

INFLUENCE OF COLLECTOR PARAMETERS ON LOAD FRACTION AND ECONOMIC VIABILITY

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(Received for publication 7 June 1985)

1. INTRODUCTION

Research and development efforts continue to be directed at improving the performance of flat-plate collectors. To arrive at guidelines to decide on the cost effectiveness of an improvement in collector design, the changes in collector parameters resulting from the design change have to be related to the change in a load fraction supplied by solar energy. For this it is essential that the collector array is considered as part of a complete system and the analysis must relate the changes in collector parameters to changes in annual load fraction supplied by the system. Though it is straightforward to evaluate these changes using methods like f-chart[1] or $\bar{\phi}$, f-chart[2, 3, 4], it is difficult to generalize because of location dependency and dependence on the reference load fraction itself.

A section of the present article deals with the development of a model for predicting the annual load fraction. Specific objectives are that the model be capable of (i) a simple analytic representation, (ii) determining location dependent constants without requiring preprocessed data, and (iii) estimating location and collector dependent constants when collector parameters change, thus facilitating the prediction of change in the load fraction.

Annual models reported in the literature by Lameiro and Bendt[5], Ward[6] and Barley and Winn[7] are correlations obtained from f-chart[1] results. Lameiro and Bendt's model requires six location dependent constants, and Barley and Winn's model requires three location dependent constants, which have been tabulated for a number of U.S. locations. Ward's correlation is based on a rather limited number of nine locations, and Huck[8] has pointed out that Ward's correlation is somewhat location dependent. Balcomb and Hedstrom[9] have tabulated the values of a parameter that can be directly used to calculate the area required for an annual load fraction of 0.25, 0.50 and 0.75 for 85 U.S. locations for space heating systems. However, these values are applicable for specific air and water collectors. This information has been presented in a linear form by Clark[10].

For the purpose of the present investigation,

these annual models have not been found suitable, since (i) the empirical constants in the correlations cannot be simply obtained for locations which are not tabulated, and (ii) it is tedious to estimate these constants explicitly as collector parameters change.

2. ANNUAL MODEL

For a particular system, location, collector design, the annual model proposed by Clark[10] can be expressed as

$$(UA)_h(F/A_c) = a - (a/b)F \quad (1)$$

where $(UA)_h$ is the building loss factor and a and b are empirical constants. It is apparent that the model breaks down at high solar fraction. The model would be suitable for evaluating changes in annual load fraction as collector parameters change at any location provided that a and b can be independently correlated to meteorological data and collector parameter and the load fraction is below 90 percent.

From eqn (1), considering the limit $A_c \rightarrow 0$, it follows:

$$(UA)_h(F/A_c)_{A_c \rightarrow 0} = a \quad (2)$$

Using the f-Chart correlations[1], summing over all the months, replacing the load L by $(UA)_h DD$ and considering the limit, $A_c \rightarrow 0$, it follows

$$(UA)_h(F/A_c)_{A_c \rightarrow 0} = \frac{F_R [1.029(\bar{\tau}\alpha)H_{YT} - 0.065U_L \Delta T_Y \Delta t_y]}{DD} \quad (3)$$

where

H_{YT} = the annual radiation per unit area on a tilted surface (J/m^2)

$\Delta T_Y = 100 - \bar{T}_{ay}$, \bar{T}_{ay} is the annual average ambient temperature ($^{\circ}C$)

Δt_y = number of seconds in a year

DD = annual degree days ($^{\circ}C$ -days)

$(\bar{\tau}\alpha)$ = effective (energy weighted) yearly transmittance-absorptance product.

For simplicity a variable ξ_1 is introduced as an approximation to the RHS of eqn (3).

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$$\xi_1 = F_R[(\tau\alpha)_n H_Y - 2.05 U_L \Delta T_Y] / DD \quad (4)$$

where H_Y is the annual radiation on a horizontal surface in MJ/m². The constant 2.05 absorbs the number of seconds in a year and conversion factors when U_L is expressed in W/m²°C. Combining eqns (2) and (3) and using eqn (4), it can be expected.

$$a \sim \xi_1 \quad (5)$$

FCHART 4.1 was used to determine a for ten locations for space heating systems. The values of various parameters employed are: $F_R U_L = 4.23$ W/m²°C, $F_R(\tau\alpha)_n = 0.44$, no. of covers = 1, $\phi - s = 0$, $\epsilon C_{\min} = 1200$ W/°C, storage capacity = 350 kJ/°Cm² and $(UA)_h = 275$ W/°C. A plot of a vs ξ_1 for liquid-based systems showed that eight out of ten locations were well correlated to ξ_1 ; the deviation for the other two locations (Los Angeles and San Francisco) is too large. The problem with the variable ξ_1 is that it does not contain information on the monthly load distribution. After trial and error, the variable ξ was found to provide a good correlation.

$$\xi = \xi_1 (\overline{DD} / DD_m)^2 \quad (6)$$

where \overline{DD} is the annual average of monthly degree days, and DD_m is the maximum degree days in a particular month in the year.

A plot of a against ξ is shown in Fig. 1 for liquid-based systems. A straight line fit between a and ξ gives

$$a = 61\xi \quad (\text{for liquid-based systems}) \quad (7)$$

$$a = 53\xi \quad (\text{for air-based systems}) \quad (8)$$

Rewriting eqn (1) for $F = 1$, it follows

$$b = \frac{a}{a - [(UA)_h / A_c]_{F=1}} \quad (9)$$

The collector area required for $F = 1$ can be estimated by recognizing that the annual load fraction will be equal to unity if the monthly load fraction is one for a month c ('coldest') when the ratio of available radiation to load is a minimum. Using the f-chart correlations with the first two terms only and introducing the simplifications similar to those used in defining ξ_1 (eqn (4)), b can be expressed to be,

$$b \sim a/(a - B) \quad (10)$$

where B is given by

$$B = F_R[(\tau\alpha)_n \overline{H}_c - 5.61 U_L \Delta T_c] / (2.78 DD_c) \quad (11)$$

† The heat exchanger and duct heat loss correction factor can be introduced through the factor F_R . See [1].

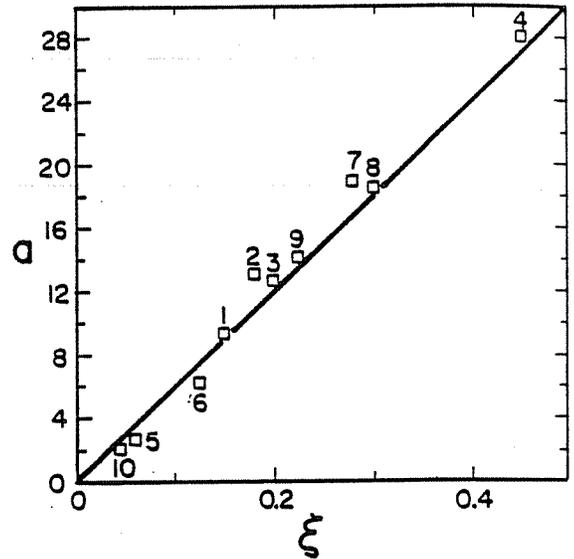


Fig. 1. Correlation for liquid based systems. Locations:

- | | |
|-----------------|------------------|
| 1. Dallas | 6. Memphis |
| 2. Houston | 7. Phoenix |
| 3. Lake Charles | 8. San Francisco |
| 4. Los Angeles | 9. Tallahassee |
| 5. Madison | 10. Winnepeg |

Numerical constants appearing in defining B are consistent when a is expressed in W/m²°C, \overline{H}_c , the average daily horizontal radiation in the month c , in kJ/m², DD_c in °C-days for the same month, and $\Delta T_c = 100_{\text{ref}} - \overline{T}_{ac}$ in °C where \overline{T}_{ac} is average ambient temperature in this month.

The intercept b determined from FCHART 4.1 is correlated to be

$$b = 0.97\eta + 0.2 \quad (\text{for liquid-based systems}) \quad (12)$$

$$b = 0.91\eta + 0.3 \quad (\text{for air-based systems}) \quad (13)$$

where

$$\eta = a/(a - B) \quad (14)$$

Using eqns (7) and (8) for liquid-based systems and eqns (8) and (13) for air-based systems, a and b have been calculated for ten new locations. The locations are Albuquerque, Atlanta, Bakersfield, Bismark, Boston, Charleston (WV), Denver, El Paso, Glasgow and Seattle. The annual load fraction has been predicted using eqn (1) and has been compared against FCHART 4.1† results. Values of the various parameters employed are the same as those employed in obtaining the correlation for a and b . The comparison is shown in Fig. 2 for liquid-based systems; similar results were obtained for air-based systems. It is found that in the range, $0.3 < F < 0.8$, the prediction for liquid-based systems is almost always higher than the FCHART 4.1 value. However, the rms error is less than 4 percent for

† FCHART 4.1 uses f-chart correlations for air-based systems and ϕ , f-chart for liquid-based systems.

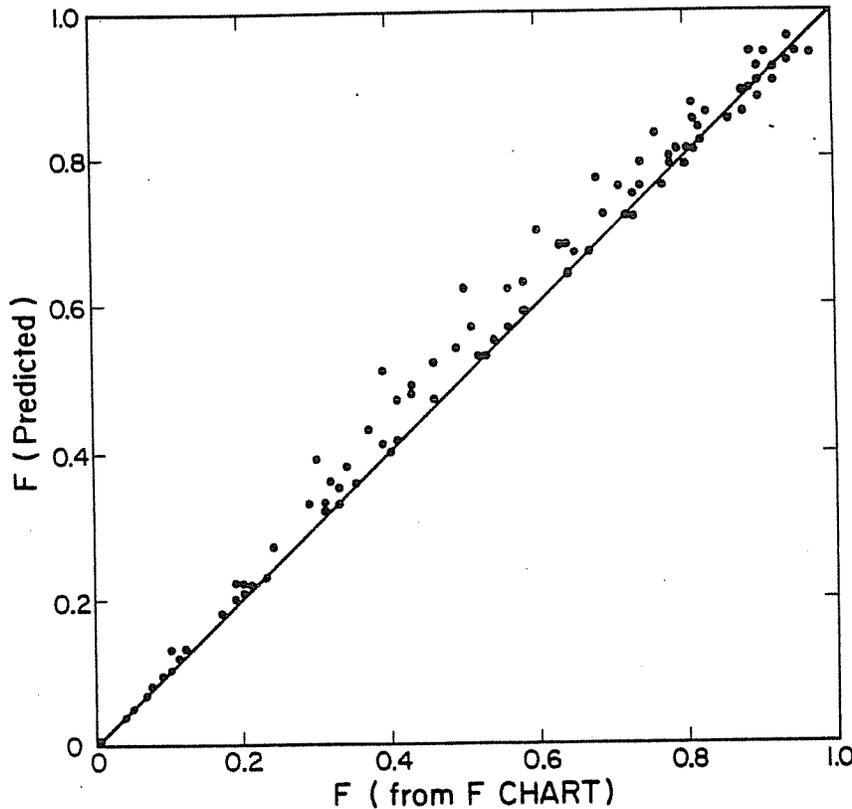


Fig. 2. Comparison of predicted annual load fraction with FCHART predictions for liquid based systems.

liquid-based systems and is less than 3 percent for air-based systems. Though the results have been validated for one set of collector parameters, the correlations for a and b can be used for other values also, by calculating ξ and η for the new set of collector parameters.

The criterion for 'coldest' month in the northern hemisphere may be satisfied either in the month of January or December. It is found that either month can be used, and the error in predicting b and, hence, F is small. Also, the method is not restricted to degree-day heating load; user supplied loads can be employed.

3. INFLUENCE OF CHANGES IN COLLECTOR PARAMETERS

A collector array of area A_c meets a load fraction F_1 , when the collector parameters are F_R , U_L and $(\tau\alpha)_n$. Let a_1 and b_1 be the corresponding constants in the annual model given by eqn (1). When the collector parameters change to $F_R + \Delta F_R$, $U_L + \Delta U_L$ and $(\tau\alpha)_n + \Delta(\tau\alpha)_n$, a_1 , b_1 and F_1 change by Δa , Δb and ΔF . $\Delta F/F_1$ can be expressed as

$$\frac{\Delta F}{F_1} = [Z - (F_1/b_2)(Z - P)]/[1 + (F_1/b_2)(Z - P)], \quad (15)$$

where

$$Z = \Delta a/a_1 \quad \text{and} \quad P = \Delta b/b_1 \quad (16)$$

Equation (15) is a general relationship which expresses the ratio of change in load fraction to reference load fraction.

A comparison of values of $\Delta F/F_1$ obtained from eqn (15) and FCHART 4.1 is shown in Table 1 for three locations, Albuquerque, Los Angeles and Madison at $F_1 = 0.25, 0.50$ and 0.75 for $\Delta F_R = 0.13$, $\Delta U_L = -2$ and $\Delta(\tau\alpha)_n = 0.08$. The agreement is quite good.

4. ECONOMIC EVALUATION

Life-cycle savings (LCS) in terms of the economic parameters P_1 and P_2 (11, 12) for a reference system meeting a load fraction F_1 is given by

$$\text{LCS} = P_1 C_F L F_1 - P_2 (C_{A1} A_c + C_E) \quad (17)$$

If F_1 changes by ΔF as a result of changes in F_R , U_L or $(\tau\alpha)_n$, the cost per unit area of the collector can change by ΔC_A . ΔC_A can be expressed as

$$\Delta C_A = a_1 P_1 C_F L (1 - F_1/b_1)(\Delta F/F_1) / [P_2 (UA)_h] \quad (18)$$

Table 1. Comparison of fractional change in load fraction

Location	Albuquerque			Los Angeles			Madison		
	ΔF_R	ΔU_L	$\Delta F/F_1$	ΔF_R	ΔU_L	$\Delta F/F_1$	ΔF_R	ΔU_L	$\Delta F/F_1$
	0.13	-2	0.08	0.13	-2	0.08	0.13	-2	0.08
F_1	CHART	CHART	PRED	CHART	CHART	PRED	CHART	CHART	PRED
0.25	0.21	0.20	0.07	0.11	0.19	0.06	0.07	0.10	0.10
0.50	0.15	0.15	0.09	0.09	0.16	0.05	0.06	0.09	0.12
0.75	0.09	0.11	0.06	0.08	0.10	0.04	0.05	0.07	0.07

Table 2. Evaluation of collectors with selective absorbers

Location	Albuquerque				Los Angeles				Madison			
	Selective 1		Selective 2		Selective 1		Selective 2		Selective 1		Selective 2	
	F_1	$\Delta F/F_1$	ΔC_A	$\Delta F/F_1$								
0.25	0.19	96	0.12	60	0.20	118	0.12	72	0.30	94	0.24	77
0.50	0.16	63	0.10	40	0.14	64	0.09	37	0.29	67	0.20	46
0.75	0.12	34	0.08	22	0.13	43	0.09	30	0.20	29	0.17	24

Equation (18) gives the additional cost that can be incurred on the unit area of the collector for changes in F_R , U_L and $(\tau\alpha)_n$, resulting in a change of ΔF in the annual load fraction, yet yielding the same life-cycle savings as that of a reference system.

5. EXAMPLE

Comparison of a tube and fin type of collector with a non-selective absorber and single glass cover has been made with a collector of the same geometry, but coated with two types of selective coatings. Values of the parameters are as follows:

	F_R	U_L ($W/m^2\text{ }^\circ\text{C}$)	$(\tau\alpha)_n$	$\frac{\dot{m}C_p}{A_c}$ ($W/m^2\text{ }^\circ\text{C}$)
1. Non-selective	0.78	8	0.83	63
2. Selective 1	0.89	4	0.80	63
3. Selective 2	0.89	4	0.75	63

$\Delta F/F_1$ has been calculated for $F_1 = 0.25, 0.5$ and 0.75 . For three locations (Albuquerque, Los Angeles and Madison) the total fractional change in load fraction is the algebraic sum of its fractional changes due to F_R , U_L and $(\tau\alpha)_n$. Using eqn (18), ΔC_A has been calculated and results are summarized in Table 2. These results show that the additional cost of a collector with a selective absorber can be as high as $\$118/m^2$ at a location like Los Angeles, provided the reduction in $(\tau\alpha)_n$ due to selectivity is 0.03 only (for a reduction of 0.08, the cost differential is $\$72/m^2$). Table 2 also shows that the cost differential drastically reduces, if the reference system meets a high load fraction. In Madison, sacrificing 0.08 in $(\tau\alpha)_n$, on a reference system meeting a load fraction of 0.75, gives a benefit of only $\$24/m^2$, though the loss coefficient has drastically reduced.

6. CONCLUSIONS

The annual model proposed for space heating systems predicts the load fraction with tolerable errors and offers the following advantages: (i) the constants in the annual model can be predicted with a minimum of meteorological information; annual

horizontal radiation, annual average ambient temperature, annual heating load and data for the coldest month and (ii) the constants can be explicitly evaluated when any of the collector parameters change.

Even though the model predicts the annual load fraction with an rms error of 4 percent, the changes in load fraction are predicted very well. Thus, the methodology can be employed to evaluate sensitivity of load fraction to uncertainty in collector parameters.

Changes in collector parameters lead to large differences in additional cost that can be incurred per unit area of the collector from location to location. Thus collector parameter targets and the associated cost per unit area of the collector is highly location dependent.

Acknowledgement—V. V. Satyamurty expresses his gratitude to Professor J. A. Duffie for excellent encouragement during the course of present investigation. The authors thank Professor S. A. Klein for his critical review of the manuscript.

NOMENCLATURE

- a constant in the annual model, W/m^2C
- A_c area of the collector, m^2
- b constant in the annual model
- B value of $(UA)_h/A_c$ at $F = 1$, given by eqn (15)
- C_A collector area dependent cost $\$/m^2$
- C_E fixed cost of the system, $\$$
- C_F cost of auxiliary energy, $\$/GJ$
- DD annual degree days, C day
- DD monthly average of annual degree days ($= DD/12$), C day
- DD_c degree days in the coldest month C day
- DD_m maximum degree days in a particular month in the year, C day
- F annual load fraction met by the solar system
- F_1 annual load fraction met by the reference system
- F_R collector heat removal factor
- \bar{H}_c monthly average daily radiation on a horizontal surface in the coldest month, kJ/m^2
- \bar{H}_T monthly average daily radiation incident on the collector surface, J/m^2
- H_{YT} total annual radiation incident on the collector surface, J/m^2
- H_Y total annual radiation on a horizontal surface, MJ/m^2
- L monthly heating load, J ; also annual heating load, GJ
- LCS life cycle savings
- P_1 an economic parameter, ratio of life cycle fuel cost savings to the first year fuel cost savings

- P_2 an economic parameter, ratio of life cycle expenditure incurred because of the additional capital investment to the initial investment
- $\frac{s}{T_a}$ slope of the collector
- \bar{T}_a monthly average ambient temperature, C
- \bar{T}_{ac} monthly average ambient temperature in the coldest month, C
- \bar{T}_{ay} annual average ambient temperature, C
- $(UA)_h$ building heat loss factor, W/C
- U_L collector overall heat loss coefficient, W/m²C

Greek Symbols

- Δa change in a
- Δb change in b
- ΔC_a change in collector area dependent cost, \$/m²
- ΔF change in load fraction
- ΔF_R change in heat removal factor
- ΔU_L change in overall heat loss coefficient
- $\Delta(\tau\alpha)_n$ change in transmittance-absorptance product at normal incidence
- Δt_y number of seconds in a year
- ϵC_{\min} heat exchanger effectiveness-minimum capacity product
- ϕ latitude of the location
- $(\tau\alpha)$ effective (energy weighted) yearly transmittance-absorptance product
- $(\tau\alpha)_n$ transmittance-absorptance product at normal incidence

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