S. A. Klein

W. A. Beckman

Solar Energy Laboratory,
University of Wisconsin-Madison,
Madison, Wis. 53706

Review of Solar Radiation Utilizability

1 Introduction

Solar radiation utilizability is defined as the fraction of the solar radiation incident on a surface that exceeds a specified threshold or critical level. Utilizability is a statistic of solar radiation data that is analogous to degree-days, a statistic of ambient temperature data.

Utilizability depends on the distribution of solar radiation. This dependence is illustrated in Fig. 1 in which two three-day sequences of solar radiation appear. Sequence A consists of three average days while sequence B consists of a clear, an overcast, and an average day. The sequences were chosen to have the same total solar radiation. However, the daily utilizability for critical level $I_{c,1}$ (i.e., the ratio of the shaded area to the total under the curves) is greater for sequence B than for sequence A. The effect of radiation distribution becomes more pronounced at higher critical levels. For example, the utilizability corresponding to critical level $I_{c,2}$ is zero for sequence A but significantly greater than zero for sequence B.

The utilizability concept was originally developed to calculate the thermal performance of flat-plate solar collectors. In recent years, the utilizability concept has also been applied to systems with concentrating collectors, as well as to passive and photovoltaic systems. For flat-plate and concentrating collectors, the curves of Fig. 1 can be interpreted as the radiation absorbed on the collector aperture. The area below the critical level represents the absorbed radiation that is necessary to overcome collector losses. The radiation above this critical level is then the "utilizable" portion of the absorbed radiation. For a passively heated direct gain house, the curves of Fig. 1 represent the energy transmitted through the window and the critical level represents the building losses. The transmitted solar radiation below the critical level can be used to offset these building losses. The energy above the critical level is in excess of the load. It is potentially "unutilizable" and must either be dumped by ventilation or drawing a shade, or stored for later use by increasing the temperature of the building mass. For a photovoltaic power system, Fig. 1 represents the output of the cells and the critical level represents the electrical load on the system. The energy below the critical level can be used to meet the load. The energy above the critical level must either be stored in a battery, dissipated in a resistor, or sold to a utility.

In these three examples, the critical level was considered constant. This is not the usual operating condition for these solar systems. In fact, in the more usual case, the critical level is not constant. In this paper, this apparent limitation will be discussed for each of these systems, along with a review of the utilizability correlations that have been developed.

2 Development

The utilizability concept was originally developed by Whillier [1] to simplify the calculations needed in the evaluation of flat-plate solar collector performance. Though not limited to this application, a review of his work provides both a historical perspective and a clear application of utilizability.

The concept of utilizability follows directly from the Hottel-Whillier [2, 3] equation which relates the rate of useful energy collection, q_u, to the design parameters of the collector and its operating conditions.

$$q_u = A[F_R(\tau\alpha)I_T - F_R U_L(T_i - T_a)]^+ \tag{1}$$

The collector parameters, $F_R(\tau\alpha)$ and $F_R U_L$, can be determined from theory [4] or from standard collector tests [5]. The superscript + is used to indicate that only positive values of the quantity in brackets are considered. In practice, a controller would be employed to prevent fluid circulation whenever the solar radiation is not sufficient to overcome thermal losses from the collector.

Equation (1) can be rearranged into

$$q_u = AF_R(\tau\alpha)[I_T - I_c]^+ \tag{2}$$

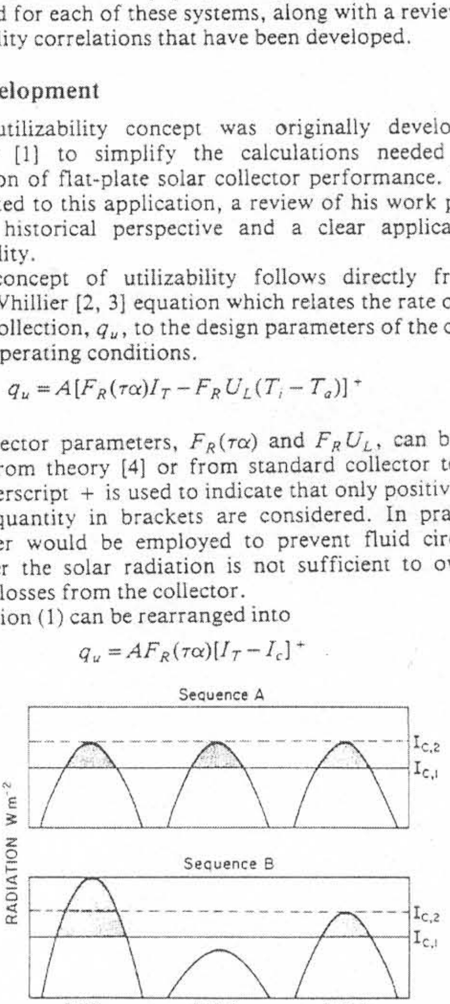

Fig. 1 Effect of radiation distribution on daily utilizability (from reference [16])

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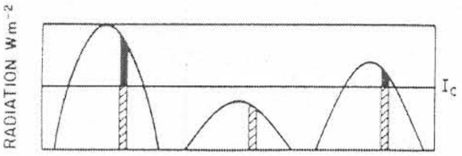

Fig. 2 Utilizability on an hourly basis

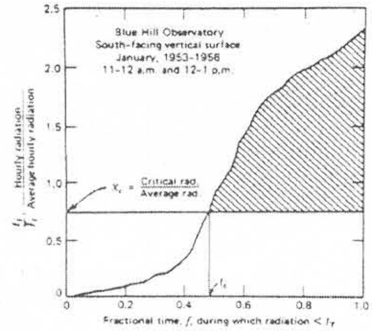

Fig. 3 Cumulative frequency for hourly radiation on a vertical south-facing surface in Blue Hill, Mass. (from reference [4], adapted from reference [6])

where I_c is the critical radiation level defined as

$$I_c = \frac{F_R U_L}{F_R(\tau\alpha)}(T_i - T_a) \tag{3}$$

In this application, I_c is the radiation level needed to maintain the collector plate at the fluid inlet temperature.

Equation (2) can be used to calculate $\bar{Q}_i$, the long-term average energy collection for a specified period, e.g., 9-10 AM in January. This calculation is made by approximating I_T to be the average solar radiation over time period Δt. (Δt is ordinarily 1 hour due to the lack of long-term measurements of solar radiation at shorter time intervals.) In this case,

$$\bar{Q}_i = AF_R(\tau\alpha)\sum^N [I_T - I_c]^+ \Delta t / \sum^N \Delta t \tag{4}$$

where N represents a large number of radiation observations sufficient to represent long-term average conditions. On an

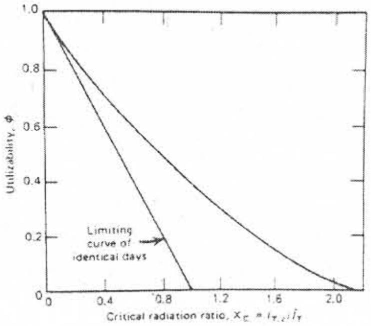

Fig. 4 Utilizability curve derived by numerical integration of Fig. 3 (from reference [4], adapted from reference [6])

hourly basis (i.e., $\Delta t = 1$ hour), utilizability is defined as the fraction of the long-term hourly radiation, $\bar{I}_T$, which is above the critical level and can be written as

$$\phi = \frac{\sum^N [I_T - I_c]^+}{\sum^N I_T} = \frac{\sum^N [I_T - I_c]^+}{N\bar{I}_T} \tag{5}$$

ϕ is the ratio of the shaded area to the sum of the shaded and cross-hatched areas for the radiation sequence in Fig. 2. ϕ is a radiation statistic, dependent on the critical level, time of day, location, and surface orientation. In terms of ϕ, equation (4) becomes

$$\bar{Q}_i = AF_R(\tau\alpha)\bar{I}_T\phi \tag{6}$$

If sufficient radiation data are available, ϕ can be determined as a function of I_c for each hour, month, and location directly from equation (5). To simplify the calculations, Whillier prepared cumulative frequency distributions of hourly solar radiation in dimensionless form as shown in Fig. 3 for a vertical surface in Blue Hill, Mass. for the hour pairs 11-12 AM and 12-1 PM. He defined the dimensionless critical level X_c, as the ratio of the critical level to the average hourly radiation. ϕ, the fraction of the radiation above the critical level, is represented by the shaded area in Fig. 3 (the total area under the curve in unity). Whillier determined ϕ by numerical integration and showed that plots of ϕ versus $I_c/\bar{I}_T$ (called ϕ-curves) could be produced for a given month,

Nomenclature

A = collector or array area
D = energy dumped from building to prevent overheating
$\bar{D}_i$ = long-term average hourly energy dissipation from a photovoltaic system
$D_{\max}$ = upper limit for energy dumping
E = auxiliary energy required by a building
$\bar{E}_i$ = long-term monthly average photovoltiac array output
$E_{\max}$ = maximum auxiliary energy required by a building
$E_{\min}$ = minimum auxiliary energy required by a building
F_R = collector heat removal factor
$\bar{H}_{COLL}$ = monthly average daily radiation on collector during collector operating period
$\bar{H}_T$ = monthly average daily radiation per unit area on a tilted plane
I = solar radiation per unit area on a horizontal surface
I_B = beam radiation component per unit aperture area
I_c = critical level
I_d = diffuse component of I
I_{DR} = diffuse and ground-reflected radiation per unit aperture area
I_R = radiation per unit area incident on the receiver of a concentrating solar collector
I_T = instantaneous (or hourly average) solar radiation per unit area on a tilted plane
$I_{T,MAX}$ = maximum possible value of I_T
$\bar{I}_R$ = long-term average value of I_R
$\bar{I}_T$ = long-term average of I_T
$\bar{K}$ = monthly clearness index
$\bar{K}'$ = modified $\bar{K}$ to account for collector tilt
L = monthly building energy requirement
$\bar{L}_i$ = long-term average hourly electric load

location, collector orientation, and time of day. A ϕ-curve for vertical south-facing surfaces in Blue Hill, Mass. is shown in Fig. 4. Whillier also showed that, over a long term, solar radiation is ordinarily symmetric about solar noon so that it is only necessary to determine ϕ for each hourly interval from solar noon, rather than for each hour of the day. This simplification does not hold in locations which, for example, typically have morning fog, or large mountains to the east or west.

Whillier's method produced ϕ-curves which were independent of hour, but dependent on month, location, and orientation. Liu and Jordan [6, 7] were able to generalize Whillier's ϕ-curve method using the results of their statistical analyses of solar radiation data. They introduced the clearness index, $\bar{K}$, the ratio of the monthly average daily radiation on a horizontal surface to the maximum possible (i.e., the extraterrestrial) radiation on the surface.[1] Liu and Jordan showed that the cumulative distribution of daily radiation on a horizontal surface depends primarily on the clearness index. They also showed that the ratio of the average hourly to daily horizontal radiation depends only on the hours from solar noon and the day length. With these results, Liu and Jordan constructed ϕ-curves for daily clearness indices of 0.3, 0.4, 0.5, 0.6, and 0.7 which were independent of month and location. A generalized ϕ-curve for $\bar{K}=0.3$ is shown in Fig. 5. They also developed a method to (approximately) include the effect of tilt into the ϕ-curves for surfaces facing directly toward the equator by providing curves for a range of values of $\bar{R}_b$, the ratio of daily beam radiation on the tilted surface to that on a horizontal surface.

3 Utilizability Correlations

Utilizability is the ratio of solar energy quantities, and as a result, the time period over which it is defined must be specified. Whillier and Liu and Jordan correlated what will be referred to here as hourly utilizability, the long-term average fraction of the solar radiation incident on a surface for a month[2] during an hourly period. Correlations for daily and annual utilizability have also been developed. Daily utilizability, designated $\bar{\phi}$, is defined for the period between sunrise and sunset each month, whereas annual utilizability applies to radiation for the entire year.

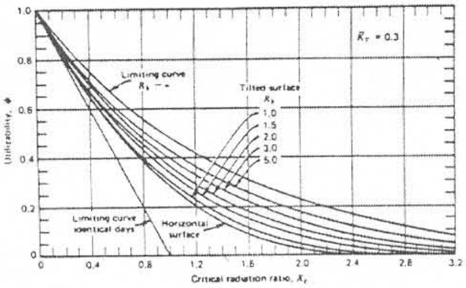

Fig. 5 Generalized ϕ-curve for $\bar{K}=0.3$ (from reference [4], adapted from reference [6])

Hourly Correlations. Although the generalizations made by Liu and Jordan offer significant advantages over the ϕ-curve method proposed by Whillier, they still have several disadvantages. Use of the generalized ϕ-curve method requires significant computational effort. The calculations are difficult to automate in that analytical expressions for the ϕ-curves are not available. In addition, the method used to incorporate the effect of surface orientation was approximate and limited to surfaces facing directly toward the equator.

The defining equation for ϕ can be written in terms of the probability distribution of insolation levels, $P(I_T)$.

$$\phi = \frac{\int_{I_c}^{I_{T,MAX}} (I_T - I_c) P(I_T) dI_T}{\int_0^{I_{T,MAX}} I_T P(I_T) dI_T} \tag{7}$$

This definition of ϕ is consistent with that given in equation (5) if N is sufficiently large to accurately represent the cumulative distribution of radiation.

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[1] Liu and Jordan used the symbol $\bar{K}_T$ and called it the monthly average "cloudiness index." Since this ratio approaches 1 with increasing clearness, the term "clearness index" seems more appropriate. The subscript T was meant to indicate total radiation (i.e., beam, diffuse, and ground-reflected). The subscript T is currently used in solar energy literature to indicate "tilted" surface. Since the clearness index is defined in terms of horizontal radiation, the subscript has been deleted.

[2] A month was judged to be a period short enough such that the effect of earth-sun geometry on solar radiation quantities could be approximated using a single average day.

Nomenclature (cont.)

N = number of radiation observations sufficient to represent long-term average conditions
$P(I_T)$ = probability distribution of I_T
q_u = rate of useful energy collection
$\bar{Q}$ = monthly average daily energy collection
$\bar{Q}_i$ = long-term average energy collection for hourly period i
Q_{LS} = monthly energy loss from storage tank
Q_s = monthly energy removed from storage
Q_u = monthly useful energy collection
$\bar{R}_b$ = monthly average ratio of beam radiation on tilted surface to that on a horizontal surface
S = monthly solar energy transmitted through glazings
$\bar{t}$ = monthly average collector operating time
T_a = ambient temperature
T_b = building balance temperature
T_i = collector fluid inlet temperature
UA = building overall energy loss coefficient
U_L = collector overall energy loss coefficient
β = surface tilt from horizontal
Δt = time interval of radiation data, generally 1 hour
ΔU = change in internal energy of the storage tank
$\bar{\eta}$ = average photovoltaic array efficiency (including the efficiency of power conditioning equipment)
$\bar{\phi}$ = daily utilizability
ϕ = hourly utilizability
ϕ_{CPR} = utilizability as defined by Collares-Pereira and Rabl [19]
ρ = ground reflectance
$(\tau\alpha)$ = effective transmittance-absorptance product
$\overline{(\tau\alpha)}$ = monthly-average value of $(\tau\alpha)$

Table 1 Daily utilizability values for beam radiation on a two-axis tracking surface

		$\bar{K}$	$I_c = 40$ W/m^2		$I_c = 125$ W/m^2		$I_c = 500$ W/m^2	
			$\bar{\phi}_{NI}$	$\bar{\phi}_{\mathrm{Clark}}$	$\bar{\phi}_{NI}$	$\bar{\phi}_{\mathrm{Clark}}$	$\bar{\phi}_{NI}$	$\bar{\phi}_{\mathrm{Clark}}$
Madison								
	Jan.	0.44	0.89	0.90	0.71	0.73	0.17	0.17
	July	0.55	0.91	0.93	0.75	0.80	0.23	0.22
	Oct.	0.50	0.92	0.92	0.77	0.77	0.26	0.23
Albuquerque								
	Jan.	0.64	0.94	0.94	0.81	0.83	0.34	0.42
	July	0.70	0.94	0.95	0.82	0.84	0.37	0.44
	Oct.	0.71	0.94	0.95	0.83	0.85	0.38	0.46
Seattle								
	Jan.	0.30	0.89	0.83	0.73	0.61	0.30	0.14
	July	0.59	0.92	0.94	0.79	0.81	0.32	0.36
	Oct.	0.44	0.91	0.90	0.76	0.74	0.29	0.27

Liu and Jordan [7] showed that the probability distribution of daily total radiation for a horizontal surface depends primarily on the clearness index. At the time of their study, the computational facilities and available radiation data were rather limited. Later studies by Theilacker [8] and Bendt, et al. [9] with substantially more data confirmed Liu and Jordan's conclusion with the largest errors occurring in coastal locations such as Miami, Fla. and Seattle, Wash. Liu and Jordan also suggested that hourly radiation distributions are nearly identical to daily distributions. Theilacker has found hourly distributions to display greater variability than daily distributions for the same clearness index value. However, the hourly and daily distributions are similar enough to allow accurate values of ϕ to be obtained using the daily radiation probability distributions. Analytical expressions for these probability distributions have been developed by Cole [10], Bendt, et al. [9], and Hollands and Huget [11, 12].

Huget [11] has rewritten equation (7) in terms of the clearness index and performed the integration analytically using a curve-fit to Liu and Jordan's radiation probability distributions. The resulting equation for ϕ is algebraically complicated, but it is suitable for computer implementation and applicable for surfaces of any orientation.

Clark, et al. [13] have used an alternate method to develop analytical expressions for hourly utilizability. Rather than curve-fit the probability distribution and then integrate, they directly correlated values of ϕ obtained by numerical integration of many years of hourly radiation data. The functional form of the correlation was chosen to have a value of 1 and slope of $-\bar{I}_T$ at $I_c = 0$ and a value of 0 and slope of 0 at $\bar{I}_c = I_{T,MAX}$.[3] Like Huget's, the Clark correlation is applicable for surfaces of any orientation. An advantage of the Clark correlation over Huget's is that it is algebraically simpler. Clark has compared utilizability values obtained using his correlation with values obtained from Huget's correlation and from numerical integration of long-term (15–23 years) hourly radiation data for Madison, Wis., Albuquerque, N. Mex., and Seattle, Wash. Thousands of utilizability values were compared for different surface orientations and critical levels. These comparisons demonstrated that the Clark and Huget correlations are of similar accuracy. Both compare to the numerical integration results with a root mean square error of about 5 percent.

Equation (7) defines utilizability in terms of I_T, the global (beam, plus diffuse) radiation on the surface. Concentrating collectors, however, do not utilize the diffuse (and ground-reflected) radiation components as efficiently as the beam component which suggests that equations (6) and (7) may not be appropriate for concentrating collectors. An appropriate definition for concentrators would result if I_T in equations (6) and (7) were replaced by I_R, the radiation per unit aperture area that would impinge on the receiver of a concentrating collector having perfect optical efficiency. I_R can be written as

$$I_R = I_B + f I_{DR} \tag{8}$$

where

I_B is the beam radiation per unit aperture area

I_{DR} is the diffuse and ground reflected radiation per unit aperture area. If the diffuse and ground-reflected radiation are assumed to be isotropic, then

$$I_{DR} = I_d(1 + \mathrm{Cos}\ \beta)/2 + \rho I(1 - \mathrm{Cos}\ \beta)/2 \tag{9}$$

f is a diffuse radiation factor equal to $1/C$ (where C is the concentration ratio) for isotropic ground and sky radiation.

The radiation incident on a flat-plate receiver, I_T, is also given by equation (8) with $f = 1$.

Hollands and Huget [12] provide a means for directly including f in their method for calculating ϕ, which makes it directly applicable to concentrating collectors. However, utilizability is not strongly dependent on f, and as a result, the utilizability obtained in terms of global radiation (from, for example, Clark's correlation) provides a reasonably accurate estimate of the utilizability appropriate to concentrating collectors.

This can be seen from the data in Table 1 which were obtained for an optically perfect two-axis tracking surface which receives only (and all) beam radiation (i.e., $f = 0$). The utilizability values in this table are radiation-weighted daily values, designated $\bar{\phi}$ and formally defined in the next section. $\bar{\phi}_{\mathrm{Clark}}$ was obtained as the beam radiation-weighted daily average of hourly utilizability values from Clark's correlation. The radiation weighting is done as indicated in equation (10) with $\bar{I}_R$, the average hourly beam radiation, replacing $\bar{I}_T$. $\bar{\phi}_{NI}$ was obtained by numerical integration of equation (11) (with I_R replacing I_T) for 23 years in Madison and Albuquerque, and 15 years in Seattle. I_R was obtained from horizontal global radiation data using the geometrical relationships for beam radiation given in reference [4] and the Erbs [14] diffuse fraction correlation. At low critical levels, the $\bar{\phi}_{NI}$ and $\bar{\phi}_{\mathrm{Clark}}$ values are in good agreement for all months in all three locations. Differences arise at higher critical levels particularly when the clearness index, $\bar{K}$, is low as for January in Seattle. These comparisons (along with others we have made) indicate that utilizability correlations developed in terms of global radiation are useful for concentrating collectors provided that both $\bar{\phi}$ and $\bar{K}$ are greater than about 0.4. This is not a severe limitation since concentrating collectors are seldom used when $\bar{K}$ is less than 0.4.

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[3] This functional form meets the necessary criteria for utilizability correlations put forth in reference [27].

Daily Correlations. There are applications in which it is appropriate to consider the critical level to be a constant value for all hours of the day. In this case, a radiation-weighted daily utilizability, $\bar{\phi}$, can be defined as the monthly average fraction of the solar radiation above the critical level during the period between sunrise and sunset. $\bar{\phi}$ can be evaluated using hourly utilizability as a radiation-weighted average of $\bar{\phi}$ for all daylight hours.

$$\bar{\phi} = \frac{\sum^{\text{day}} \bar{I}_T \phi}{\sum^{\text{day}} \bar{I}_T} \tag{10}$$

However, methods for directly evaluating $\bar{\phi}$ with less computation effort have been developed.

If sufficient data were available for every location of interest, $\bar{\phi}$ could be evaluated numerically from equation (11).

$$\bar{\phi} = \frac{\sum^{\text{days}} \sum^{N} [I_T - I_c]^+}{\sum^{\text{days}} \sum^{N} I_T} = \frac{\sum^{\text{days}} \sum^{N} [I_T - I_c]^+ \Delta t}{N \bar{H}_T} \tag{11}$$

$\bar{H}_T$ is the monthly average daily total radiation on a tilted surface and can be calculated from horizontal data as described, for example, by Klein and Theilacker [15]. Klein [16], has developed a correlation for $\bar{\phi}$ by curve-fitting values calculated in this manner. Rather than using actual radiation data for one or several locations, hourly radiation data were generated using the statistical information given in Liu and Jordan [7]. Hourly radiation data generated in this manner accurately represent long-term average conditions. Values of $\bar{\phi}$ were generated for all 12 months and for a broad range of critical levels, surface orientations, and clearness indices. The resulting correlation is a function of a dimensionless critical level (defined as the ratio of the critical level to the radiation at noon on the average day of the month), a geometry factor (which incorporates the effects of collector orientation, location and time of the year), and $\bar{K}$, the monthly average daily clearness index. The correlation is limited to surfaces facing directly toward the equator.

Theilacker [8] has proposed a simplification to Klein's correlation which also improves its overall accuracy. Theilacker reports a 2.7 percent standard deviation between his correlation and 8400 $\bar{\phi}$ values obtained by numerical integration of hourly radiation data. Theilacker's correlation is also limited to surfaces facing directly toward the equator. However, he has developed correlations for east and west-facing vertical surfaces as well as for overhang-shaded vertical surfaces facing the equator. Tables containing monthly values of $\bar{\phi}$ calculated using Theilacker's correlation for a vertical south-facing surface in 244 locations have been prepared by Klein, et al. [17].

Collares-Pereira and Rabl [18, 19] have developed monthly average daily utilizability correlations for five collector types: flat-plate, compound parabolic concentrator (*CPC*), east-west tracking, polar tracking, and two-axis tracking. Because radiation for *CPC* collector types must be within the acceptance angle of the collector as well as above the critical level to be useful, Collares-Pereira and Rabl define utilizability in a slightly different manner than that given by equation (11). Their monthly average daily utilizability, $\bar{\phi}_{CPR}$, is the fraction of the radiation incident on the surface while the collector is operating, which is above the critical level. The collector operating time may be dictated by thermal or optical considerations.

Two steps are needed to calculate utilizability with the Collares-Pereira and Rabl method. First, it is necessary to calculate $\bar{H}_{COLL}$, the monthly average radiation incident on the surface during collector operating period. Simple equations are given for $\bar{H}_{COLL}$ for each collector type in terms of the collector "cut-off" time. However, an iterative calculation procedure is needed to determine the cut-off time. The utilizability, $\bar{\phi}_{CPR}$, is then found from a correlation for each collector type involving $\bar{H}_{COLL}$, the cut-off time, the critical level, and the monthly average clearness index. Collares-Pereira and Rabl report an overall accuracy of 5 percent for useful energy calculated with their procedure. The Collares-Pereira and Rabl utilizability definition is related to the Klein definition by

$$\bar{\phi} \bar{H}_T = \bar{\phi}_{CPR} \bar{H}_{COLL} \tag{12}$$

Adnot et al., [20] calculate the total radiation during a month that is above the critical level using cumulative frequency curves of solar radiation. The cumulative frequency curves differ from the Liu and Jordan [7] curves in that they are for daily radiation on a tilted surface, and they must be constructed for each month, location, and orientation. Adnot's curve-fit to the daily cumulative frequency curve is then integrated analytically to obtain $\bar{\phi}$.

Lunde [21] has proposed two methods for calculating monthly average daily utilizability for surfaces facing the equator. First, he has prepared tables for a few locations that give the monthly total radiation above seven critical levels. These tables were constructed by numerically integrating hourly radiation on tilted surfaces for several years. Second, he correlates $\bar{\phi}$ to critical level with a simple exponential relation involving a single factor, which Lunde correlates to the monthly clearness index and the incidence angle of the sun on the surface at solar noon.

Evans, Rule, and Wood [22] have developed an empirical relationship for $\bar{\phi}$ in terms of flat-plate collector parameters rather than in terms of a critical radiation level. Their relationship, designed for collectors facing directly toward the equator, is algebraically very simple, and it is reported to be of similar accuracy to the equations of Klein [15]. The relationship is a quadratic function of $(T_i - T_a)/\bar{K}'$ with coefficients that depend on the collector parameters $F_R U_L$ and $F_R(\tau\alpha)$. $\bar{K}$ is a modified monthly clearness index that accounts for the effects of collector tilt. The authors show that using the monthly average daytime ambient temperature in place of the 24-hour average temperature does not result in a significant improvement in the accuracy of their correlation.

The Evans, Rule, and Wood correction is specific for applications involving flat-plate solar collectors; it cannot be used for other applications in which only a critical radiation level is available. However, these investigators have developed an additional correlation for $\bar{\phi}$ which is a quadratic function of critical level with coefficients dependents on $\bar{K}'$. In comparing these correlations, they show that the correlation based on critical level has a slightly larger standard error (0.017 versus 0.013) for $\bar{\phi}$ values greater than 0.6, and a slightly smaller standard error (0.028 versus 0.040) for $\bar{\phi}$ values less than 0.3. For $\bar{\phi}$ values between 0.3 and 0.6, both correlations exhibit a standard error of about 0.03.

Table 2 provides a comparison of $\bar{\phi}$ values calculated by various methods for several critical levels, surface tilts, and months in Madison, Wis. Tables 3 and 4 provide the same comparisons for Albuquerque, N.Mex. and Seattle, Wash. These locations were chosen to cover a broad range of climate types. The column labeled $\bar{\phi}_{NI}$ gives the utilizability value obtained by summing hourly radiation data for 23 years in Madison and Albuquerque and 15 years in Seattle as indicated in equation (11). Only hourly horizontal radiation data were available [23]; tilted surface radiation data were calculated from the horizontal data using the method described in reference [4] with the diffuse fraction correlation developed by Erbs, et al. [14]. In these tables, $\bar{\phi}_{\text{Klein}}$, $\bar{\phi}_{CPR}$, $\bar{\phi}_{\text{Lunde}}$, and $\bar{\phi}_{ERW}$ refer to the results obtained using the monthly

Table 2 Comparison of $\bar{\phi}$ values for Madison, Wis.

			$I_c = 125$ W/m²						$I_c = 500$ W/m²					
Month	Tilt	$\bar{K}$	$\bar{\phi}_{NI}$	$\bar{\phi}_{\text{Klein}}$	$\bar{\phi}_{CPR}$	$\bar{\phi}_{\text{Lunde}}$	$\bar{\phi}_{ERW}$	$\bar{\phi}_{\text{Clark}}$	$\bar{\phi}_{NI}$	$\bar{\phi}_{\text{Klein}}$	$\bar{\phi}_{CPR}$	$\bar{\phi}_{\text{Lunde}}$	$\bar{\phi}_{ERW}$	$\bar{\phi}_{\text{Clark}}$
Jan.	0	0.44	0.47	0.47	0.49	0.50	0.60	0.48	0.00	0.01	0.00	0.06	0.06	0.00
July	0	0.55	0.74	0.75	0.75	0.70	0.75	0.74	0.22	0.22	0.28	0.24	0.27	0.22
Oct.	0	0.50	0.62	0.62	0.63	0.57	0.68	0.62	0.06	0.07	0.06	0.11	0.16	0.06
Jan.	43	0.44	0.67	0.66	0.66	0.65	0.69	0.68	0.15	0.12	0.22	0.18	0.18	0.17
July	43	0.55	0.72	0.73	0.75	0.69	0.73	0.73	0.21	0.19	0.29	0.24	0.26	0.22
Oct.	43	0.50	0.73	0.71	0.72	0.70	0.73	0.72	0.24	0.17	0.27	0.24	0.25	0.23
Jan.	90	0.44	0.65	0.65	0.65	0.64	0.70	0.67	0.14	0.11	0.21	0.17	0.20	0.15
July	90	0.55	0.50	0.52	0.58	0.52	0.64	0.52	0.00	0.02	0.00	0.07	0.11	0.00
Oct.	90	0.50	0.68	0.66	0.67	0.64	0.71	0.67	0.16	0.11	0.22	0.16	0.20	0.14

Table 3 Comparison of $\bar{\phi}$ values for Albuquerque, N. Mex.

			$I_c = 125$ W/m²						$I_c = 500$ W/m²					
Month	Tilt	$\bar{K}$	$\bar{\phi}_{NI}$	$\bar{\phi}_{\text{Klein}}$	$\bar{\phi}_{CPR}$	$\bar{\phi}_{\text{Lunde}}$	$\bar{\phi}_{ERW}$	$\bar{\phi}_{\text{Clark}}$	$\bar{\phi}_{NI}$	$\bar{\phi}_{\text{Klein}}$	$\bar{\phi}_{CPR}$	$\bar{\phi}_{\text{Lunde}}$	$\bar{\phi}_{ERW}$	$\bar{\phi}_{\text{Clark}}$
Jan.	0	0.64	0.66	0.65	0.67	0.60	0.68	0.65	0.07	0.08	0.06	0.13	0.17	0.06
July	0	0.70	0.80	0.80	0.80	0.81	0.81	0.79	0.35	0.34	0.35	0.44	0.41	0.33
Oct.	0	0.71	0.74	0.74	0.75	0.69	0.76	0.74	0.20	0.19	0.20	0.23	0.30	0.20
Jan.	35	0.64	0.80	0.76	0.76	0.73	0.76	0.78	0.30	0.23	0.29	0.29	0.30	0.29
July	35	0.70	0.79	0.79	0.80	0.77	0.79	0.79	0.33	0.31	0.33	0.39	0.39	0.31
Oct.	35	0.71	0.81	0.80	0.80	0.80	0.80	0.80	0.36	0.31	0.35	0.40	0.39	0.36
Jan.	90	0.64	0.79	0.74	0.75	0.70	0.76	0.76	0.27	0.19	0.25	0.24	0.30	0.25
July	90	0.70	0.50	0.52	0.56	0.52	0.65	0.49	0.00	0.01	0.0	0.07	0.12	0.00
Oct.	90	0.71	0.75	0.74	0.75	0.69	0.76	0.75	0.22	0.19	0.22	0.23	0.30	0.22

Table 4 Comparison of $\bar{\phi}$ values for Seattle, Wash.

			$I_c = 125$ W/m²						$I_c = 500$ W/m²					
Month	Tilt	$\bar{K}$	$\bar{\phi}_{NI}$	$\bar{\phi}_{\text{Klein}}$	$\bar{\phi}_{CPR}$	$\bar{\phi}_{\text{Lunde}}$	$\bar{\phi}_{ERW}$	$\bar{\phi}_{\text{Clark}}$	$\bar{\phi}_{NI}$	$\bar{\phi}_{\text{Klein}}$	$\bar{\phi}_{CPR}$	$\bar{\phi}_{\text{Lunde}}$	$\bar{\phi}_{ERW}$	$\bar{\phi}_{\text{Clark}}$
Jan.	0	0.29	0.31	0.36	0.33	0.44	0.55	0.33	0.00	0.01	0.00	0.04	0.00	0.00
July	0	0.58	0.75	0.75	0.75	0.71	0.75	0.74	0.25	0.23	0.29	0.25	0.29	0.23
Oct.	0	0.44	0.55	0.55	0.56	0.53	0.64	0.55	0.04	0.04	0.06	0.08	0.11	0.02
Jan.	47	0.29	0.56	0.51	0.54	0.60	0.62	0.56	0.16	0.06	0.13	0.13	0.09	0.07
July	47	0.58	0.74	0.74	0.76	0.72	0.75	0.74	0.26	0.20	0.30	0.27	0.29	0.24
Oct.	47	0.44	0.68	0.67	0.68	0.67	0.71	0.68	0.22	0.13	0.23	0.20	0.20	0.18
Jan.	90	0.29	0.58	0.49	0.53	0.59	0.63	0.53	0.18	0.05	0.14	0.12	0.10	0.06
July	90	0.58	0.56	0.56	0.62	0.55	0.66	0.56	0.02	0.03	0.00	0.09	0.14	0.02
Oct.	90	0.44	0.65	0.62	0.64	0.62	0.69	0.63	0.17	0.08	0.19	0.15	0.17	0.11

utilizability correlation of Klein-Theilacker [8], Collares-Pereira and Rabl [19], Lunde [21] and Evans, Rule and Wood [22]. Also shown are the results obtained from equation (10) using the Clark [13] hourly utilizability correlation.

The $\bar{\phi}$ values obtained by numerical integration of many years of hourly radiation data are presumably the most accurate values. Of the five correlation methods investigated, the Clark correlation most closely reproduces these values. This is not surprising in that the Clark correlation is applied on an hourly basis and thus requires the most computational effort of the five methods. Significant computational effort is also required by the Collares-Pereira and Rabl correlation, since it involves an iterative procedure. However, the resulting $\bar{\phi}$ values are comparable in accuracy to the Klein correlation, which requires less computation. The major advantage of the Collares-Pereira and Rabl correlation method is that it explicitly involves solar time and can thereby be applied to concentrating collectors which have optical cut-off times or to shaded receivers in which the hours of direct irradiation can be specified. As shown earlier, the Clark correlation can also be applied for concentrating collectors. The Lunde and ERW methods are easiest to use since they do not involve the complex trigonometric relations used in the other methods. The overall accuracy of both of these methods is good, but for the cases represented in Tables 2–4, not as good as that obtained from the other methods.

Applications requiring $\bar{\phi}$ almost always require estimates of radiation on inclined surfaces as well. To estimate radiation on inclined surfaces, it is necessary to evaluate trigonometric factors such as the ratio of monthly radiation on a tilted surface to that on a horizontal surface. Thus, the overall computational effort required in a particular application may not be significantly reduced by using the Lunde or ERW correlations.

Annual Correlations. Rabl [24] has developed empirical correlations for the average yearly total energy above a specified critical level for flat-plate and concentrating solar collectors. His correlations can be used to determine a yearly average utilizability by dividing the total energy above the critical level by the total energy for a critical level of zero. Rabl's correlations are derived from hourly calculations using meteorological data for 26 stations in the United States. The correlations are second-order polynomials in terms of critical level, latitude, and yearly average insolation. Rabl's correlations are limited to applications in which the critical level can be assumed to be a constant value for the entire year. However, Rabl shows that small variations of the critical level about its mean yearly value have only a small effect on the annual energy collected. It may not be possible, however, to accurately determine the mean yearly critical level for some systems, particularly those having energy storage.

Gordon and Zarmi [25, 26] have developed a relationship for annual utilizability appropriate for flat-plate and concentrating types of solar collectors. Their relationship differs from Rabl's in that they arrive at it analytically by using the radiation probability distribution developed by Bendt et al. [9]. To simplify the integration and allow an analytical result, they replace the annual calculation by a calculation for one representative day having a daylength of 12 hours. A further approximation, strictly correct only for very clear climates, results in a very simple expression for annual utilizability which "fortuitously" retains excellent accuracy in all climate types. Gordon and Zarmi compare their utilizability relation with the empirical relations of Rabl [24] and find good agreement for all collector types.

Gordon and Zarmi point out that their expression for utilizability is of a functional form which satisfies the necessary criteria for all utilizability expressions. These criteria, which are explained in greater detail in a separate paper [27] can be summarized as follows:

$$(a)\ \phi|_{I_c=0}=1$$

$$(b)\ \phi|_{I_c=I_{T,MAX}}=0$$

$$(c)\ \left.\frac{d\phi}{dI_c}\right|_{I_c=0}=(1-p)$$

$$(d)\ \left.\frac{d\phi}{dI_c}\right|_{I_c=I_{T,MAX}}=0$$

$$(e)\ \frac{d^2\phi}{dI_c^{\,2}}\geq 0$$

where p is a constant characterizing the collector type. p is 0 for flat-plate collectors which may receive beam radiation during all daylight hours.

4 Applications

The utilizability concept was originally developed to simplify the calculation of flat-plate solar collector performance. However, utilizability is a statistic of solar radiation data, and its use is not tied to this application. In recent years, the utilizability method has been used in performance calculations of a wide range of solar energy systems.

The utilizability concept requires that the critical level be constant over the period of analysis. This feature poses a limitation on the utilizability method when applied to solar energy system performance calculations. The actual critical level in most solar energy systems, particularly those having energy storage, varies from hour to hour and from day to day. The yearly utilizability correlations of Rabl [24] and Gordon and Zarmi [25, 26] are most restrictive in this sense in that they require the critical level to be a constant value for the entire year. Daily correlations require the critical level to be constant for every hour in the month, but it may vary from one month to the next. The hourly correlations require the critical level to be a constant for each hourly period (e.g., 9-10 AM), but it may differ for other hourly periods as well as for each month.

Because of this limitation, the utilizability method is often used only to calculate an upper or lower bound on system performance, for which a constant critical level is appropriate. Correlations of computer simulation results are then used with the performance bounds obtained using the utilizability method to calculate system performance.

Thermal Performance of Active Solar Collector Systems. The critical level for solar collectors is defined as the radiation level required to maintain the collector plate at the fluid inlet temperature. Radiation must be above this level to produce a useful energy gain. A relationship for the critical level in terms of standard flat-plate collector parameters is given in equation (3). An analogous equation can be written for concentrating collectors.

The collector parameters, $F_R U_L$ and $F_R(\tau\alpha)$ are relatively constant for a particular hour over a period such as a month. The transmittance-absorptance product, $(\tau\alpha)$, varies with solar incidence angle and thereby is a function of solar time. However, an energy weighted monthly average daily transmittance-absorptance product, $(\overline{\tau\alpha})$, can be evaluated (28) so that $F_R\ (\overline{\tau\alpha})$ can be assumed constant for a monthly period.

In general, the critical level for active solar collectors varies with time, primarily because of the variation in the collector fluid inlet temperature. There are, however, several situations in which the critical level can be assumed to be constant or nearly constant for a period such as a month and the utilizability method can be directly applied to calculate $\bar{Q}$, the monthly average daily energy collection.

$$\bar{Q}=AF_R(\overline{\tau\alpha})\bar{H}_T\bar{\phi} \tag{13}$$

One example of a system for which this equation is directly applicable is an air-based solar heating system with no thermal storage. Room temperature air is circulated through the collectors whenever solar energy can be collected. As a result, the collector fluid inlet temperature and thus the critical level is nearly constant, and an estimate of the monthly energy collection can be obtained from equation (13). These calculations assume that all of the energy collected is needed at the time it is available and thus no energy dumping occurs. Dumping rarely occurs when the collected energy represents a small fraction of the load.

Equation (13) can also be used to design active solar systems having long-term (i.e., seasonal) energy storage. An energy balance on the storage tank over a one-month period can be written

$$\Delta U=Q_u-Q_{LS}-Q_s \tag{14}$$

where

ΔU is the change in internal energy of the storage, which is the product of the mass of storage, its specific heat, and its temperature change during the month

Q_u is the monthly useful energy supplied to the tank from the solar collector system

Q_{LS} is the energy loss from storage for the month

Q_s is the energy removed from storage for the load during the month

If an appropriate critical level could be determined, Q_u would be the product of N, the number of days, and $\bar{Q}$, the monthly average daily useful energy collection from equation (13). An appropriate critical level is difficult to determine for systems having short-term energy storage, since the tank temperature variation (and therefore, the critical level) is strongly coupled to weather conditions for individual days. For long-term storage, however, the tank temperature variation is damped and the critical level is more nearly constant. Braun et al. [29] show that for systems having more than 200 liters of water storage per square meter of flat-plate collector area, the assumption of a constant critical level evaluated at the average tank temperature results in performance estimates that compare well with detailed simulations.

Perhaps the most useful application for utilizability in active system design is to determine the limits on system performance. For example, the maximum energy collection will occur when the collector fluid inlet temperature is always at its minimum. For space heating, the minimum temperature is approximately 20°C, whereas for solar air conditioning with an absorption chiller, the minimum useful temperature may be 80°C. An estimate of the maximum system performance can be used to determine the feasibility of a proposed system from a thermal or economic standpoint.

When the critical level varies considerably during a month (as it generally does in systems having short-term thermal energy storage), equation (13) cannot be used to accurately estimate system performance, since an average critical level and thus an average utilizability cannot be determined. Estimates of the performance of such systems can often be obtained from a design method developed by correlating the results of detailed simulations. One example of a design method is the *f*-Chart [30] method, for active space and domestic hot water systems. More general design methods have been developed for these and other applications. One example of a general design method is the $\bar{\phi}$*f*-Chart method

[31, 32]. The $\bar{\phi}f$-Chart method can be applied to determine the performance of space heating, air conditioning, and industrial process heating systems with flat-plate or concentrating solar collectors.

The $\bar{\phi}f$-Chart method consists of two steps. The first step is the estimation of the maximum useful energy collection. This calculation is made using equation (13) with $\bar{\phi}$ evaluated at a critical level corresponding to the minimum collector fluid inlet temperature. The second step corrects the maximum performance estimate to account for variations in the critical level and storage energy losses. The correction factor, developed from simulation results, depends primarily on the storage capacity and the fraction of the load supplied by the system.

Other design methods that make use of the utilizability function have been developed for solar space and water heating systems [33, 34], industrial process heating systems with no storage [35], and single-pass open-loop industrial process hot water systems [36].

Collector Operating Time. Mitchell, et al. [37] have shown that the utilizability function can provide an estimate of the collector operating time in active systems as well as an estimate of thermal performance. Their reasoning is as follows.

Consider a situation in which the critical level is constant at a value I_c over a monthly period. The average daily useful energy collection is given by equation (13). The monthly average daily collector operating time, $\bar{t}$, is the number of hours during the month in which the solar radiation exceeds the critical level, I_c, divided by the number of days in the month. Now assume the critical level is raised by a small amount, ΔI_c. The reduction in average daily useful energy collections that would result would be given approximately by the product of $F_R(\overline{\tau\alpha})A\Delta I_c$ and $\bar{t}$. In the limit, as I_c approaches zero, $\bar{t}$ can be found from

$$\bar{t} = \lim_{I_c \to 0} \frac{AF_R(\overline{\tau\alpha})\bar{H}_T\bar{\phi}|_{I_c} - AF_R(\overline{\tau\alpha})\bar{H}_T\bar{\phi}|_{I_c+\Delta I_c}}{AF_R(\overline{\tau\alpha})\Delta I_c}$$

$$= -\bar{H}_T \frac{d\bar{\phi}}{dI_c}\bigg|_{I_c} \tag{15}$$

Passive Systems. It is possible to analytically derive lower and upper limits on the performance of direct-gain, and collector-storage wall systems by considering the building to have zero and infinite storage capacity, respectively. These limits are easily demonstrated for direct-gain systems.

An energy balance on a direct-gain system for a period such as a month can be expressed

$$E = [L - S - D]^+ \tag{16}$$

where

- E is the monthly auxiliary energy needed to keep the building at the desired temperature.
- L is the load, i.e., the auxiliary energy the building would require if the direct-gain glazings were opaque and the buildings were maintained at the thermostat set temperature.
- S is the monthly solar energy transmitted through the glazings and absorbed within the building.
- D is the solar energy that is dumped (i.e., removed) from the building to prevent overheating.

The superscript + equation (16) is used to indicate that only positive values are considered.

Dumping can occur in two ways. First the indoor temperature of the building will rise above the set temperature whenever the solar gains are in excess of the load. In this case, the losses from the building will increase due to the increased temperature difference between the building and its surroundings. This increased energy loss, which is not considered in L, is one form of dumping. Another form would be deliberate dumping by opening a window, pulling a shade, or operating an exhaust fan in order to maintain the temperature within the comfort zone.

Consider a direct-gain building having infinite thermal storage capacity. In this case, the temperature of the indoor space is constant. Whenever the auxiliary requirement, E, is greater than zero, D is zero, since energy in excess of the load can be stored without penalty until it can be used. Thus, the lower limit on auxiliary energy use E_{min}, is then

$$E_{min} = [L - S]^+ = [L - A\bar{H}_T(\overline{\tau\alpha})N]^+ \tag{17}$$

where A is the glazing area and N is the number of days in the month.

The upper limit on auxiliary energy use occurs for a building that has zero thermal storage capacity. In this case, all absorbed solar energy in excess of the instantaneous load must be dumped. The monthly total energy which must be dumped can be evaluated by summing the positive values of the difference between the hourly absorbed solar radiation and the load. Thus,

$$D_{max} = A(\tau\alpha) \sum^{N}_{\text{day}} \sum [I_T - I_c]^+ \Delta t \tag{18}$$

where

$$I_c = \frac{(UA)(T_b - T_a)}{A(\overline{\overline{\tau\alpha}})} \tag{19}$$

In equation (19), (UA) is the building overall energy loss coefficient, and T_b and T_a are the indoor balance[4] and ambient temperatures, respectively. D_{max} can be expressed in terms of $\bar{\phi}$, the monthly average daily utilizability and the upper limit of auxiliary energy use can then be written as

$$E_{max} = L - (1 - \bar{\phi})\bar{H}_T(\overline{\tau\alpha})AN \tag{20}$$

The actual auxiliary energy use will lie between the limits of E_{min} and E_{max}. Monsen et al. [38, 39] have derived correlations for the actual auxiliary use in direct-gain and collector-storage wall buildings in terms of these physical limits. In a passive solar application, the utilizability function provides an estimate of the solar energy that must be dumped. As a result, Monsen has aptly named this design approach the "unutalizability" method.

Monsen's method can also be applied for active collection-passive storage systems, i.e., solar space heating systems that do not have a storage component other than the building structure itself [40].

Photovoltaic Systems. In any photovoltaic system, there may be times when the electrical energy produced by the array is greater than that needed to satisfy the electrical demand, i.e., the load. This excess energy may be fed back to the utility, sent to a storage battery, or dissipated. The amount of excess energy is affected by the capacity of the array relative to the load and by the distribution of the load during daytime hours.

The efficiency of a photovoltaic array operating at its maximum power point is nearly a linear function of cell operating temperature. Solar radiation affects the efficiency indirectly in that it affects the cell temperature. Methods of estimating the monthly average array efficiency have been developed by Siegel, et al. [41], Evans [42], and Clark [43].

The long-term monthly average array output for a particular hourly period (e.g., 10–11 AM) can be expressed in terms of the average efficiency, $\bar{\eta}$,

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[4] The balance temperature is the indoor temperature minus the ratio of the internal energy gains to (UA).

$$\bar{E}_i = A\bar{I}_T\bar{\eta} \tag{21}$$

where A is the array area. A critical radiation level, I_c, can be defined as the radiation level at which the electrical energy production is just equal to $\bar{L}_i$, the monthly average load for the hour.

$$I_c \frac{\bar{L}_i}{A\bar{\eta}} \tag{22}$$

Radiation above the critical level will result in energy production greater than the load. The fraction of the radiation above the critical level is ϕ, the hourly utilizability defined by equation (5).

In terms of utilizability, the monthly average hourly energy in excess of the average hourly load is $\bar{D}_i$, given by

$$\bar{D}_i = \bar{E}_i\phi \tag{23}$$

If $\bar{L}_i$ (and therefore the critical level) is constant for all daylight hours, the daily utilizability function, $\bar{\phi}$, can be used to evaluate the monthly average daily excess energy. If $\bar{L}_i$ is not constant, then equation (23) can be summed for each hour of sunlight.

In utility feedback systems, $\bar{D}_i$ is the monthly average hourly energy which is fed back into the utility. In a battery storage system, $\bar{D}_i$ is the excess energy that is sent to the storage battery; the portion of this energy that can be effectively stored and used later can be estimated from a correlation developed by Clark [43]. In a stand-alone system, $\bar{D}_i$ is the energy that must be dissipated.

5 Conclusion

The utilizability concept was originally developed for estimating the performance of flat-plate solar collectors over long time periods. In this application, low critical radiation levels are desirable in that they result in high utilizability values and thus efficient collection of solar energy. For this reason, early correlations of the utilizability function (e.g., the Liu and Jordan [6], Klein [16], and Collares-Pereira and Rabl [19] correlations) were designed to provide accurate estimates of the utilizability at low to intermediate critical levels.

It is now apparent that the utilizability concept is useful for other applications such as passive and photovoltaic system analysis. In these latter applications, high critical levels and thus low utilizability (unutilizability) values are desirable in that they imply a better match of collectable solar energy to the load. Utilizability correlations that retain acceptable accuracy at high critical levels are needed for these applications. A complicating factor is that as the critical level increases, utilizability becomes more sensitive to the distribution of solar radiation. At high critical levels, utilizability may be sensitive to variations in the solar radiation at time intervals shorter than one hour. These factors place a limit on the accuracy achievable with a location-independent correlation. To improve accuracy, it may be necessary to introduce one or more additional parameters (in addition to the clearness index) to characterize the local solar climate.

The utilizability function also contains information about operating times through its derivative with respect to critical level. None of the existing correlations were developed with the concern to accurately represent both the utilizability function and its derivative and so the estimation of operating time is perhaps not as accurate as it could be.

Existing utilizability correlations provide a simple and elegant means for estimating the long-term effect of solar radiation on any solar process. Improvements in the utilizability correlations are possible and should be pursued.

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