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# Simplified Dimensionless Relations for Heat Loss from Basements

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## ABSTRACT

A method is developed for calculating the monthly and annual values of heat loss from basement walls and floors. The overall wall and floor conductances are calculated first. These are determined from a two-dimensional finite element analysis. The numerical values from this analysis are correlated in terms of the governing nondimensional parameters. These conductance values are then combined with a representative ground temperature based on local ambient temperature profiles. The results of this approach compare favorably with those in the ASHRAE Handbook-1981 Fundamentals and more recent studies. The correlations presented are suitable for use with programmable calculators or microcomputers.

## INTRODUCTION

The determination of the proper insulation level for basements has become increasingly important. In the past, the heat loss from basements was considered negligible, and insulation of other parts of the house was considered more important. As a result, the portion of the total heat loss from a house due to the basement has become larger. A need exists for a rapid and accurate method for calculating the amount of heat loss from basements.

The advent of the programmable calculator and the affordable microcomputer has made it possible for architects and engineers to perform energy and heat-loss calculations that were virtually impossible only a few years ago. With this new hardware has come a demand for simplified but accurate algorithms for energy calculations that may readily be programmed. Equations and correlations are preferable to tabular values.

Although well-established and simplified methods exist for calculating the heat loss from above-ground structures (ASHRAE 1981), determining the heat loss from a basement or underground structure is somewhat more complicated. This heat loss is a three-dimensional phenomenon, with heat conducted radially away from the basement and vertically toward the ground and deep soil. Several two-dimensional methods have been developed to approximate the loss from basements. These include the circular heat-flow path method (Latta and Boileau 1969), the double heat-flow path method (Wang 1979), two-dimensional finite element or finite difference programs (Wang 1979; Shipp and Broderick n.d.), and, more recently, a shape-factor method (Mitalas 1983). The heat-flow path methods and the shape-factor method are time consuming and tedious when parameters are being varied. The numerical techniques require extensive computer facilities that are not usually available to architects and engineers. The shape-factor method of Mitalas (1983) is accurate, but it requires considerable time to calculate yearly loss. Furthermore, it is restrictive with respect to the relationship between floor and wall insulation thickness and the soil thermal conductivity allowed. This paper presents a method of calculating basement heat loss in which correlations are developed in terms of the governing nondimensional parameters. These closely approximate the results of two-dimensional finite element calculation programs but do not require extensive computational time or effort.

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## METHOD OF ANALYSIS

The method of calculating heat flow rate from a basement is based on the following conduction equation:

$$q = U_f A_f (T_b - T_{gf}) + U_w A_w (T_b - T_{gw}) \tag{1}$$

where q is the basement heat loss, U is the basement wall or floor and overall ground conductance, A is the basement wall or floor area, $T_b$ is the basement space temperature, and $T_g$ is an effective ground temperature. In order to calculate q, all of the variables on the right-hand side of equation 1 must be obtained. The areas are readily determined, and the space temperature is either known or determined in terms of the house temperature. The overall wall and floor conductances and the effective ground temperature must be established. In the method developed here, the conductances are defined as the steady-state values for the basement-ground-ambient air combination. The effective ground temperature accounts for energy storage in the ground and seasonal temperature variations. It is the temperature that gives the actual heat flow when combined with the steady-state conductance values. The advantage of this approach is that the steady-state conductances are readily calculated in terms of governing parameters.

The overall heat-transfer coefficients and the effective ground temperature were calculated using a finite element conduction program (Myers 1978). Values of U and $T_g$ generated by this program were correlated as functions of readily available physical parameters in nondimensional form. The next two sections describe how values of U and $T_g$ were generated and tabulated.

### Calculation of U

In order to calculate conductances for the floor and walls of the basement, an array of elements was set up as shown in figure 1. The right-hand and bottom elements along the boundary were specified to be at a constant temperature, $T_c$. The conductance, U, does not depend on temperatures, and an arbitrary constant temperature, $T_c$, is used for calculating these steady-state conductances only. The conductances calculated in this manner will then be used with the effective ambient and ground temperatures in order to calculate heat loss.

Convective boundary conditions were established between the ground surface and the ambient at $T_c$ and between the inside of the basement and the basement air at $T_b$. The addition of wall or floor insulation is treated as an additional thermal resistance on the basement floor or walls. This assumes that insulation has negligible capacitance and that little vertical heat flow occurs within the insulation. The walls were taken as conventional concrete walls. The steady state heat flow, $q_{ss}$, was computed using the finite element program. The conductances $U_w$ and $U_f$ were then calculated from

$$U_w = \frac{q_{ss,w}}{A_w(T_b - T_c)} \tag{2}$$

$$U_f = \frac{q_{ss,f}}{A_f(T_b - T_c)} \tag{3}$$

Values of $U_w$ and $U_f$ were determined over a wide range of floor and wall insulation values. These results were expressed in terms of dimensionless resistance and geometry ratios as

$$\frac{U_w D}{k_s} = f\left(\frac{D}{R_w k_s}, \frac{D}{R_f k_s}\right) \tag{4}$$

$$\frac{U_f D}{k_s} = f\left(\frac{D}{R_w k_s}, \frac{D}{R_f k_s}, \frac{W}{D}\right) \tag{5}$$

The wall and floor resistances ($R_w$ and $R_f$) are the sums of the thermal resistances of the concrete wall, insulation, wall coverings, and inside film coefficient. The ratios ($U_wD/k_s$) and $U_fD/k_s$) are nondimensional overall wall and floor conductances, respectively, and include ground resistance. The ratios ($D/R_wk_s$) and ($D/R_fk_s$) are nondimensional wall and floor resistances, respectively, and do not include the ground resistance. The ratio (W/D) is a geometric ratio for the basement width and depth. These five nondimensional parameters allow convenient generalization of these results to a wide range of situations. A least squares Box algorithm (Ambrose 1970) was used to determine the appropriate functional relationships between the groups.

## Calculation of $T_g$

As previously discussed, $T_g$ is an effective ground temperature with no readily apparent physical meaning. It is that ground temperature that gives the correct heat flow and it depends on the seasonal ambient temperature profile. If the soil had no capacitance, $T_g$ would always be equal to the current ambient temperature, and the heat flow at any time could be calculated directly using equation 1. However, the soil does have capacitance and, therefore, both the current ambient temperature and the ambient temperature history affect the value of $T_g$.

It was decided to describe the ambient temperature profile in terms of a few readily available parameters. The long-term daily average temperatures are known to closely follow a sine wave pattern, with the mean value of the sine wave equal to the yearly average temperature. For different locations, the mean and the amplitude will vary, but the pattern formed by the long-term daily average temperatures may always be approximated as a sine wave. This allows the pattern of daily average temperatures to be predicted for any location. The only parameters required to describe such an ambient temperature curve are the yearly average temperature (the mean of the sine wave), the highest and lowest long-term daily average temperatures (the amplitude of the sine wave), and the phase angle (the day of the year that the sine wave weather pattern first exceeds the mean value). The period of any long-term daily average temperature curve is always one year.

The expected temperature curve of daily average temperatures can therefore be obtained. However, diurnal variations in temperature may have to be taken in to account. Erbs (et al. 1983) found that the average diurnal variation in ambient temperature can be expressed in terms of a series of sinusoids. The average is the daily average value for the month, and the amplitude depends on the amount of solar radiation striking the ground surface.

The effective temperature was found using the sinusoidal variation of diurnal and daily averge temperatures in the finite element program to calculate heat flow. It was found that the daily heat flows, using a constant daily average temperature, were essentially the same as those computed using the diurnal temperature variation. Therefore, daily heat-flow rates were calculated using a constant daily average temperature for each day. Daily heat flows were computed for each day of the year. The effective ground temperature for each day of the year was then computed using

$$T_{g,w} = T_b - \frac{q_{d,w}}{U_w A_w} \qquad (6)$$

$$T_{g,f} = T_b - \frac{q_{d,f}}{U_f A_f} \qquad (7)$$

where $T_{g,w}$ and $T_{g,f}$ are the daily effective ground temperatures for the basement wall and floor, and $q_{d,w}$ and $q_{d,f}$ are the average daily heat-flow rates for the basement wall and floor computed from the simulations.

The effective ground temperature calculated in this manner varies sinusoidally over the course of a year. It is written as

$$T_g = \overline{T}_a + B_g \sin\left(\frac{(n-n_i)(360)}{365} - \phi\right) \qquad (8)$$

where $\bar{T}_a$ is the yearly average temperature, $B_g$ is the amplitude of the effective ground temperature curve, $\phi$ is the phase lag between the effective ground temperature curve and the ambient temperature curve (in degrees), $n_i$ is the day number on which the ambient temperature curve crosses the mean value, and n is the day number on which the heat flow is to be calculated. The term $n_i$ reflects the fact that ambient temperature is out of phase with the Gregorian calendar, while $\phi$ reflects the phase difference between the ground temperature at any depth and ambient temperature.

The amplitude and phase lag are expressed as functions of the amplitude and phase lag of the daily average temperature curve and the Fourier modulus. The Fourier modulus is the nondimensional time and is defined as

$$Fo = \alpha\theta/D^2 \tag{9}$$

where $\theta$ is the period (one year), $\alpha$ is the soil thermal diffusivity, and D is the basement depth. The amplitudes and phase were found to be correlated as follows for the walls:

$$\phi_w = f(Fo) \tag{10}$$

$$(B_{g,w}/B_a) = f(Fo) \tag{11}$$

For the floor

$$\phi_f = f(Fo) \tag{12}$$

$$(B_{g,f}/B_a) = f(Fo) \tag{13}$$

where $B_a$ is the amplitude of the ambient temperature curve. This must be determined from the monthly values for a specific location.

Equation 8 together with equations 6 and 7 can be used to compute daily heat-flow rates; however, this often provides more information than is required. The effective ground temperature equation may be integrated to obtain the average effective ground temperature over any period of time. The integrated relation is

$$\bar{T}_g = \bar{T}_a - \frac{B_g(365)^2}{2\pi(n_2-n_1)360}\left[\cos\left(\frac{(n_2-n_i)}{365}\,360 - \phi\right) - \cos\left(\frac{(n_1-n_i)}{365}\,360 - \phi\right)\right] \tag{14}$$

where $n_2$ is the first day number after the time period of interest ends, and $n_1$ is the day number on which the time period of interest begins. The value of $n_i$ is found to be approximately 110 days for U.S. locations. Using the above method, the expected average heat-flow rate can be calculated for any period of time in any place.

## RESULTS

The nondimensional overall conductances for the wall and floor are given in figures 2 and 3, respectively, as functions of the corresponding nondimensional surface resistances. The results show that conductances decrease as the surface resistance increases, as expected.

The equation describing the wall conductance is

$$\frac{U_w D}{k_s} = C_1 \left(\frac{D}{R_w k_s}\right)^{C_2} + C_3 \tag{15}$$

<u>where</u> $C_1 = -0.23 \ \ln \left[\frac{D}{R_f k_s} + 0.0078\right] + 3.3$

$C_2 = 0.1584$

$C_3 = -2.568 + 0.176 \ \ln \left[\frac{D}{R_f k_s} + 0.0078\right]$

and for the floor

$$\frac{U_f D}{k_s} = \left[C_4 \left(\frac{D}{R_f k_s}\right)^{C_5} + C_6\right] f\left(\frac{W}{D}\right) \tag{16}$$

<u>where</u> $C_4 = 0.029 \ \ln\left(\frac{D}{R_w k_s} + 0.63\right) - 0.45$

$C_5 = -0.27$

$C_6 = -0.055 \ \ln \left(\frac{D}{R_w k_s} + 0.63\right) + 0.809$

and the f(W/D) function is

$$f\left(\frac{W}{D}\right) = C_7 \ \ln \left(\frac{D}{R_f k_s}\right) + C_8 \tag{17}$$

<u>where</u> $C_7 = 0.3764 \ (W/D)^{-1.02} - 0.0832$

$C_8 = \left(-0.0968 \ (W/D)^{-0.83} + 0.0298\right) \ln \left(\frac{D}{R_w k_s}\right) + 1.61 \left(\frac{W}{D}\right)^{-0.39} + 0.08$

These relations are valid for nondimensional floor and wall surface resistances ranging from 0.025 to 4.

Neither of the overall conductances was found to be stongly affected by the ambient conductance nor the depth to the water table. The wall conductance is not a function of the geometric relationship between floor and wall. However, the floor coefficient is a strong function of W/D. This relationship is expressed mathematically in equation 17 and is illustrated in figure 4.

The amplitude and phase lag of the effective ground temperature for the wall can be expressed as

$$\frac{B_{g,w}}{B_a} = -0.035 \ Fo^{-0.37} + 1.01 \tag{18}$$

$$\phi_w = 22. \ Fo^{-0.54} - 0.68 \tag{19}$$

and for the floor

$$\frac{B_{g,f}}{B_a} = -0.73 \ Fo^{-0.172} + 1.12 \tag{20}$$

$$\phi_f = 289. \ Fo^{-0.104} - 176 \tag{21}$$

The Fourier modulus is a function of the basement depth and the thermal diffusivity of the soil. As the thermal diffusivity of the soil increases (i.e., if the soil is wet), the Fourier modulus increases; therefore, the amplitude of the effective ground temperature increases and the lag decreases. Increasing the basement depth has the opposite effect, since the Fourier modulus decreases proportional to the inverse of the depth squared.

The relationship between the ambient temperature and the effective ground temperature is illustrated in figure 5 for Madison, WI, for an 8.2 ft (2.5 m) deep basement. The amplitude of the effective ground temperature for the floor is approximately 70% of that of the wall. This demonstrates the damping effect of the soil. Furthermore, the effective ground temperature of the floor lags that of the wall by about 60°. This lag is the result of the time required for ambient temperature variations to penetrate the 8.2 ft (2.5 m) of soil.

Figure 6 illustrates the results of a comparison of the monthly average basement heat loss by the method described here with the shape-factor method of Mitalas (1983). With the exception of the insulation levels, the basement is identical to that used by Mitalas. It is 30.2 ft x 27.9 ft x 5.74 ft (9.2 m x 8.5 m x 1.75 m) below grade, located in Ottawa, Ontario, Canada. The insulation levels correspond to type 25 of Mitalas. The total yearly energy loss for this basement and two others is tabulated in table 1. As shown in figure 6, the total monthly heat loss predicted by each of the two methods compares quite favorably, particularly during the crucial winter months. The predicted phase lags are virtually identical.

Both methods show an interaction between wall loss and floor insulation. As floor insulation level is decreased, obviously the energy loss through the floor increases. However, the wall energy loss decreases slightly, presumably because the increased floor loss warms the soil, particularly that near the bottom portion of the wall.

For any of the three cases shown in table 1, the predicted wall energy loss is very close. In fact, for floor insulation resistance 0.62 Btu/hr$\cdot$ft$^2\cdot$°F (3.52 m$^2\cdot$°K/W), wall, floor, and total energy losses are remarkably close. With decreasing floor insulation, the predicted values are not as close. The shape-factor method predicts a greater floor loss than does the dimensionless parameter method, particularly for the case of no added floor insulation.

## EXAMPLE

An example of a basement calculation will be presented and compared to one using the method of ASHRAE (1981). The basement is 7 ft (2.1 m) deep, 28 ft (8.54 m) wide, and 42 ft (12.8 m) long. The soil has a thermal conductivity of 0.8 Btu/hr$\cdot$ft$\cdot$°F (1.38 W/m$\cdot$°C) and a volumetric thermal capacity of 24 Btu/ft$^3\cdot$°F (1.6 MJ/m$^3\cdot$°C). The inside wall is insulated with one inch of insulation, and the overall R-value of the wall, including the surface films and the concrete wall, is 5. hr$\cdot$ft$^2\cdot$°F/Btu (0.88 m$^2\cdot$°C/W). The floor is bare, and the overall resistance due to concrete and surface film is 1.5 hr$\cdot$ft$^2\cdot$°F/Btu (0.26 m$^2\cdot$°C/W).

For Madison, as shown in figure 5, the annual average temperature is 45°F (7.3°C) and the amplitude $B_a$ is 25.2°F (14°C). The amplitude was obtained by plotting the monthly average temperature over the year, fitting a sine curve to the values, and determining the amplitude. The annual cycle initiates on April 20 ($n_1$ = 110 days); this appears to be common to most U.S. locations. The basement is heated at a temperature of 68°F (20°C). The energy loss in February will be determined.

The nondimensional wall resistances are:

$$\frac{D}{R_w k_s} = 1.75 \quad \text{and} \quad \frac{D}{R_f k_s} = 5.83$$

From equations 15 and 16, the overall conductances for wall and floor are:

$$\frac{U_w D}{k_s} = 0.905 \quad \text{and} \quad \frac{U_f D}{k_s} = 0.512$$

The corresponding overall conductance values become $U_w$ = 0.103 Btu/hr·ft$^2$·°F and 0. Btu/hr·ft$^2$·°F.

The Fourier modulus is $Fo = (\alpha\ \theta/D^2) = 5.96$

From equations 18 through 21, the wall and floor amplitude and phase lags are:

$$B_{g,w} = 20.9°F \quad \text{and} \quad \phi_w = 8.7°$$

$$B_{g,f} = 14.7°F \quad \text{and} \quad \phi_f = 64.0°$$

The effective ground temperatures for February are calculated from equation 14 to be

$$T_{g,w} = 25.2°F \quad \text{and} \quad T_{g,f} = 33.2°F$$

The average energy loss rate for February is 4320 Btu/hr from the wall and 2415 Btu through the floor for a total of 6735 Btu/hr. These results compare favorably with the des value obtained using the ASHRAE (1981) method of 5154 Btu/hr and 1457 Btu/hr for the wall floor, respectively, for a total of 6611 Btu/hr. Since the highest predicted heat loss is the month of February, it should be close to the design value.

## CONCLUSIONS

A dimensionless parameter method for calculating basement heat loss is described above. algorithm is simple, straightforward, and well suited for use on a programmable calculator a microcomputer. In this last respect the method described here differs from previo published methods (Latta and Boileau 1969; Wang 1979; Shipp and Broderick n.d.; Mit 1983).

Furthermore, the method allows any combination of wall and floor insulation levels soil thermal conductivity. The dimensionless parameter method can be extended to any loca using readily obtainable weather information.

## NOMENCLATURE

$A$ = area (ft$^2$, m$^2$)
$B_a$ = amplitude of expected daily average temperature curve (°F, °C)
$B_g$ = amplitude of effective ground temperature curve (°F, °C)
$D$ = basement depth (ft, m)
$Fo$ = Fourier modulus; $\alpha\theta/D^2$ (dimensionless)
$H$ = distance from ground surface to water table (ft, m)
$h_{amb}$ = convection coefficient at ground surface (Btu/hr·ft$^2$·°F, W/m$^2$·°K)
$k_s$ = soil thermal conductivity (Btu/hr·ft·°F, W/m·°K)
$R_f$ = thermal resistance of basement floor (ft$^2$·°F·hr/Btu, m$^2$·°C/W)
$R_w$ = thermal resistance of basement wall (ft$^2$·°F·hr/Btu, m$^2$·°C/W)
$\overline{T}_a$ = yearly average temperature (°F, °C)
$T_b$ = basement base temperature (°F, °C)
$T_c$ = reference temperature for calculating U (°F, °C)
$T_g$ = effective ground temperature (°F, °C)
$\overline{T}_g$ = average effective ground temperature (°F, °C)
$U_f$ = basement floor overall heat transfer coefficient (Btu/hr·ft$^2$·°F, W/m$^2$·°C)
$U_w$ = basement wall overall heat transfer coefficient (Btu/hr·ft$^2$·°F, W/m$^2$·°C)
$W$ = minimum basement width (ft, m)

$\alpha$ = soil thermal diffusivity ($ft^2/hr$, $m^2/s$)

$\theta$ = time (hr, s)

$\rho c$ = soil volumetric heat capacity ($Btu/ft^3 \cdot °F$, $J/m^3 \cdot °C$)

$\phi$ = phase lag between ambient temperature curve and effective ground temperature curve (deg)

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TABLE 1

Comparison of Two Methods of Computing Total Yearly Energy Loss for Three Different Floor Insulation Levels (Wall Insulation Resistance = 3.52 $m^2 \cdot °K/W$)

| R | | A | B | C |
|---|---|---|---|---|
| 0 | Wall | 4.2 | 4.4 | .95 |
| | Floor | 7.7 | 11.6 | .66 |
| | Total | 11.9 | 16.0 | .74 |
| 1.76 | Wall | 4.9 | 5.0 | .98 |
| | Floor | 5.1 | 6.3 | .81 |
| | Total | 10.0 | 11.3 | .89 |
| 3.52 | Wall | 5.2 | 5.2 | 1.0 |
| | Floor | 3.9 | 4.4 | .89 |
| | Total | 9.0 | 9.6 | .94 |

R - floor insulation resistance, $m^2 \cdot °K/W$

A - yearly energy loss computed with the dimensionless parameter method, GJ

B - yearly energy loss computed with the shape-factor method of Mitalas (1983), GJ

C - ratio of yearly energy loss of method A to method B

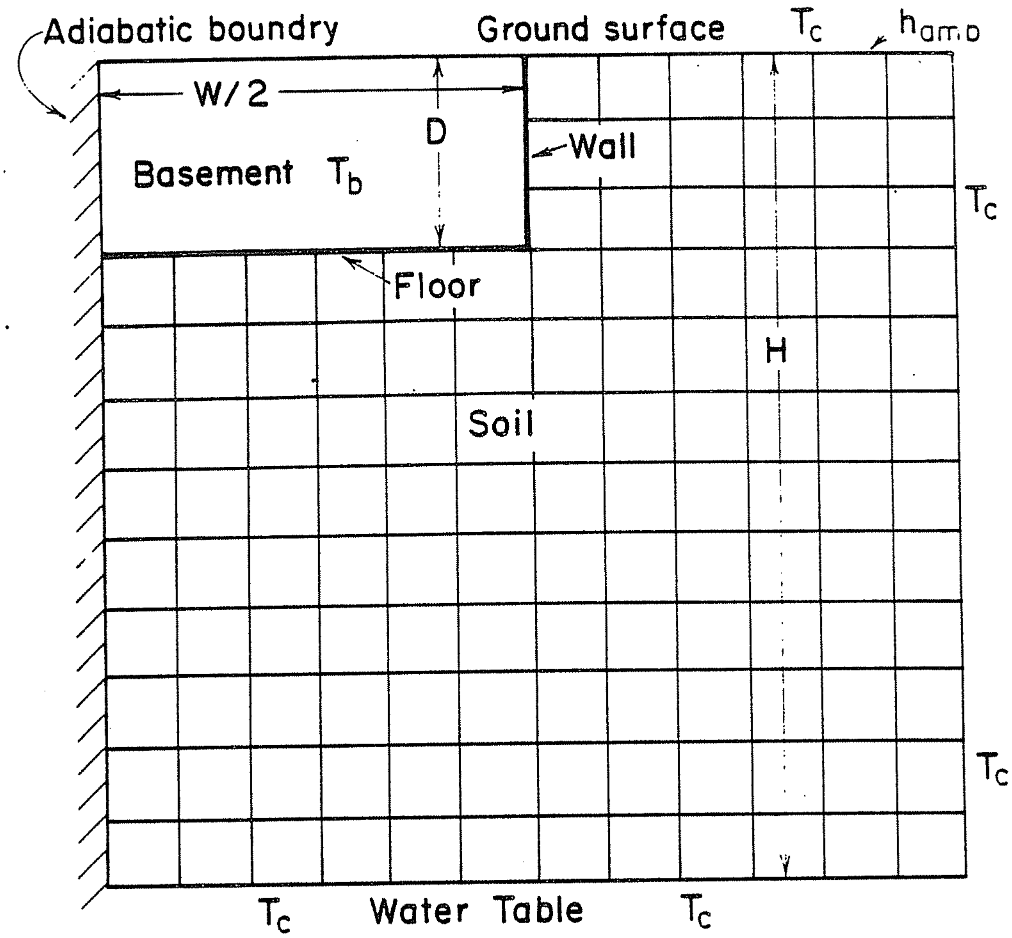


*Figure 1.* *Schematic representation of the array of elements used to calculate $U_w$ and $U_f$.*

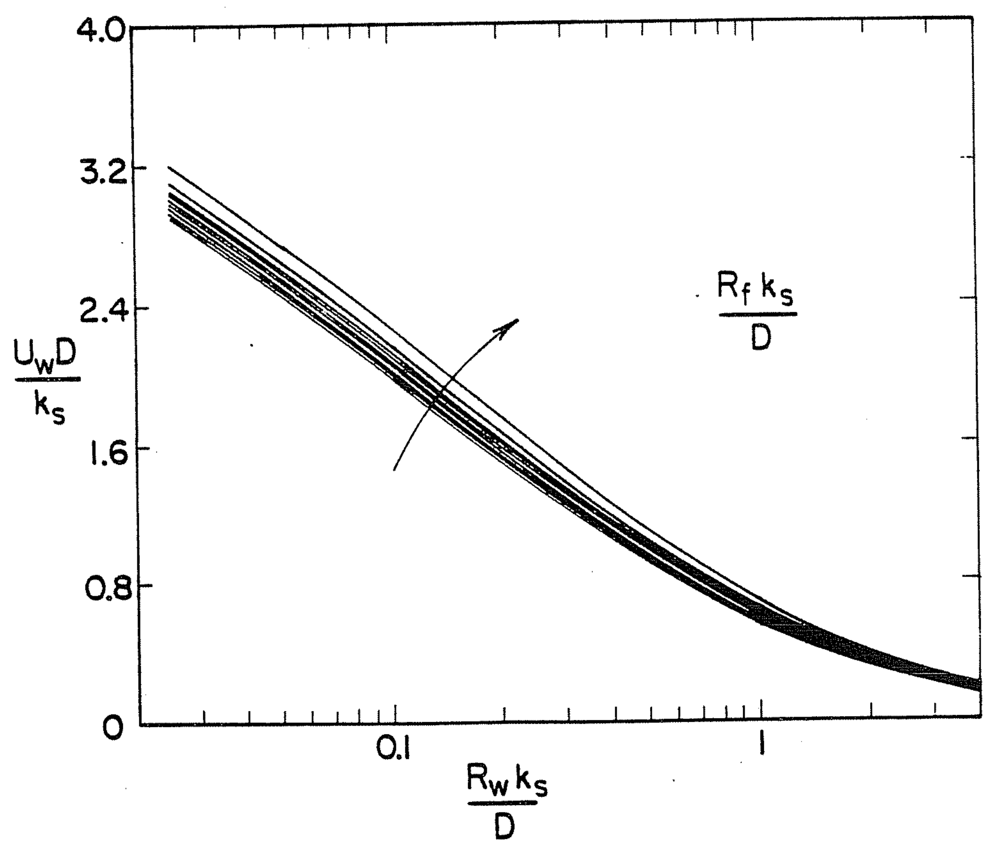


*Figure 2.* *Dimensionless wall overall conductance as a function*

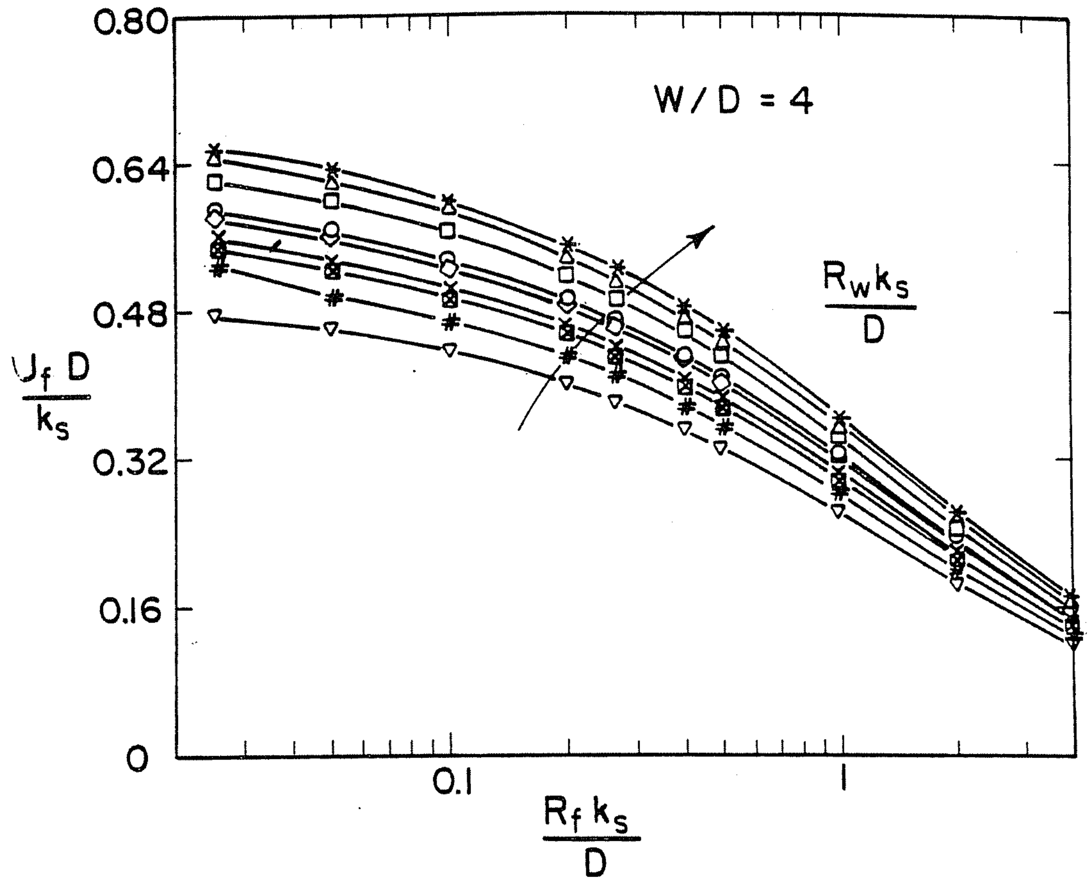


*Figure 3. Dimensionless floor overall conductance as a function of dimensionless floor resistance with dimensionless wall resistance as a parameter. W/D = 4.*

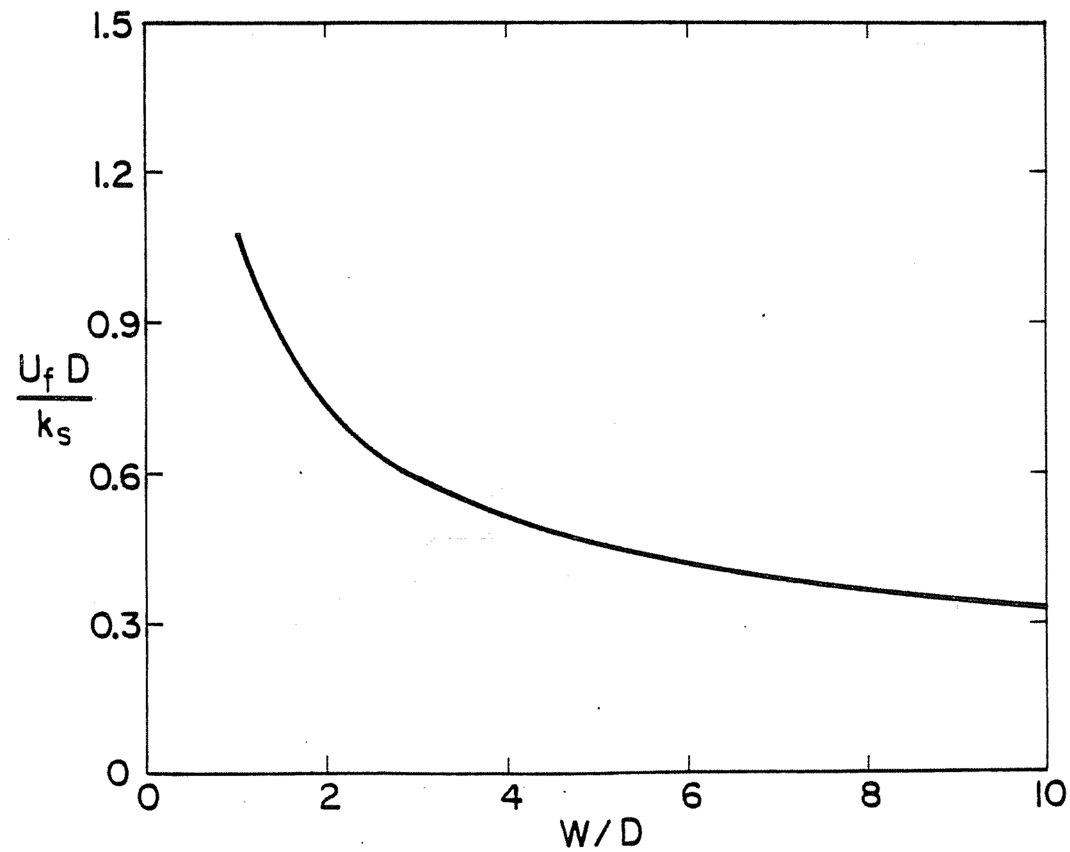


*Figure 4. Dimensionless floor conductance as a function of basement geometry for $R_w k_s/D = 0.57$, $R_f k_s/D = 0.17$.*

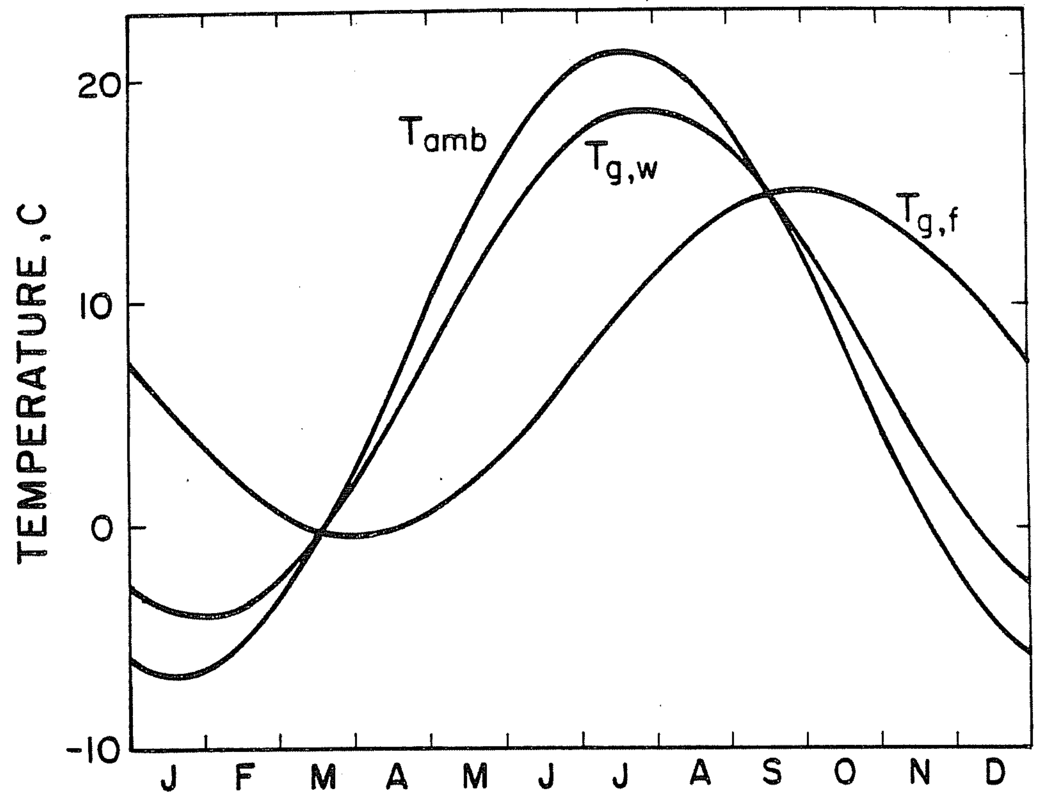


Figure 5. Ambient temperature profile and effective ground temperatures for Madison, Wisconsin, $T_a$ = 45°F (7.3°C), basement W/D = 4.

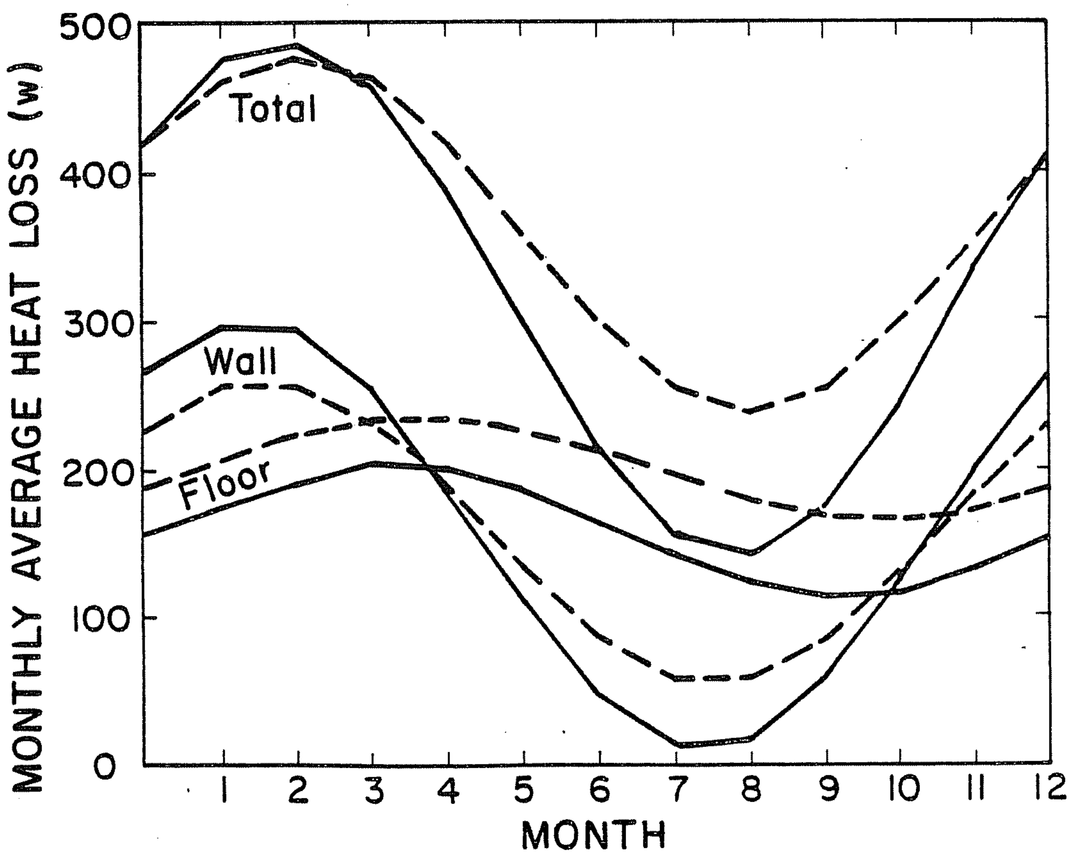


Figure 6. Monthly average basement heat loss in W. Wall insulation resistance 3.52 $m^2K/w$, floor insulation resistance 1.76 $m^2K/w$. Solid line, calculated with dimensionless parameter method. Dashed line calculated with shape factor method of Mitalas (5).