An Explanation for Observed Compression Ratios in Internal Combustion Engines

S. A. Klein
Mechanical Engineering Department,
University of Wisconsin-Madison,
Madison, WI 53706

Comparisons of the compression ratios, efficiencies, and work of the ideal Otto and Diesel cycles are presented at conditions that yield maximum work per cycle. The compression ratios that maximize the work of the Diesel cycle are found always to be higher than those for the Otto cycle at the same operating conditions, although the thermal efficiencies are nearly identical. The compression ratios that maximize the work of the Otto and Diesel cycles compare well with the compression ratios employed in corresponding production engines.

Introduction

The air-standard Otto and Diesel cycles are used to describe the major processes occurring in internal combustion engines. The two cycles differ only in the energy input process and provide limiting cases for actual engine operation. Thermodynamic analysis shows the efficiency of both cycles to increase with increasing compression ratio. The implication of this result is that the higher the compression ratio, the better. However, spark-ignition internal engines typically operate at a compression ratio less than 10, whereas compression-ignition engines may operate at a compression ratio of 15 or more. An explanation for this difference, offered in most mechanical engineering thermodynamic textbooks, is that the compression ratio in spark-ignition engines is limited to about 10 to prevent pre-ignition of the fuel-air mixture. Compression-ignition engines do not have this limitation and can thereby operate at higher compression ratio, and consequently, higher efficiency. There is, however, another explanation for the observed compression ratios in internal combustion cycles, which is the objective of this paper.

Maximum Work of the Air-Standard Otto Cycle

Although the thermal efficiency of the Otto cycle increases with increasing compression ratio, there is a compression ratio that maximizes the net work per cycle, as seen in Fig. 1. An analytical expression for this optimum compression ratio can easily be derived for the air-standard Otto cycle assuming a constant specific heat ratio. Referring to the T-s diagram in Fig. 2(a), W_{net}, the net work of the Otto cycle per unit mass of gas in the cylinder, is

$$W_{\text{net}} = W_{1\text{-}2} + W_{3\text{-}4} = C_v(T_1 - T_3 - T_2 - T_4) \quad (1)$$

Assuming processes 1-2 and 3-4 to be isentropic

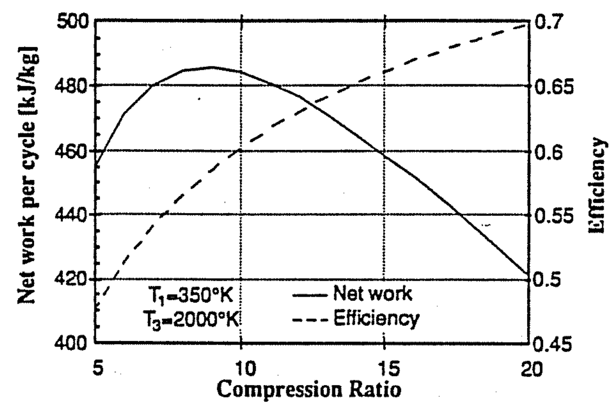

Fig. 1 Net work per cycle (per unit mass of gas) and efficiency versus compression ratio for the air-standard Otto cycle with $k = 1.4$

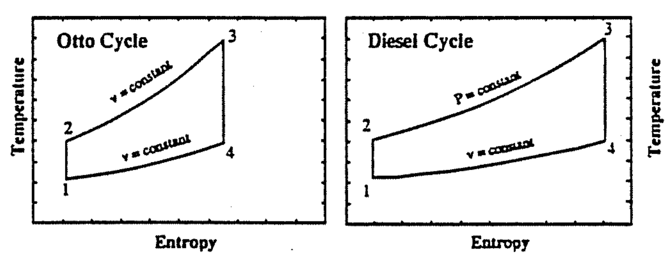

Fig. 2 Temperature-entropy representations of the air-standard (a) Otto and (b) Diesel cycles operating at maximum net work (drawn to scale) between the same temperature limits

$$T_2 = T_1 r^{k-1} \quad (2)$$

$$T_4 = T_3 / r^{k-1} \quad (3)$$

where r is the compression ratio $= v_1/v_2 = v_4/v_3$.

Eliminating T_2 and T_4 using Eqs. (2) and (3),

$$W_{\text{net}} = C_v[T_1(1 - r^{k-1}) + T_3(1 - 1/r^{k-1})] \quad (4)$$

Contributed by the Advanced Energy Systems Division for publication in the JOURNAL OF ENGINEERING FOR GAS TURBINES AND POWER. Manuscript received by the Advanced Energy Systems Division June 22, 1990; revision received September 24, 1990.

T_1, C_v, and k are constants in the following analysis. If T_3 is also assumed constant, the net work depends only on the compression ratio. Setting dW_{net}/dr to zero results in the following expression for r_{max}, the compression ratio that maximizes the net work:

$$r_{max} = (T_1/T_3)^{-1/(2k-2)} \tag{5}$$

The thermal efficiency, η, at maximum net work is

$$\eta_{max} = 1 - \frac{1}{r^{k-1}} = 1 - \sqrt{\frac{T_1}{T_3}} \tag{6}$$

Remarkably, the efficiency at maximum net work for the Otto cycle has the same form as the efficiency derived by Curzon and Alhborn (1975) for the heat transfer limited Carnot cycle as first observed by Leff (1987). It is also worth noting that, when the cycle operates at maximum net work,

$$T_2 = T_4 = \sqrt{T_1 T_3} \tag{7}$$

Maximum Work of the Air-Standard Diesel Cycle

An analysis similar to that presented for the Otto cycle can be made for the air-standard Diesel cycle represented in Fig. 2(*b*). Assuming T_1, C_v, and k to be constants, the net work of the Diesel cycle can be expressed as

$$W_{net} = W_{12} + W_{23} + W_{34} = C_v(T_1(1 - kr^{k-1}) + kT_3 - T_1^{1-k} T_3^k r^{k(1-k)}) \tag{8}$$

As with the Otto cycle, there is a compression ratio that maximizes the net work. Assuming T_3 to be constant, the maximum net work compression ratio for the Diesel cycle is

$$r_{max} = \left(\frac{T_1}{T_3}\right)^{\frac{k}{(1-k^2)}} \tag{9}$$

Again, at maximum net work, $T_2 = T_4$. The thermal efficiency of the Diesel cycle at maximum net work is

$$\eta_{max} = 1 - \frac{(r^{k-1} - 1)}{k(r^{(k^2-1)/k} - r^{k-1})} \tag{10}$$

Although Eq. (10) appears to be quite different from the corresponding efficiency for the Otto cycle (Eq. (6)), the efficiency of the Diesel cycle at maximum net work is very close, but not exactly equal, to $1 - \sqrt{T_1/T_3}$.

Comparison of the Air-Standard Otto and Diesel Cycles for Fixed Cycle Temperatures

If the maximum cycle temperature, T_3, is assumed to be independent of the compression ratio and k and C_v are known constants, the maximum work per cycle and the corresponding efficiency and compression ratio are functions only of the ratio T_3/T_1. Figure 3 presents the work, compression ratio, and thermal efficiency of the Diesel cycle divided by the corresponding quantities for the Otto cycle as a function of T_3/T_1 for $k = 1.40$. The compression ratio that maximizes the work of the Diesel cycle is always higher than that for the Otto cycle

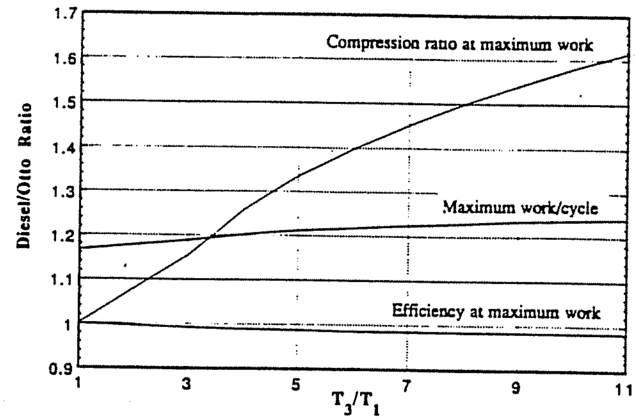

Fig. 3 Diesel/Otto cycle ratios of the maximum work/cycle, compression ratio, and thermal efficiency at maximum work as a function of the maximum/minimum cycle temperatures for $k = 1.40$

at the same value of T_3/T_1. The efficiencies of the two cycles, operated at maximum net work, are very similar, with the Otto cycle having a slightly higher efficiency. The Diesel cycle, however, enjoys significantly higher net work per cycle for the same value of T_3/T_1, as evident from the *T–s* diagrams in Fig. 2. Similar results have been presented by Leff (1987).

Heat Transfer Considerations

In a real engine, T_3 could not be expected to remain constant as the compression ratio is varied, as assumed in Fig. 1 and in the derivation of Eqs. (5) and (9). Increasing the compression ratio increases T_2, which results in higher values of T_3, assuming the energy release during combustion is unchanged. There are, however, additional relationships between T_3 and the compression ratio, which result from a consideration of heat transfer to the cylinder walls.

Internal combustion engines combust fuel with air in near-stoichiometric proportions. The combustion process is obviously not adiabatic since the maximum temperature in the cycle is far below the adiabatic combustion temperature. The actual heat transfer processes occurring within the cylinder are quite complicated. Regardless, the cylinder wall heat transfer will increase with increasing compression ratio since the difference between the mass-averaged gas and cylinder wall temperatures increase. For a given bore to stroke ratio, increasing the compression ratio also increases the peak gas density (which increases the gas to cylinder wall heat transfer coefficient) and slightly decreases the surface area available for heat transfer.

As a first approximation, the heat transfer to the cylinder walls is assumed to be a linear function of the difference between the average gas and cylinder wall temperatures during the energy release process 2–3. The wall temperature is further assumed to be constant as in Mozurkewich and Berry (1982) and Hoffman et al. (1985). In this case, Q_{23}, the energy transferred to the gas in the cylinder during combustion for the Otto cycle, can be represented

$$Q_{23} = C_v(T_3 - T_2) = \alpha - \beta(T_3 + T_2) \tag{11}$$

where α and β are constants. If the combustion process were

Nomenclature

C_v = constant-volume specific heat of working fluid (assumed as air)

k = ratio of constant pressure to constant volume specific heat (assumed to be 1.4)

r = compression ratio

Q_{23} = energy added to gas during process 2–3 as a result of combustion

T_i = temperature at state i

W = work per unit mass of gas in the cylinder per cycle

α = heat transfer constant used in Eqs. (11)–(14)

β = heat transfer constant used in Eqs. (11)–(14)

η = thermal efficiency defined as the ratio of the net work to energy input

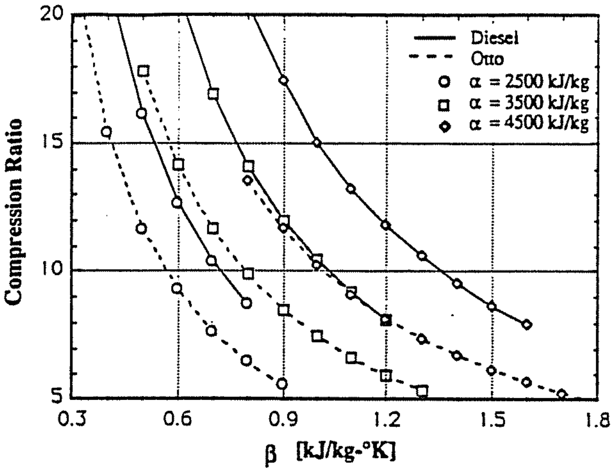

Fig. 4 Comparison of the compression ratios of the Diesel (solid lines) and Otto (dotted lines) at maximum net work for various values of α and β with T_1 = 350 K and k = 1.4

adiabatic ($\beta = 0$), the heat transfer to the gas would be equal to α, the heat release during combustion. Fuels used in internal combustion engines release about 50,000 kJ/kg. The air–fuel ratio for stoichiometric conditions is approximately 15. As a result, α should be approximately 3500 kJ/kg air. Values of β must range from about 0.3 to 1.8 in order to achieve maximum gas temperatures of 1500 to 3000 K, as observed in practice.

Eliminating T_3 from Eq. (4) using Eq. (11)

$$W_{\text{net}} = C_v\left(T_1(1-r^{k-1}) + \frac{(\alpha+(C_v-\beta)T_1 r^{k-1})(1-r^{1-k})}{C_v+\beta}\right) \tag{12}$$

Setting dW_{net}/dr to zero results in the compression ratio at maximum power for the Otto cycle with heat transfer considerations.

$$r_{\max} = \left(\frac{\alpha}{2T_1\beta}\right)^{1/(2k-2)} \tag{13}$$

It can again be shown that, at this compression ratio, T_2 is equal to T_4.

Considering heat transfer to the cylinder walls for the Diesel cycle with the same heat transfer model used for the Otto cycle,

$$Q_{23} = C_p(T_3 - T_2) = \alpha - \beta(T_3 + T_2) \tag{14}$$

A simple analytical expression for the compression ratio at maximum net work that does not involve T_3 is not apparent. However, the maximum net work compression ratio for the Diesel cycle can be determined numerically.

The compression ratios that result in maximum work for the Otto and Diesel cycles are plotted in Fig. 4 as a function of α and β with T_1 = 350 K and k = 1.4. The values of β for each value of α are such that the maximum gas temperature lies in the range of 1500 to 3000 K, as observed in actual engines. For given values of α and β, the compression ratio that maximizes the work of the Diesel cycle is always higher than that of the Otto cycle.

Conclusions

The air-standard Otto and Diesel cycles are very simplistic and cannot possibly represent the complex processes occurring in internal combustion engines. Furthermore, compromises in the performance of modern engines are made in order to meet emission standards. Nevertheless, the results observed in Figs. 3 and 4 show that the maximum work compression ratios for the Otto cycle are significantly lower than those for the Diesel cycle, as observed in practice. Further, these compression ratios compare reasonably well with the compression ratios employed in production engines.

Power output is surely a major design consideration in the determination of the compression ratio for internal combustion engines. The point of this paper is that the choice of compression ratio in internal combustion engines is dictated to some extent by the desire to maximize engine power output. Preignition is certainly a design consideration in spark-ignition engines. However, the characteristics of gasoline are an economic consequence of engine power–compression ratio tradeoffs. The textbook explanation of factors limiting the compression ratio of spark-ignition engines misses this point.

References

Curzon, F. L., and Alhborn, B., 1975, "Efficiency of a Carnot Engine at Maximum Power Output," *Am. J. Phys.*, Vol. 43, pp. 22–24.

Hoffman, K. H., Watowich, S. J., and Berry, R. S., 1985, "Optimal Paths for Thermodynamic Systems: The Ideal Diesel Cycle," *J. Appl. Phys.*, Vol. 58(6), pp. 2125–2134.

Leff, H. S., 1987, "Thermal Efficiency at Maximum Work Output: New Results for Old Heat Engines," *Am. J. Phys.*, Vol. 55, pp. 602–610.

Mozurkewich, M., and Berry, R. S., 1982, "Optimal Paths for Thermodynamic Systems: The Ideal Otto Cycle," *J. Appl. Phys.*, Vol. 53(1), pp. 34–42.