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THEORETICAL LIMITS FOR STORAGE OF ENERGY IN BUILDINGS

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Abstract—The mass incorporated in the structure of a building has long been used to store energy and offset heating requirements. The amount of mass to employ has been a subject of debate. Existing passive solar design manuals in the United States give recommendations for the amount of mass to employ in different building types, and establish optimum levels for different designs. However, an evaluation of the maximum benefits of building mass on energy consumption has not been performed.

In this paper, two limits for the effect of building mass on energy consumption are formulated. The limit for maximum energy consumption is based on there being no mass available in the building for energy storage. The limit for minimum energy consumption is based on there being sufficient mass present to allow excess energy gains to be used at any time during the time period of interest. These limits allow the maximum economic benefit of building mass to be determined. Examples of the application of these limits are presented.

1. INTRODUCTION

The role of mass in the storage of energy in buildings has long been a subject of interest. For buildings with large glazing areas, there if often energy available from solar and internal gains during the daytime that is greater than the daytime heating requirements. This energy may be stored in the building structure and furnishings, and released at night to offset the building heat loss. In a similar manner, building mass may allow the structure to remain cool during daytime in summer and reduce air conditioning requirements.

The *Passive Solar Heating Design Manual*[1] gives guidelines for the level of structural mass to incorporate in buildings. These levels were chosen to prevent excessive temperature swings in the occupied space. There are not, however, established studies that definitively establish the maximum effect of mass on building energy consumption. In this paper, the maximum benefit of building mass for the storage of thermal energy is determined. There are two limits on building energy consumption which are derived analytically. The thermal performance of buildings both with typical and with recommended amounts of building mass are evaluated and compared to these limits. These results are helpful in the assessment of the further research into structural energy storage.

2. THEORETICAL LIMITS FOR BUILDING ENERGY USE

The theoretical limits for energy use in buildings can be developed from the heating energy requirements of a building zone (a portion of a building that is controlled by a single thermostat). The limits will be developed for both heating and cooling energy. These limits are based on the following two concepts:

- Maximum Energy Use. All instantaneous energy gains are useful in offsetting instantaneous losses *only* at the time that the gains are available.
- Minimum Energy Use. All instantaneous energy gains in excess of the instantaneous losses can be stored and then released to offset those losses occurring at *any* time during the period (day, month, or year).

The limit of maximum energy use assumes that gains are used only when there are building losses; that is, when the ambient temperature is less than that of the building space. These gains directly reduce the auxiliary energy required to maintain the building zone at the thermostat temperature. During time periods when the gains exceed the losses, the excess of the gains over the losses is not useful and is "dumped" to the environment. This is the situation for a building with no mass; that is, with zero thermal storage capacity.

The limit of minimum energy use assumes gains can be used at any time within the time period of interest. If the losses are greater than the gains, the gains offset the auxiliary energy requirement. If the gains exceed the losses, the excess is stored for use at any time during the time period. If the time period is a month, for example, the excess gains present during any time of the month can be used to offset the losses that occur at any other time in the month. This situation would occur in a building with a large enough mass to store energy over a time period such as a month, but not large enough for storage to be carried over from month to month.*

For both of these limits, the building temperature at all times will be the same as the thermostat setting. For the maximum energy (zero thermal capacity) building, all gains in excess of the building load are

*In reality, excess gains that occur near the end of a month cannot be used to offset auxiliary energy requirements that occur near the beginning of a month. However, this is essentially equivalent to assuming excess gains near the end of one month are carried into the next month.

dumped immediately to the environment, and the building temperature remains constant. For the minimum energy (large thermal capacity) building, all excess energy is stored, but the capacity is so large that the building temperature does not change. For actual buildings which have finite capacity, excess energy that is stored in the building mass would raise the interior temperature. Depending on the temperature rise, some of the excess energy may have to be dumped to maintain comfort in the interior.

The heating requirements for a building generally decrease with an increase in the number of the days of storage. For zero storage capacity, the heating requirements are maximum. "One-day" storage allows excess daytime energy gains to be utilized that night and reduces purchased heating requirements. Similarly, "one-week" storage allows excess gains available during the week to be used during that week, and reduces the heating requirement over that for one-day storage. Further reductions in purchased heating requirements occur as storage is increased to "monthly" storage. The greatest savings occur for "annual" storage in which summer gains are used to offset winter loads. In the development of the relations presented here, the time period is general. However, for the numerical results, a time period corresponding to "monthly" storage is used. This is in the range of storage levels proposed for many buildings, and provides a reasonable measure of the benefits of storage.

Relations for the limits of energy consumption are based on the instantaneous energy requirements for a building. At any instant of time, the rate at which energy must be supplied to maintain the zone of a building at the room temperature is given by:

$$a = [UA(T_r - T_a) - s - g]^+ \qquad (1)$$

where

a = The rate at which energy must be supplied to the space. This energy can be supplied from a combination of auxiliary energy or heat released from any stores in the building
UA = Building zone overall conductance-area product
T_r = Building thermostat (room) temperature
T_a = Ambient temperature
s = Instantaneous solar gains
g = Instantaneous internal gains

The superscript + denotes that only positive values of the term in parentheses contribute to the heating energy use. If the gains exceed the losses, the term in parentheses is negative and there is the possibility to store energy in the building mass.

Equation (1) implies that there is no thermal storage in the external walls of the structure so that the instantaneous heat loss is directly proportional to the difference between the interior and the ambient temperatures. This is consistent with good design since the most efficient use and control of storage occurs if the store is insulated from the environment to provide maximum use of the storage capacity in offsetting losses.

The balance temperature concept can be employed to simply eqn (1). The balance temperature is the ambient temperature for which the gains and losses balance, and there is no heating energy requirement. Solving eqn (1) for the balance temperature yields

$$T_b = T_r - (s + g)/UA \qquad (2)$$

The heating energy requirement, eqn (1), can then be written as

$$a = [UA(T_b - T_a)]^+ \qquad (3)$$

The *maximum* energy consumption occurs when there is no storage capacitance in the zone. All of the energy that is supplied for heating is from an auxiliary energy source. Equation (3) then becomes

$$a_{H,\max} = [UA(T_b - T_a)]^+ \qquad (4)$$

The maximum energy consumption is obtained by integrating eqn (4).

$$A_{H,\max} = \int_0^N [UA(T_b - T_a)]^+ dt \qquad (5)$$

Where N is the length of the time period in units consistent with the units of UA. In order to evaluate the integral, the variations of balance and ambient temperature over the month must be known.

The *minimum* energy consumption occurs when all of the excess heat gains are stored and used to offset heat losses in the chosen time period. The instantaneous rate at which energy may be stored, c, is given by

$$c = [s + g - UA(T_r - T_a)]^+ \qquad (6)$$

Using the balance temperature, the rate of energy storage can be written as

$$c = [UA(T_a - T_b)]^+ \qquad (7)$$

The total energy available from storage is obtained by integrating eqn (7).

$$C = \int_0^N [UA(T_b - T_a)]^+ dt \qquad (8)$$

The minimum auxiliary energy requirement for the time period is the difference between the maximum auxiliary energy requirement as given by eqn (5) and the energy available from storage as given by eqn (8), or

$$A_{H,\min} = A_{H,\max} - C \qquad (9)$$

or, in terms of integrals

$$A_{H,\min} = \int_0^N UA(T_b - T_a)^+ dt - \int_0^N UA(T_a - T_b)^+ dt \quad (10)$$

The two integrals may be combined and the conductance-area product factored out. The auxiliary energy requirement can be written as

$$A_{H,\min} = \int_0^N UA[(T_b - T_a)^+ - (T_a - T_b)^+]dt \quad (11)$$

The subtraction of the positive values of the term $(T_a - T_b)^+$ can be written as the addition of the negative values of the term $(T_b - T_a)^-$. Thus the auxiliary energy requirement becomes

$$A_{H,\min} = \int_0^N UA[(T_b - T_a)^+ + (T_a - T_b)^-]dt \quad (12)$$

The addition of the positive and negative values of the term $(T_b - T_a)$ equals the sum of all of the values. However, only positive values of the integral contribute to an auxiliary energy requirement. The minimum auxiliary energy requirement becomes

$$A_{H,\min} = \left[\int_0^N UA(T_b - T_a)dt \right]^+ \quad (13)$$

The two limits may be simplified to yield insight into the meaning of the limits. For a constant value of the building conductance-area product UA over the time period, the expression for the maximum energy consumption, eqn (5) can be written as

$$A_{H,\max} = UA \int_0^N (T_b - T_a)^+ dt \quad (14)$$

The integral is the integration over the time period of the positive difference between the instantaneous balance and ambient temperatures and is termed the "degree-hours."* Using the term degree-hours, eqn (14) can be written as

$$A_{H,\max} = UA\, DH_H \quad (15)$$

where DH_H are the heating degree-hours for the time period.

*To be dimensionally correct, the time unit of UA must be hours. The evaluation of this term requires that the balance and ambient temperature variation with time be known. The integral in eqn (14) is often evaluated approximately using average values over a period such as a full day. When average temperatures for a day are used and when the time period is a month, the result is what is known as degree-days for the month.

The expression for the minimum energy consumption, eqn (13), for the condition of a constant conductance-area product becomes

$$A_{H,\min} = UA\left[\int_0^N (T_b - T_a)dt \right]^+ \quad (16)$$

The integral can be directly evaluated by integrating both balance and ambient temperatures over the time period directly. This yields the difference between the average temperature times the number of hours in the time period. The minimum energy consumption is then given by

$$A_{H,\min} = UA(\bar{T}_b - \bar{T}_a)^+ N \quad (17)$$

The maximum possible reduction in energy use over the time period due to storage is then the difference between the two limits as given by

$$\Delta A_{H,\text{storage}} = A_{H,\max} - A_{H,\min}$$

or

$$\Delta A_{H,\text{storage}} = UA[DH_H - (\bar{T}_b - \bar{T}_a)^+ N] \quad (18)$$

The annual difference is obtained by summing eqn (18) over all time periods in the year.

Equations (15), (17), and (18) are written for heating energy; a parallel development holds for the limits on sensible cooling energy. The maximum and minimum sensible cooling energy requirements are given by

Maximum sensible cooling energy

$$A_{C,\max} = UA\, DH_c \quad (19)$$

Minimum sensible cooling energy

$$A_{C,\min} = UA(\bar{T}_a - \bar{T}_b)^+ N \quad (20)$$

where the cooling degree-hours are based on the difference between the ambient and balance temperatures. The maximum energy savings for sensible cooling is

$$\Delta A_{C,\text{storage}} = UA[DH_c - (\bar{T}_a - \bar{T}_b)^+ N] \quad (21)$$

3. CONDITIONS UNDER WHICH STORAGE IS IMPORTANT

The simplified form for maximum savings in auxiliary heating energy, eqn (18), allows a ready determination of the conditions under which storage is important. The energy savings will be significant if

$$DH_H \gg (\bar{T}_b - \bar{T}_a)^+ N \quad (22)$$

Using weather statistics, it can be shown that the monthly degree-hours are significantly greater than the difference in average temperatures times the number of hours only when the average balance temperature is within about 9°C (16°F) of the average ambient temperature (2,3). When these average temperatures differ by more than this amount, the degree-hours are essentially equal to the difference in average temperature times the number of hours.

Thus, storage will significantly reduce energy consumption when the average balance temperature is close to the average ambient temperature. This means that the solar gains, internal gains, building loss coefficient, and mass need to be considered in combination. If the building design and location is such that the average balance temperature is much higher than the average ambient temperature, most of the gains are used, there is little excess energy, and building mass will not be important in reducing energy use. If on the other hand, the average balance temperature is close to the average ambient temperature, energy storage can significantly reduce energy consumption.

4. EXAMPLES OF APPLICATION OF LIMITS

A set of houses based on the design house from the Passive Solar Heating Design Manual[1] is used to illustrate the limits of thermal storage and the effect of storage on energy use. The design chosen is example 16–2, with a direct gain system similar to DGA1. It is a well-constructed house with 146 m^2 (1575 ft^2) of floor area and 25 m^2 (270 ft^2) of south-facing double-glazing. The overall house loss coefficient, which includes the direct gain system, is 167 W/C (317 Btu/hr-°F). The load to collector ratio (LCR) is 4.6 W/C-m^2 (19.4 Btu/°F-day-ft^2). The storage capacity associated with the direct gain system is 612 kJ/m^2C (30 Btu/ft^2-°F) per unit of projected area of the south-facing glazing. In addition, it is assumed that the remainder of the house has a storage capacity representative of conventional construction. This is approximately a capacity of 81.6 kJ/°C (4 Btu/°F) per square meter (ft^2) of floor area (3,4).

The passive design manuals did not allow a ready determination of the limits of energy use and of the heating requirements for typical residential construction. The program F-LOAD[6] was used for all calculations for consistency. The results of the F-LOAD calculations agreed closely with those of the *Passive Solar Heating Design Manual*[1] for the residential passive structures used as examples. The limits of energy use are a fundamental part of the F-LOAD program.

The maximum and minimum daily average heating requirements are shown as a function of the daily average temperature for each month of the year in Fig. 1 for Madison, Wisconsin. The monthly values of energy consumption for the example house with the recommended amount of storage capacity for the direct gain system are also shown.

The minimum energy limit, which is the condition

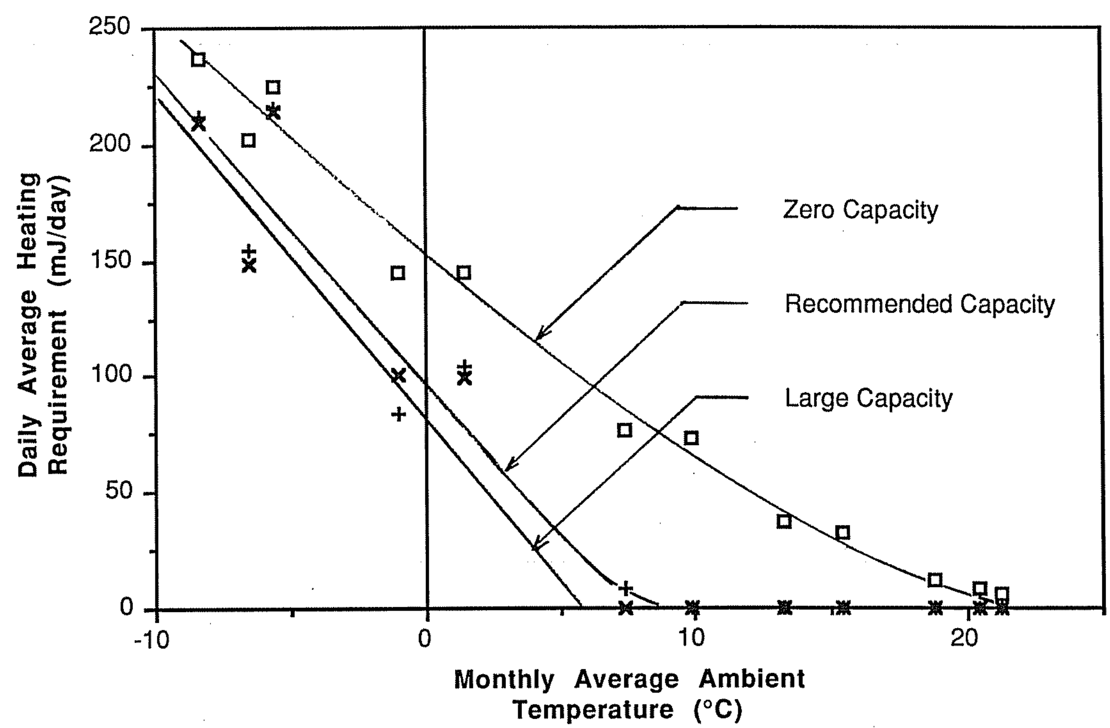

Fig. 1. Daily average heating requirements as a function of monthly average ambient temperatures for a passive house in Madison, Wisconsin.

for large storage capacity, is essentially a linear function of monthly average temperature. This is consistent with eqn (17), which shows that the energy use should be proportional to the difference between monthly average balance and ambient temperatures with a slope equal to the overall conductance-area product. The use of actual weather data produces different solar gains for each month, and a corresponding different balance temperature, and thus all of the points do not lie exactly on the same line. The average balance temperature is given by the intersection of the line with the abcissa. For this building in Madison, the average balance temperature is about 6°C.

The maximum energy limit is also shown. For winter months, the heating energy requirement is essentially linear with ambient temperature. For warmer months, the average balance temperature is greater than the monthly average temperature, but there are still some heating degree-days. Thus, even in summer, there is some heating requirement for the zero capacitance house. The two limits converge at low winter temperatures since the difference between the average balance and ambient temperature is large.

As shown in Fig. 1, the difference between the maximum and minimum monthly energy consumption is small during the months when the heating requirements are large, and large when the heating requirements are small. The effect of storage is most pronounced when there are relatively small heating requirements.

The annual auxiliary energy limits for this house are given in Table 1, and are useful in determining the maximum economic value of storage. The difference in annual energy consumption between the two limits is 14.5 GJ. At a representative delivered energy cost of $15/GJ the maximum amount that storage is worth in reducing heating requirements is about $200 annually.

A moderate amount of thermal mass reduces the energy requirement over that for the zero capacity, maximum energy consumption limit. The performance of the example house with the average storage capacity as recommended by the Passive Design Handbook is labeled "recommended capacity" in Fig. 1. This recommended construction corresponds to a distributed total capacity level in the building of about 185 kJ/C per m^2 of floor area. This level is about twice the thermal capacity value of ordinary homes.

The annual heating requirement for the passive house with the recommended amount of storage is given in Table 1. The annual energy use with recommended storage is reduced by 13.4 GJ, or about 35%, over that of the zero capacity building. The energy consumption of the passive house with a storage capacity equal to that of a typical house is also given. It is seen that the storage capacity present in the typical walls, floor and furnishings also reduces the energy consumption significantly over the maximum energy limit. The additional storage capacity to bring the house to the recommended range saves an additional 2.2 GJ annually; the added storage associated with the direct gain system is worth about $30 annually in Madison.

To illustrate the importance of the interplay between gains and storage, the reference design building was modified to increase the loss coefficient. The thermal resistance of all of the wall elements were halved, and the limits were determined. This produces a building that is somewhat typical of conventional residences in Madison. As shown in Fig. 2, the difference between the minimum and maximum energy consumption limits is about the same as for the passive house. It is only during the warm spring and fall months that capacity has a significant effect. During winter, essentially all of the gains are used to offset the heating loads and essentially none are stored for later use. As shown in Table 1, the difference between the two limits is 12.0 GJ, or 17% of the total

Table 1. Annual energy consumption values GJ/year

	Passive house Madison	Conventional house Madison	Passive house Albuquerque	Extreme passive house Columbia
Minimum energy limit	23.8	59.0	0	8.8
Recommended capacity	24.9	61.1	2.2	10.7
Typical capacity	27.1	64.4	11.2	13.9
Maximum energy limit	38.3	71.0	23.4	25.1

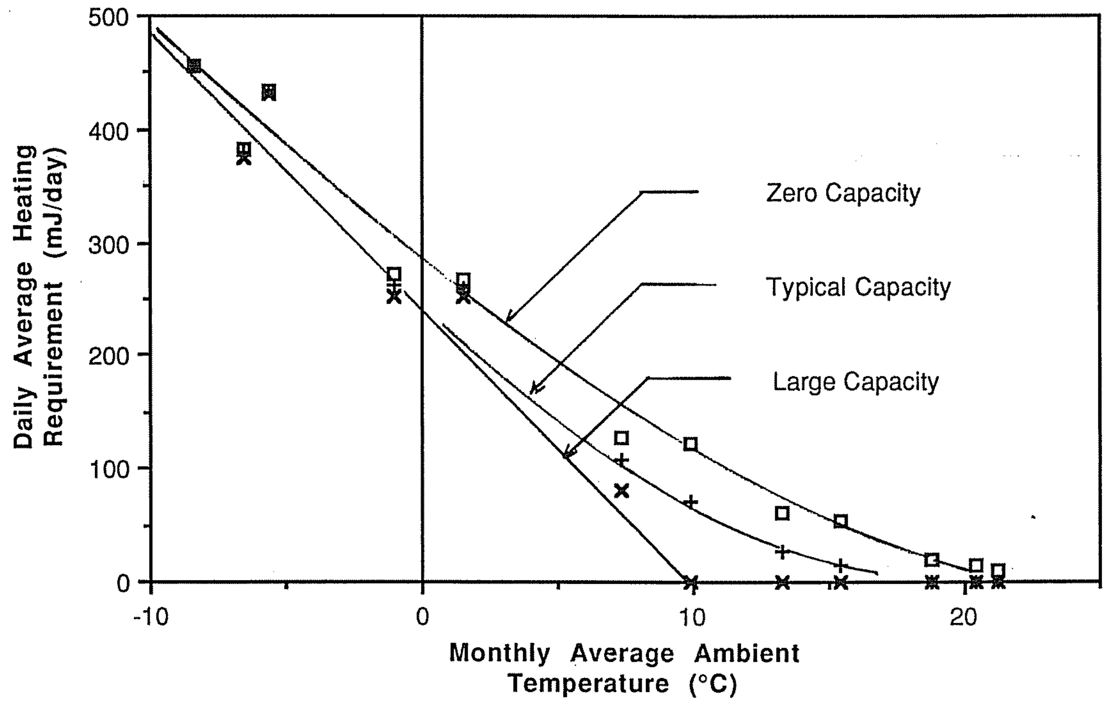

Fig. 2. Daily average heating requirements as a function of monthly average ambient temperatures for a conventional house in Madison, Wisconsin.

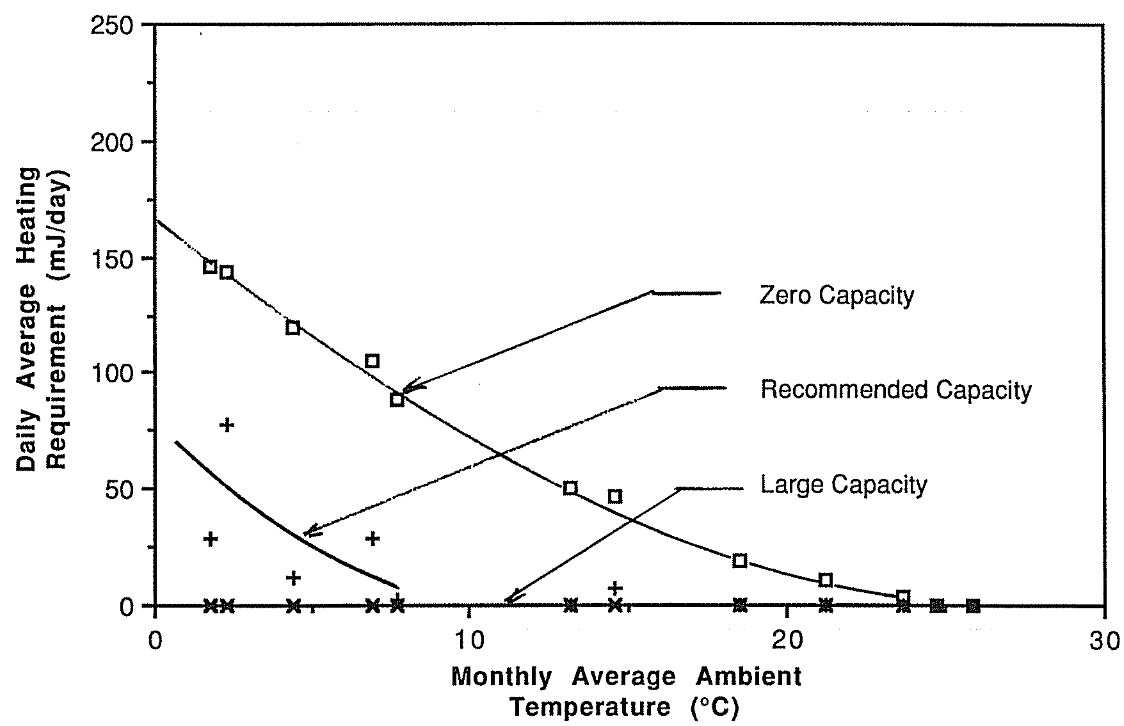

Fig. 3. Daily average heating requirements as a function of monthly average ambient temperatures for a passive house in Albuquerque, New Mexico.

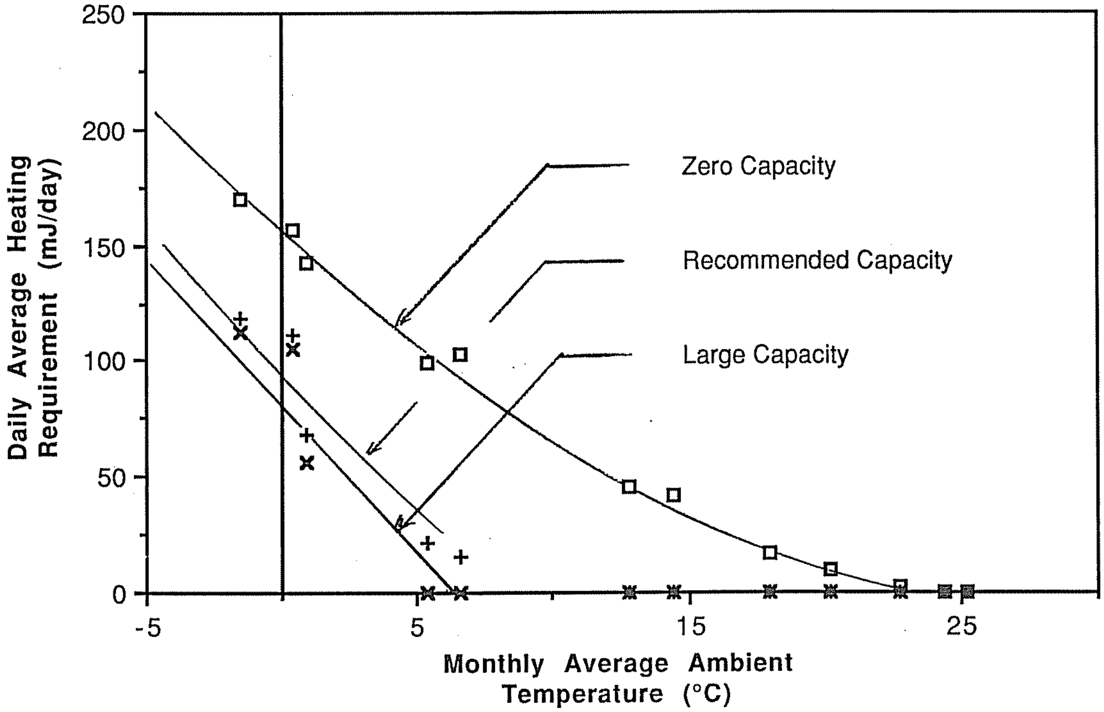

Fig. 4. Daily average heating requirements as a function of monthly average ambient temperatures for a passive house in Columbia, Missouri.

heating energy. There is a maximum potential benefit of storage of about $180. The benefit of increasing storage over that typically found in residences to a higher, recommended value is again small (3.3 GJ) for this situation. The difference in energy consumption between the house with the typical storage and the zero storage limit is 6.6 GJ, which amounts to about $100 annually.

Albuquerque, New Mexico, is a better location for passive heating, and, potentially, there is more benefit to storage. The energy consumption for the limits and for the building with recommended storage are shown in Fig. 3. A large thermal capacity building has no heating requirement since the balance temperature is about −3°C, which is below any of the average monthly temperatures.

The difference in annual energy consumption between the zero and large capacity limits as given in Table 1 is 23.4 GJ. The difference between the building with the recommended value of storage and the maximum limit is 21.2 GJ. Thus, in Albuquerque the annual economic value of the storage in the residence is worth about $320 per year, which is about 50% more than in Madison. The increase in storage level from that found in typical construction to that recommended for passive buildings is also significant. As shown in Table 1, the capacity associated with typical construction reduces energy consumption to about one-half of the maximum limit, and the additional storage as recommended reduces the consumption to nearly the maximum limit. The value of the added recommended storage (9.0 GJ) is worth about $135 annually.

Madison and Albuquerque are extreme climates, and a more moderate location was chosen to illustrate the effects of storage. The heating requirements for Columbia, Missouri, are shown in Figure 4 and presented in Table 1. In this climate, there is a significant difference between the two limits, and storage is again beneficial. The presence of typical storage reduces the energy consumption 11.2 GJ annually over the maximum limit, and the increase to the recommended value reduces the energy consumption another 3.2 GJ. Although the levels of heating energy are less than those for Madison, the benefits of storage are quite similar.

These economic considerations are only for the impact of storage on energy consumption. Storage of thermal energy in the structure mass is essential to maintaining livable conditions inside the building. For the passive design considered, storage prevents the interior temperature rise due to solar gains from reaching unacceptable levels. Comfort levels are maintained only through adequate thermal storage.

5. CONCLUSIONS

The following conclusions can be drawn from this study:

1. It is possible to analytically derive expressions for the limits on building energy consumption. These limits give the maximum possible thermal benefit of including storage either in the building mass or as a separate component. These limits allow a ready determination of the economic benefit of storage.

2. For storage to have a significant effect on energy consumption, it is important that the average balance temperature be close to that of the ambient during the heating season. If the difference between these two temperatures is greater than about 9°C (15°F), then storage can have no significant effect.
3. For the locations and houses studied, the difference between the minimum and maximum limits is 12 to 24 GJ annually. This indicates the order of magnitude of maximum heating energy savings due to storage that could be achieved for typical residential dwellings.
4. For the buildings studied, the presence of the storage capacity found in typical residential construction reduces energy consumption from the maximum energy limit by 6 to 12 GJ annually. Added storage to bring the building capacity to the recommended level for passive houses reduces consumption another 2 to 10 GJ annually. Thus, the value of the capacity found in the construction materials of these example homes is worth about \$100 to \$200 annually in heating cost reduction. The value of added storage to bring the buildings to the levels recommended for passive houses is \$30 to \$150 annually.

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