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AN EFFECTIVENESS MODEL OF LIQUID-DESICCANT SYSTEM HEAT/MASS EXCHANGERS

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Abstract—This paper describes a computationally efficient model for packed-bed, liquid-desiccant heat and mass exchangers. The model is derived from an effectiveness model of a cooling tower. The paper includes the development and derivation of the effectiveness model, comparisons with experimental data and with a finite-difference model.

1. INTRODUCTION

Air conditioning is traditionally accomplished with vapor-compression equipment. A disadvantage of vapor-compression conditioning is that the air must be cooled below its dewpoint in order to provide dehumidification. As a result, the conditioning equipment must normally operate at a colder temperature than is necessary to meet the sensible part of the load. Hybrid liquid-desiccant air-conditioning systems, which separate the sensible and latent loads, have been proposed as alternatives to the traditional systems. A liquid-desiccant system uses low-grade heat, such as that provided by solar collectors or a waste heat stream, to provide dehumidification, and can possibly provide cooling at a lower cost. Additional advantages are more precise humidity control and possible bactericidal effects.

Computer simulations can be used to study the performance of liquid-desiccant systems relative to traditional systems for the expected range of operating conditions. Computer simulations rely on models of the system components, which are evaluated thousands of times in a yearly simulation. As a result, an important characteristic of such models is their calculational efficiency.

Several types of computer models exist for liquid-desiccant heat and mass exchangers. Finite-difference models[1,2] require few assumptions, but involve extensive calculations. Empirical models[3] have been formulated based on experimental data, but they are limited to the equipment and range of conditions for which the data were taken. A manufacturer[4] provides a computationally simple model, but it relies on a factor that depends (in some unknown fashion) on the mass flow rates and the size of the heat/mass exchanger.

A computationally simple effectiveness model has recently been developed for cooling towers[5]. Because of the combined heat and mass transfer, liquid-desiccant heat and mass exchangers are analogous to cooling towers. In this paper, the cooling tower effectiveness model is modified so as to be applicable to liquid-desiccant components. This liquid-desiccant model is shown to compare well with a finite-difference model, as well as with experimental data.

2. DERIVATION

The liquid-desiccant chamber is filled with packing material. Solution drips down from the top, wetting the packing material, while air is blown through from the bottom in a counter current arrangement. Heat transfer occurs because of a temperature difference between the air and the desiccant solution, while mass transfer results because the water vapor pressure above the solution differs from the partial pressure of the water vapor in the air.

A schematic of the liquid-desiccant chamber is shown in Figure 1 along with the nomenclature for a differential element. Energy and water balances on the differential element can be written:

$$\dot{m}_s dh_s + h_s d\dot{m}_s = \dot{m}_a dh_a \tag{1}$$

$$d\dot{m}_s = \dot{m}_a d\omega_a \tag{2}$$

Integrating Equation 2 from the bottom of the element to the top of the chamber results in:

$$\dot{m}_s = \dot{m}_{s,i} - \dot{m}_a(\omega_{a,0} - \omega_a) \tag{3}$$

A volumetric heat transfer coefficient, h_C, based on heat transfer surface area per unit volume, A_V, can be defined with an air-side heat transfer equation:

$$\dot{m}_a dh_a = h_C A_V dV(T_s - T_a) + h_{v,T_s}\dot{m}_a d\omega_a \tag{4}$$

where the enthalpy of water vapor at solution temperature, T_s, is the sum of the vapor enthalpy at 0°C and the product of the vapor specific heat and the solution temperature [°C].

$$h_{v,T_s} = h_{v,O} + c_{p,v}T_s \tag{5}$$

The mass transfer coefficient, h_D, is defined by an air-side mass transfer equation:

$$\dot{m}_a d\omega_a = h_D A_V dV(\omega_{T_s,\text{sat}} - \omega_a) \tag{6}$$

The reference states where enthalpy is equal to zero are taken to be dry air at 0°C and liquid water at 0°C. The enthalpy of the desiccant solution is the

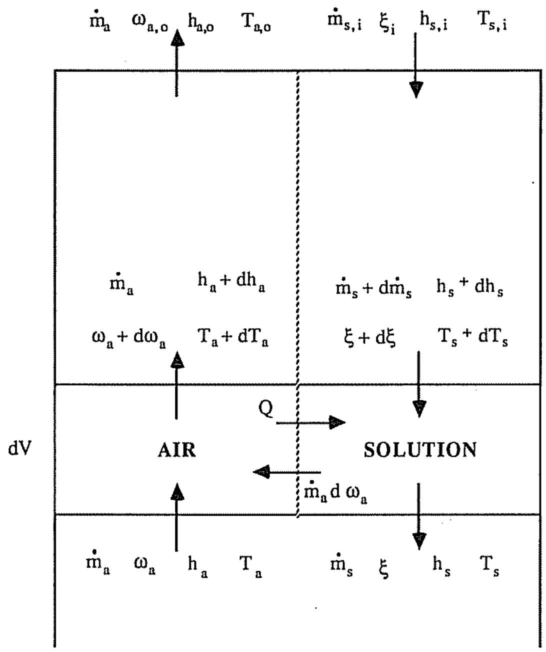

Fig. 1. Schematic of liquid-desiccant chamber used to derive effectiveness model.

sum of the solution enthalpy at its concentration, ξ, and 0°C and the product of the solution specific heat and temperature.

$$h_s = c_{p,s}T_s + h_{O,\xi} \tag{7}$$

$$dh_s = c_{p,s}dT_s \tag{8}$$

The enthalpy of moist air is the sum of two products: the moist air specific heat and air temperature, and the humidity ratio and the enthalpy of water vapor at 0°C.

$$h_a = c_{p,m}T_a + \omega_a h_{v,O} \tag{9}$$

The Lewis number and NTU are defined:

$$\mathrm{Le} = h_C/h_D c_{p,m} \tag{10}$$

$$\mathrm{NTU} = \frac{h_D A_V V_T}{\dot{m}_a} \tag{11}$$

The preceding relations and definitions form the basis of the heat/mass exchanger model. However the presentation is simplified by the following algebraic manipulations.

Substitute eqn (2) into (1) to yield:

$$dh_s = \frac{1}{\dot{m}_s}(\dot{m}_a dh_a - h_s \dot{m}_a d\omega_a) \tag{12}$$

Combine eqns (3) and (12):

$$dh_s = \frac{\dot{m}_a dh_a - h_s \dot{m}_a d\omega_a}{\dot{m}_{s,i} - \dot{m}_a(\omega_{a,o} - \omega_a)} \tag{13}$$

Divide the numerator and denominator of eqn (13) by the air mass flow rate:

$$dh_s = \frac{dh_a - h_s d\omega_a}{\dot{m}_{s,i}/\dot{m}_a - (\omega_{a,o} - \omega_a)} \tag{14}$$

Combine Equations 4, 6, 10 and simplify to give:

$$\dot{m}_a dh_a = h_D A_V dV[c_{p,m}\mathrm{Le}(T_s - T_a) + h_{v,T_s}(\omega_{T_s,\mathrm{sat}} - \omega_a)] \tag{15}$$

Substitute eqn (8) into (14):

$$dT_s = \frac{1}{c_{p,s}}\left(\frac{dh_a - c_{p,s}T_s d\omega_a}{\dot{m}_{s,i}/\dot{m}_a - (\omega_{a,o} - \omega_a)}\right) \tag{16}$$

An algebraic equivalent of eqn (15) can be written[6]:

$$\dot{m}_a dh_a = h_D A_V dV \mathrm{Le}\left[(h_{T_s,\mathrm{sat}} - h_a) + \left(\frac{1}{Le} - 1\right)h_{v,s}(\omega_{T_s,\mathrm{sat}} - \omega_a)\right] \tag{17}$$

Combining both eqns (17) and (6) with the definition of NTU:

$$\frac{dh_a}{dV} = \frac{\mathrm{NTU\ Le}}{V_T}\left[(h_{T_s,\mathrm{sat}} - h_a) + \left(\frac{1}{\mathrm{Le}} - 1\right)h_{v,s}(\omega_{T_s,\mathrm{sat}} - \omega_a)\right] \tag{18}$$

$$\frac{d\omega_a}{dV} = \frac{\mathrm{NTU}}{V_T}(\omega_{T_s,\mathrm{sat}} - \omega_a) \tag{19}$$

The heat/mass transfer processes in the conditioner and regenerator of a liquid desiccant unit are similar to those occurring in a cooling tower. In his analysis of a cooling tower, Merkel[7] suggests that the change in the liquid mass flow rate in the chamber can be neglected and that the Lewis number can be assumed to be approximately equal to one. With these additional assumptions eqns (16) and (18) become

$$\frac{dT_s}{dV} = \frac{1}{c_{p,s}}\left(\frac{(dh_a/dV)\dot{m}_a}{\dot{m}_{s,i}}\right) \tag{20}$$

$$\frac{dh_a}{dV} = \frac{\mathrm{NTU}}{V_T}(h_{T_s,\mathrm{sat}} - h_a) \tag{21}$$

A saturation specific heat can be defined as the derivative of the saturated air enthalpy with respect to temperature at solution conditions:

$$C_{\mathrm{sat}} = \frac{dh_{T_s,\mathrm{sat}}}{dT_s} \tag{22}$$

A capacitance ratio, m^*, can be defined analogous to the capacitance ratio used in sensible heat exchangers:

$$m^* = \frac{\dot{m}_a C_{\text{sat}}}{\dot{m}_{s,i} c_{p,s}} \qquad (23)$$

With these definitions, Equation 20 becomes:

$$\frac{dh_{T_s,\text{sat}}}{dV} = m^* \frac{dh_a}{dV} \qquad (24)$$

If C_{sat} is assumed to be constant over the operating conditions of the chamber, eqns (21)–(24) are exactly analogous to those for a sensible heat exchanger. A solution for these equations can be expressed in terms of the familiar heat exchanger counterflow effectiveness relations:

$$\epsilon = \frac{1 - e^{-\text{NTU}(1-m^*)}}{1 - m^* e^{-\text{NTU}(1-m^*)}} \qquad (25)$$

The solution of Equations 21 and 24 for the air outlet enthalpy gives:

$$h_{a,o} = h_{a,i} + \epsilon(h_{T_s,\text{sat}} - h_{a,i}) \qquad (26)$$

In order to determine the outlet humidity ratio, an 'effective' heat and mass transfer process is assumed in which the solution stream is at a constant 'effective' temperature that gives the correct air outlet enthalpy. With this assumption, integration of eqn (21) yields an 'effective' saturation enthalpy of

$$h_{T_s,\text{sat,eff}} = h_{a,i} + \frac{h_{a,o} - h_{a,i}}{1 - e^{-\text{NTU}}} \qquad (27)$$

Using this enthalpy value along with the condition of saturation fixes a corresponding effective humidity ratio, $\omega_{T_s,\text{sat,eff}}$. Equation 19 can then be integrated to find the outlet air himidity ratio:

$$\omega_{a,o} = \omega_{T_s,\text{sat,eff}} + (\omega_{a,i} - \omega_{T_s,\text{sat,eff}})e^{-\text{NTU}} \qquad (28)$$

The steps for solving for the outlet states of a liquid-desiccant chamber using the effectiveness model are

1. Determine the value of NTU for the system and conditions (eqn 11).
2. Calculate the saturation specific heat for the range of conditions expected (eqn 22).
3. Calculate the capacitance ratio, m^* (eqn 23).
4. Calculate the effectiveness (eqn 25).
5. Calculate the air outlet enthalpy (eqn 26).
6. Use an energy balance to calculate the solution outlet enthalpy.
7. Find the effective saturation enthalpy (eqn 27).
8. Use this enthalpy and a saturated condition to find the effective saturation humidity ratio ($\omega_{T_s,\text{sat,eff}}$)
9. Find the air outlet humidity ratio (eqn 28)
10. Use mass balances and the known states to calculate solution outlet flow rate, concentration and temperature, and air outlet temperature

The derivation presented above assumes Lewis number to be unity. However, the effect of non-unity values of Lewis number can be approximately considered by defining NTU* to be the product of Le and NTU for use in eqn (18). The term (1/Le − 1) must be dropped for simplification but the effect of Lewis number in this term is small. A similar adjustment was used by Jefferson[8] in assuming an effective Nusselt number that included the effects of the Lewis number. NTU* replaces NTU in eqns (25) and (27). The use of a non-unity Lewis number in the effectiveness model is demonstrated in the comparison with experimental data in section 4.

3. COMPARISONS WITH A FINITE-DIFFERENCE SOLUTION

The finite-difference model used for comparison is derived from the basic differential equations that govern heat and mass transfer in the liquid-desiccant chamber, combined with mass and energy balances. The basic equations follow those developed by Factor and Grossman[2]. The differential equations were numerically integrated starting from the air-inlet side of the chamber. Since the solution condition is not known at the position of the air inlet, guess values of the temperature and concentration at this point must be assumed. Iteration proceeds until the calculated solution inlet conditions match the actual inlet state. The finite-difference model depends on both NTU and Le as input parameters. A Lewis number of one was assumed in the following comparison between the finite-difference model and effectiveness model.

The main objective of the comparison of these two models is to investigate the validity of the assumptions of (a) linear variation of saturation enthalpy with temperature; and (b) neglecting the water loss term from the solution energy balance. These are the only assumptions in the effectiveness model made in addition to those of the finite-difference model. In this comparison, 16 cases were examined, with the following parameters varied from low to high values: air inlet temperature (15 to 35°C), solution inlet temperature (15 to 35°C), air inlet humidity ratio (0.001 to 0.01 at 15°C and 0.003 to 0.03 at 35°C), and solution concentration (10 to 40%). In each comparison, NTU is varied from 0.01 to 10.0. The mass flow rate ratio (air flow rate/solution flow rate) was held at 0.5. Increasing this value required additional interations in the finite-difference model but did not result in any additional error between the two models. Figures 2 and 3 show typical results. These figures show that there is no appreciable difference between the results of the two models. Similar comparisons for non-unity Lewis numbers are presented in [6] which indicate that the Lewis number effect can be accounted for in the effectiveness model.

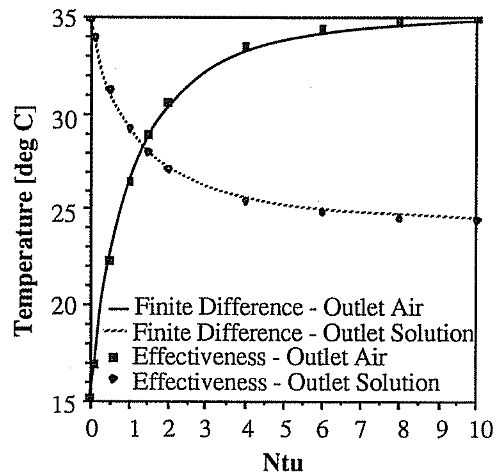

Fig. 2. Comparison of the outlet air and solution temperature calculated by the finite difference and effectiveness models (T_{ai} = 15°C, $T_{s,i}$ = 35°C, $\omega_{a,i}$ = 0.01, LiCl concentration = 0.10 lb/lb solution).

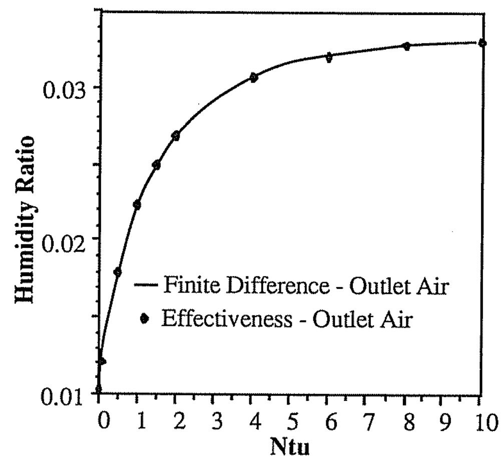

Fig. 3. Comparison of the outlet air humidities calculated by the finite difference and effectiveness models. (Same inlet conditions as for Fig. 2).

The non-linearity of saturation enthalpy with temperature increases with increasing temperature. At high temperatures, such as those which may occur in regenerators, or in situations in which there are large temperature differences between the inlet and outlet of the heat and mass exchanger, it may be necessary to repeat steps 3–10 of the calculation procedure with an improved estimate of the saturation specific heat based on the previously calculated outlet conditions. Even if an iterative solution is employed, however, assumption of a constant value of saturation specific heat will introduce some error. One approach for reducing this error is to divide the heat and mass exchanger into two or more sections and apply the effectiveness model to each section. Another approach involves choosing a better linearization than a straight line between the inlet and outlet conditions. These correction procedures are considered for cooling tower applications by Braun[6].

4. COMPARISONS WITH EXPERIMENTAL DATA

Data from the Science Museum of Virginia (SMVA) were received from the TVA monitoring program[4,9]. There were inconsistencies in much of

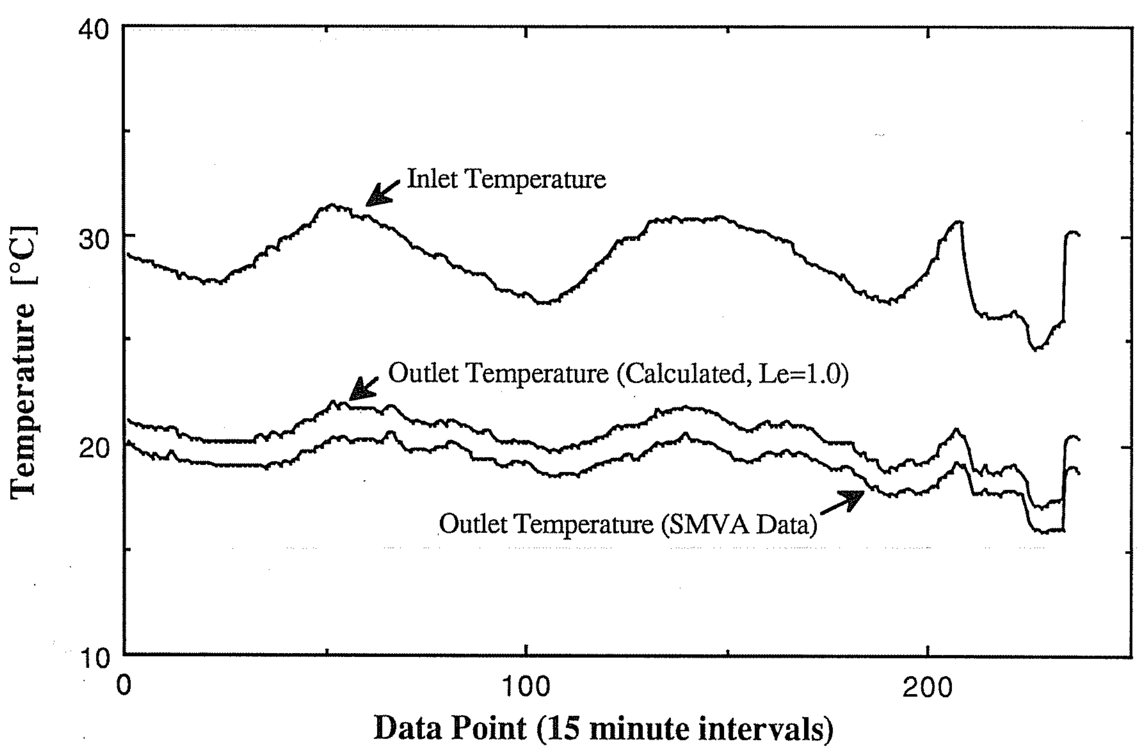

Fig. 4. Comparison of SMVA air outlet temperatures with calculated values with the effectiveness model for conditions of July 10–12, 1987 (Le = 1.0).

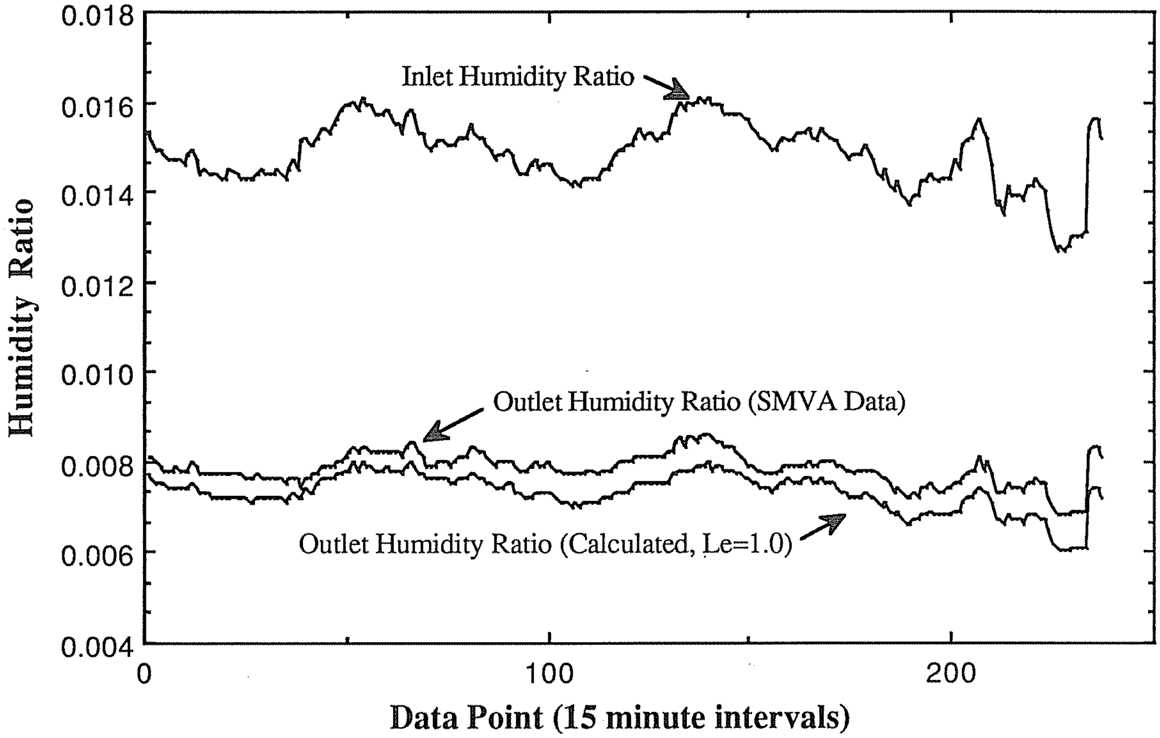

Fig. 5. Comparison of SMVA air outlet humidities with calculated values with the effectiveness model for conditions of July 10–12, 1987 (Le = 1.0).

these data. However, the measurements of the conditioner (dehumidifier) performance for July 10–12, 1987 have approximate closure on mass and energy balances. An average air side NTU was calculated from these data to be 1.86. This value of NTU was then used in the effectiveness model to compare its results with the experimental data.

Figures 4 and 5 show the comparison of effectiveness model results with SMVA data. The overall agreement with SMVA data is not as close as with

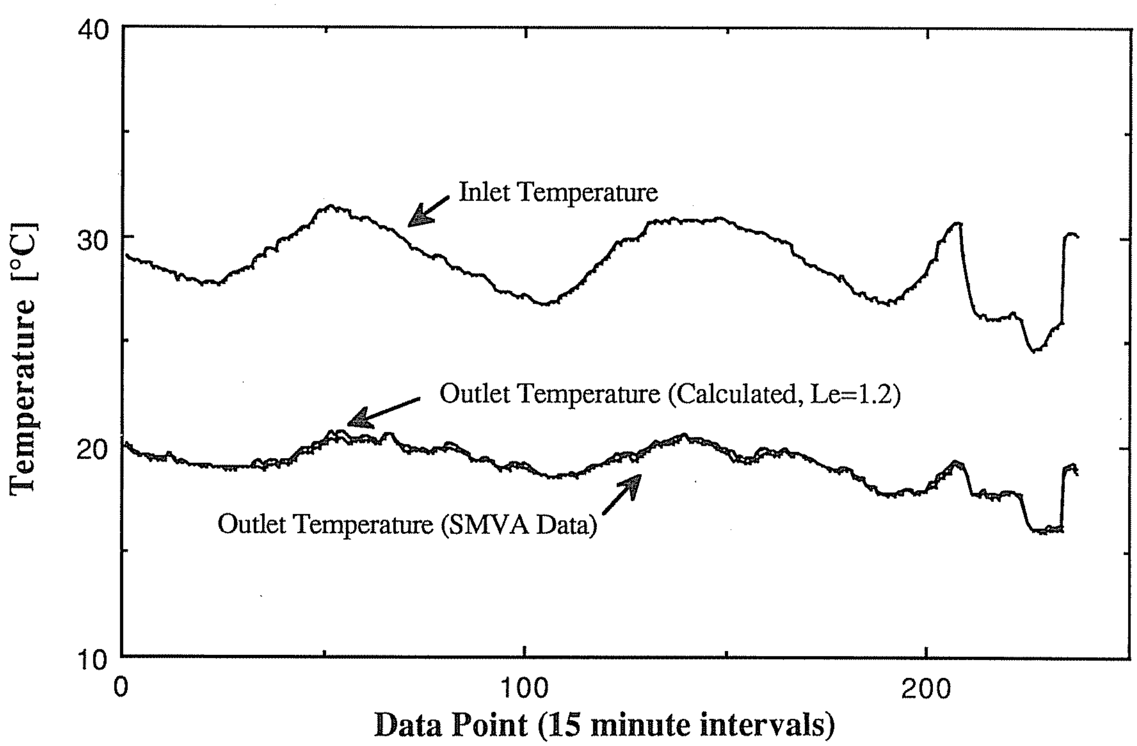

Fig. 6. Comparison of SMVA air outlet temperatures with calculated values with the effectiveness model for conditions of July 10–2, 1987 (Le = 1.2).

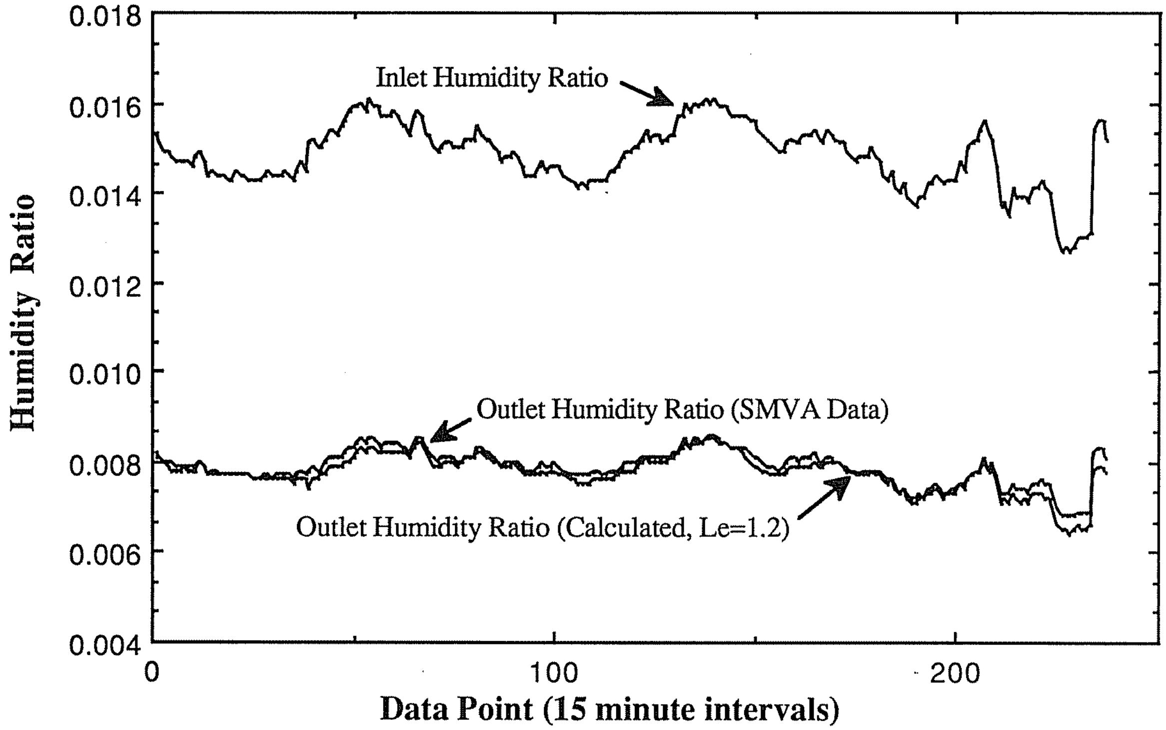

Fig. 7. Comparison of SMVA air outlet temperatures with calculated values with the effectiveness model for conditions of July 10–12, 1987 (Le = 1.2).

the finite-difference model. The enthalpy changes of both the air and solution given by the effectiveness model match the SMVA data, indicating that the average value of NTU chosen is applicable over the range of data considered. The calculated and measured solution temperature changes are almost precisely the same. However, the calculated air outlet temperature and outlet humidity ratio do not agree precisely with SMVA data. The disparity is probably a result of several factors. Based on the frequency of the violations of the first and second laws of thermodynamics in some of the SMVA data, the disagreement may be due to experiment error. Some of the difference may also be explained by the use of a unity value for the Lewis number. In his analysis of the SMVA conditioner, Buschulte[1] used a curve fit to determine a Lewis number of 1.2. Figures 6 and 7 show that the use of this value of Lewis number gives almost perfect agreement with the experimental data.

Lenz, *et al.*[3,11], at Colorado State University (CSU) have prodouced experimental liquid desiccant data using LiBr salt solution. A set of 7 data points were used in this comparison. Table 1 shows these points and the effectiveness model results for the same inlet conditions and effectiveness calculated at each point. The last column shows the percent difference between the total change in enthalpy of the solution and the total change in enthalpy of the air for the experimental data. The change in air temperature is so low for the first four points that any comparison in results is inconclusive. For the last three data points, the effectiveness model underpredicts air temperature change by about 20%. The calculated outlet air humidity ratios are slightly off, with some points low,

Table 1. Comparison of effectiveness model with 1987 CSU liquid-desiccant data

	Air Temperatures [deg C]			Air Humidity Ratios [kg/kg dry air]			Solution Temperatures [deg C]			[% error]
Calculated Effectiveness	inlet	outlet model	outlet CSU	inlet	outlet model	outlet CSU	inlet	outlet model	outlet CSU	energy balance
0.282	33.0	32.3	35.3	0.0185	0.0142	0.0130	27.8	34.3	35.9	19.65
0.293	31.4	31.0	33.7	0.0195	0.0158	0.0147	28.0	33.0	33.9	16.03
0.290	34.2	32.9	33.9	0.0194	0.0160	0.0156	28.1	33.1	34.2	17.69
0.380	35.4	34.8	35.7	0.0187	0.0153	0.0149	32.0	36.6	36.7	1.06
0.265	35.8	33.7	32.5	0.0152	0.0138	0.0143	28.0	31.5	31.9	9.50
0.271	36.1	33.8	32.2	0.0148	0.0135	0.0141	27.7	31.3	31.8	11.52
0.385	36.1	32.7	31.5	0.0162	0.0144	0.0149	27.7	31.4	32.5	23.08

and some high. Solution outlet temperatures come close to matching. The CSU data show 1 to 23% lack of closure on energy balances. Further data from CSU are required to draw any major conclusions.

5. CONCLUSIONS

An effectiveness model of liquid-desiccant heat and mass exchangers has been developed. The resulting equations are exactly analogous to effectiveness equations for sensible (or dry) heat exchangers. The assumptions critical to its derivation that differ from more detailed models are Lewis number of 1; a linear relationship for saturation enthalpy with temperature; and negligible change in solution flow rate. However, the effects of Lewis number not equal to 1 may be considered in an approximate manner. Comparisons of the effectiveness model with a finite-difference model show excellent agreement. Comparisons with available experimental data show reasonable agreement, with differences believed to be due to experimental error. A Lewis number of 1.2 best compares with the data from the Science Museum of Virginia. The effectiveness model can be used in simulations to predict the performance of a hybrid liquid-desiccant air-conditioning system.

NOMENCLATURE

- A_v transfer area/unit volume (1/m)
- c_p specific heat (kJ/kg–°C)
- C_{sat} saturation specific heat (kJ/kg–°C)
- ϵ air side effectiveness
- h specific enthalpy (kJ/kg)
- h_D mass transfer coefficient (kg/s–m^2–$\Delta\omega$)
- h_C heat transfer coefficient (kW/m^2–°C)
- Le Lewis number
- $\dot{m}$ mass flow rate (kg/s)
- m^* capacitance ratio
- NTU number of transfer units
- T temperature (°C)
- V_T packing material volume (m^3)
- ω humidity ratio (kg/kg dry air)
- ξ solution concentration (weight fraction)

Subscripts

- a air
- eff effective value
- i inlet
- m moist air (used as subscript to specific heat)
- o outlet
- s solution
- $T_{s,sat}$ air in equilibrium with solution at solution temperature
- T_s at solution temperature
- v water vapor
- 0 at 0 degrees C.

REFERENCES

1. T. K. Buschulte, "Analysis of Hybrid Liquid-Desiccant Systems," M.S. Thesis in Chemical Engineering, University of Wisconsin–Madison (1984).
2. H. M. Factor, and G. Grossman, A packed bed dehumidifier/regenerator for solar air conditioning with liquid desiccants, *Solar Energy*, **24**, 541–550 (1980).
3. T. G. Lentz, et al., "Open Cycle Absorption Cooling Studies," DOE report SAN-11927-43 (1986).
4. G. Meckler, "Data Collection and Model Development of Liquid-Desiccant Integrated HVAC System," Science Museum of Virginia, Richmond, VA (1984,1985).
5. J. E. Braun, "Methodologies for the Design and Control of Chilled Water Systems," Ph.D. Thesis in Mechanical Engineering, University of Wisconsin–Madison (1988).
6. D. I. Stevens, "Analysis of Liquid-Desiccant Systems and Component Modelling," M.S. Thesis in Mechanical Engineering, University of Wisconsin–Madison (1988).
7. F. Merkel, "Verdunstungskuehlung," VDI Forschungsarbeiten, No. 275, Berlin (1925).
8. C. P. Jeffreson, "Prediction of Breakthrough Curves in Packed Beds," AIChE Journal, **18**(2), 409 (1972).
9. D. J. Chaffin, Tennessee Valley Authority, personal communication (1988).
10. J. L. Threlkeld, Thermal Environmental Engineering, Prentice Hall, New York, Second Edition (1970).
11. T. G. Lenz, Colorado State University, personal communication (1987).