and ambient temperature effects taken in this study is to establish a new weather variable, sol-air degree-days. The distributions of solar radiation and ambient temperature are incorporated in sol-air temperature statistics, providing a direct measure of the interaction of these variables. Heating and cooling sol-air degree-days replace traditional degree-days in the estimation of heating and cooling loads. What follows is a description of the sol-air concept, a discussion of how the sol-air temperature is used in the estimation of building loads and a set of relationships for the estimation of sol-air heating and cooling degree-days. The use of the method is demonstrated through example problems and results are provided to indicate the relative importance of solar radiation and ambient temperature in determining the heating and cooling loads for a building.

2. SOL-AIR TEMPERATURE CONCEPT

The fraction of the incident solar radiation, I_T, which is absorbed by an opaque wall of unit area is equal to its solar absorptance, α_s. The coefficient h_0 includes both convective and long-wave radiative heat transfer between the wall surface and the surroundings. It is assumed that the surroundings are at the local ambient temperature, T_a. The energy flow rate at the outer wall surface, q_{in}, is given by

$$q_{in} = h_0(T_a - T_w) + I_T\alpha_s \tag{1}$$

where T_w is the surface temperature of the wall. The sol-air temperature [3] is defined as

$$T_{sa} \equiv T_a + \frac{I_T\alpha_s}{h_0}. \tag{2}$$

The sol-air temperature is a fictitious temperature of the surroundings that yields the same wall heat transfer rate without absorbed solar radiation as actually occurs with absorbed solar radiation. The sol-air temperature is not purely a meteorological variable, since it is a function of the surface geometry, the surface property α_s, and the heat transfer coefficient h_0. Each distinct surface may have a different sol-air temperature at a particular instant in time. The sol-air temperature is the forcing function for heat loss or gain at the outer surface of the wall.

3. SOL-AIR DEGREE-DAYS

The rate of energy transfer at the outer surface of an opaque wall is proportional to the difference between the sol-air temperature and the outer wall surface temperature. The degree-day concept extends the analysis by the assumption that the rate of heat transfer through the wall is equal to the product of the wall conductance, UA, and the difference between the sol-air and inside air temperature. On an instantaneous basis this assumption will not always be satisfied due to energy storage within the wall. However, degree-days provide reasonable estimates of integrated heating–cooling loads when the sol-air temperature is always either less than or greater than the inside air temperature or when thermal storage within the walls and interior is small relative to the size of the load. Over an extended period of time, such as a month, the integrated heating load for the month is the product of UA (assumed to be constant) and the sol-air heating degree-days for the month, D_{SH}, given by

$$D_{SH}(T_b) = \int_{t_1}^{t_2} (T_b - T_{sa}(t))^+ \, dt \quad (3)$$

where T_b is the value of T_{sa} at which heating is first required, t_1 and t_2 are the starting and ending times for the month, and the superscript '+' signifies negative values of the enclosed quantity are set equal to zero. The integration indicated in eqn (3) can be transformed into an integration over the sol-air temperature

$$D_{SH}(T_b) = N \int_{T_{sa,min}}^{T_b} (T_b - T_{sa})^+ P(T_{sa}) \, dT_{sa} \quad (4)$$

where N is the number of days in the month, $P(T_{sa})$ is the probability density function for T_{sa} and $T_{sa,min}$ is the smallest value of T_{sa} for which $P(T_{sa})$ is finite.

$P(T_{sa})$ is a complex function of a number of variables, some of which are stochastic in nature and others which are deterministic. If the analysis is restricted to a particular hour of the day, and if h_0 is assumed to be constant, $P(T_{sa})$ is much easier to describe. For a given surface I_T is only a function of the hourly clearness index, k, defined as the ratio of the horizontal radiation to the extraterrestrial radiation [5]. The probability distribution of the hourly sol-air temperature is the result of variations in the ambient temperature and atmospheric clearness for that hour. Equation (5) becomes

$$D_{SH,h}(T_b) = \frac{N}{24} \int_{T_{ah,min}}^{T_{ah,max}} \int_0^{k_{max}} \left[\left(T_b - T_{ah} - \frac{\alpha_s I_T}{h_0} \right)^+ P(k \mid T_{ah}) \, dk \right] P(T_{ah}) \, dT_{ah} \quad (5)$$

where $P(k \mid T_{ah})$ is the conditional probability density function for k, $T_{ah,min}$ is the minimum ambient temperature, $T_{ah,max}$ is the maximum ambient temperature, k_{max} is the maximum clearness index, and subscript h indicates that the analysis is for an hour of the day.

The probability density for a value of k at a given value of T_{ah} is $P(k \mid T_{ah})$. The dependence of $P(k \mid T_{ah})$ on both k and T_{ah} incorporates any correlation that may exist between solar radiation and ambient temperature. An analytical expression for $P(k \mid T_{ah})$ is not available. If k and T_{ah} were independent, $P(k \mid T_{ah})$ would simply be $P(k)$. Since functions are available for $P(k)$ [6] and $P(T_{ah})$ [7], this simplification will be made. The consequences of assuming k and T_{ah} are independent is demonstrated by comparisons with long-term hourly data in a later section. Equation (5) can be rearranged to yield

$$D_{SH,h}(T_b) = \frac{N}{24} \int_{T_{ah,min}}^{T_{ah,max}} \left\{ \int_0^{k_{max}} \left[(T_b - T_{ah})^+ - \frac{\alpha_s I_T}{h_0} \right] P(k) \, dk - \int_{k_c}^{k_{max}} \left[(T_b - T_{ah})^+ - \frac{\alpha_s I_T}{h_0} \right] P(k) \, dk \right\} P(T_{ah}) \, dT_{ah} \quad (6)$$

where k_c is the value of k for which $\alpha_s I_T / h_0$ is equal to $(T_b - T_{ah})^+$. This rearrangement permits $\alpha_s I_T / h_0$ to be brought outside of the '+', which simplifies the integration over k. The second integral is necessary because $\alpha_s I_T / h_0$ may be larger than $(T_b - T_{ah})^+$, and according to eqn (5), the integrand in the first integral must be set equal to zero when this is true. Integrating

$$D_{SH,h}(T_b) = D_{H,h}(T_b) - \frac{N \alpha_s \bar{I}_T}{24 h_0} \left(1 - \int_{T_{ah,min}}^{T_{ah,max}} \theta(I_c) P(T_{ah}) \, dT_{ah} \right) \quad (7)$$

where

$$I_c = \frac{h_0}{\alpha_s} (T_b - T_{ah})^+$$

and $\phi(I_c)$, referred to as the utilizability function, is the fraction of the total hourly incident radiation for the month which occurs at intensities greater than I_0, the critical level. $\bar{I}_T$ is the monthly-average hourly tilted surface radiation, which can be estimated from horizontal data in the manner described in Ref. [5].

The remaining integral in eqn (7) cannot be evaluated analytically using the functions available for $\phi(I_0)$ and $P(T_{ah})$. As an approximation, T_{ah} is assumed to be always equal to $\bar{T}_{ah}$, the monthly-average value of T_{ah}. This approximation is equivalent to assuming that the mean value of ϕ over the range of critical levels encountered is equal to the utilizability at the mean critical level. The validity of this assumption is discussed in the next section. With this approximation, an analytic expression for the sol-air heating degree-days during an hourly period can be obtained

$$D_{SH,h}(T_b) = D_{H,h}(T_b) - \frac{N\alpha_s\bar{I}_T}{24h_0}(1 - \phi(\bar{I}_c)) \quad (8)$$

where

$$\bar{I}_c = \frac{h_0}{\alpha_s}(T_b - \bar{T}_{ah})^+$$

A similar development leads to a relationship for the sol-air cooling degree-days

$$D_{SC,h}(T_b) = D_{c,h}(T_b) + \frac{N\alpha_s\bar{I}_T}{24h_0} \cdot \phi(\bar{I}_c) \quad (9)$$

where $\bar{I}_c$ is defined above.

The evaluation of eqn (8) or eqn (9) requires values of hourly ambient temperature degree-days, hourly utilizability, and monthly-average solar radiation. Numerous relationships are available for the estimation of hourly utilizability [6, 8] and degree-days [7, 9, 10]. The degree-day method developed by Erbs *et al.* [7] only requires the monthly-average daily temperature, and can be used to obtain values of $\bar{T}_{ah}$. Since for many locations only the monthly-average daily clearness index, $\bar{K}$, is available, it is necessary to estimate the hourly clearness index from the relationship

$$\bar{k} = (r_T/r_d)\bar{K} \quad (10)$$

where r_T is the ratio of the monthly-average hourly horizontal radiation to the monthly-average daily horizontal radiation and r_d is the ratio of the monthly-average hourly extraterrestrial radiation to the monthly-average daily extraterrestrial radiation. Collares-Pereira and Rabl provide analytical realtionships for r_T and r_d [11].

4. COMPARISON OF MODEL TO LONG-TERM DATA

Equations (8) and (9) are based on a number of assumptions. Additional relationships are required to use these equations, introducing other sources of uncertainty. Long-term hourly SOLMET data [12] (an average of 20 years at each location) for nine U.S. locations were used to generate sol-air heating and cooling degree-days. Base temperatures between 1 and 20°C for heating and 10 and 29°C for cooling were considered. Degree-days for hours when the sun was above the horizon were summed to obtain 'daylight' sol-air degree-days. A ground reflectance of 0.2 was used for all months, and the diffuse radiation was modeled as isotropic [5]. The value of α_s was 0.9, and h_0 was 34 W m^{-2} °C^{-1} for heating degree-days and 23 W m^{-2} °C^{-1} for cooling degree-days in the following comparisons.

Six surface orientations were chosen to test the effect of orientation on the accuracy of the method: vertical surfaces facing the four compass directions, a horizontal surface, and a south-facing surface with a slope equal to the latitude. In addition, a surface which receives no solar radiation was included to demonstrate the relative importance of solar radiation.

The hourly utilizability relationships of Clark [8] and the ambient temperature and degree-day relationships of Erbs *et al.* [7] were used. Both $\bar{T}_{ah}$ and $\bar{k}$ were estimated from the monthly-average daily values. The hourly estimates were summed to yield estimated 'daylight' sol-air heating and cooling degree-days for the same locations, surface orientations, and base temperatures as determined using the hourly meteorological data.

Figure 1 is a comparison of measured and estimated sol-air heating degree-days for the six lo-

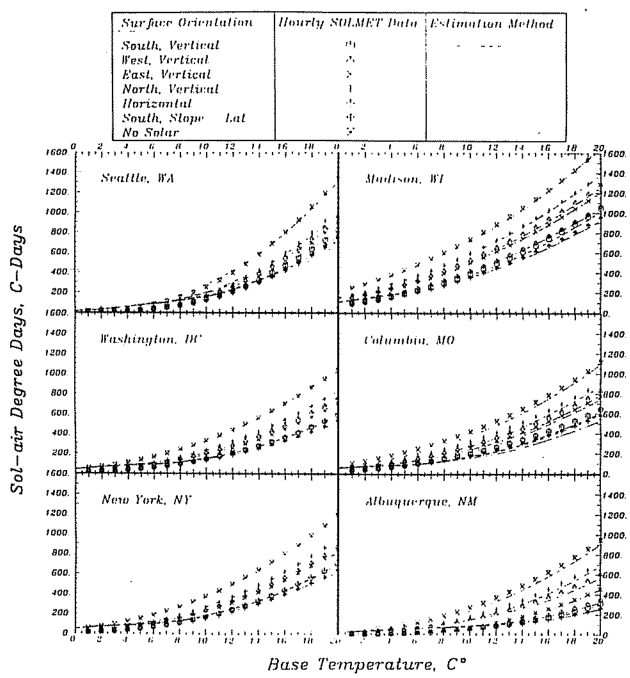

Fig. 1. A comparison of sol-air heating degree-days calculated from long-term hourly data to sol-air heating degree-days estimated from values of monthly-average ambient temperature and solar radiation.

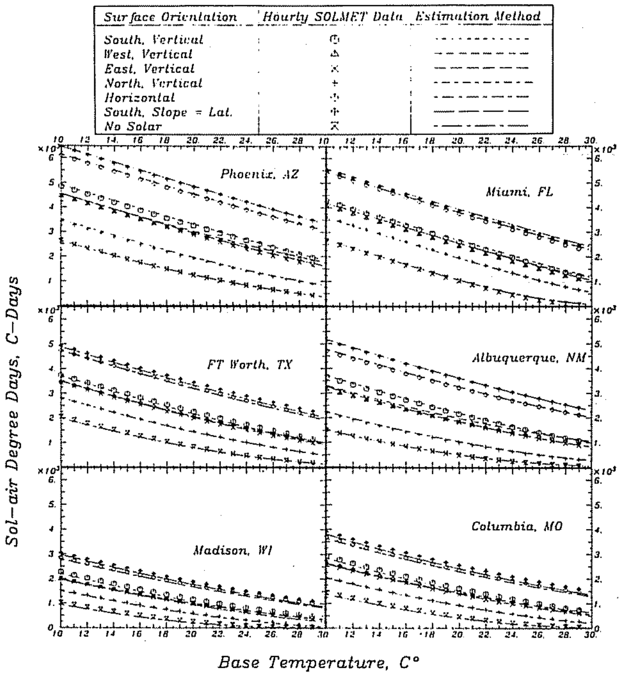

Fig. 2. A comparison of sol-air cooling degree-days calculated from long-term hourly data to sol-air cooling degree-days estimated from values of monthly-average ambient temperature and solar radiation.

cations with the largest heating degree-day totals. The no solar curve indicates the accuracy of the ambient temperature degree-day estimates using Erbs *et al.*'s [7] relationships. In Columbia and Albuquerque, the ambient degree-days are slightly underpredicted, which causes a slight underprediction of the sol-air degree-days for these locations for most surfaces. In Albuquerque, a compensating error in the utilizability estimation occurs for the surfaces which receive the most solar radiation. A more systematic error which shows up in all locations at low base temperature values is the overprediction of sol-air degree-days. The error can be attributed to the use of an average critical level ($\bar{I}_c$), based on the monthly-average temperature, to evaluate ϕ. In all cases, however, the differences between the measured and estimated sol-air degree-days are small in magnitude.

Figure 2 is a comparison of measured and estimated sol-air cooling degree-days for the six locations with the largest cooling degree-day totals. The lowermost curve indicates the accuracy of the ambient temperature degree-day estimate. For a value of α_s of 0.9, solar radiation can have a substantial effect on the cooling load. In general, the agreement between measured and estimated sol-air degree-days is comparable to the agreement between measured and estimated ambient degree-days. Again, the errors are small in size.

5. APPLICATIONS

The relationships presented so far are only useful for a single surface. Buildings are composed of a number of distinct surfaces having different orientations and surface properties. In addition to the solar radiation which is absorbed on outer surfaces, solar radiation is also transmitted through glazings, and is absorbed internally. Energy may be flowing in through some of the surfaces while at the same time other surfaces are losing energy to the environment. To properly account for the net effect solar radiation and ambient temperature have on the heating and cooling loads, it is necessary to calculate the sol-air heating and cooling degree-days for the entire structure.

Equation (5) can be expanded into a more general form which includes all building surfaces and the transmission of solar energy through glazings.

$$D_{\mathrm{SH,h}} = \frac{N}{24} \int_{T_{\mathrm{ah,min}}}^{T_{\mathrm{ah,max}}} \int_0^{k_{\max}} \left[\left(T_b - T_{\mathrm{ah}} - \sum_{j=1}^{n_s} \frac{(UA)_j \alpha_{sj} I_{\mathrm{T}sj}}{h_0 (UA)_0} - \sum_{i=1}^{n_w} \frac{(\tau\alpha)_i A_{gi} I_{\mathrm{Tw}i}}{(UA)_0} \right)^+ \cdot P(k \mid T_{\mathrm{ah}})\, \mathrm{d}k \right] P(T_{\mathrm{ah}})\, \mathrm{d}T_{\mathrm{ah}} \tag{11}$$

where n_w is the number of windows, n_s is the number of surfaces, $(\tau\alpha)_i$ is the effective transmittance–absorptance product for the room–window system, A_{gi} is the glazing area, $(UA)_j$ is the thermal conductance for each surface, and $(UA)_0$ is the overall building conductance. If the glazings absorb a significant fraction of the incident solar radiation, they must be included in both the window and surface summations. After some rearrangement, the simplifications made for a single surface can be applied and the integrals evaluated, yielding

$$D_{\mathrm{SH,h}}(T_b) = D_{\mathrm{H,h}}(T_b) - Z \frac{N}{24} \bar{I}_{\mathrm{T0}} (1 - \phi(\bar{I}_{\mathrm{c0}})) \tag{12}$$

where

$$\bar{I}_{\mathrm{c0}} = \frac{(T_b - \bar{T}_{\mathrm{ah}})^+}{Z}$$

$$\bar{I}_{\mathrm{T0}} = \frac{\sum_{j=1}^{n_s} (UA)_j \alpha_{sj} \bar{I}_{\mathrm{T}sj} + h_0 \sum_{i=1}^{n_w} (\tau\alpha)_i A_{gi} \bar{I}_{\mathrm{Tw}i}}{Z (UA)_0 h_0}$$

and

$$Z = \frac{\sum_{j=1}^{n_s} (UA)_j \alpha_{sj} + h_0 \sum_{j=1}^{n_s} (\tau\alpha)_i A_{gi}}{(UA)_0 h_0}.$$

If surface dependent properties are required in the estimation of ϕ, they should be weighted averages of the properties for the various surfaces. The utilizability for an 'effective' surface which is a linear combination of the actual surfaces receiving solar radiation is $(\bar{I}_{c0})$. Sol-air cooling degree-days are found using the relationship

$$D_{SC,h(T_b)} = D_{C,h}(T_b) + Z(N/24)\bar{I}_T\phi(\bar{I}_{c0}) \quad (13)$$

where Z, $\bar{I}_{T0}$ and $\bar{I}_{c0}$ are defined above.

A solar air heating system which has no thermal storage draws in room air and returns it directly to the heated space. Solar gains from a no-storage system can be treated in the same manner as direct gains through a window, with one important difference. When the radiation incident on the collector is less than the critical level of the collector, the solar system will not operate. There may be certain hours of the day for some months when the collector does not operate at all, even though the sun is shining. The following equation must be solved for $I_{Tc}(k'_{c0})$ to determine whether the collector will provide useful gains

$$\sum_{j=1}^{ns} (UA)_j\alpha_{sj}I_{Tsj}(k'_{c0}) + h_0 \sum_{i=1}^{nw} (\tau\alpha)_i A_{gi} I_{Twi}(k'_{c0}) + A_c F_R(\tau\alpha)_c h_0 I_{Tc}(k'_{c0}) = h_0 UA_0(T_b - T_{ah})^+ + h_0 A_c F_R U_L(T_r - T_{ah})^+ \quad (14)$$

where I_{Tc} is the radiation incident on the collector surface, F_R is the collector heat removal factor, U_L is the collector loss coefficient, $(\tau\alpha)_c$ is the effective transmittance–absorptance product for the collector (for a more complete definition of F_R, $(\tau\alpha)_c$, and U_L see Ref. [5]) and T_r is the interior air temperature. If the condition

$$I_{Tc}(k'_{c0}) < \frac{U_L}{(\tau\alpha)_c}(T_r - T_{ah})^+ \quad (15)$$

is satisfied, the collector system will not run for the hour of the month under consideration, and sol-air degree-days can be calculated using eqn (12).

When eqn (15) is not satisfied, the collector will be able to supply useful gains to the structure for at least part of the month. If eqn (11) is modified to include the energy delivery by the collectors, the expression for the sol-air heating degree-days becomes

$$D_{SH,h} = D_{H,h}(T_b) - \frac{N}{24} Z'\bar{I}'_{T0}(1 - \phi(\bar{I}_{c0})) + \frac{NA_cF_R(\tau\alpha)_c}{24(UA)_0}\bar{I}_{Tc}(1 - \phi(\bar{I}_{cc})) \quad (16)$$

where

$$\bar{I}'_{c0} = \frac{(T_b - \bar{T}_{a,h})^+}{Z'} + \frac{A_cF_RU_L}{Z'(UA)_0}(T_r - \bar{T}_{ah})^+$$

$$\bar{I}_{cc} = \frac{U_L}{(\tau\alpha)_c}(T_r - \bar{T}_{ah})^+$$

$$Z' = Z + \frac{A_cF_R(\tau\alpha)_c}{(UA)_0}$$

$$\bar{I}_{T0} = \frac{\left(\sum_{j=1}^{ns}(UA)_j\alpha_{sj}\bar{I}_{Tsj} + h_0\sum_{i=1}^{nw}(\tau\alpha)_iA_{gi}\bar{I}_{Twi} + A_cF_R(\tau\alpha)_ch_0\bar{I}_{Tc}\right)}{Z'(UA)_0h_0}$$

Equation (16) is very similar to eqn (12), but it contains an additional term to account for incident radiation which occurs at levels below the collector critical level. Since the collectors will not contribute to the building cooling load, no new expression is required for $D_{SC,h}$.

Cooling may be required even when the ambient temperature is significantly less than the room temperature due to internal and solar gains. In many structures it is possible to use outside air to meet part or all of the cooling load when this is the case. However, traditional cooling degree-days and the sol-air cooling degree-days presented so far cannot account for the reduction in the cooling load made possible by the use of outside air. Sol-air degree-days which include the effects of ventilation will be referred to as ventilated sol-air degree-days, $D_{SCV,h}$.

The rate at which energy can be removed from the room by ventilation with outside air is given by the product of the air capacitance rate, $\dot{m}C_p$, and the inside–outside temperature difference. An expression similar to eqn (11) can be written for ventilated sol-air cooling degree-days as

$$D_{SCV,h} = N\int_{T_{ah,min}}^{T_r}\int_0^{k,max}\left(T_{ah} + ZI_{T0} - T_b - \frac{\dot{m}c_p(T_r - T_{ah})}{(UA)_0}\right)^+ P(k)\,dk\,P(T_{ah})\,dT_{ah} + \int_{T_r}^{T_{ah,max}}\int_0^{k,max}(T_{ah} + ZI_{T0} - T_b)P(k)\,dk\,P(T_{ah})\,dT_{ah}. \quad (17)$$

The ambient temperature distribution function of Erbs *et al.* [7] is required to evaluate eqn (17). Essentially the same simplifications that were used for a single surface can be applied, and an expression is obtained for $D_{SCV,h}$

$$D_{SCV,h} = \left(\frac{N^{3/2}\sigma_m}{48}\right)\left(\frac{\dot{m}c_p + (UA)_0}{(UA)_0}\right) \times \left[\frac{\ln\cosh(\beta h_b^*) - \ln\cosh(\beta h_r^*)}{\beta} + (h_r^* - h_b^*)\tanh(\beta h_r^*)\right] + \frac{N}{24}Z\bar{I}_{T0}\phi(\bar{I}''_{c0}) + D_{Ch}(T_r) + \frac{N}{24}(1 - Q(h_r))(T_r - T_b)$$

where

$$h_r^* = \frac{(\bar{T}_{ah} - T_r)}{\sqrt{N}\,\sigma_m}$$

$$h_b^* = \frac{(\bar{T}_{ah} - T_b')}{\sqrt{N}\,\sigma_m}$$

$$h_r = \frac{(T_r - \bar{T}_{ah})}{\sqrt{N}\,\sigma_m}$$

$$T_b' = \frac{(UA)_0}{(\dot{m}c_p + (UA)_0)} T_b + \frac{\dot{m}c_p}{(\dot{m}c_p + (UA)_0)} T_r$$

$$\bar{I}_{c0}'' = \frac{(T_b + \dot{m}c_p(T_r - \bar{T}_{ah}')/(UA)_0 - \bar{T}_{ah})}{Z}$$

$$\bar{T}_{ah}' = \frac{\sigma_m\sqrt{N}[\beta h_r \tanh(\beta h_r) - \ln\cosh(\beta h_r) - 0.693]}{\beta(\tanh(\beta h_r)^{-1}} + \bar{T}_{ah}$$

and

$$Q(h_r) = \frac{1 + \tanh(\beta h_r)}{2}$$

and where $\beta = 1.698$, Q is the cumulative distribution function for ambient temperature, and σ_m is the standard deviation of $\bar{T}_a$.

6. SAMPLE CALCULATIONS

Sol-air heating and cooling degree-days will be estimated for a building located in Madison, Wisconsin. Use of the relationships will be demonstrated by calculating the sol-air heating degree-days for the hour from 11 A.M. to noon for the month of November. The structure consists of four walls and a pitched roof with an attic. Three of the walls contain windows. The building is 18.3 m long, 9.1 m wide, and 3 m high. The infiltration rate was assumed to be one air change per hour, and conductivities for the walls, roof and windows were obtained from ASHRAE [3] for a typical insulated frame construction. The basic building description is summarized in Table 1.

Values for $\bar{K}$ and $\bar{T}_a$ were taken from Ref. [13]. For November, $\bar{K}$ is 0.40 and $\bar{T}_a$ is 1.5°C. Using the relationships of Erbs *et al.* [7], estimates of $\bar{T}_{ah}$ and $D_{H,h}$ are 3.2 and 16.1°C days, respectively. The monthly-average hourly tilted surface radiation for each surface was calculated using the methods provided in Sections 2.15 and 2.16 of Ref. [5]. Table 2 presents the results for the six distinct surface orientations of this building. The data in Tables 1 and 2 yield a value for $\bar{I}_{T0}$ of 253 W m^{-2}; $\bar{I}_{c0}$ is equal to 367 W m^{-2}, Z is 0.035°C m^2 W^{-1}, and the ratio of $\bar{I}_{T0}$ to $\bar{I}$ is equal to 0.92. The hourly utilizability, ϕ, was estimated using the method of Clark [8] to be 0.38. The sol-air heating degree-days for the hour are found by use of eqn (12) to equal 9.2°C days.

A computer program was written to repeat the set of calculations outlined above for the remaining hours and months. The daily results for each month are presented in Table 3. The effect of solar radiation is to reduce the heating degree-day total for the year by nearly 600°C days, or 15% of the annual

Table 1. Building description for sample calculation

Surface characteristics				
Surface slope (deg)	Surface azimuth (deg)	Conductance (W K^{-1})	Solar absorptance	Surface description
90	0	22.0	0.6	South wall
90	0	21.7	0.05	South window
90	90	11.0	0.6	West wall
90	90	6.6	0.05	West window
90	180	25.6	0.6	North wall
90	−90	11.0	0.6	East wall
90	−90	6.6	0.05	East window
18.3	0	23.9	0.9	South roof
18.3	180	23.9	0.9	North roof

Window characteristics			
Window slope (deg)	Window azimuth (deg)	Window area (m^2)	$(\overline{\tau\alpha})$
90	0	7.80	0.7
90	90	2.32	0.7
90	−90	2.32	0.7

Infiltration conductance = 170.4 W K^{-1}
Heating base temperature = 16°C
Cooling base temperature = 19°C
Outside surface convection coefficient, h_o = 34 W m^{-2} K^{-1} (heating) or 22 W m^{-2} K^{-1} (cooling)
$(UA)_o$ = 322.7 W K^{-1}

Table 2. Monthly-average hourly tilted surface radiation

Surface orientation	I_T (W m^{-2})
South vertical	351
North vertical	96
West vertical	96
East vertical	132
South roof, slope = 18.3°	342
North roof, slope = 18.3°	182

heating load. While this represents a significant solar gain, the effect of solar radiation on cooling loads is much more pronounced. The cooling degree-day total for the year increases by more than 800°C days when solar radiation is accounted for. The cooling load based on sol-air degree-days is nearly four times as large as the cooling load based on ambient temperature degree-days.

The effect of neglecting solar radiation absorbed on the outer surfaces was tested by setting the solar absorptance to zero for each surface. The sol-air heating degree-day total changed only slightly, increasing by 90°C days. However, the sol-air cooling degree-day total was more strongly affected, decreasing by 360°C days. If the south glazing area is increased to 15 m^2, the sol-air heating degree-day total drops by 150°C days, which represents a 5% reduction in the heating load, but the sol-air cooling degree-day total jumps by 500°C days, which is 44% of the cooling load. Neglecting solar radiation entering through the east and west glazings has the effect of increasing the sol-air heating degree-day total by 90°C days and decreasing the sola-air cooling degree-day total by 230°C days.

The south-facing glazing was replaced by a wall and a vertical no-storage air collector system. All remaining building parameters were set equal to the base case values given in Table 1. Replacing the south window with an insulated wall changes the house loss conductance to 304.6 W °C^{-1}, a 6% reduction. The insulating effect of the collectors, particularly important when there is incident solar radiation and the collectors are warm, was neglected to simplify the analysis. The values used for $F_R(\overline{\tau\alpha})$ and $F_R U_L$ were 0.7 and 6 W m^{-2} $\overline{°C^{-1}}$, respectively.

Table 3. Normal and sol-air degree-days for a structure in Madison, Wisconsin

Month	D_H (°C days)	D_{SH} (°C days)	D_C (°C days)	D_{SC} (°C days)
Jan	757	674	0	0
Feb	630	529	0	0
Mar	528	416	1	9
Apr	266	192	3	44
May	122	98	17	121
Jun	32	30	63	216
Jul	15	15	107	277
Aug	20	20	92	253
Sep	77	69	27	152
Oct	205	164	7	69
Nov	437	373	1	4
Dec	670	607	0	0
Yr	3759	3187	317	1146

It was first necessary to determine which hours of the day for each month the collector would operate at least some of the time. This was accomplished by solving eqn (14) numerically and testing the inequality given by eqn (15). Sol-air heating degree-days were estimated with eqn (12) for the hours when the collectors would not operate, and eqn (16) was used for the remaining hours. A net collector area of 7.8 m^2 (the size of the original south window) resulted in an annual sol-air heating degree-day total of 3299°C days. This total is higher than for the direct gain case because the collectors can only turn on when the incident radiation is above the collector critical level. However, the reduction in the house $(UA)_0$ is larger than the increase in D_{SH} and in Madison the annual heating load for the no-storage collector system is 2% smaller than the annual heating load for the direct gain system. It should be noted that no night insulation was employed in the direct gain system.

The value of $F_R U_L$ was reduced from 6 to 4.5 W m^{-2} °C, with the result that the annual sol-air heating degree-day total decreased to a value of 3272°C days. When the original value of $F_R U_L$ was used but the collector area was doubled, the annual D_{SH} total was found to be 3210°C days. An estimate of the sol-air cooling degree-days was obtained for the no-storage collector house by eliminating the south window and by neglecting the effect of the collector on the cooling load due to the south wall. The annual sol-air cooling degree-day total of 771°C days is nearly 400°C days less than the D_{SC} total for the direct gain system. However, it should also be noted that no shading was employed in the direct gain system.

The potential for reducing the house cooling load by ventilation with outside air was also investigated. Equation (18) was used to estimate ventilated sol-air cooling degree-days for the house described in Table 1. For a ventilation rate of 5 air changes h^{-1}, the annual sol-air cooling degree-day total drops to 530°C days, which is less than half of the cooling degree-day total when ventilation is not considered. Increasing the ventilation rate to 10 air changes h^{-1} reduces the annual total to 340°C days, while a ventilation rate of 50 air changes h^{-1} results in an annual cooling degree-day total of 290°C days. A room temperature of 22°C was used to generate these results. For the highest ventilation rate, the ventilated sol-air cooling degree-day total is actually less than the traditional cooling degree-day total because of the 'free' cooling which takes place when the ambient temperature is between 19 and 22°C.

7. CONCLUSIONS

Solar radiation is an important variable in determining the heating and cooling loads for buildings.

Transmission through glazings and absorption by opaque surfaces are both important mechanisms for solar gains. In many climates, the cooling load is increased substantially by the absorption of solar energy. As a result, cooling degree-days based on ambient temperature data do not always provide a good measure of the cooling load and have not been used as widely as heating degree-days. Sol-air degree-days combine the effects of solar radiation and ambient temperature, providing a more fundamental measure of the driving potential for building loads.

Sol-air degree-days are a building specific weather statistic. As such, they provide a basic measure of the heating and cooling loads for a building. The thermal performance of different building constructions can be compared with a relatively simple set of calculations. Since the only inputs required are the monthly-average daily temperatures and solar radiation, the relationships presented are useful at a large number of locations. The actual building load may be influenced by other factors, such as building capacitance and control strategies. It may be necessary to correct for the effects of these variables. Sol-air degree-days represent a first step in the development of more comprehensive design procedures for the estimation of building loads.

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