Effects of Irreversibility and Economics on the Performance of a Heat Engine

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Previous investigators have shown that an internally reversible Carnot cycle, operating with heat transfer limitations between the heat source and heat sink at temperatures T_H *and* T_L, *achieves maximum power at an efficiency equal to* $1-\sqrt{T_L/T_H}$ *independent of the heat exchanger transfer coefficients. In this paper, optimization of the power output of an internally irreversible heat engine is considered for finite capacitance rates of the external fluid streams. The method of Lagrange multipliers is used to solve for working fluid temperatures which yield maximum power. Analytical expressions for the maximum power and the cycle efficiency at maximum power are obtained. The effects of irreversibility and economics on the performance of a heat engine are investigated. A relationship between the maximum power point and economically optimum design is identified. It is demonstrated that, with certain reasonable economic assumptions, the maximum power point of a heat engine corresponds to a point of minimum life-cycle costs.*

Introduction

The maximum thermal efficiency of a thermodynamically reversible heat engine is commonly used in thermodynamic texts to define the upper limit of performance of heat engines. This efficiency is determined solely by the temperatures of the heat source and heat sink. To achieve this maximum efficiency, however, heat transfer between the cycle and the external thermal energy source or sink would have to occur reversibly, i.e., isothermally. For finite heat exchange surface areas, the requirement of isothermal heat transfer necessarily results in zero heat transfer rates. Both the rates of heat transfer to and from the cycle, and thus their difference (which is the power), must be zero. Any totally reversible power cycle achieves the highest possible efficiency, but produces zero power. Thus, the maximum efficiency of an ideal engine provides an upper limit on the efficiency of power production, but it does not provide a realistic design goal.

In order to produce power with finite-sized equipment, temperature differences must exist between the thermal source/sink and the cycle. El-Wakil (1962) and Curzon and Alborn (1975) considered an internally reversible Carnot cycle in which the heat transfers to and from the cycle were assumed to be a linear function of these temperature differences. They found that the cycle exhibited maximum power at an efficiency equal to $1-\sqrt{T_L/T_H}$, independent of the heat exchanger characteristics, where T_H and T_L are temperatures of the isothermal heat source and sink, respectively. This same maximum power efficiency relation also applies to the air standard Brayton and Otto cycles, and (approximately) to the Diesel cycle (Leff, 1987; Klein, 1991).

The cycle considered by El-Wakil (1962) and Curzon and Alborn (1985) operated between an isothermal heat source and sink. Practical heat engines do not operate between constant temperature thermal reservoirs, but rather they transfer energy to and from flowing streams which have finite capacitance rates (i.e., mass flow rate-specific heat products). Internally reversible heat engines operating with finite capacitance rate streams were investigated numerically by Ondrechen et al. (1983) and Wu (1988). In this paper, an analytical solution is obtained for this situation, which is then generalized for the case of internal cycle irreversibilities.

Some economic aspects of the maximum power problem have been discussed by Curzon and Ahlborn (1975) and Bejan (1988). They have noted that the efficiencies of actual power plants are reasonably close to the maximum power efficiency of internally reversible Carnot-like heat engines operating over the same temperature extremes. This agreement seems to be a mathematical coincidence since power plant designs are dictated by economics in addition to thermodynamics and heat transfer. The major contribution of this paper is to analytically demonstrate that there are circumstances for which the maximum power operating point of a heat engine corresponds to operation at minimum life-cycle costs.

Power Optimization

The heat engine considered in this study is modeled as a closed system in which the working fluid passes through a continuous set of thermodynamic states and returns to its original state. The first law of thermodynamics for this situation can be expressed:

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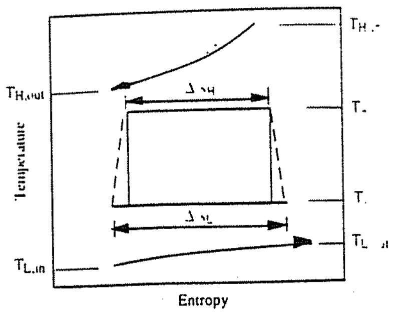

Fig. 1 Heat engine with irreversible expansion and compression coupled to heat source and heat sink with finite capacitance rates

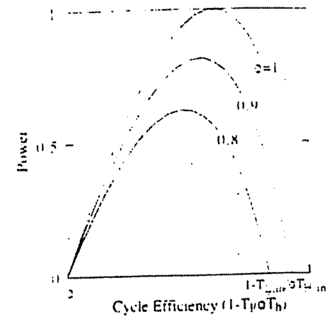

Fig. 2 Power/efficiency trade-offs for a heat engine showing the effects of internal irreversibility

$$\dot{W} = \dot{Q}_H - \dot{Q}_L \tag{1}$$

where $\dot{Q}_H$ is the rate at which heat is supplied for a heat engine cycle, $\dot{Q}_L$ is the rate at which heat is rejected, and $\dot{W}$ is the power output.

For cyclic operation, the second law of thermodynamics requires that

$$\oint \delta Q/T \leq 0. \tag{2}$$

Consider a heat engine represented in Fig. 1 which operates between two constant temperatures, T_h and T_l, as in the Carnot cycle. The energy source for the heat engine is a hot fluid stream having a finite capacitance rate $\dot{C}_H$ and inlet temperature at $T_{H,\mathrm{in}}$. The hot fluid stream leaving the cycle is discharged to the surroundings at an outlet temperature of $T_{H,\mathrm{out}}$. The energy sink is a cold fluid stream having a capacitance rate of $\dot{C}_L$ and an inlet temperature of $T_{L,\mathrm{in}}$. Because of finite heat-transfer coefficients, the cycle operates between $T_h (< T_{H,\mathrm{in}})$ and $T_l (> T_{L,\mathrm{in}})$ to provide temperature differences between the cycle and the source and sink streams. An entropy balance on the cycle operating at steady-state requires that

$$\frac{\dot{Q}_H}{T_h} - \frac{\dot{Q}_L}{T_l} \leq 0. \tag{3}$$

Equation (3) is the well-known Clausius inequality. The left-hand side of Eq. (3) will be less than zero wherever there is any thermodynamic internal irreversibility such as friction, pressure drops, internal heat transfer, etc., associated with the power production between temperatures T_h and T_l. An irreversibility factor (ϕ) is defined so that the Clausius inequality can be written as an equality:

$$\frac{\dot{Q}_H}{T_h} - \phi \frac{\dot{Q}_L}{T_l} = 0. \tag{4}$$

The cycle efficiency is defined as

$$\eta = \frac{\dot{W}}{\dot{Q}_H} \tag{5}$$

and can be written in terms of the irreversibility factor as

$$\eta = 1 - \frac{T_l}{\phi T_h}. \tag{6}$$

ϕ is equal to one when the cycle is internally reversible and less than one when the cycle is internally irreversible. Recognizing that the cycle efficiency must be greater than zero requires that $\phi \geq T_l/T_h \geq T_L/T_H$, which sets a lower limit for ϕ.

The rate at which heat is supplied and rejected can be expressed as:

$$\dot{Q}_H = \dot{C}_H \epsilon_H \ (T_{H,\mathrm{in}} - T_h) \tag{7}$$

$$\dot{Q}_L = \dot{C}_L \epsilon_L \ (T_l - T_{L,\mathrm{in}}) \tag{8}$$

where ϵ_H and ϵ_L are the hot-side and cold-side heat exchanger effectiveness factors, respectively (Kays and London, 1964). Since the working fluid is assumed to undergo isothermal heat transfer as in phase change processes, the effectiveness relationships simplify to

$$\epsilon_H = 1 - \exp(-\mathrm{NTU}_H) \tag{9}$$

$$\epsilon_L = 1 - \exp(-\mathrm{NTU}_L). \tag{10}$$

NTU_H and NTU_L are the number of transfer units of the hot and cold-side heat exchanger, respectively.

$$\mathrm{NTU}_H = UA_H/\dot{C}_H \tag{11}$$

$$\mathrm{NTU}_L = UA_L/\dot{C}_L \tag{12}$$

where UA_H and UA_L are the hot and cold-side heat exchanger conductances (total heat-transfer coefficient-area product), respectively.

Nomenclature

- $\dot{C}$ = capacitance rate (mass flow rate-specific heat product), kW/K
- c = unit cost of capacitance rate, dollars/(kW/K)
- c' = unit cost of heat exchange conductance, dollars/(kW/K)
- C_E = engine investment costs which are independent of heat exchanger area, in dollars
- E = initial equipment investment, in dollars
- F = first year operating cost, in dollars
- LCC = life-cycle cost in dollars
- NTU = number of transfer units, $\mathrm{NTU} = UA/\dot{C}$
- $\dot{Q}$ = rate of heat transfer, kW
- P_1 = factor relating life-cycle operating cost to first year operating cost
- P_2 = factor relating life-cycle expenditures incurred by additional capital investment to the initial cost
- s = entropy, kJ/kg K
- T = temperature, K
- UA = heat exchanger conductance (total heat transfer coefficient-area product), kW/K
- $\dot{W}$ = power, kW
- ϵ = heat exchanger effectiveness
- η = cycle efficiency
- η^* = maximum power efficiency
- λ = Lagrange multiplier
- τ = capacitance rate ratio, $\dot{C}_L/\dot{C}_H$
- ϕ = factor relating the entropy change during heat rejection and heat addition

Subscripts

- H = heating fluid, heat source, hot-side heat exchanger
- h = high, heater
- L = cooling fluid, heat sink, cold-side heat exchanger
- l = low
- out = outlet
- in = inlet
- max = maximum

Figure 2 shows the relationship between the power and efficiency for a heat engine with fixed heat exchanger sizes and external stream conditions. These curves were obtained by solving Eqs. (1), (4), (6), (7), and (8) simultaneously to determine $\dot{W}$, $\dot{Q}_H$, $\dot{Q}_L$, T_h, and T_l for known values of ϕ, η, $T_{H,\text{in}}$, $T_{L,\text{in}}$, $\dot{C}_H$, $\dot{C}_L$, ϵ_H, and ϵ_L. Different curves correspond to different values of ϕ. As shown in Fig. 2, internal irreversibilities decrease both the maximum power and the efficiency at the maximum power.

The purpose of the following analysis is to determine an analytical expression for maximum power and the efficiency corresponding to maximum power. Using Eqs. (7) and (8), the energy and entropy balances can be rewritten as

$$\dot{W}=\dot{C}_H\epsilon_H(T_{H,\text{in}}-T_h)-\dot{C}_L\epsilon_L(T_l-T_{L,\text{in}}) \tag{13}$$

$$g(T_h,T_l)=\frac{\dot{C}_H\epsilon_H(T_{H,\text{in}}-T_h)}{T_h}-\phi\frac{\dot{C}_L\epsilon_L(T_l-T_{L,\text{in}})}{T_l}=0 \tag{14}$$

where g is an entropy constraint function which is used in the following optimization analysis.

The values of T_h and T_l, which result in maximum power, are most easily found using the method of Lagrange multipliers. A Lagrange multiplier, λ, is defined such that

$$\partial\dot{W}/\partial T_h=\lambda\,\partial g/\partial T_h \tag{15}$$

$$\partial\dot{W}/\partial T_l=\lambda\,\partial g/\partial T_l. \tag{16}$$

Evaluating the partial derivatives allows Eqs. (15) and (16) to be written as

$$1=\lambda\,T_{H,\text{in}}/T_h^2 \tag{17}$$

$$1=\lambda\,\phi\,T_{L,\text{in}}/T_l^2. \tag{18}$$

Solving Eqs. (17) and (18) leads to the following relation for the unknown cycle temperatures:

$$\frac{T_l}{T_h}=\sqrt{\frac{\phi\,T_{L,\text{in}}}{T_{H,\text{in}}}}. \tag{19}$$

The efficiency at the maximum power point, η^*, is then

$$\eta^*=1-\sqrt{\frac{T_{L,IN}}{\phi T_{H,\text{in}}}}. \tag{20}$$

The maximum power for the simple heat engine coupled to source and sink streams with finite capacitance rates is

$$\dot{W}_{\max}=\frac{\phi\,\dot{C}_L\epsilon_L}{\phi\,\dot{C}_L\epsilon_L+\dot{C}_H\epsilon_H}\,\dot{C}_H\epsilon_H[\sqrt{T_{H,\text{in}}}-\sqrt{T_{L,\text{in}}/\phi}]^2. \tag{21}$$

Equation (20) shows that the efficiency of the simplified cycle at maximum power depends only on the inlet temperatures of the source and sink streams, and the irreversibility factor ϕ. If ϕ is 1, the efficiency reduces to the relationship of El-Wakil (1962) and Curzon and Ahlborn (1975).

Economic Optimization

The goal of the economic optimization is to find values of the system design variables (temperatures, equipment sizes, etc.) which minimize the life-cycle cost of providing a specified power output. The life-cycle cost includes the total cost of building and operating the plant over its expected lifetime, accounting for inflation and other economic factors. Minimizing the life-cycle cost necessarily involves a trade-off between the operating and capital cost.

Economic Analysis. The life-cycle cost of a power plant can be expressed as the sum of two contributions: one associated with operating cost and the other with capital cost. The life-cycle cost is conveniently represented by the P_1-P_2 life cycle cost method of Duffie and Beckman (1991) for which

$$\text{LCC}=P_1F+P_2E, \tag{22}$$

where LCC is the life-cycle cost, F is the first year operating energy cost, E is the initial equipment investment, P_1 is a factor relating life-cycle operating cost to first year cost, and P_2 is a factor relating life-cycle expenditures incurred by additional capital investment to the initial investment.

P_1 incorporates all factors which affect the first year operating cost. P_2 incorporates all costs which are proportional to the initial investment. These factors allow for variations of annual expenses with time (e.g., inflation). P_1 is a function of the number of years of the economic analysis, fuel price inflation rate, interest rate, and income tax rate. P_2 is a function of the down payment, payment on principal, tax deductions, property tax, maintenance, depreciation, salvage value, and tax credit. Specific relations for P_1 and P_2 in terms of these economic factors are provided in Duffie and Beckman (1980). In general, P_1 is on the order of the number of years of the analysis and P_2 is on the order of unity.

The total operating cost can be approximated as the sum of the operating costs to supply energy to and reject energy from the cycle. In the following analysis, the external source and sink streams inlet temperatures are assumed to be fixed. The operating cost related to energy supply (including resource utilization and pumping cost) is assumed to be directly proportional to the capacitance rate of the hot stream, $\dot{C}_H$. The operating cost of heat rejection (e.g., pumping costs) is assumed to be directly proportional to $\dot{C}_L$.

The equipment investment cost for a specified design power can be divided into costs associated with the heat exchangers and costs associated with equipment which is independent of heat exchanger area (e.g., pumps, turbines, etc.). The heat exchanger cost is assumed to be a linear function of the heat exchanger conductances and independent of the cycle temperatures T_h and T_l. The cost per unit heat exchanger conductance for the heat-temperature heat exchanger (boiler) may differ from that for the low-temperature heat exchanger (condenser). With these assumptions, Eq. (22) can be written as

$$\text{LCC}=P_1[c_H\dot{C}_H+c_L\dot{C}_L]+P_2[c_H'\,UA_H+c_L'\,UA_L+C_E] \tag{23}$$

where

c_H = unit cost of the heat source capacitance rate, dollars/(kW/K)

c_L = unit cost of the heat sink capacitance rate, dollars/(kW/K)

c_H' = unit cost of hot-side heat exchange conductance, dollars/(kW/K)

c_L' = unit cost of cold-side heat exchange conductance, dollars/(kW/K)

C_E = engine investment costs which are independent of heat exchanger area (pumps, turbines..; etc.), dollars.

The energy balance, Eq. (13), and the life cycle cost, Eq. (23), can be combined into the following equation:

$$\text{LCC}=\frac{\dot{W}P_1[c_H+\tau c_L]+\dot{W}P_2[c_H'\text{NTU}_H+\tau c_L'\text{NTU}_L]}{[1-\exp(-\text{NTU}_H)](T_{H,\text{in}}-T_h)-\tau[1-\exp(-\text{NTU}_L)](T_l-T_{L,\text{in}})}+P_2C_E \tag{24}$$

where

$$\tau=\dot{C}_L/\dot{C}_H.$$

The entropy balance constraint function, Eq. (14), can be rewritten as

$$g(T_H,T_l,\tau,\text{NTU}_H,\text{NTU}_L)=\frac{[1-\exp(-\text{NTU}_H)](T_{H,\text{in}}-T_h)}{T_h}-\phi\frac{\tau[1-\exp(-\text{NTU}_L)](T_l-T_{L,\text{in}})}{T_l}=0. \tag{25}$$

The purpose of the following analysis is to determine the minimum life-cycle costs associated with providing power at a fixed specified rate. C_E is assumed to be a function of T_h. P_1 and P_2 are fixed for a given economic environment. The source and sink inlet temperatures, $T_{H,\text{in}}$ and $T_{L,\text{in}}$ are assumed to be constant. The internal irreversibility factor, ϕ, is assumed to be constant. The economic optimization is then determined by minimizing LCC with respect to T_h, T_l, τ, NTU_H, and NTU_L subject to the entropy balance constraint function given by Eq. (25). The method of Lagrange undetermined multipliers will again be used which involves the following partial derivatives:

$$\partial \text{LCC}/\partial T_h = \lambda(\partial g/\partial T_h) \tag{26}$$

$$\partial \text{LCC}/\partial T_l = \lambda(\partial g/\partial T_l) \tag{27}$$

$$\partial \text{LCC}/\partial \tau = \lambda(\partial g/\partial \tau) \tag{28}$$

$$\partial \text{LCC}/\partial \text{NTU}_H = \lambda(\partial g/\partial \text{NTU}_H) \tag{29}$$

$$\partial \text{LCC}/\partial \text{NTU}_L = \lambda(\partial g/\partial \text{NTU}_L). \tag{30}$$

Using Eqs. (24) and (25), the partial derivatives in Eqs. (26) through (30) can be written as

$$\frac{J}{Y^2}\left[1+\frac{Y^2}{J}P_2(\partial C_E/\partial T_h)\right] = -\lambda\frac{T_{H,\text{in}}}{T_h^2} \tag{31}$$

$$\frac{J}{Y^2} = -\lambda\phi\frac{T_{L,\text{in}}}{T_l^2} \tag{32}$$

$$\frac{\epsilon_L(T_l-T_{L,\text{in}})J - YK}{Y^2} = -\lambda\phi\frac{\epsilon_L(T_l-T_{L,\text{in}})}{T_l} \tag{33}$$

$$\frac{\exp(-\text{NTU}_H)(T_{H,\text{in}}-T_h)J - Y\dot{W}P_2c_H'}{Y^2} = \lambda\frac{\exp(-\text{NTU}_H)(T_{H,\text{in}}-T_h)}{T_h} \tag{34}$$

$$\frac{\exp(-\text{NTU}_L)(T_l-T_{L,\text{in}})J - Y\dot{W}P_2c_L'}{Y^2} = \lambda\phi\frac{\exp(-\text{NTU}_L)(T_l-T_{L,\text{in}})}{T_l} \tag{35}$$

where

$$J = \dot{W}P_1[c_H+\tau c_L] + \dot{W}P_2[c_H'\text{NTU}_H + \tau c_L'\text{NTU}_L] \tag{36}$$

$$Y = \epsilon_H(T_{H,\text{in}}-T_h) - \tau\epsilon_L(T_l-T_{L,\text{in}}) \tag{37}$$

$$K = \dot{W}(P_1c_L + P_2c_L'). \tag{38}$$

Solving Eqs. (6), (31), and (32), the following result is obtained:

$$\eta = 1-\sqrt{\frac{(1+\xi)T_{L,\text{in}}}{\phi T_{H,\text{in}}}} \tag{39}$$

where

$$\xi = \frac{Y^2P_2(\partial C_E/\partial T_h)}{J} = \frac{P_2\dot{W}(\partial C_E/\partial T_h)}{\dot{C}_H(\text{LCC}-C_E)}. \tag{40}$$

Equation (39) provides the efficiency of a power plant supplying a specified power $\dot{W}$ in a manner which minimizes the LCC. For the case where $\xi \approx 0$ (e.g., the derivative of C_E with respect to T_h is small or C_E is independent of temperature), the efficiency at the minimum life-cycle cost reduces to

$$\eta = 1-\sqrt{\frac{T_{L,\text{in}}}{\phi T_{H,\text{in}}}}. \tag{41}$$

With the economic assumptions listed above, the efficiency at which the LCC is minimized is exactly the same as the efficiency obtained in Eq. (20) for a heat engine operating at maximum power. A rationalization of this result is provided in the following section.

Rationalization of the Relation Between the Minimum LCC and the Maximum Power. In this section a simplified economic analysis is considered to help understand the relation between the minimum life-cycle cost and the maximum power. In this simplified economic optimization, the only operating cost is the energy supply which again is assumed to be directly proportional to $\dot{C}_H$. The inlet temperature of the energy supply stream is assumed to be $T_{H,\text{in}}$. The only capital costs are those associated with heat exchangers.

The case where the heat exchanger conductances are kept constant will be considered first. In this case, the capital cost is fixed. The goal thus is to minimize the operating cost of producing a specified power, $\dot{W}$; the operating cost is directly proportional to the heat source capacitance rate.

Figure 3 shows the relationship between the power and efficiency for a heat engine with fixed heat exchanger sizes and external streams conditions. Different curves correspond to

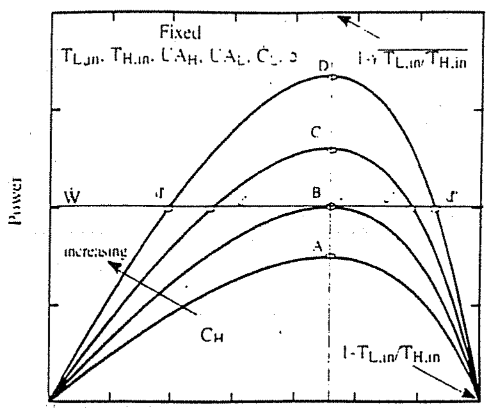

Fig. 3 A number of cycles with different efficiencies produce the same amount of power

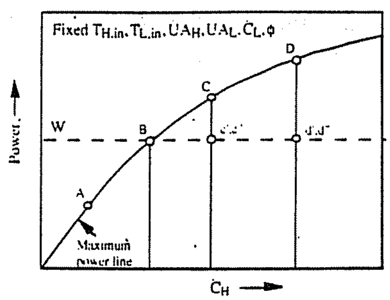

Fig. 4 Power versus heat source capacitance rate

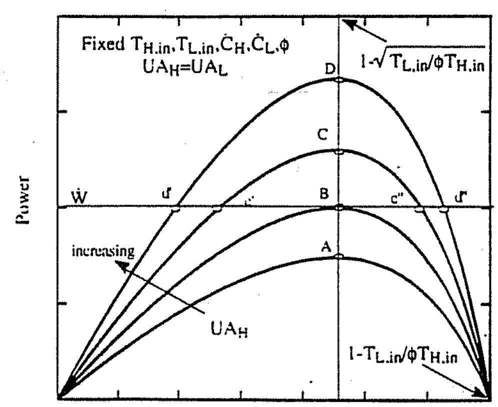

Fig. 5 A number of cycles with different efficiencies producing the same amount of power

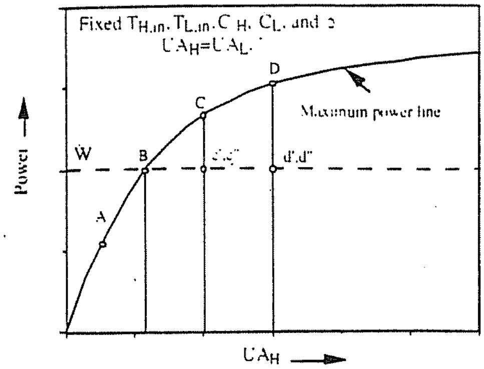

Fig. 6 Power versus hot-side heat exchanger conductance

different values of $\dot{C}_H$. For example, points C, c', and c'' correspond to a power plant with the same heat source capacitance rate and heat exchanger sizes. A number of cycles which have different cycle efficiencies can produce the same amount of power. For example, points B, c', c'', d', and d'' all produce the same power with different cycle efficiencies. Points A, B, C, and D are the maximum power points for different energy supply flow rates. Since the heat exchanger conductances, the sink capacitance rate, and the heat source/sink inlet temperatures are specified, the only possible way to increase the maximum power is by increasing the heat source capacitance rate as shown in Fig. 3. At point d'', the capacitance rate $\dot{C}_H$ must be greater than that of point c'' which is greater than at point B.

Figure 4 is a plot of the maximum power versus the source capacitance rate, $\dot{C}_H$. Points above line $ABCD$ exceed the maximum power of the cycle. However, it is possible to operate on or under the maximum power line. Operating at conditions c', c'', B, d', d'', all satisfy the design power; however, operating point B has the minimum source capacitance rate as shown in Figs. 3 and 4. In Fig. 3, all points to the left or to the right of the maximum power point, B, correspond to curves with larger values of $\dot{C}_H$ and a result, higher operating costs. Operating at B then minimizes the operating cost and is the optimum.

Consider the case where the heat source capacitance rates are constant. The goal in this case is to minimize the cost of the heat exchangers while supplying a specified power. The cost of heat exchangers is assumed to be proportional to the heat exchanger conductances, UA_H and UA_L. As shown in Figs. 5 and 6, a number of cycles, which have different efficiencies, can produce the same amount of power. Using the same logic as just presented for Figs. 3 and 4, it is clear that point B satisfies the design power with minimum heat exchanger conductances, and thus with minimum investment costs.

A heat engine operating at maximum power corresponds to the minimum operating cost when the capital cost is held constant, and also to the minimum capital cost when the operating cost is held constant, assuming that operating costs are linearly related to $\dot{C}_H$ and the investment costs are linearly related to heat exchanger UAs, respectively.

Discussion and Conclusions

This paper shows how heat transfer and economic constraints affect the design of a heat engine. A relationship between the maximum power point and economically optimum design has been identified as determined by life-cycle economic considerations. The economic assumptions for which heat engine power optimization and economic optimization yield the same optimum cycle efficiency are: (*i*) the cost of equipment is independent of temperature, (*ii*) operating costs are linearly related to capacitance rates of the external streams ($\dot{C}_H$ and $\dot{C}_L$), (*iii*) the irreversibility factor, ϕ, is constant, and (*iv*) the heat exchanger costs are linearly related to heat exchanger conductances (UA_H and UA_L).

These assumptions allow the system performance and the economics equations to be solved analytically. The solution clearly shows the relation between the maximum power point and minimum life-cycle costs. If these assumptions were relaxed, a numerical solution could be used to determine the conditions of minimum life-cycle costs. The sensitivity of the relationship of maximum power and minimum life-cycle costs to these assumptions need to be explored.

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