# PROCEDURES FOR PREDICTING THE PERFORMANCE OF AIR-TO-AIR HEAT PUMPS IN STAND-ALONE AND PARALLEL SOLAR-HEAT PUMP SYSTEMS

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ABSTRACT

In this thesis, methods are proposed for predicting the per-
formance of air-to-air heat pumps in parallel solar-heat pump
systems, and in stand-alone systems. The procedure for predicting
the performance of parallel systems requires as inputs the frac-
tion of the space and water heating load met by solar energy, and
the fraction of the load that would have been met by the same heat
pump operating without a solar system. The procedure combines
these results in a way which accounts for the interaction of the
solar system and the heat pump, and yields the performance of the
combined system. When the results from this procedure are compared
to those from detailed simulations, the standard deviation of the
prediction errors are within 1.5% of the load.

The procedure proposed for predicting the performance of stand-
alone systems involves replacing the ambient temperature bin data
required for the bin method with a continuous generalized distri-
bution. The heat pump and load energy rates are then integrated
over this distribution. When the results from the generalized
distribution method are compared with those from the bin method,
the standard deviation of the prediction errors are again within
1.5%.

# ACKNOWLEDGEMENTS

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## NOMENCLATURE

COP      Heat pump heating coefficient of performance

FATM      Fraction of the load met by energy absorbed from the atmosphere by the heat pump

FAUX      Fraction of the load met by auxiliary energy

FPUR      Fraction of the load met by purchased energy

FSOL      Fraction of the load met by solar energy

FW      Fraction of the load met by heat pump work

MPF      Monthly performance factor of heat pump and auxiliary

$N_{TOT}$      Total number of hours in the month

QATM      Energy absorbed from the atmosphere by the heat pump

QAUX      Auxiliary energy

QDEL      Energy delivered to the load by the heat pump.

QL      Space heating load

QSOL      Energy supplied to the load by the solar system

$T_{ave}$      Monthly average ambient temperature

$\Delta T_s$      Temperature spread of the generalized distribution

W      Work input to the heat pump

WL      Domestic hot water load

Subscripts

o      Value is for stand-alone system

b      Value is for a particular bin

A dot above a term means that term is an energy rate.

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1. INTRODUCTION

Since the early days of solar research in this country, there has been an interest in combining heat pumps with solar systems. This interest has grown through the years, nourished by the idea that the purchased energy savings realized by the combined system will be greater than the savings from either system by itself. There are three basic types of solar-heat pump system which have been proposed to meet this goal. These systems are delineated on the basis of the source of energy to the evaporator. In the first system, called a series system, the evaporator draws energy only from solar thermal storage. The second type of system is called a parallel system, and draws energy into the evaporator from the outdoor atmosphere. The third type of system, a dual source system, is a hybrid of the series and parallel systems in which the heat pump can draw energy from either the thermal storage or the atmosphere.

Many studies have been done on the thermal performance and economic feasibility of these systems. In general, these studies can be divided into 3 groups on the basis of the technique used to determine the thermal performance of the heat pump. The earliest reports, such as those by Löf and by Jordan and Threlkeld (references 1-4), used design point conditions or a fixed performance factor to predict the heat pump performance. This approach has the disadvantage that it is very difficult to know

*a priori* exactly how the heat pump and solar system will interact in a given configuration, and how the interaction will affect their performance.

More recently, a second group of studies (5-14) have used the results of computer simulations to determine the performance of various types of solar-heat pump systems. Detailed simulations account for the interaction of the heat pump and the solar system in much greater detail. This has led to non-intuitive results regarding the relative efficacy of various configurations (19). The disadvantage of simulations is that they are complex and often prohibitively expensive to use for routine design work.

The third group of studies (15-17) has dealt with the performance of experimental systems. These are extremely important, but are limited in scope and quite expensive to carry out. Physical experiments require long periods of time before they produce useful results, and the results are dependent on the particular meteorological conditions experienced by the system.

None of these approaches is satisfactory for use on a routine basis, and a need exists for simple design procedures which will accurately predict the long term average performance of the various types of solar heat-pump systems.

One such procedure has been published for parallel solar-heat pump systems. In 1978, T. E. Audit (25) proposed a design procedure based on a modified bin method. The procedure which is pro-

posed here represents an improvement over Audit's method in terms of
flexibility, ease of computation, quantity of weather data required,
and generalization of results.

This thesis contains two major sections. In the first, a new
method for predicting the performance of parallel solar-heat pumps
is proposed. This method requires as inputs the fraction of the
space and water heating load met by the solar part of the system,
and the fraction of the load which would have been met by the heat
pump in a stand-alone system (without any solar contributions). The
method combines these values in a way which accounts for the inter-
action of the two systems, and yields results for the combined
system. When the results predicted by this method are compared
to the values from detailed simulations, the standard deviations
of the prediction errors are found to be within 1.5% of the load.

The second major section of this thesis discusses two methods
for predicting the performance of stand-alone heat pumps. The
first of these methods to be discussed is the ASHRAE bin method (20).
Then an alternative method which replaces the detailed ambient
temperature data required for the bin method with a simple general-
ized ambient temperature distribution is presented. The results
predicted by the generalized distribution method are compared to
the results from the bin method, and the standard deviations of
the prediction errors are again within 1.5% of the load. One
advantage of the generalized distribution method is that since it

requires only the monthly average ambient temperatures, it is particularly compatible with the f-chart method.

Overall, the procedures presented here should fill the need for a simple design procedure which will accurately predict the thermal performance of air-to-air heat pumps, either in stand-alone or parallel systems. Once the thermal performance is established, these heat pump systems can be compared directly with conventional fossil fuel or solar-only systems on an economic basis by using standard economic analyses.

## 2. PREDICTION OF PARALLEL SYSTEM PERFORMANCE

### 2.1 Introduction

In this section, a method for predicting the performance of parallel solar-heat pump systems is developed. This involves determining each of the energy flows into and out of the system. The method capitalizes upon established procedures for finding the solar contribution and the performance of a stand-alone heat pump system, and combines the results from these procedures in a manner which takes into account the solar system and heat pump interaction. The predicted results are compared to those from detailed simulations, and are found to agree quite well.

### 2.2 System Description

A schematic of a parallel solar heat pump system is shown in Figure 2.1. A conventional air or liquid solar system is used as



Figure 2.1  Parallel Solar-Heat Pump System

the primary source for direct space and domestic hot water heating. The air-to-air heat pump serves as the primary space heating auxiliary. A conventional electric or fossil fuel energy source is used as the secondary auxiliary for space and hot water heating. Previous studies by Freeman et al. (19) have shown that this control strategy minimizes the purchased energy requirement.

Since the heat pump in this system absorbs energy only from the ambient air, the storage temperature generally remains in the range where solar energy can be supplied directly to the load. This is in contrast to a series or dual-source system where the energy absorbed by the heat pump often draws the storage temperature down well below the value required for direct solar heating.

If the solar part of the system were not present, the remaining components would be a conventional stand-alone air-to-air heat pump system. In this stand-alone heat pump system the heat pump is the primary source for space heating, while the fossil fuel or electric auxiliary energy is used as the space heating backup and for the domestic hot water load.

## 2.3 Performance Evaluation

The energy required to meet the space heating load (QL) in the parallel system comes from four possible sources: direct solar heating (QSOL); energy absorbed from the atmosphere by the heat pump (QATM); work into the heat pump (W); and auxiliary energy (QAUX). An energy balance on the system for some period relates

these terms by

$$QSOL + QATM + W + QAUX = QL \qquad (2.1)$$

The energy balance can be written in terms of the load fractions by dividing through by the space heating load QL

$$FSOL + FATM + FW + FAUX = 1 \qquad (2.2)$$

where the prefix F indicates a fraction of the load. In order to predict the thermal performance of the system, each of the terms in Equation (2.1) or (2.2) must be evaluated in terms of known parameters and weather information.

## 2.3.1 Solar Contribution

Since the air or liquid solar system is always the primary energy source, it operates independently of the heat pump. That is, the heat pump operation has no effect on the performance of the solar system. The fraction of the space and water heating load met by solar energy can then be predicted directly by a method such as f-chart (18).

## 2.3.2 Heat Pump Contribution

The fraction of the space heating load carried by energy absorbed from the atmosphere (FATM) is not only a function of the heat pump performance characteristics and the ambient temperature

patterns, but also depends on the interaction between the solar system and the heat pump. This interaction consists of two aspects. First, the solar system meets some portion of the load and reduces the amount of energy the heat pump is required to deliver. Secondly, the heat pump in combination with the solar system may operate during different times of the day than would a stand-alone heat pump. This would change the distribution of ambient temperatures that the heat pump would use as a source, and thus would change the average COP at which it operates. This change in the COP would, in turn, change the ratio of non-purchased to purchased energy.

The load met by the heat pump and auxiliary (i.e., the load not met by the solar system) is given by

$$QL - QSOL = QATM + W + QAUX \qquad (2.3)$$

The fraction of this load (QL-QSOL) that is met by energy absorbed from the atmosphere can be written as

$$\frac{QATM}{QL-QSOL} = \frac{QATM + (W + AUX) - (W + AUX)}{QATM + W + QAUX} \qquad (2.4)$$

The performance of a given heat pump operating in combination with a solar system can be related to the performance of that same heat pump operating as a stand-alone machine through the monthly performance factor MPF, defined as

$$MPF = (QATM + W + QAUX)/(W + QAUX) \qquad (2.5)$$

Substituting the definition of MPF, Equation (2.5), into Equation (2.4) yields an expression for the atmospheric fraction in terms of the monthly performance factor.

$$\frac{QATM}{QL-QSOL} = 1 - \frac{1}{MPF} \qquad (2.6)$$

Equations (2.3) and (2.6) apply to both parallel solar heat pump systems and to stand-alone systems. The quantities for a stand-alone heat pump system will be denoted by the subscript o. The atmospheric fraction for the parallel system is obtained by dividing Equation (2.6) through by the total space heating load and re-arranging to yield

$$FATM = (1 - \frac{1}{MPF}) \frac{QL-QSOL}{QL} \qquad (2.7)$$

Equation (2.6) also applies to the stand-alone system. In this case however, QSOL equals zero and Equation (2.7) becomes

$$FATM_o = (1 - \frac{1}{MPF_o}) \qquad (2.8)$$

In both of these expressions the prefix F again refers to the fraction of the total space heating load. The relationship between FATM and $FATM_o$ is found by taking the ratio of Equations (2.7) and (2.8).

$$FATM = FATM_o \frac{(MPF - 1)}{MPF} \frac{MPF_o}{(MPF_o - 1)} (1 - FSOL) \qquad (2.9)$$

It will be demonstrated later that the monthly performance factor

MPF varies only slightly with the fraction of the load supplied by

the solar system. Physically this means that the major effect of

the solar system is only to reduce the load that the heat pump must

meet. With the assumption that MPF is independent of FSOL, MPF

and $MPF_o$ are equal and Equation (2.9) becomes

$$FATM = FATM_o \ (1 - FSOL) \tag{2.10}$$

The quantity $FATM_o$ is readily calculated using a procedure such as

the ASHRAE bin method (20) or the generalized distribution method

discussed in Chapter 3. FATM can then be found directly from Equa-

tion (2.10).

The underlying premise of Equation (2.10) is shown graphically

in Figure 2.2, which is a plot of a fraction of the load met by non-

purchased energy vs collector area for a particular system in Colum-

bia, Missouri. The premise is that the energy absorbed from the

atmosphere by the heat pump always represents the same fraction of

the load that is not met by the solar system. For example, in the

stand-alone case, none of the load is met by solar energy and the

heat pump contributes the fraction $FATM_o$ of the total load. In

the case of the parallel system, the solar system reduces the load

that the heat pump and auxiliary must meet by the fraction FSOL.

The assumption is that the heat pump contributes the same fraction

$FATM_o$ of the reduced load $(1 - FSOL)$.
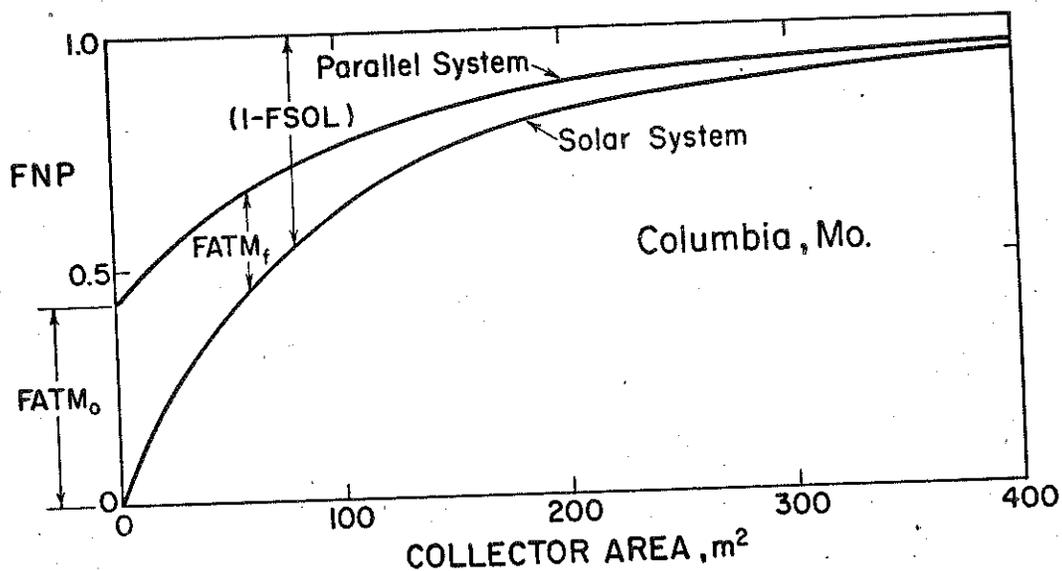
Figure 2.2  Fraction of the load met by non-purchased energy vs.
collector area for the month of February in Columbia, MO.

The fraction of the load that is supplied by the work input

to the heat pump is evaluated by using the monthly heating co-

efficient of performance COP, defined as

$$COP = \frac{QATM + W}{W} \qquad (2.11)$$

Dividing through the right side of Equation (2.11) by QL and re-

arranging gives the relationship between FATM and FW

$$FW = FATM/(COP - 1) \qquad (2.12)$$

The work fraction for a parallel system, FW, can be related

to that for a stand-alone system, $FW_o$, by writing Equation (2.12)

for both cases and substituting the results into Equation (2.10).
The work fraction is then given by

$$FW = FW_o \frac{(MPF - 1)MPF_o}{(MPF_o - 1)MPF} \frac{(COP_o - 1)}{(COP - 1)} (1 - FSOL) \qquad (2.13)$$

It will be demonstrated in a later section that the COP also varies
only slightly with FSOL. This is consistent with the earlier obser-
vation that MPF varies only slightly with FSOL. With the assumption
that both COP and MPF are independent of FSOL, Equation (2.13) reduces
to

$$FW = FW_o (1 - FSOL) \qquad (2.14)$$

Again, $FW_o$ is readily calculated using the ASHRAE bin method or the
generalized distribution method, and thus FW is readily calculated
from Equation (2.14).

## 2.3.3 Auxiliary Contribution

The nature of the control strategy for the parallel solar-heat
pump system requires that auxiliary energy meet any of the load not
met by contributions from the solar system or the heat pump. Once
FSOL, FATM and FW have been determined, the fraction of the load met
by auxiliary energy can be found by rearranging Equation (2.2).

$$FAUX = 1 - FSOL - FATM - FW \qquad (2.15)$$

Using Equations (2.10) and (2.14), FAUX can also be expressed in terms of the stand-alone values

$$FAUX = (1 - FATM_o - FW_o)(1 - FSOL) \qquad (2.16)$$

### 2.3.4 Purchased Energy

It is also possible to combine FAUX and FW into a single term which represents the fraction of the load met by all forms of purchased energy, FPUR. Using Equations (2.10), (2.14) and (2.16) this can be expressed as

$$FPUR = (1 - FATM_o)(1 - FSOL) \qquad (2.17)$$

### 2.4 Verification of Procedure

The procedure outlined above and the assumptions used in its development were tested using TRNSYS (21) simulations of parallel solar heat pump systems. Simulations were first run in 9 U.S. locations over all combinations of two heat pumps, two house UA values, and two annual solar fractions. The parameters used in these simulations are listed in Table 2.1, and the heat pump performance characteristics (adapted from 22) are shown in Figure 2.3. In addition, several other runs were made to test the influence of the house thermal capacitance and heat pump low temperature cutoff point on the accuracy of the prediction procedure.

TABLE 2.1

PARAMETERS USED IN SIMULATED SYSTEMS

Heat Pump Sizes:  3 ton and 5 ton

House UA Values:  297 W/C and 535 W/C

Approximate Annual FSOL:  0.30 and 0.60

Domestic Hot Water Load:  284 Kg/day delivered at 60C
from the main's temperature
for each city, RANN daily
profile (21).

Locations:  Madison, WI          Columbia, MO

Albuquerque, NM      Seattle, WA

New York, NY          Charleston, SC

Santa Maria, CA

Washington, DC

Bismark, ND
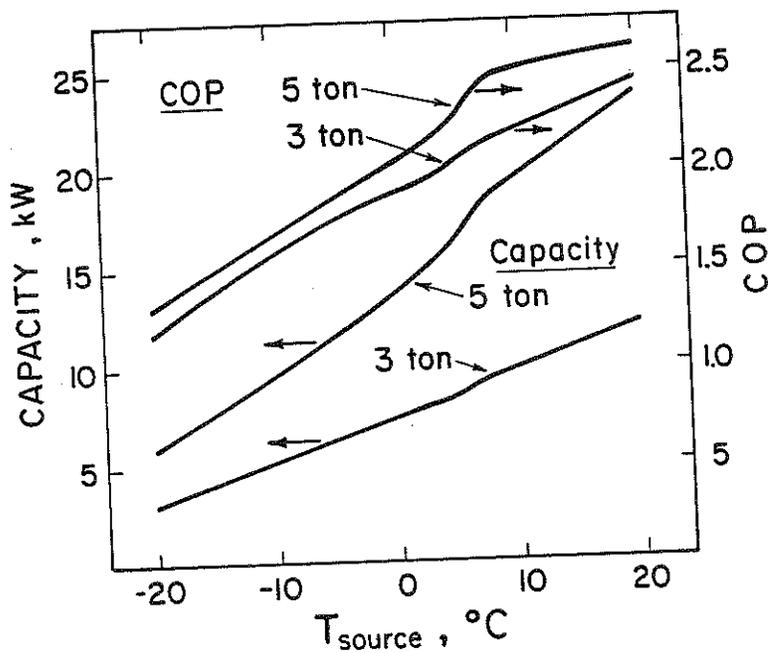


Figure 2.3  Heat Pump Characteristics

In order to find the stand-alone heat pump performance values needed for Equations (2.10), (2.14), (2.16) and (2.17), the hour-by-hour ambient temperature data used in the simulations were collated into bin data form in accordance with the ASHRAE method. This bin data was then used with the house UA value and the heat pump performance data to perform a bin method analysis of the stand-alone system (see Section 3.2). The values from both the simulations and the bin method were then used to test the assumption that MPF and COP are independent of FSOL, and to determine the accuracy of the values predicted by Equations (2.10), (2.14), (2.16) and (2.17).

2.4.1 Variation of Monthly Performance Factors and Coefficient of Performance with Fraction by Solar

Values of the monthly performance factor MPF and heating coefficient of performance COP from the TRNSYS simulations were compared to the stand-alone values $MPF_o$ and $COP_o$ to determine their variation with FSOL. It was found that in most cases both of these factors tended to decrease somewhat with increasing FSOL. Some examples of this variation are shown in Figure 2.4. The magnitude of the change was fairly small, usually within 5% of the stand-alone value for values of FSOL up to 0.80 or 0.90. At very high values of FSOL the differences sometimes became larger, but this is not important because when the solar system is carrying practically all of the load the heat pump contributions are necessarily very small.
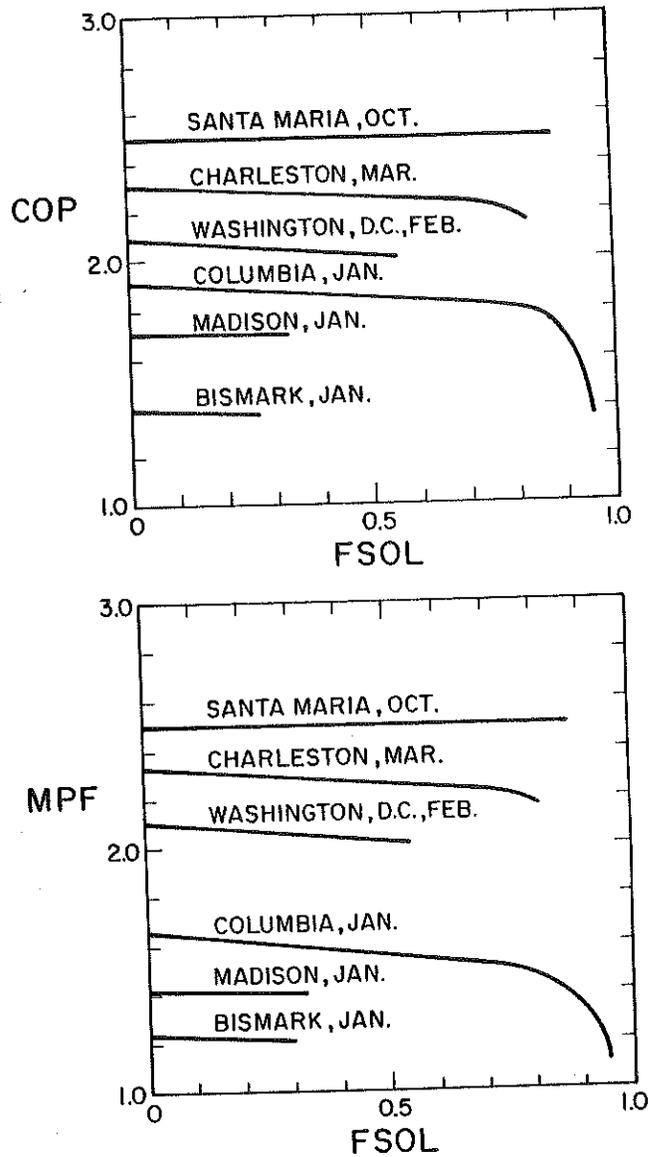
Figure 2.4  MPF and COP vs. fraction by solar.

The effect of this variation on the predicted value of FATM

can be seen by examining Equation (2.9), which shows that FATM

is dependent on MPF through the term

$$\frac{(MPF - 1)MPF_o}{(MPF_o - 1)MPF} = \frac{(1 - \frac{1}{MPF})}{(1 - \frac{1}{MPF_o})} \qquad (2.18)$$

Since this term is less than one for $MPF_o > MPF$, setting it equal to one in Equation (2.10) tends to over-predict FATM. This effect will be particularly noticable at high values of FSOL, which correspond to lower values of FATM.

Similarly, examination of Equation (2.13) shows that FW is dependent on both COP and MPF through the terms

$$\frac{(COP - 1)}{(COP_o - 1)} \frac{(MPF - 1)MPF_o}{(MPF_o - 1)MPF} \qquad (2.19)$$

As just noted, the term which involves MPF is usually less than one. However, since COP also tends to decrease with FSOL ($COP_o > COP$), the other term is usually greater than one and the product of these terms can, in general, vary on either side of one.

2.4.2 Serendipity

There are several fortuitous factors which tend to improve the prediction accuracy of this method despite the variation in COP and MPF with FSOL. The first of these is that the variation in COP and MPF from the stand-alone values does not produce a corresponding error in the seasonal energy fractions. The difference is largest when FSOL is the largest, but under these conditions the heat pump is contributing only a small fraction of the load. Thus, when the

energy values are totalled over the entire season, the seasonal values will be more accurate than the values for any month with large solar contribution.

Another factor which further mitigates the effect of the variation in MPF and COP is that the term FSOL in the preceding equations is based on the solar contribution to the space heating load only. The influence of the solar energy which is used to meet the additional load imposed by the domestic hot water system is not accounted for. However, in a solar system design procedure such as f-chart, the solar contribution to these loads is not known individually, and the value of FSOL that is available represents the solar contribution to both the space and water heating loads. Because in almost every U.S. location with a significant heating load the water mains temperature is lower than the room temperature, the effect of including the solar contribution to the water heating load is to lower the effective minimum temperature at which solar energy will be useful, and thus to increase the fraction by solar. This effect causes the term (1 − FSOL) in Equations (2.10), (2.14), (2.16) and (2.17) to be slightly smaller than it would be if FSOL were based on the space heating load only. This, in turn, tends to counteract the effect of assuming constant values of COP and MPF. The net effect is that very little error is introduced by assuming the same heat pump performance in stand-alone and parallel systems.

### 2.4.3 Accuracy of Predictions

The fractions of the space and water heating loads carried by solar as determined in the simulations were used in equations (2.10), (2.14) and (2.17) with the values of $FATM_o$ and $FW_o$ from the bin method to predict the values of FATM, FW and FPUR. Figures 2.5, 2.6 and 2.7 show the seasonal values of FATM, FW and FPUR from the simulations plotted against the predicted values. The diagonal line represents perfect agreement between the simulations and the predicted values.
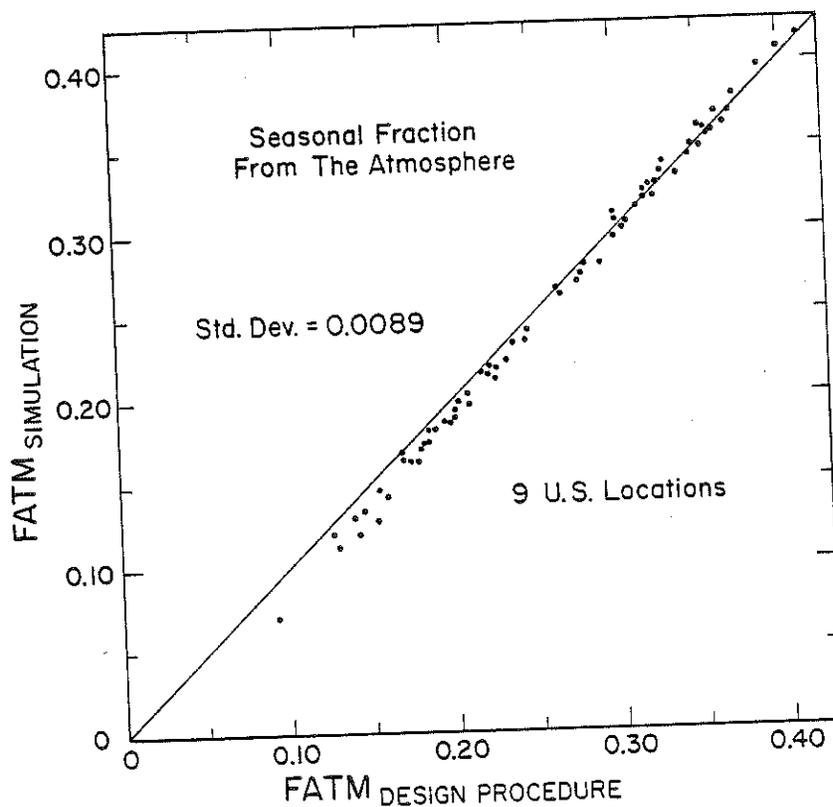


Figure 2.5   Seasonal values of FATM from simulation vs. predicted values.
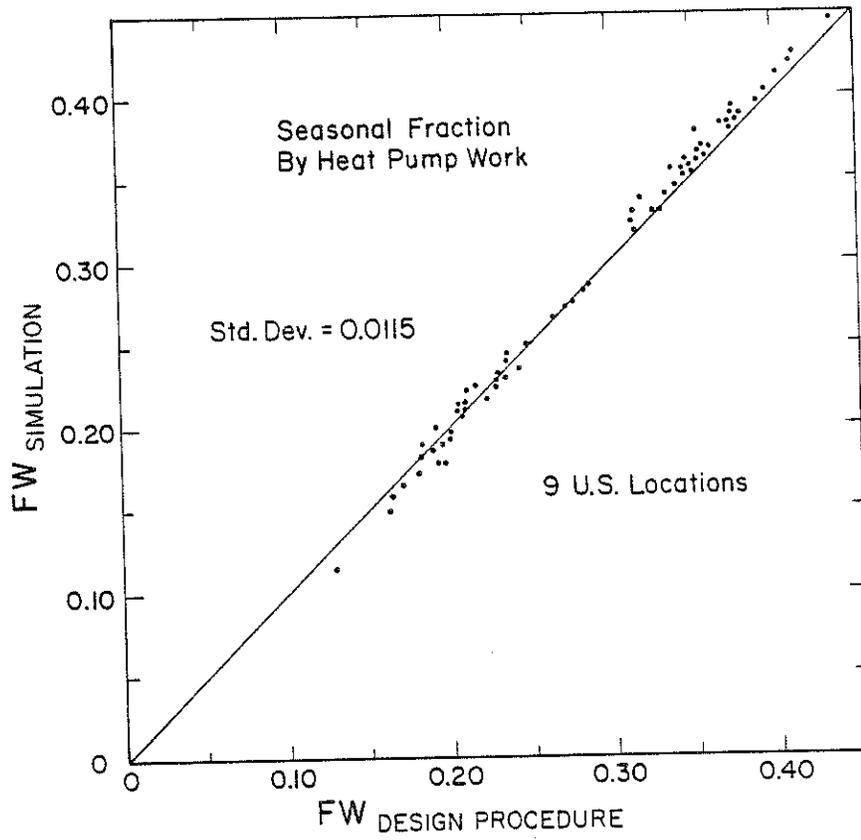
Figure 2.6   Seasonal values of FW from simulation vs.
            predicted values
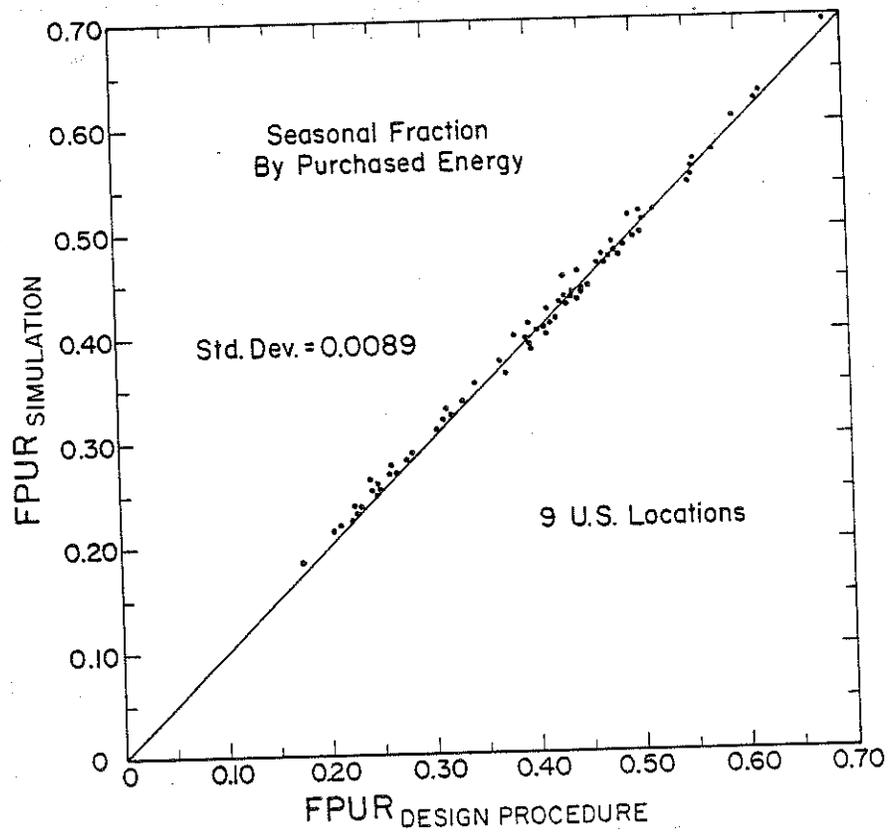
Figure 2.7   Seasonal values of FPUR from simulation vs.
             predicted values

The points on these graphs represent the results from 9 U.S. locations for all combinations of the heat pumps, house UA values and fractions by solar shown in Figure 2.3 and Table 2.1. The 9 locations were chosen to represent a wide range of climates in which the use of heat pumps might be considered. The heat pumps and house UA values are typical of those encountered in residential applications.

The agreement between the simulation values and the predicted values is quite good. There is a slight tendency to overpredict FATM at low values of FATM which is in agreement with the explanations given in section 2.4.1.

The standard deviations of the prediction errors on both a monthly and a seasonal basis are listed in Table 2.2.

TABLE 2.2

Standard Deviation of Predicted Errors

|  | FATM | FW | FPUR |
|---|---|---|---|
| Monthly (432 points) | 0.011 | 0.013 | 0.011 |
| Seasonal (72 points) | 0.009 | 0.012 | 0.009 |

These standard deviations are less than 1.5% of the total load, which is within the accuracy with which the system parameters are known. The errors in the seasonal values are smaller than those for the monthly values, which is again consistent with earlier explanations.

### 2.4.4 House Capacitance and Low-Temperature Cut-off

Tests were also run to determine the sensitivity of the prediction method to the value of the house capacitance and to the heat pump low temperature cutoff point. It was found that the house capacitance has only a slight effect on the accuracy of the predicted values. Prediction errors are listed in Table 2.3 for various values of the house capacitance on systems in Bismarck, N.D. and Columbia, MO. The prediction error is defined as the value from the simulation minus the predicted value. Thus a negative prediction error represents an overprediction. The capacitance values range from 0 to 100,000 KJ/°C. The latter value represents an extremely massive house. There is very little difference as the house capacitance varies, despite the fact that the bin method, which was used to determine the stand-alone heat pump values, always assumes a zero capacitance load.

The heat pump low temperature cutoff point was found to have a significant effect on the accuracy of the predicted value. The existence of the low temperature cutoff is caused by the fact that the heat pump COP decreases with decreasing temperature. At some temperature the compressor can no longer maintain a significant refrigerant flow rate at the required pressure drop across the expansion value, and the COP approaches one. Rather than continuing to run the heat pump with a COP at or below one, most manufacturers will turn the heat pump off and let the auxiliary energy source carry the load.

TABLE 2.3

FATM Prediction Errors vs House Capacitance

Bismark, N.D.

House Capacitance (KJ/°C)

| Month | 0 | 50,000 | 100,000 |
|-------|------|--------|---------|
| OCT | -.009 | -.005 | -.006 |
| NOV | -.019 | -.018 | -.017 |
| DEC | -.016 | -.017 | -.016 |
| JAN | -.012 | -.013 | -.011 |
| FEB | -.036 | -.034 | -.032 |
| MAR | -.024 | -.022 | -.019 |

Columbia, MO

House Capacitance (KJ/°C)

| Month | 0 | 50,000 | 100,000 |
|-------|------|--------|---------|
| OCT | -.011 | .010 | .007 |
| NOV | .004 | .011 | .009 |
| DEC | .001 | .003 | .006 |
| JAN | -.002 | .004 | .005 |
| FEB | -.001 | .004 | .006 |
| MAR | -.001 | .004 | .006 |

The rationale behind Equation 2.10 is that if the heat pump always meets the same fraction of the load not met by the solar system, the energy contributed by the heat pump in a parallel system

can be found by a direct ratio to the stand-alone case. This can be illustrated by rewriting Equation 2.10 using the definitions of FATM and FSOL.

$$\frac{QATM}{QL - QSOL} = \frac{QATM_o}{QL} \qquad (2.20)$$

In order for this ratio to be valid, however, the heat pump must be able to operate to meet any of the load not met by the solar system. When part of the load occurs below the low-temperature cutoff point, the heat pump cannot always operate to meet the load, and the portionality is no longer valid.

The larger the fraction of the load which occurs below the cut-off point, the worse the predictions will be. This can be seen in Table 2.4, where the monthly prediction errors are listed for various values of the cutoff temperature for systems in Columbia, Mo. and Bismarck, N.D. The only months which are significantly affected are those with minimum ambient temperatures well below the cutoff temperature.

The importance of this effect is dependent on both the climate and the particular heat pump. Fortunately most of the available air-to-air heat pumps have cutoff temperatures of $-18°C$ ($0°F$), which is low enough to avoid serious prediction errors in all but the most severe of climiates. For example, when this procedure was used with a heat pump cutoff temperature of $-20°C$ in Bismarck, N.D., which has minimum ambient temperatures below $-28°C$, the maximum monthly error was less than 5%.

TABLE 2.4

FATM Prediction Errors for Various Cutoff Temperatures

Columbia, MO

| Month | Minimum Ambient Temp. (°C) | Heat Pump Cutoff Temperature | | |
|---|---|---|---|---|
| | | -20°C | -10°C | 0°C |
| OCT | 0 | -.011 | -.011 | -.014 |
| NOV | - 8 | .004 | .004 | -.052 |
| DEC | -18 | .001 | -.003 | -.032 |
| JAN | -20 | -.002 | -.012 | -.023 |
| FEB | -20 | -.001 | -.004 | -.040 |
| MAR | -10 | -.001 | -.001 | -.044 |

Bismarck, N.D.

| Month | Minimum Ambient Temp. (°C) | Heat Pump Cutoff Temperature | |
|---|---|---|---|
| | | -20°C | -10°C |
| OCT | - 6 | .001 | .001 |
| NOV | -20 | -.001 | -.009 |
| DEC | <-28 | -.010 | -.014 |
| JAN | <-28 | -.004 | -.012 |
| FEB | <-28 | -.018 | -.024 |
| MAR | -16 | -.011 | -.021 |

2.5  Summary of Design Method

The design method consists of the following steps.

1. Determine FSOL

   Using a procedure such as the f-chart method, determine the monthly solar contribution to the space and water heating loads, FSOL.

2. Determine $FATM_o$ and $FW_o$

   Apply either the bin method or the generalized distribution method (Section 3) to determine the monthly fractions of the space heating load met by energy from the atmosphere $FATM_o$, and heat pump work $FW_o$, for a stand-alone system.

3. Calculate FATM, FW and FAUX

   Use the value of FSOL from Step 1 and the values of $FATM_o$ and $FW_o$ from Step 2 in Equations 2.10, 2.14 and 2.16 to find the fractions of the space heating load met by energy from the atmosphere, heat pump work, and auxiliary energy for the parallel system.

$$FATM = FATM_o \ (1 - FSOL) \qquad (2.10)$$

$$FW = FW_o \ (1 - FSOL) \qquad (2.14)$$

$$FAUX = (1 - FATM_o - FW_o)(1 - FSOL) \qquad (2.16)$$

4. Calculate QATM, W and AUX

   Using the value of the space heating load QL used in Step 1, determine the following energy quantities.

$$QATM = FATM \cdot QL \qquad (2.21)$$

$$W = FW \cdot QL \qquad (2.22)$$

$$AUX = FAUX \cdot QL \qquad (2.23)$$

5.  Determine the auxiliary energy supplied to the hot water load QAUXW by

$$QAUXW = WL \ (1 - FSOL) \qquad (2.24)$$

where WL is the value of the hot water load used in Step 1.

6.  Repeat Steps 1 through 5 for each month to determine the annual values. The annual fractions for any of the energy sources can be determined by

$$F_{annual} = \frac{\Sigma \ Q_i}{\Sigma \ QL_i} \qquad (2.25)$$

where $Q_i$ is any of the monthly energy values and $QL_i$ is the monthly load value.

## 2.6 Example Problem

As an example of the use of this procedure, the performance of a parallel-solar heat pump system will be deteremined. The system consists of a liquid based solar system with collector area of 41.5 $m^2$ located in Madison, Wis. The heat pump is the 3 ton unit with characteristics as plotted in Figure 2.3. The steps in this example are numbered in accordance with the steps listed in the summary, Section 2.5

1. The space heating load for January in Madison is QL = 19.59 GJ, and the water heating load is WL = 2.03 GJ. From f-chart it is determined that for this system in January the fraction of the space and water heating loads met by solar is FSOL = 0.324.

2. From the bin method: $FATM_o$ = .293 and $FW_o$ = .420. (The ambient temperature bin data for January in Madison is shown in Figure 3.2a.)

3. The atmospheric, heat pump work, and auxiliary fractions are:

$$FATM = FATM_o (1 - FSOL) = .293 (1 - .324) = .198$$

$$FW = FW_o (1 - FSOL) = .420 (1 - .324) = .284$$

$$FAUX = (1 - FATM_o - FW_o)(1 - FSOL) = (1 - .293$$

$$- .420)(1 - .324) = .194$$

4. The atmospheric, work, and auxiliary energy flows are

$$QATM = .198 \cdot 19.59 = 3.88 \ GJ$$

$$W = .284 \cdot 19.59 = 5.56 \ GJ$$

$$QAUX = .194 \cdot 19.59 = 3.80 \ GJ$$

5. The auxiliary energy for the domestic hot water is

$$QAUXW = 2.03 (1 - .324) = 1.37 \ GJ$$

For this month the fraction of the total space and water heating loads that is met by purchased energy is

$$\frac{5.56 + 3.80 + 1.37}{19.59 + 2.03} = 0.496$$

when this procedure is carried out for all 12 months, the fraction of the total space and water heating load met by purchased energy is 0.301.

This example illustrates that once the appropriate values are obtained from the solar system design method and the bin method, the procedure is readily carried out. Unfortunately, doing month by month bin method calculations for an entire heating season is not only tedious, but also requires a substantial amount of data. A simplified approach to this determination is presented in Chapter 3.

## 2.7 Conclusions

The procedure developed in this chapter represents a simple method of predicting the long term thermal performance and economic viability of parallel solar heat pump systems. The accuracy of the seasonal purchased energy predictions is within about 1.3% of the total load, which is well within the uncertainties introduced by solar system parameters and heat pump performance data.

The procedure requires the value of the solar contribution to the space and water heating loads from a solar system design procedure such as the f-chart method. It also requires the stand-alone heat pump performance values which can be found with a method such as the bin method or with the generalized distribution method. Both of these methods are discussed in Chapter 3.

The procedure tends to predict less accurately in climates where there are a large number of hours in which the ambient temperature is less than the heat pump low-temperature cutoff point. The cutoff temperatures on most heat pumps available today, however, are low enough that this method should give good results for virtually all U.S. locations for which air-to-air heat pumps would be considered.

# 3. PREDICTION OF STAND-ALONE HEAT PUMP PERFORMANCE

## 3.1 Introduction

This chapter deals with the prediction of the fraction of the space heating load met by a stand-alone air-to-air heat pump system in a given climate. The traditional means of predicting energy use for this system is the bin method, in which the average number of hours at a given ambient temperature is used in conjunction with the heat pump and load characteristics to determine both monthly and seasonal energy use. The bin method, however, requires a substantial amount of ambient temperature data for every location. An alternative method which replaces the weather data used in the bin method with a continuous generalized ambient temperature distribution and requires only the monthly average ambient temperatures will be proposed. It will be shown that the results from the generalized distribution compare quite well with those from the bin method for virtually all locations tested. Because the generalized distribution requires less temperature data than the bin method, it is more compatible with solar design methods such as the f-chart method.

A schematic of the stand-alone heat pump system is shown in Figure 3.1. There are three sources of energy to meet the load in this system: QATM is the energy absorbed from the atmosphere by the evaporator of the heat pump; W is the compressor work done by the heat pump; and QAUX is the auxiliary energy supplied by a conventional electric or fossil fuel backup.
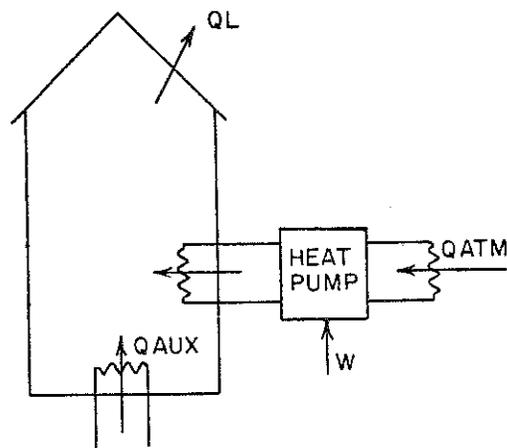
Figure 3.1  Stand-alone heat pump system

The energy balance for this system over any period of time is

$$QL = QATM + W + QAUX \qquad (3.1)$$

or, dividing through by the space heating load,

$$1 = FATM + FW + FAUX \qquad (3.2)$$

where the prefix F again refers to a fraction of the space heating load.  The next two sections will show how these terms are predicted using both the bin method and the generalized distribution method.

3.2  The Bin Method

The bin method is recommended by ASHRAE (20) and has been used for years as the most convenient way of predicting the monthly and seasonal performance of stand-alone air-to-air heat pumps.  In the

bin method the ambient temperature scale is divided into narrow "bins" (commonly 5°F), and the number of hours in which the prevailing ambient temperature falls into each bin is recorded. When the number of hours of occurrence are known for all the temperature bins, a histogram of the hours of occurrence as a function of the temperature bin can be constructed. Three examples of this type of histogram which were constructed from the SOLMET TMY weather data (23) are shown in Figure 3.2. These three months were chosen to indicate the type of variations commonly encountered in the actual bin data. The overall shape of the histograms is quite irregular, and they are only rarely symmetric around the monthly average temperature. In addition, there is little similarity between any two months in a given location, or between any two locations in a given month.

In order to perform a bin method analysis, the heat pump heat delivery rate $\dot{Q}DEL$, the heat pump work input rate $\dot{W}$, and the space heating load rate $\dot{Q}L$ must all be known as a function of ambient temperature. An example of this type of information is shown for a 2 ton heat pump and a simple degree-day load in Figure 3.3. Both $\dot{Q}DEL$ and $\dot{W}$ decrease with decreasing ambient temperature. The rate at which energy is absorbed from the atmosphere by the heat pump $\dot{Q}ATM$ is defined as the difference between $\dot{Q}DEL$ and $\dot{W}$, and also decreases with decreasing temperature. The load rate $\dot{Q}L$, however, increases with decreasing ambient temperature. The temperature at which $\dot{Q}DEL$ equals $\dot{Q}L$ is called the balance point. At temperatures
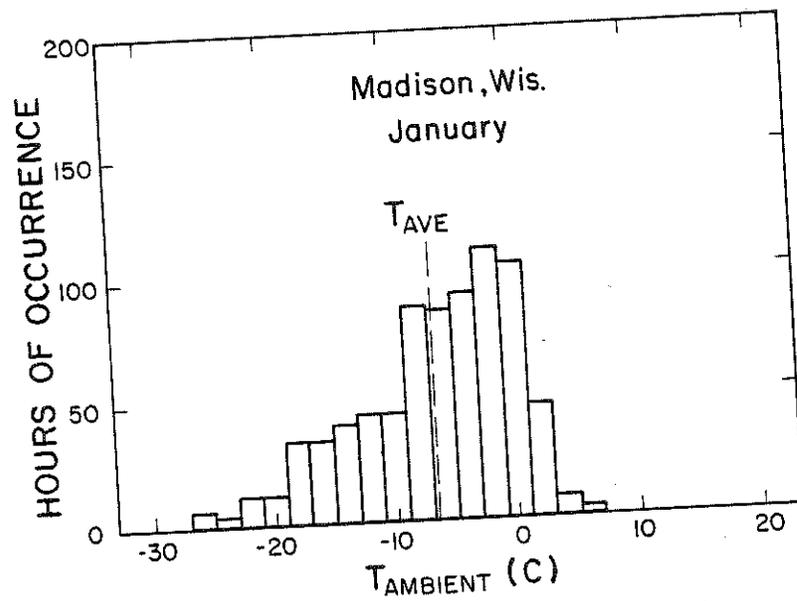
Figure 3.2a   Ambient temperature bin data for Madison, WI in January
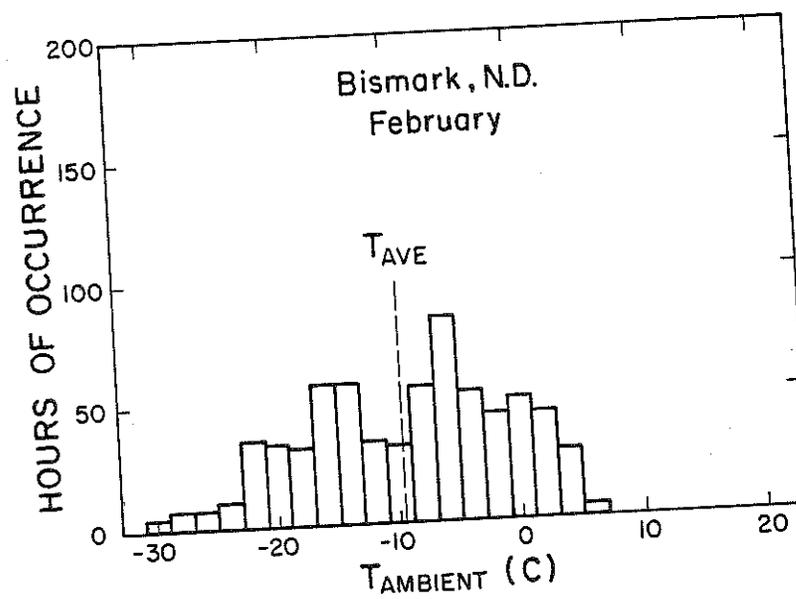


Figure 3.2b   Ambient temperature bin data for Bismarck, N.D. in
February
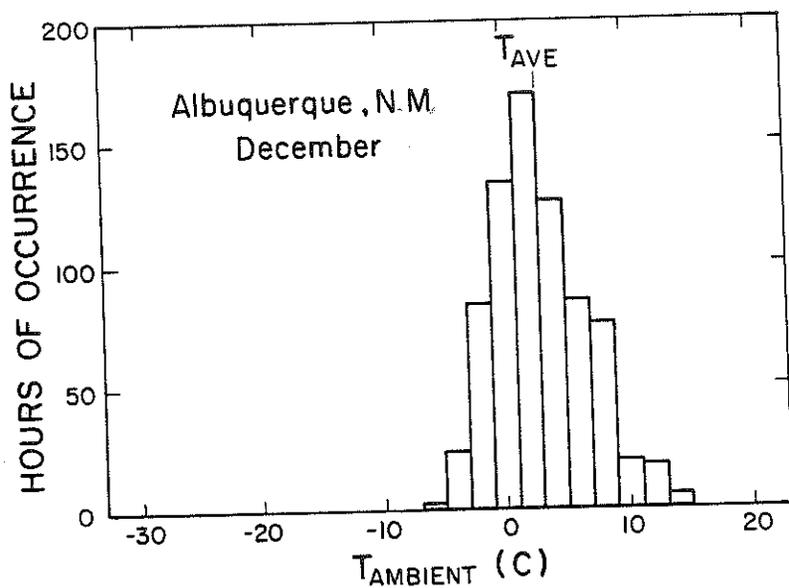
Figure 3.2c   Ambient temperature bin data for Albuquerque, N.M.
in December
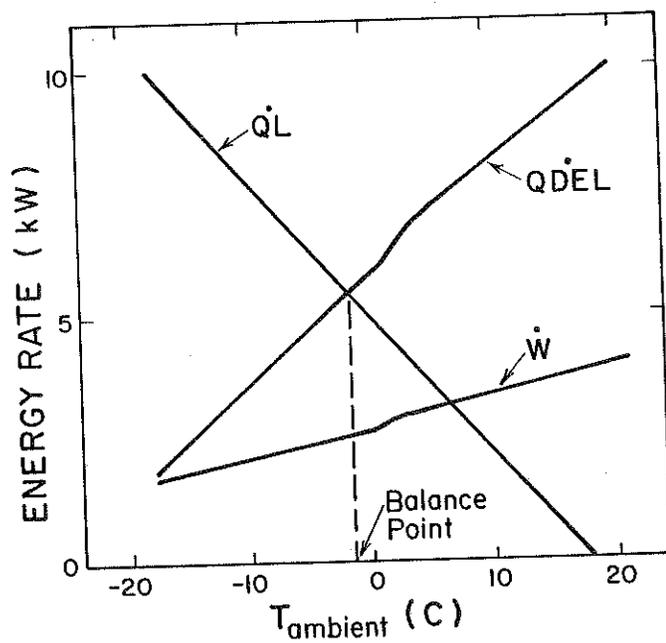


Figure 3.3   Load and heat pump performance rates vs. ambient
temperature

above this value the heat pump has excess capacity, while below this value the heat pump cannot meet all of the load and auxiliary energy must be supplied.

The heat pump work input data given by the manufacturer normally represents the electric work input either to the compressor motor, or to the heat pump unit as a whole (including fans, crankcase heaters, etc. In the analysis presented here, all of the work input energy is assumed to enter the heated space. That is, losses from motors and the compressor are included in the total energy delivered by the heat pump. If this is not the case, the manufacturer's work input data should be adjusted to more accurately reflect the amount of work input which actually reaches the heated space.

For example, if the manufactuer's data represented the total electric work input to an outdoor unit which includes both the evaporator fan and the compressor motor, the fan input would first be subtracted, and the remaining value could be multiplied by the combined motor and compressor efficiency. If the manufacturer's data is adjusted in this manner, the final compressor work values should be corrected back to reflect the total purchased energy after the analysis is completed.

The first step in the bin method is to determine the values of $\dot{Q}DEL$, $\dot{W}$, and $\dot{Q}L$ at the average temperature of each bin. At all temperatures below the balance point, the heat pump operates continuously. Therefore, for each of the bins below the balance point the energy delivered by the heat pump $QDEL_b$ and the work input to the

heat pump $W_b$ can be determined by

$$QDEL_b = \dot{QDEL}_b \cdot HRS_b \tag{3.3}$$

$$W_b = \dot{W}_b \cdot HRS_b \tag{3.4}$$

where the subscript b refers to the values associated with bin b, and $HRS_b$ is the number of hours of occurrence for that bin. The amount of auxiliary energy required for each bin $QAUX_b$ can be found by

$$QAUX_b = (\dot{QL}_b - \dot{QDEL}_b) \cdot HRS_b \tag{3.5}$$

At temperatures above the balance point, the heat pump capacity is larger than the load, and the heat pump must cycle on and off. In the bin method it is assumed that in this situation the heat pump always delivers just enough energy to meet the load, and that the work input to the heat pump is reduced proportionally. Thus for all bins above the balance point $QDEL_b$ and $W_b$ can be found by

$$QDEL_b = \dot{QL}_b \cdot HRS_b \tag{3.6}$$

$$W_b = \frac{\dot{QL}_b}{\dot{QDEL}_b} \cdot \dot{W}_b \cdot HRS_b \tag{3.7}$$

Since at these temperatures the heat pump can always meet the full load, no auxiliary energy is required.

When the load, the energy delivered by the heat pump, the work input to the heat pump, and the auxiliary energy required have been

determined for each bin, the monthly values can be found by adding up the values for all bins. If required, the amount of energy absorbed from the atmosphere QATM can be determined at this point by subtracting the total work input from the total energy delivered by the heat pump. Also, the values of FATM and FW are now readily determined by dividing through by the total space heating load. An example of the calculations involved in the bin method is given in Appendix A, and a program which carries out the bin method calculations is listed in Appendix B.

Although somewhat tedious to do by hand, the bin method is fairly easy to carry out. This method does, however, have a major disadvantage. The number of hours of occurrence in each temperature bin must be available for each location in which the bin method is to be used. A further restriction applies in the case of parallel solar heat pumps, where the bin data must be available in a monthly form. Although some data does exist in this form (24), there are many locations for which this level of detailed data is not available. In addition, the rather large quantity of data which must be stored and input to the bin method for each location makes this method unwieldy for small computers or programmable hand-held calculators.

In order to avoid the problems associated with the bin method, a new method has been developed which requires only the monthly average ambient temperature. This new method is discussed in the following section.

3.3   The Generalized Distribution Method

3.3.1   The Generalized Distribution

Comparison of Figure 3.2 a, b and c with one another demon-
strates that the histograms of ambient temperature occurrence vary
widely by both month and location.  It has not been possible to con-
struct from commonly available data, such as average ambient temper-
atures and number of degree-days, functions which will closely
approximate the actual histogram for a given month in a particular
location.  However, it has been found that the dependence of the
system performance on the shape of the actual distribution is weak
enough to allow the use of a single continuous distribution instead
of the actual bin distributions.  A diagram of the continuous gener-
alized distribution employed in this procedure is shown in Figure
3.4.  Since the distribution is continuous, the units on the ordin-
ate are hours per degree of temperature, instead of hours of occur-
rence as for the bin distribution.  The distribution $N(T)$ is cen-
tered at the monthly average ambient temperature, and decreases
linearly on either side.

In order to apply this distribution to a particular situation,
two parameters must be found.  These are the maximum frequency $N_{max}$,
and the spread of the distribution, $\Delta T_s$.  The value of $N_{max}$ can be
found in terms of $\Delta T_s$ since the integral of $N(T)$ over the whole
distribution must be equal to the total number of hours in a month
$N_{TOT}$.
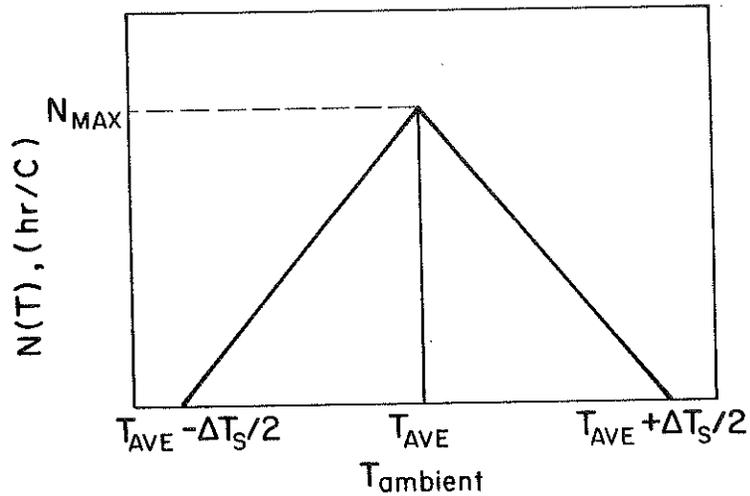
Figure 3.4  A generalized ambient temperature distribution

$$\int_{T_{ave} - \Delta T_s/2}^{T_{ave} + \Delta T_s/2} N(T)\,dT = N_{TOT} \qquad (3.8)$$

or, for the triangular distribution chosen,

$$N_{max} = 2\ N_{TOT}/\Delta T_s \qquad (3.9)$$

The equation for N(T) then becomes

$$N(T) = \frac{2\ N_{TOT}}{\Delta T_s} \left(1 - \frac{2}{\Delta T_s} \mid T - T_{ave} \mid \right) \qquad (3.10)$$

The second parameter, $\Delta T_s$, is difficult to determine.  Comparison of the results from the generalized distribution against those from the bin method have shown that the temperature spread

which gives the minimum prediction error is an unknown function of both the actual temperature spread for the month and the balance point temperature. Since the actual temperature spread is not generally available for all locations, the results presented here were found by using a $\Delta T_s$ of 32°C for all situations. This value was chosen as one which gave acceptable results for a wide variety of U.S. climates. Since the actual monthly values of the temperature spread range from about 15°C in Seattle to over 50°C in Bismarck, the 32°C value represents about the middle of the range.

It will be shown that using a single value of the temperature spread in the generalized distribution method gives results which have a standard deviation within 1.5% of the load in a wide variety of climates, despite the variations in actual temperature spreads between climates. There is however, one type of situation in which the difference between the actual temperature spread and the temperature spread used in the generalized distribution becomes quite important. This situation will be discussed in detail in Section 3.4.

### 3.3.2 Comparison of the Generalized Distribution to Bin Data Histograms

Since the generalized distribution is continuous over the ambient temperature scale, direct comparison to the bin distribution in Figure 3.2 is not possible. If, however, the generalized distribution were integrated piecewise over small temperature ranges the

size of the temperature bins, the result would be an equivalent bin
distribution. A histogram of this bin distribution for a month with
744 hours and a temperature spread of 32°C is shown in Figure 3.5.
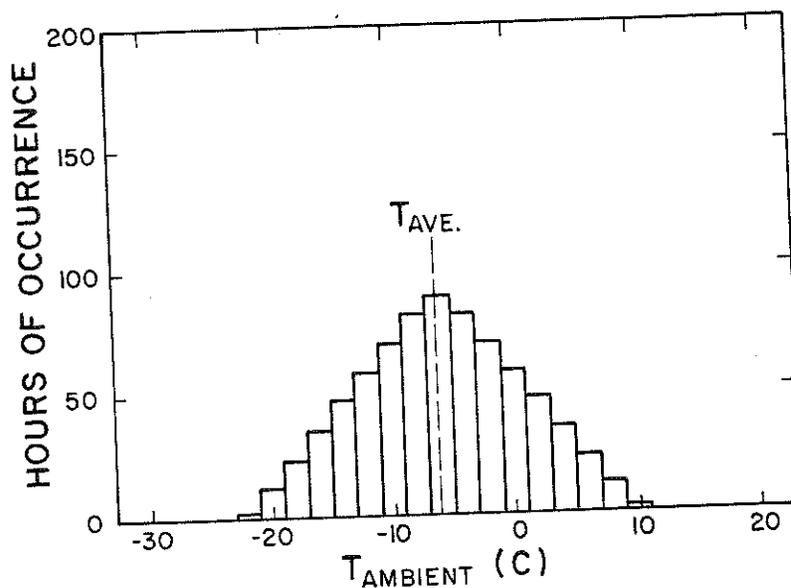


Figure 3.5 Equivalent bin distribution derived from generalized
distribution.

In comparison with distributions in Figure 3.2, this distribution
is always symmetric about the average temperature, and the size of
the bins decreases linearly on either side. For a fixed value of
$\Delta T_s$, the number of hours in each bin is fixed by the total number
of hours in the month.

### 3.3.3 Consideration of Other Distribution Shapes

Since the system performance predictions are fairly insensitive
to the exact shape of the frequency distribution, several other

ALASKA

NOME
(>6000)

MEDFORD
(2739)

BISMARK
(5024)

ALBANY
(3826)

MADISON
(4294)

WASHINGTON,D.C.
(2346)

ELY
(4297)

COLORADO
SPRINGS
(3343)

DODGE
CITY
(2803)

COLUMBIA
(1142)

ALBUQUERQUE
(2381)

MEMPHIS
(1793)

CHARLESTON
(1131)

HUNTSVILLE
(1578)

LAKE CHARLES
(811)
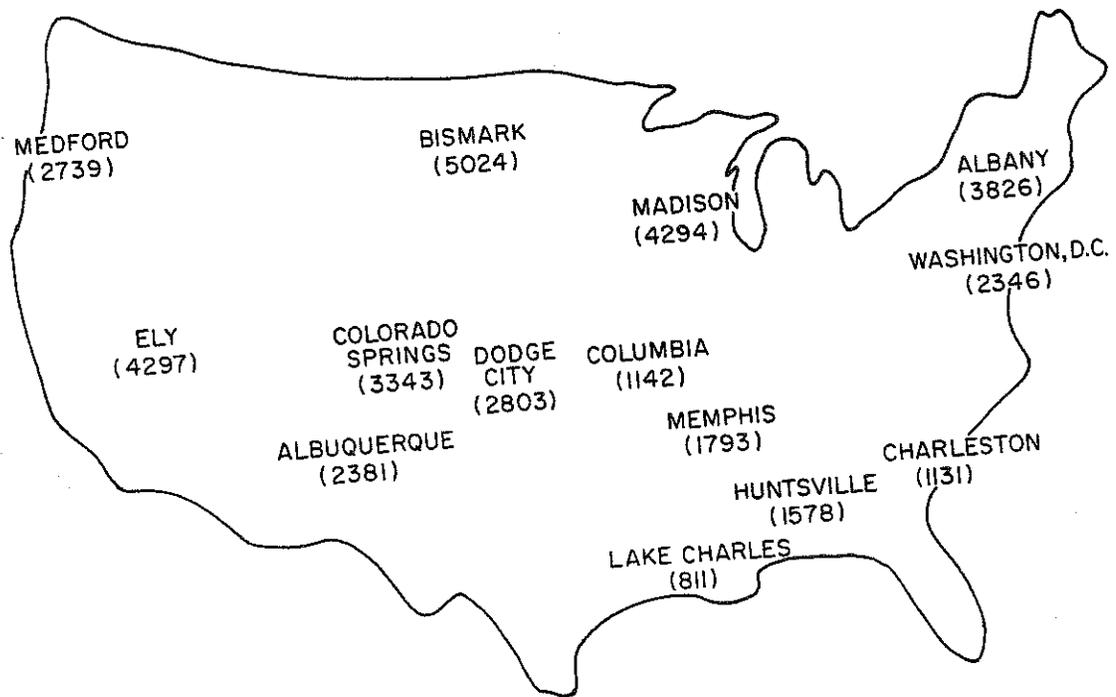
shapes besides the triangular distribution shown here will give equally good results. For example, the square of a cosine distribution with a temperature spread of 35°C which is centered about the average ambient temperature will give results which are essentially identical to those listed here for the triangular distribution with a temperature spread of 32°C. A normal Gaussian distribution could be used, but has the disadvantage that the tails extend to infinity in either direction while the actual data has finite upper and lower bounds.

In fact, almost any symmetrical distribution, including a uniform distribution, will work reasonably well in locations which have average ambient temperatures close to the middle of the range of heat pump operating temperatures. At the upper and lower ends of this range, however, the actual shape and size of the distribution becomes more important, as discussed in Section 3.3.1. The triangular distribution which was used here was chosen as a very simple distribution which gave excellent results over a wide range of climate types.

## 3.3.4 Integrating Over the Generalized Distribution

In order to evaluate the monthly energy quantities with the generalized distribution, the system performance rates must be integrated over the distribution. Several methods are available for doing this, with the most straightforward approach being a numerical integration. Since the heat pump performance data is usually

given as discrete points, interpolation is required. This method

normally requires a computing machine to carry out the repetitive

steps of the integration procedure.

The computer program which was used to generate the generalized

distribution results is listed in Appendix C. Figure 3.6 is useful

in visualizing the basic process followed here, which is to integrate

the energy delivered by the heat pump, the energy absorbed from the
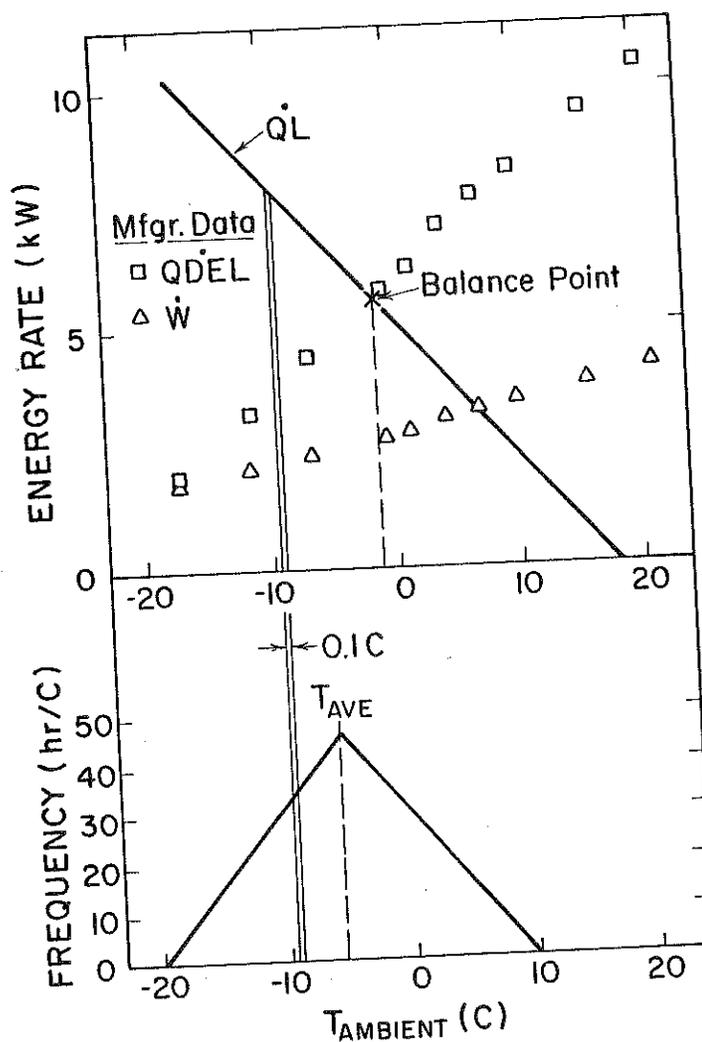


Figure 3.6    Heat pump and load characteristics plotted with the
generalized distribution.

atmosphere, the work input to the heat pump, the auxiliary required and the space heating load individually over the triangular distribution. At the beginning of each integration step the heat pump performance rates, the frequency from the generalized distribution N(T), and the load rate are found for the new temperature. Then the energy delivered by the heat pump is compared to the load to determine if the temperature is above or below the balance point. If the load is larger than the energy delivered, the temperature is below the balance point and auxiliary energy is required. If the load is less than the energy delivered, the temperature is above the balance point and the heat pump operates only a fraction of the time.

After each of the rates have been multiplied by the frequency of occurrence, the integrated value for that step is determined. Since the functions to be integrated are smooth and well behaved, a simple trapezoidal integration algorithm with a step size of 0.1°C is adequate. A running total of the integrated values is maintained. After the final integration step, the totals of the integrated values of QATM and W are divided by the total for the space heating load to determine FATM and FW, and these values are printed out.

There are other techniques for integrating the heat pump performance rates over the generalized distribution, and several approximations that can be made to simplify the calculations. Several of these variations will be discussed in Section 3.5.

3.4  Comparison of Results

In order to compare the results predicted by the generalized distribution to the results from the bin method, calculations were made with both methods for 14 U.S. locations with all combinations of the heat pump and load data listed in Table 3.1.  The locations for which the comparisons were made are also shown in Figure 3.7.

TABLE 3.1

Heat Pumps:  2 Ton (Figure 3.3), 3 Ton and 5 Ton (Figure 2.3)

House UA:  278, 556 and 833 w/°C

Locations:  Albany, NY            Ely, NV

Albuquerque, NM       Huntsville, AL

Bismarck, ND          Lake Charles, LA

Charleston, SC        Madison, WI

Colorado Springs, CO  Medford, OR

Columbia, MO          Memphis, TN

Dodge City, KA        Nome, AK

The bin method program listed in Appendix B was used with 10 year average bin data from reference (24), and calculated both the energy quantities and the monthly average temperature.  These average temperatures were then used as the center points for the generalized distribution with a temperature spread of 32°C, and the system energy rates were numerically integrated over the monthly temperature ranges using the computer program listed in Appendix C.  In

both the bin method and the generalized distribution method the

energy values for the 6 month heating season were summed up to give

the seasonal values.

ALASKA

NOME
(>6000)

MEDFORD
(2739)

BISMARK
(5024)

MADISON
(4294)

ALBANY
(3826)

WASHINGTON, D.C.
(2346)

ELY
(4297)

COLORADO
SPRINGS
(3343)

DODGE
CITY
(2803)

COLUMBIA
(1142)

ALBUQUERQUE
(2381)

MEMPHIS
(1793)

CHARLESTON
(1131)

HUNTSVILLE
(1578)

LAKE CHARLES
(811)

Figure 3.7  Locations in which the generalized distribution method
was tested.  Values in parentheses represent average
number of °C-days.

The seasonal values of FATM and FW from the generalized dis-
tribution are plotted against those from the bin method in Figures
3.8 and 3.9.

The standard deviations of the prediction errors are listed
in Table 3.2 on both a monthly and a seasonal basis.
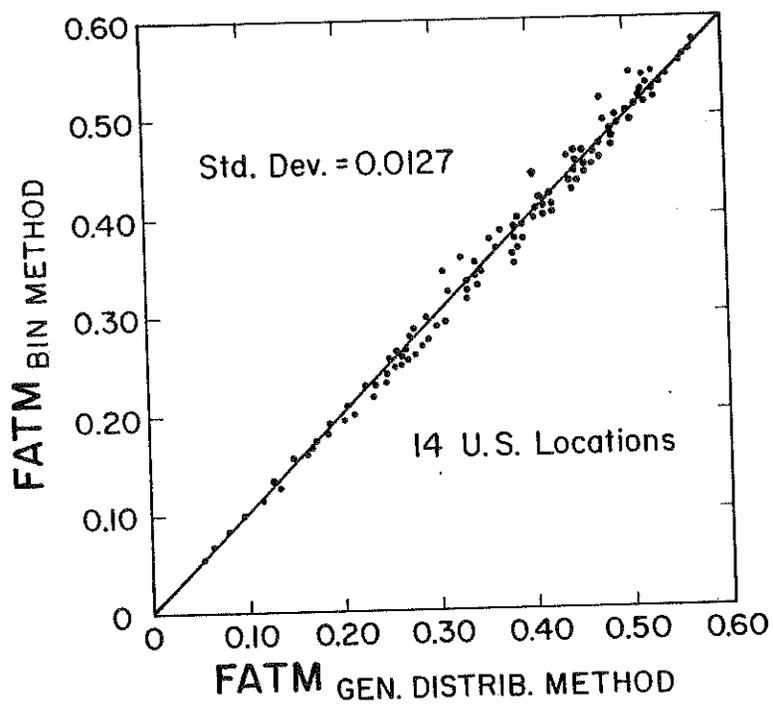


Figure 3.8   Seasonal values of FATM from the bin method vs.
those from the generalized distribution.
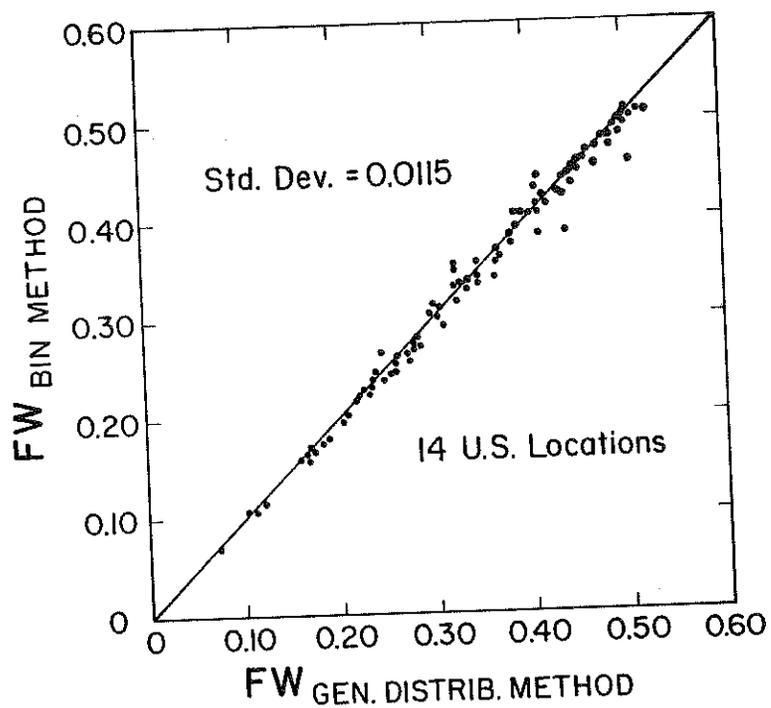
Figure 3.9 Seasonal values of FW from the bin method vs. those from the generalized distribution.

TABLE 3.2

Standard Deviations of Prediction Errors
Using Manufacturer's Heat Pump Performance Data

|  | FATM | FW | FPUR |
|---|---|---|---|
| Monthly (756 points) | .014 | .013 | .014 |
| Seasonal (126 points) | .013 | .012 | .013 |

The standard deviation of the prediction errors are within 1.5% of

the space heating load, indicating that overall the results from the

generalized distribution agree quite well with those from the bin

method.

As mentioned in Section 3.3.1 however, there is a situation in which the generalized distribution with a universal value for the temperature spread can significantly mispredict the values of FATM and FW. This situation arises because the temperature range over which the heat pump operates is limited. The upper bound on this range is the maximum temperature at which there is a space heating load, normally about 18°C (65°F). Since there is no space heating load at or above this temperature, there is no need for the heat pump to operate. The lower temperature bound on the range of heat pump operation is the heat pump low temperature cutoff, the temperature below which the heat pump is turned off. An explanation of this is given in Section 2.4.4

The size of the temperature spread used in the generalized distribution is most critical when the monthly average temperature is close to one of the limits of heat pump operation. If this occurs in a location where the actual temperature spread is sig-nificantly different from the temperature spread used in the gen-eralized distribution, there will be some hours which are either incorrectly included in, or incorrectly excluded from, the heat pump operating time.

In the range of U.S. climates studied, two climate extremes that gave rise to these problems were found. The first of these is a climate in which the heating season is characterized by ambient temperatures which are generally quite low but also vary quite

widely on a monthly basis. This weather is typical of the very northern parts of the U.S. In these climates the actual monthly temperature spreads may be as large as 50°C. Using the generalized distribution with a temperature spread of 32°C causes some hours to be incorrectly classified as heat pump operating time, as shown for a month in Bismarck, N.D. in Figure 3.10.
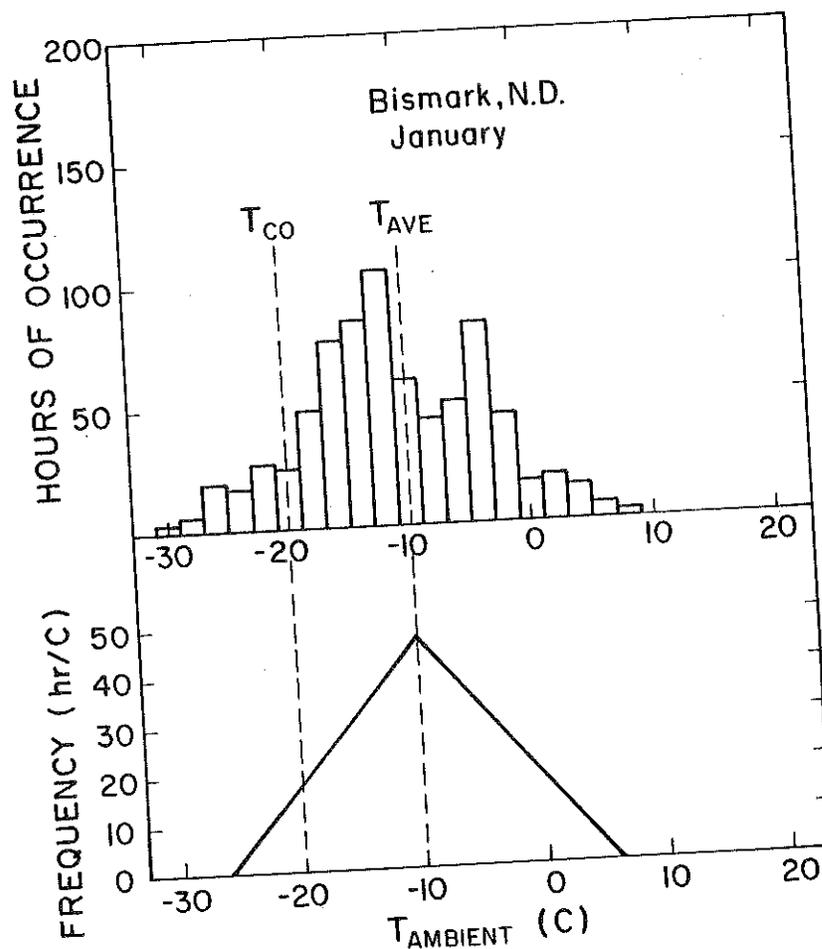


Figure 3.10   Bin distribution and generalized distribution for a month with low average temperature and large temperature spread.

There are two effects associated with this incorrect classification. First, with the temperature spread used in the generalized distribution the heat pump will be computed to operate more hours than would have been the case with the actual distribution. This tends to increase the value of both FATM and FW.

The second effect is that since the average temperature at which the heat pump will be calculated to operate is lower in the generalized distribution than in the actual distribution, the coefficient of performance will be lower. This tends to increase the value of FW and decrease the value of FATM. Thus in this situation we would expect the generalized distribution to overpredict the value of FW, but to predict the value of FATM more accurately.

The second type of climate in which the generalized distribution can mispredict is one in which the ambient temperatures are quite mild and quite uniform over a month, which is typical of some areas in the Pacific Northwest. In this climate the temperature spread of 32°C is too large, and some hours are incorrectly classified as being above the maximum temperature at which there is a space heating load. This incorrect classification again has two effects. Since with the temperature spread used in the generalized distribution the heat pump is assumed to operate fewer hours overall, the predicted values of FATM and FW will tend to be too low. In addition, the average temperature at which the heat pump operates will be lower with the generalized distribution. This tends to decrease the heat pump coefficient of performance, causing FW to be

somewhat larger and FATM to be still smaller. The generalized

distribution will tend to underpredict the value of FATM, while pre-

dicting the value of FW more accurately.

Since air-to-air heat pumps are rarely considered for applica-

tions in extremely cold climates, the overprediction of FW in cli-

mates such as Bismarck, N.D. and Nome, AK is probably of little

importance. A more important situation from the standpoint of

general applicability of heat pumps is the climate with very mild

and uniform temperatures. In the locations tested here this type

of climate was represented by Medford, Oregon. For the nine systems

tested (3 heat pumps and 3 house UA values) in Medford, the stand-

ard deviations of the prediction errors on a seasonal basis are

0.033 and 0.020 for FATM and FW respectively. These values are

significantly higher than the values for all 14 locations which

are listed in Table 3.2. In addition, the generalized distribu-

tion method with a universal temperature spread always overpre-

dicted FATM and occasionally overpredicted FW in this location.

This is consistent with the explanation given above.

Overall then, the generalized distribution method with a uni-

versal temperature spread predicted the values of FATM and FW quite

accurately in the majority of locations tested. The mispredictions

in Medford, however, point out a possible need for flexibility in

the value of the temperature spread used in some locations. This

is a difficult problem. The main advantage of the generalized

distribution method is that less data is required than for the bin

method. Therefore if some method of adjusting the temperature spread in these locations is to be developed, it must also rely on commonly available weather data such as the monthly average temperature and the number of degree-days.

## 3.5 Seasonal Integrations and Straight Line Heat Pumps

Although the method presented in the preceding section is the most accurate way of using the generalized distribution, it is also the way which requires the most calculation. It is possible to use the generalized distribution in ways which simplify the calculations at the expense of slightly reduced accuracy.

If only seasonal energy values are required, a single seasonal integration can be substituted for the monthly integrations. That is, the heat pump performance rates can be integrated using a seasonal distribution. The seasonal distribution is formulated in much the same manner as the monthly distribution, except that the total number of hours $N_{TOT}$ must now represent the total number of hours in the heating season, and the temperature spread must be increased.

The results from seasonal integrations with a universal temperature spread of 39°C were compared against the results from the bin method for all of the locations and parameters listed in Table 3.1. The standard deviations of the prediction errors (126 points) were 0.019 for FATM and 0.015 for FW. These values are slightly higher than the standard deviations listed in Table 3.2 for the monthly in-

tegrations, indicating that the seasonal integrations give somewhat less accurate results. However, the less accurate results may be sufficient for some applications. The advantage of this simplification is that only a single integration is required, as opposed to several monthly integrations.

Examination of the performance characteristics of air-to-air heat pumps (e.g., Figure 2.3 and 3.3) reveals that over most of the temperature range the performance varies linearly with ambient temperature. However, there is commonly a nonlinear decrease in the energy delivery rate just above 0°C, caused by the necessity of removing excessive frost build-up from the outdoor coils.

If this defrost dip is ignored, the variation in the performance of the heat pump with temperature would be well described by a straight line. The advantage in using a straight line heat pump model is that it eliminates the necessity of storing and interpolating between the discrete data points customarily supplied by the manufacturer.

The results from the generalized distribution method using a straight line model for the heat pump characteristics were compared to the results from the bin method using the manufacturer's heat pump data. The comparisons were again made for all the locations and parameters listed in Table 3.1. The standard deviations of the prediction errors are listed for monthly, seasonal (sum of the monthly values), and seasonal integration values in Table 3.3.

TABLE 3.3

Standard Deviations of the Prediction Errors
Using Straight Line Heat Pump Models

|  | FATM | FW | FPUR |
|---|---|---|---|
| Monthly (756 points) | .017 | .017 | .017 |
| Seasonal (126 points) | .016 | .018 | .016 |
| Seasonal Integration (126 points) | .021 | .020 | .021 |

The standard deviations using the straight line heat pump with

monthly integrations are slightly under 2% of the load, while those

from the seasonal integration are just over 2%. Comparison of the

values in Table 3.3 to those in Table 3.2 demonstrates that the

standard deviation of the prediction errors with the straight line

heat pump are about 0.5% higher than the comparable values using

the manufacturer's data.

In terms of the numerical integration, the advantage of using

the straight line heat pump is that instead of storing all the manu-

facturer's performance data and interpolating between points on each

step of the integration, the heat pump performance rates can be

quickly and easily generated within the program itself. This fact

would be particularly useful on a hand held calculator, where stor-

age space is quite limited. A second, more general, advantage of

using the straight line heat pump approximation with the generalized

distribution is that since the load, the heat pump characteristics,

and the weather data are all represented by analytic expressions,

the integration over the generalized distribution can be written in

closed analytical form. Unfortunately, because of factors such as the difference in the heat pump performance above and below the balance point, the resulting expressions are only piecewise integrable. As a result, they are more cumbersome than the numerical integration procedure. The set of expressions which result from these integrations are listed in Appendix D.

Overall, the results presented in this section demonstrate that the more detailed the generalized distribution calculations are, the closer the results agree with those from the bin method. For example, when each monthly average temperature is used in finding the seasonal values, the results agree more closely than when only a seasonal average temperature is used. Similarly, when the manufacturer's heat pump performance data is used, the results from the generalized distribution are more accurate than when a straight line approximation to the heat pump data is used. Clearly, there is a trade-off between ease of computation and accuracy.

## 3.6 Summary of Generalized Distribution Method

The basic steps involved in using the generalized distribution method are as follows:

1. Determine the space heating load

   Use the degree-day method or similar means to determine the space heating load QL.

2. Determine FATM and FW

   Use a program such as that listed in Appendix C with the monthly

average ambient temperature to determine FATM and FW.

3. Calculate QATM and W

Substitute the values from steps 1 and 2 into the following

expressions to get QATM and W

$$QATM = FATM \cdot QL \qquad (3.11)$$

$$W = FW \cdot QL \qquad (3.12)$$

4. Calculate QAUX

Use the values from steps 1 and 3 to calculate QAUX with the

following expression.

$$QAUX = QL - QATM - W \qquad (3.13)$$

5. Repeat steps 1 through 4 for each month to determine the

annual values.

The annual fractions of the load met by any of the sources

can be found by

$$F_{ANNUAL} = \frac{\Sigma Q_i}{\Sigma QL_i} \qquad (3.14)$$

where $Q_i$ is the energy contributed to the load in month i, and

$QL_i$ is the load in month i.

3.7 Example Problem

The example problem involves finding stand-alone heat pump

values which could have been used in the example problem from Chapter

2. The system consists of the 3 ton heat pump whose characteristics

are plotted in Figure 2.3, and the calculations will be carried out

for the month of January in Madison, WI.

1.  For the month of January the space heating load is QL = 19.59 GJ.

2.  Using the monthly average temperature of −6.6°C, temperature
    spread of 32°C and a step size of 0.1°C in the program listed
    in Appendix C, the values of FATM and FW are found to be 0.285
    and 0.420 respectively (compared to 0.293 and 0.420 from the bin
    method).

3.  The energy absorbed from the atmosphere and the work are

    $$QATM = 0.285 \cdot 19.59 = 5.58 \text{ GJ}$$
    $$W = 0.420 \cdot 19.59 = 8.23 \text{ GJ}$$

4.  The auxiliary energy required for space heating is

    $$QAUX = 19.59 - 5.58 - 8.23 = 5.78 \text{ GJ}$$

    The fraction of the space heating load met by purchased  energy is

    $$\frac{8.23 + 51.78}{19.59} = .715$$

5.  When steps 1 through 4 are repeated for the 6 month heating
    season, the seasonal fraction by purchasing energy is 0.642
    (compared to 0.627 from the bin method).

## 3.8 Conclusions

The generalized distribution appears to be an acceptable substitute for the bin method. With overall standard deviations under 1.5%, the generalized distribution method using a universal temperature spread predicts the heat pump performance values to within the accuracy with which the system parameters are known for virtually all locations in which heat pumps are normally considered.

The methods of using the generalized distribution which require the least calculation effort are the least accurate. It is more accurate to use the manufacturer's heat pump data than to use a straight line approximation, and it is more accurate to sum the monthly values over the season than to do a single seasonal integration. The straight line approximation and the seasonal integration may, however, be quite useful for application to programmable hand held calculators in situations where high accuracy is not required.

The primary advantage of the generalized distribution method is that only the monthly average temperatures are required, as opposed to the temperature bin data for the bin method. The generalized distribution is quite compatible with the f-chart method, since no additional weather data is required beyond that already necessary for f-chart.

4.   CONCLUSIONS

The procedures presented here represent fairly simple methods for predicting the performance of air-to-air heat pumps, either in stand-alone systems or in parallel solar-heat pump systems.   The results predicted by the generalized distribution agree well with those from the bin method, giving standard deviations of prediction errors less than 1.5%.   The standard deviations of the errors associated with the equations and assumptions used in predicting parallel system performance are within 1.3%.

The overall accuracy of the prediction method for parallel systems is dependent not only on the accuracy of the equations and assumptions themselves, but also on the accuracy of the methods used to obtain the solar system and stand-alone heat pump performance values.   A schematic of procedures and information flows for determining the thermal performance of the parallel system is shown in Figure 4.1.

SOLAR SYSTEM

| f-chart Method |
| OR |
| other solar system design procedures |

FSOL
QL
WL

FATM$_o$
FW$_o$

STAND-ALONE HEAT PUMP

| generalized distri-bution method |
| OR |
| bin method |

parallel system method

QATM
W
QAUX
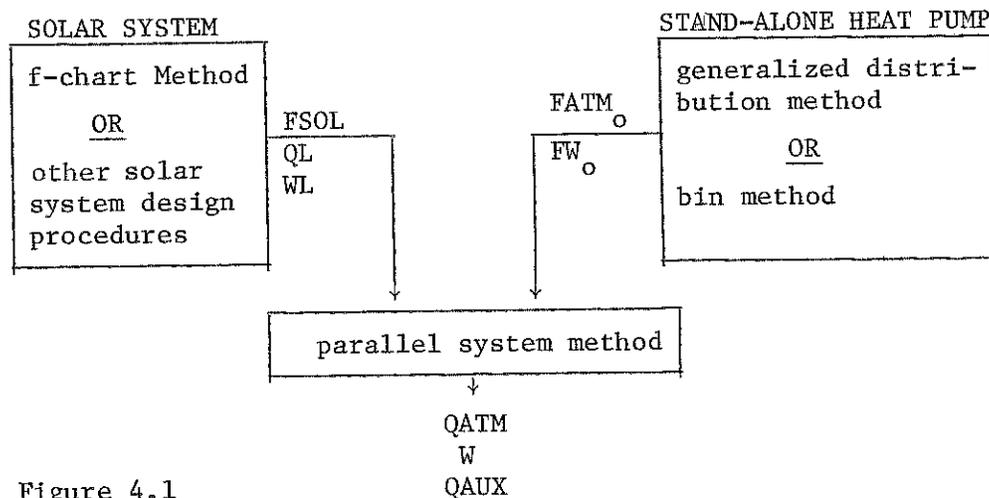
Figure 4.1

There are several options available for finding the values required as inputs to the parallel system method. The generalized distribution method is compatible with the f-chart method since the ambient temperature data required is the same in both methods. However, any solar system design method which gives an accurate value for the fraction of the space and water heating load met by solar energy could be used with the parallel system prediction method. Similarly the bin method could be used in place of the generalized distribution method to generate the required stand-alone heat pump performance values. The choice of methods to be used in a particular situation depends on the type of weather data and the computational resources which are available.

There are also several options available in terms of using the generalized distribution. The most accurate method is to do monthly integrations with manufacturer's heat pump performance data. However, in situations where ease of computation is important, seasonal integrations or a straight line heat pump approximation could be used with some loss of accuracy. Again the choice of technique depends on the availability of weather data and computational resources.

This flexibility in the available techniques should allow the user to tailor a procedure which will simultaneously be compatible with his resources and give accurate predictions of the thermal performance. Once the thermal performance is known, it will be

possible to establish the economic viability of these heat pump systems relative to conventional fossil-fuel systems and to solar-only systems using standard economic analyses.

APPENDIX A:   BIN METHOD EXAMPLE

As an example of the calculations involved in the bin method, the values of $FATM_o$ and $FW_o$ used in the example problem in Chapter 2 will be determined.   The example involves the 3 ton heat pump whose performance characteristics are shown in Figure 2.3.   The bin data used is for Madison, WI in January, and is shown in Figure 3.2. The house UA value is 1.070 MJ/hr °C (297 w/°C).

A work sheet for the bin method calculations is given in Table A-1.   The first two columns show the center temperature and the hours of occurrence for each bin.   The space heating load rate $\dot{Q}L$, the heat pump energy delivery rate $\dot{Q}DEL$, and the work input rate $\dot{W}$ are tabulated for each of the temperatures listed in the first column.

The fourth column shows the fraction F of the time that the heat pump must operate.   For temperatures above the balance point F is given by the ratio of $\dot{Q}L$ to $\dot{Q}DEL$, while for temperatures below the balance point the heat pump must run continuously and F is always equal to one.   For this example, the balance point temperature is −4.1 C.

Once these values have been established, the values of QDEL, W, and QL for each bin are calculated by

$$QDEL_b = \dot{Q}DEL \cdot F \cdot HOURS \qquad (A.1)$$

$$W_b = \dot{W} \cdot F \cdot HOURS \qquad (A.2)$$

$$QL_b = \dot{Q}L \cdot HOURS \qquad (A.3)$$

## TABLE A-1
## BIN METHOD EXAMPLE

| TEMP C | HOURS | $\dot{Q}_L$ MJ/HR | $\dot{Q}_{DEL}$ MJ/HR | F | $\dot{W}$ MJ/HR | QDEL MJ | W MJ | AUX MJ | QL MJ |
|---|---|---|---|---|---|---|---|---|---|
| 6.0 | 3.0 | 12.84 | 31.78 | .40 | 15.39 | 38.5 | 18.7 | .0 | 38.5 |
| 4.0 | 8.5 | 14.98 | 29.72 | .50 | 14.99 | 127.3 | 64.2 | .0 | 127.3 |
| 2.0 | 46.5 | 17.12 | 28.20 | .61 | 14.58 | 796.1 | 411.4 | .0 | 796.1 |
| .0 | 103.5 | 19.26 | 26.62 | .72 | 14.14 | 1993.4 | 1058.8 | .0 | 1993.4 |
| -2.0 | 110.0 | 21.40 | 25.06 | .85 | 13.71 | 2354.0 | 1287.9 | .0 | 2354.0 |
| -4.0 | 92.0 | 23.54 | 23.54 | 1.00 | 13.29 | 2165.7 | 1222.8 | .0 | 2165.7 |
| -6.0 | 76.0 | 25.68 | 22.02 | 1.00 | 12.83 | 1673.9 | 974.9 | 277.8 | 1951.7 |
| -8.0 | 77.0 | 27.82 | 20.45 | 1.00 | 12.36 | 1575.0 | 951.8 | 567.1 | 2142.1 |
| -10.0 | 43.0 | 29.96 | 18.88 | 1.00 | 11.82 | 811.9 | 508.3 | 476.4 | 1288.3 |
| -12.0 | 44.0 | 32.10 | 17.36 | 1.00 | 11.10 | 763.9 | 488.3 | 648.5 | 1412.4 |
| -14.0 | 40.0 | 34.24 | 15.77 | 1.00 | 10.83 | 631.0 | 433.3 | 738.6 | 1369.6 |
| -16.0 | 34.0 | 36.38 | 14.22 | 1.00 | 10.48 | 483.4 | 356.4 | 753.5 | 1236.9 |
| -18.0 | 33.5 | 38.52 | 12.70 | 1.00 | 9.99 | 425.5 | 334.6 | 865.0 | 1290.4 |
| -20.0 | 12.0 | 40.66 | 11.18 | 1.00 | 9.50 | 134.2 | 114.0 | 353.7 | 487.9 |
| -22.0 | 11.5 | 42.80 | .00 | 1.00 | .00 | .0 | .0 | 492.2 | 492.2 |
| -24.0 | 4.0 | 44.94 | .00 | 1.00 | .00 | .0 | .0 | 179.8 | 179.8 |
| -26.0 | 5.5 | 47.08 | .00 | 1.00 | .00 | .0 | .0 | 258.9 | 258.9 |
| TOTALS | | | | | | 13973.8 | 8225.5 | 5611.4 | 19585.3 |

FATN=.294
FW=.420

where the subscript b refers to the energy totals for an individual bin. The auxiliary energy required for each bin below the balance point is

$$QAUX_b = QL_b - QDEL_b \qquad (A.4)$$

As an example, these calculations will be carried out for the $-10°C$ bin. There are 43 hours of occurrence for this bin. The space heating load rate is

$$\dot{Q}L = 1.070 \ [18-(-10)] = 29.96 \ MJ/hr$$

From the heat pump performance characteristics, the rate of energy delivery $\dot{Q}DEL$ is 18.88 MJ/hr. Since $\dot{Q}L$ is larger than $\dot{Q}DEL$, this temperature is below the balance point, and F is equal to one. The rate of work input to the heat pump, $\dot{W}$, is 11.82 MJ/hr at $-10°C$. The energy delivered, work input required, and load for this bin are found from Equations (A.1), (A.2), and (A.3)

$$QDEL_b = 18.88 \cdot 1.0 \cdot 43 = 811.9 \ MJ$$
$$W_b = 11.82 \cdot 1.0 \cdot 43 = 508.3 \ MJ$$
$$QL_b = 29.96 \cdot 43 = 1288.3 \ MJ$$

Since this bin is below the balance point, the auxiliary energy required is calculated with Equation (A.4)

$$QAUX_b = 1288.3 - 811.9 = 476.4 \ MJ$$

The calculations for bins above the balance point procede in very much the same manner as those shown here.

When the procedures illustrated above have been carried out
for all of the bins, the monthly totals are found by summing the
values from all of the bins.  Once this has been done, the fractions
of the load met by heat pump work and energy absorbed from the
atmosphere can be found by

$$FW = \frac{W}{QL} = \frac{8225.3}{19585.3} = .420$$

$$FATM = \frac{QDEL-W}{QL} = \frac{13973.8-8225.5}{19585.3} = .293$$

In order to find annual or seasonal values when the data is
given on a monthly basis as shown here, this procedure must be
carried out for each month.  Some data is available in which the
hours of occurrence have been tabulated on either an annual or a
seasonal basis.  The procedure for carrying out the bin method on
this data is essentially identical to the procedure shown here.

```
C          APPENDIX B
C
C
C
C***********************************************************************
C                                                                     *
C          BINMETH6 IS AN INTERACTIVE FORTRAN PROGRAM WHICH            *
C      CARRIES OUT THE BIN METHOD CALCULATIONS ON A                    *
C      MONTHLY BASIS.                                                  *
C          THE BIN DATA CAN BE ENTERED IN EITHER 5 DEGREE F OR         *
C      2 DEGREE C BINS.  BINMETH6 WILL REQUEST THE NUMBER              *
C      OF BIN DATA POINTS (<26), THE AVERAGE TEMPERATURE               *
C      OF THE HIGHEST TEMPERATURE BIN, AND THEN THE NUMBER             *
C      OF HOURS OF OCCURANCE FOR EACH OF THE BINS.  ALL                *
C      OUTPUT TEMPERATURES ARE IN DEGREES C.  ALL INPUT ENERGY         *
C      RATES MUST BE IN CONSISTENT UNITS (E.G. BTU/HR, KJ/HR,          *
C      OR KW).                                                         *
C          BINMETH6 CALLS ONE SUBROUTINE INTERP, WHICH READS           *
C      IN FOUR COLUMNS OF HEAT PUMP DATA (AMBIENT TEMPERATURE,         *
C      QATM, QDEL, AND W) ON THE FIRST CALL, AND INTERPOLATES          *
C      ON THAT DATA ON SUCCEEDING CALLS.  THE HEAT PUMP DATA           *
C      POINT TEMPERATURES SHOULD BE IN DEGREES C.                      *
C                                                                     *
C***********************************************************************
C
      DIMENSION Q(3),T(25),HRS(12,25),TOT(5),C(5),MON(12),
     1  NBIN(12),T1(12)
      DATA MON/'JAN','FEB','MAR','APR','MAY','JUN','JUL',
     1  'AUG','SEP','OCT','NOV','DEC'/
      TR=18.0
      PRINT 1000
      PRINT 1010
      READ 1100, IUNITS
  5   PRINT 1020
      READ, U
      CALL INTERP(-1000.,Q)
      PRINT 1030
      DO 6 I=1,12
      READ, NB,T1(I),(HRS(I,J),J=1,NB)
  6   NBIN(I)=NB
      PRINT 1050
C
C*** BEGIN MONTHLY LOOP ****
      DO 110 I=1,12
      HRTOT=0.0
      XNTOT=0.0
      DO 7 K=1,5
      TOT(K)=0.0
```

```
7     C(K)=0.0
      NB=NBIN(I)
C
C*** SET UP BIN TEMPS ****
      IF(IUNITS.EQ.'Y') GO TO 20
      T1(I)=(T1(I)-32)/1.8
      DO 10 J=1,NB
 10   T(J)=T1(I)-(J-1)*5./1.8
      GO TO 40
 20   DO 30 J=1,NB
 30   T(J)=T1(I)-(J-1)*2.
C
C*** BEGIN 'BIN BY BIN' LOOP ****
 40   DO 100 J=1,NB
      IF(T(J).GT.TR) GO TO 80
      CALL INTERP(T(J),Q)
      C(4)=0.0
      F=1.0
      C(5)=HRS(I,J)*U*(TR-T(J))
      C(2)=HRS(I,J)*Q(2)
      IF(C(2).GT.C(5)) GO TO 50
      C(4)=C(5)-C(2)
      GO TO 60
 50   F=C(5)/C(2)
      C(2)=F*C(2)
 60   C(3)=F*HRS(I,J)*Q(3)
      C(1)=F*HRS(I,J)*Q(1)
      DO 70 K=1,5
 70   TOT(K)=TOT(K)+C(K)
 80   XNTOT=XNTOT+(T(J)*HRS(I,J))
      HRTOT=HRTOT+HRS(I,J)
 100  CONTINUE
      TBAR=XNTOT/HRTOT
      FATM=TOT(1)/TOT(5)
      FW=TOT(3)/TOT(5)
      FAUX=TOT(4)/TOT(5)
      XMPF=(TOT(2)+TOT(4))/(TOT(3)+TOT(4))
      COP=TOT(2)/TOT(3)
      PRINT 1060 MON(I),TBAR,(TOT(K),K=1,5),FATM,FW,XMPF,COP
 110  CONTINUE
 120  PRINT 1000
      STOP
 1000 FORMAT(1X///1X,70('*'))
 1010 FORMAT('0RESULTS FROM BINMETH6'///' BIN DATA IN DEG C? (Y OR N)')
 1020 FORMAT('0ENTER HOUSE UA (KJ/HR)')
 1030 FORMAT('0ENTER THE BIN DATA FOR THE YEAR,'/1X,
     1 'STARTING WITH JAN.  FOR EACH MONTH ENTER THE NUMBER'/1X,
     1 'OF BIN DATA POINTS AND THE HIGHEST BIN TEMPERATURE,'/1X,
```

```
      1   'FOLLOWED BY THE HOURS OF OCCURRENCE FOR EACH BIN,')
1050 FORMAT('0',T5,'MONTH',T15,'TAVE',T24,'QA',T35,'QR',T47,'W',
      1   T57,'AUX',T67,'SHLOAD',T77,'FATM(SH)',T88,'FW(SH)',
      1   T97,'MPF',T105,'COP')
1060 FORMAT(1X,T6,A3,T13,F6.2,5(3X,1PE8.3),T77,0PF5.4,T88,F5.4,
      1   T96,F5.3,T104,F5.3)
1100 FORMAT(A1)
1110 FORMAT(A6)
     END




C***********************************
C                                  *
      SUBROUTINE INTERP(T,Q)
C                                  *
C***********************************
      DIMENSION D(25,4),Q(3)
      IF(T.GT.-900) GO TO 10
C*** INITIAL CALL ***
      PRINT 100
      READ, N
      READ(-,-), ((D(I,J),J=1,4),I=1,N)
      RETURN
  10  IF(T.LT.D(1,1)) GO TO 60
C*** FIND CLOSEST DATA POINTS  ***
      DO 20 I=1,N
      IF(T.LT.D(I,1)) GO TO 40
  20  CONTINUE
C*** USE LARGEST VALUES FOR TEMPS ABOVE DATA RANGE ****
      DO 30 J=2,4
  30  Q(J-1)=D(N,J)
      RETURN
C
C*** INTERPOLATE ****
  40  F=(T - D(I-1,1))/(D(I,1) - D(I-1,1))
      DO 50 J=2,4
  50  Q(J-1)=D(I-1,J) + F*(D(I,J)-D(I-1,J))
      RETURN
C*** BELOW DATA RANGE - HP OFF ****
  60  DO 70 I=1,3
  70  Q(I)=0.0
      RETURN
 100  FORMAT('0ENTER NUMBER OF HP DATA PTS AND HP DATA'/1X,
      1   '(TAMB, QATM, QDEL, AND  W)')
      END
```

```
C          APPENDIX C
C
C
C******************************************************************
C                                                                *
C            INTEG2 IS AN INTERACTIVE FORTRAN PROGRAM WHICH       *
C       INTEGRATES THE HEAT PUMP PERFORMANCE RATES               *
C       OVER THE GENERALIZED (TRIANGULAR) DISTRIBUTION           *
C       ON A MONTHLY BASIS.    INTEG2 WILL                       *
C       REQUEST 12 MONTHLY AVERAGE AMBIENT TEMPERATURES          *
C       FOR THE YEAR, BEGINNING WITH JANUARY.                    *
C            INTEG2 CALLS TWO SUBROUTINES, INTERP AND HRDIST.     *
C       INTERP READS IN FOUR COLUMNS OF HEAT PUMP DATA           *
C       (AMB.TEMP., QATM, QDEL, AND W) ON THE FIRST CALL,        *
C       AND RETURNS THE INTERPOLATED VALUES ON SUCCEEDING        *
C       CALLS.   INTERP IS LISTED IN APPENDIX B.                 *
C            THE SUBROUTINE HRDIST SETS UP THE GENERALIZED       *
C       DISTRIBUTION ON THE FIRST CALL, AND RETURNS THE          *
C       FREQUENCY (HRS) ON SUCCEEDING CALLS.                     *
C                                                                *
C******************************************************************
C
C
      DIMENSION Q(3),TBAR(12),V(5),C(5),X0(5),NM(12),MON(12)
      DATA NM/744,672,744,720,744,720,744,744,720,744,720,744/MON/
     1  'JAN','FEB','MAR','APR','MAY','JUN','JUL','AUG','SEP',
     1  'OCT','NOV','DEC'/
      PRINT 1000
      TR=18
      DT=0.1
C
C*** READ LOAD UA AND TEMP SPREAD ****
      PRINT 1010
      READ, U,TS
      CALL INTERP(-1000.,Q)
      CALL HRDIST(-1000.,XN,0,TS,0)
C
C*** FIND THE BALANCE POINT ****
      ICT=0
      T=0.0
      TO=2.0
      CALL INTERP(TO,Q)
      FO=Q(2) - U*(TR-T)
  30  IF(ICT.GT.10) GO TO 40
      ICT=ICT+1
      CALL INTERP(T,Q)
      F=Q(2) - U*(TR-T)
```

```
          T1=T
          T=T - (T-T0)*F/(F-F0)
          F0=F
          T0=T1
          IF(ABS(T-T0).GT.DT) GO TO 30
          PRINT 1020, T
          GO TO 50
  40      PRINT 1030
C
C*** START CRUNCHING ****
  50      PRINT 1040
  51      READ(-,-,END=110) (TBAR(I),I=1,12)
          PRINT 1050
C
C*** MONTHLY LOOP ****
          DO 100 K=1,12
          CALL HRDIST(-1000.,,XN,TBAR(K),TS,NM(K))
          DO 55 L=1,5
          V(L)=0.0
          C(L)=0.0
  55      XO(L)=0.0
          TMIN=TBAR(K) - TS/2.
          TMAX=TBAR(K) + TS/2.
          IF(TMAX.GT.TR) TMAX=TR
          IF(TMAX.LE.TMIN) GO TO 95
          NSTEP=INT((TMAX-TMIN)/DT)
C
C*** INNER INTEGRATION LOOP ****
          DO 90 J=1,NSTEP
          T=TMIN + J*DT
          CALL INTERP(T,Q)
          CALL HRDIST(T,HRS,TBAR(K),TS,NM(K))
          C(4)=0.0
          F=1.0
          C(2)=HRS*Q(2)
          C(5)=HRS*U*(TR-T)
          IF(C(2).GT.C(5)) GO TO 60
          C(4)=C(5)-C(2)
          GO TO 70
  60      F=C(5)/C(2)
          C(2)=F*C(2)
  70      C(3)=F*HRS*Q(3)
          C(1)=F*HRS*Q(1)
C
C*** INTEGRATE ****
          DO 80 I=1,5
          V(I)=V(I) + DT*(C(I)+XO(I))/2.
  80      XO(I)=C(I)
```

```
 90    CONTINUE
C
C*** MONTHLY PRINTING ****
 95    IF(V(5).LE.0.0) V(5)=1.0
       FATM=V(1)/V(5)
       FW=V(3)/V(5)
       PRINT 1060, MON(K), TBAR(K), FATM, FW
 100   CONTINUE
 110   PRINT 1000
       STOP
1000 FORMAT(1X///1X,70('*'))
1010 FORMAT('0ENTER LOAD UA AND TEMP SPREAD')
1020 FORMAT('0THE BALANCE POINT IS',F7.3)
1030 FORMAT('0TROUBLE IN SECTION TBAL - TOO MANY ITERATIONS')
1040 FORMAT('0ENTER 12 MONTHLY AVE TEMPS, STARTING WITH JANUARY')
1050 FORMAT('0',T5,'MONTH',T15,'TAVE',T24,'FATM',T32,'FW')
1060 FORMAT(1X,T6,A3,T13,F6.2,T24,F4.3,T31,F4.3)
       END




C*******************************************************
C                                                      *
       SUBROUTINE HRDIST(T,HRS,TAVE,TS,NM)
C                                                      *
C*******************************************************
       DATA I/0/
       IF(T.GT.-900.) GO TO 10
       IF(I.EQ.0) PRINT 100
       I=1
 5     A1=4*NM/TS/TS
       B1=TS/2.
       RETURN
 10    HRS=A1*(B1-ABS(T-TAVE))
       RETURN
 100   FORMAT(1X/' RESULTS OF NUMERICAL INTEGRATION ON ',
      1  'GENERALIZED DISTRIBUTION')
       END
```

APPENDIX D

If one assumes straight line performance characteristics for the heat pump and a straight line degree-day type load, the integration over the generalized triangular distribution can be expressed in analytical form. Unfortunately, because of factors like the difference in heat pump performance above and below the balance point and the change in the shape of the triangular distribution, the functions are only piecewise integrable. This results in a set of 4 equations for the heat pump work W, 4 equations for the auxiliary energy required QAUX, and 2 equations for the space heating load QL. A list of variable names with their definitions is given in Table D-1, and the equations are listed in Table D-4.

For any given set of heat pump parameters and ambient temperature conditions, only certain of the equations for W and QAUX are required. In addition, the limits of integration, $T_H$ and $T_L$, to be used in each equation are also dependent on the particular situation. A control map is shown in Table D-2 which lists the appropriate equations and limits of integration for W and QAUX as a function of the balance point temperature and the heat pump low temperature cutoff point.

The equations to be used and the limits of integration for the load calculations are dependent only on the ambient temperature conditions and the temperature at which the space heating load goes to zero, $T_R$. A control map for the load calculations is shown in Table D-3.

To illustrate the procedure involved in using these equations, the example problem from Chapter 3 will be set up. The system parameters and ambient temperature conditions are:

$$T_{ave} = -6.6°C$$

$$\Delta T_s = 32°C$$

$$T_{max} = -6.6+16 = 9.4°C$$

$$T_{min} = -6.6-16 = -22.6°C$$

$$N_{TOT} = 744 \text{ hrs}$$

$$T_R = 18°C$$

$$T_{co} = -20°C$$

$$T_{BAL} = -4.4°C$$

$$UA = 1070 \text{ KJ/hr}$$

The heat pump performance characteristics are

$$\dot{Q}DEL = 26251+909 \cdot T \text{ KJ/hr}$$

$$\dot{W} = 13146+353 \cdot T \text{ KJ/hr}$$

or

$$C_1 = 26251 \qquad C_2 = 909$$

$$C_3 = 13146 \qquad C_4 = 353$$

Having established all of the system parameters, the next step is to find the appropriate set of equations for W and QAUX. Since the cutoff temperature is between $T_{min}$ and $T_{ave}$ (-22.6<-20<-6.6) and the balance point is between $T_{ave}$ and $T_{max}$ (-6.6<-4.4<9.4), the equations to be solved are listed in the second column and the third row of Table D-2. From the table the equations to be eval-

uated for the work input are

Equation 3, with T evaluated from $T_{co}$ to $T_{ave}$

Equation 4, with T evaluated from $T_{ave}$ to $T_{BAL}$

Equation 2, with T evaluated from $T_{BAL}$ to $T_{max}$.

The equations and limits of integration for the auxiliary energy are

shown immediately to the right of those for the work.

Similarly, the equations to be solved for the load can be found

from Table D-3. For this example, since $T_R$ is greater than $T_{max}$

(9.4<18), they are

Equation 9, with T evaluated from $T_{min}$ to $T_{ave}$

Equation 10, with T evaluated from $T_{ave}$ to $T_{max}$

After the system parameters and the limits of integration have

been substituted into these equations and the equations have been

solved, the total monthly values can be found by summing the results

from each of the individual equations. For the example shown here,

$$W = W_3 + W_4 + W_2$$

$$QAUX = QUAX_3 + QAUX_1 + QAUX_2$$

$$QL = QL_1 + QL_2$$

Each of the equations in Table D-4 is unwieldly by itself, and

when several of them must be solved for each problem the whole pro-

cedure becomes unreasonably complex and cumbersome. However, the

fact that with the straight line approximation and the generalized

distribution such equations can be written, leads to the possibil-

ity of finding a combination of the parameters listed in D-1 which will correlate well with the heat pump system performance. If this could be done, then each of the monthly heat pump and auxiliary energy values could be read directly from a graph, or found quickly with a single equation.

TABLE D-1

Variable Definitions

$T$ = ambient temperature

$T_{ave}$ = monthly average ambient temperature

$\Delta T_s$ = temperature spread

$T_{max}$ = $T_{ave} + \Delta T_s/2$

$T_{min}$ = $T_{ave} - \Delta T_s/2$

$N_{tot}$ = total number of hours in the month.

$T_R$ = temperature at which the space heating load goes to zero

$T_{co}$ = heat pump low temperature cutoff point

$T_{BAL}$ = balance point temperature

$T_H$ = the upper integration limit

$T_L$ = the lower integration limit

The space heating load rate is defined as

$$\dot{Q}L = UA(T_R - T)$$

where UA is the building heat loss coefficient

The heat pump performance rates are defined as

$$\dot{Q}DEL = C_1 + C_2 \cdot T$$
$$\dot{W} = C_3 + C_4 \cdot T$$

where $C_1$ and $C_3$ are the intercepts and $C_2$ and $C_4$ are the slopes of the straight line heat pump performance charac-teristics.

TABLE D-2

CONTROL MAP FOR WORK AND AUXILIARY EQUATIONS

| $T_{co} \rightarrow$ / $T_{BAL} \rightarrow$ | $T_{co} < T_{min}$ W EQN | $T_L$ | $T_H$ | QAUX EQN | $T_L$ | $T_H$ | $T_{min} < T_{co} < T_{ave}$ W EQN | $T_L$ | $T_H$ | QAUX EQN | $T_L$ | $T_H$ | $T_{ave} < T_{co} < T_{max}$ W EQN | $T_L$ | $T_H$ | QAUX EQN | $T_L$ | $T_H$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T_{BAL} < T_{min}$ | 1<br>2 | $T_{min}$<br>$T_{ave}$ | $T_{ave}$<br>$T_{max}$ | (QAUX = 0) | | | ----- | | | ----- | | | ----- | | | ----- | | |
| $T_{min} < T_{BAL} < T_{ave}$ | 3<br>1<br>2 | $T_{min}$<br>$T_{BAL}$<br>$T_{ave}$ | $T_{BAL}$<br>$T_{ave}$<br>$T_{max}$ | 5 | $T_{min}$ | $T_{BAL}$ | 3<br>1<br>2 | $T_{co}$<br>$T_{BAL}$<br>$T_{ave}$ | $T_{BAL}$<br>$T_{ave}$<br>$T_{max}$ | 7<br>5 | $T_{min}$<br>$T_{co}$ | $T_{co}$<br>$T_{BAL}$ | ----- | | | ----- | | |
| $T_{ave} < T_{BAL} < T_{max}$ | 3<br>4<br>2 | $T_{min}$<br>$T_{ave}$<br>$T_{BAL}$ | $T_{ave}$<br>$T_{BAL}$<br>$T_{max}$ | 5<br>6 | $T_{min}$<br>$T_{ave}$ | $T_{ave}$<br>$T_{BAL}$ | 3<br>4<br>2 | $T_{co}$<br>$T_{ave}$<br>$T_{BAL}$ | $T_{ave}$<br>$T_{BAL}$<br>$T_{max}$ | 7<br>5<br>6 | $T_{min}$<br>$T_{co}$<br>$T_{ave}$ | $T_{co}$<br>$T_{ave}$<br>$T_{BAL}$ | 4<br>2 | $T_{co}$<br>$T_{BAL}$ | $T_{BAL}$<br>$T_{max}$ | 7<br>8<br>6 | $T_{min}$<br>$T_{ave}$<br>$T_{co}$ | $T_{ave}$<br>$T_{co}$<br>$T_{BAL}$ |
| $T_{max} < T_{BAL}$ | 3<br>4 | $T_{min}$<br>$T_{ave}$ | $T_{ave}$<br>$T_{max}$ | 5<br>6 | $T_{min}$<br>$T_{ave}$ | $T_{ave}$<br>$T_{max}$ | 3<br>4 | $T_{co}$<br>$T_{ave}$ | $T_{ave}$<br>$T_{max}$ | 7<br>5<br>6 | $T_{min}$<br>$T_{co}$<br>$T_{ave}$ | $T_{co}$<br>$T_{ave}$<br>$T_{max}$ | 4 | $T_{co}$ | $T_{max}$ | 7<br>8<br>6 | $T_{min}$<br>$T_{ave}$<br>$T_{co}$ | $T_{ave}$<br>$T_{co}$<br>$T_{max}$ |

TABLE D-3

CONTROL MAP FOR LOAD EQUATIONS

| | QL | | |
|---|---|---|---|
| | EQN | $T_L$ | $T_H$ |
| $T_{max} < T_R$ | 9 | $T_{min}$ | $T_{ave}$ |
| | 10 | $T_{ave}$ | $T_{max}$ |
| $T_{ave} < T_R < T_{max}$ | 9 | $T_{min}$ | $T_{ave}$ |
| | 10 | $T_{ave}$ | $T_R$ |
| $T_{min} < T_R < T_{ave}$ | 9 | $T_{min}$ | $T_R$ |

TABLE D-4

INTEGRATED EQUATIONS

Note that $\left.\begin{array}{c} T_H \\ \\ T_L \end{array}\right|$ means evaluated from $T_L$ to $T_H$

1. $W_1 = \dfrac{4N_{TOT}}{\Delta T_s^{\,2}} \dfrac{UA}{C_2^{\,4}} \left\{ -C_4 \cdot [\tfrac{1}{3}C_2^3 T^3 - \tfrac{1}{2}C_1 C_2^{\,2} T^2 - 5C_1^{\,2} C_2 T - C_1^3 LN(C_1+C_2 T)] \right.$

$+C_2[C_4(T_R+T_{min})-C_3] \cdot [\tfrac{1}{2}C_2^{\,2} T^2 - C_1 C_2 T - \tfrac{3}{2}C_1^{\,2} + C_1^{\,2} LN(C_1+C_2 \cdot T)]$

$+C_2^{\,2}[C_3(T_R+T_{min})-C_4 T_R T_{min}] \cdot [C_1+C_2 T - C_1 LN(C_1+C_2 T)]$

$\left. -[C_2^{\,2} C_3 T_R T_{min} LN(C_1+C_2 T)] \right\} \left.\begin{array}{c} T_H \\ \\ T_L \end{array}\right.$

2. $W_2 = \dfrac{4N_{TOT}}{\Delta T_s^{\,2}} \dfrac{UA}{C_2^{\,4}} \left\{ C_4 \cdot [\tfrac{1}{3}C_2^{\,3} T^3 - \tfrac{1}{2}C_1 C_2^{\,2} T^2 - 5C_1^{\,2} C_2 T - C_1^{\,3} LN(C_1+C_2 T)] \right.$

$-C_2[C_4(T_R+T_{max})-C_3] \cdot [\tfrac{1}{2}C_2^{\,2} T^2 - C_1 C_2 T - \tfrac{3}{2}C_1^{\,2} + C_1^{\,2} LN(C_1+C_2 T)]$

$-C_2^{\,2}[C_3(T_R+T_{max})-C_4 T_R T_{max}] \cdot [C_1+C_2 T - C_1 LN(C_1+C_2 T)]$

$\left. +[C_2^{\,3} C_3 T_R T_{max} LN(C_1+C_2 T)] \right\} \left.\begin{array}{c} T_H \\ \\ T_L \end{array}\right.$

TABLE D-4 (cont.)

3. $W_3 = \dfrac{4N_{TOT}}{\Delta T_s^2} \left[ \dfrac{1}{3}C_4 T^3 + \dfrac{1}{2}(C_3 - C_4 T_{min})T^2 - C_3 T_{min} T \right] \Bigg|_{T_L}^{T_H}$

4. $W_4 = \dfrac{4N_{TOT}}{\Delta T_s^2} \left[ -\dfrac{1}{3}C_4 T^3 - \dfrac{1}{2}(C_3 + C_4 T_{max})T^2 + C_3 T_{max} T \right] \Bigg|_{T_L}^{T_H}$

5. $QAUX_1 = \dfrac{4N_{TOT}}{\Delta T_s^2} \left\{ -\dfrac{1}{3}(UA + C_2)T^3 + \dfrac{1}{2}[UA \cdot T_R - C_1 + T_{min}(UA + C_2)]T^2 \right.$

$\left. - (UA\ T_R - C_1)T_{min}\ T \right\} \Bigg|_{T_L}^{T_H}$

6. $QAUX_2 = \dfrac{4N_{TOT}}{\Delta T_s^2} \left\{ \dfrac{1}{3}(UA + C_2)T^3 - \dfrac{1}{2}[UA\ T_R - C_1 + T_{max}(UA + C_2)]T^2 \right.$

$\left. + (UA\ T_R - C_1)T_{max}\ T \right\} \Bigg|_{T_L}^{T_H}$

7. $QAUX_7 = \dfrac{4N_{TOT}}{\Delta T_s^2} \left[ -\dfrac{1}{3}UA\ T^3 + \dfrac{1}{2}UA(T_R + T_{min})T^2 - UA\ T_R\ T_{min}\ T \right] \Bigg|_{T_L}^{T_H}$

8. $QAUX_8 = \dfrac{4N_{TOT}}{\Delta T_s^2} \left[ \dfrac{1}{3}UA\ T^3 - \dfrac{1}{2}UA(T_R + T_{max})T^2 + UA\ T_R\ T_{max}\ T \right] \Bigg|_{T_L}^{T_H}$

9. $QL_1 = \dfrac{4N_{TOT}UA}{\Delta T_s^2} \left[ -\dfrac{1}{3}T^3 + \dfrac{1}{2}(T_R + T_{min})T^2 - T_R\ T_{min}\ T \right] \Bigg|_{T_L}^{T_H}$

10. $QL_2 = \dfrac{4N_{TOT}UA}{\Delta T_s^2} \left[ \dfrac{1}{3}T^3 - \dfrac{1}{2}(T_R + T_{max})T^2 + T_R\ T_{max}\ T \right] \Bigg|_{T_L}^{T_H}$

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Columbia, Mo.
January

Seasonal Fraction
By Heat Pump Work

Std. Dev. = 0.0115

9 U.S. Locations

FW SIMULATION

FW DESIGN PROCEDURE

2.6

Seasonal Fraction From The Atmosphere

Std. Dev. = 0.0089

9 U.S. Locations

Axis labels: $FATM_{SIMULATION}$ (vertical), $FATM_{DESIGN\ PROCEDURE}$ (horizontal)

25

Seasonal Fraction
By Purchased Energy

Std. Dev. = 0.0089

9 U.S. Locations

FPUR $_{\text{SIMULATION}}$ (y-axis: 0, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70)

FPUR $_{\text{DESIGN PROCEDURE}}$ (x-axis: 0, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70)

Seasonal Fraction
From The Atmosphere

Std. Dev. = 0.0089

9 U.S. Locations

FATM SIMULATION (y-axis)

FATM DESIGN PROCEDURE (x-axis)



Seasonal Fraction
By Heat Pump Work

Std. Dev. = 0.0115

9 U.S. Locations

FW SIMULATION (y-axis)

FW DESIGN PROCEDURE (x-axis)

FW SIMULATION

0.10   0.20   0.30   0.40

FW DESIGN PROCEDURE

0   0.10   0.20   0.30   0.40

Seasonal Fraction
By Heat Pump Work

Std. Dev. = 0.0115

9 U.S. Locations

FATM SIMULATION

FATM DESIGN PROCEDURE

Seasonal Fraction
From The Atmosphere

Std. Dev. = 0.0089

9 U.S. Locations

Seasonal Fraction
By Purchased Energy

Std. Dev. = 0.0089

9 U.S. Locations

FPUR SIMULATION

FPUR DESIGN PROCEDURE

**Top chart:**

FW SIMULATION

FW DESIGN PROCEDURE

Std. Dev. = 0.0115

Seasonal Fraction
By Heat Pump Work

9 U.S. Locations

**Bottom chart:**

FATM SIMULATION

FATM DESIGN PROCEDURE

Std. Dev. = 0.0089

Seasonal Fraction
From The Atmosphere

9 U.S. Locations

COP

SANTA MARIA, OCT

CHARLESTON, MAR

WASHINGTON D.C., FEB

COLUMBIA, JAN

MADISON, JAN

BISMARK, JAN

FSOL



MPF

SANTA MARIA, OCT

CHARLESTON, MAR

WASHINGTON D.C., FEB

COLUMBIA, JAN

MADISON, JAN

BISMARK, JAN

FSOL

FATM DESIGN PROCEDURE

0.40   0.30   0.20   0.10   0

FATM SIMULATION

0.10

9 U.S. Locations

0.20

Std. Dev. = 0.0089

0.30

Seasonal Fraction
From The Atmosphere

0.40

FATM DESIGN PROCEDURE

FATM SIMULATION

9 U.S. Locations

Std. Dev. = 0.0089

Seasonal Fraction From The Atmosphere

FPUR DESIGN PROCEDURE

FPUR SIMULATION

9 U.S. Locations

Std. Dev. = 0.0089

Seasonal Fraction
By Purchased Energy

Columbia, Mo.

COLLECTOR AREA, m²

Seasonal Fraction
By Heat Pump Work

Std. Dev. = 0.0115

9 U.S. Locations

FW SIMULATION

FW DESIGN PROCEDURE

Std. Dev. = 0.0089

Seasonal Fraction
From The Atmosphere

9 U.S. Locations

FATM $_{\text{SIMULATION}}$

FATM $_{\text{DESIGN PROCEDURE}}$

Columbia, Mo.
January

MPF

COP

FSOL

Fig 3

Columbia, Mo.
January

MPF or CoP

FSOL

MPF

COP

FATM DESIGN PROCEDURE

0.40   0.30   0.20   0.10   0

9 U.S. Locations

Std. Dev. = 0.0089

Seasonal Fraction
From The Atmosphere

FATM SIMULATION

0   0.10   0.20   0.30   0.40

FW DESIGN PROCEDURE

0.40   0.30   0.20   0.10   0

FW SIMULATION

0.10   0.20   0.30   0.40

9 U.S. Locations

Std. Dev. = 0.0115

Seasonal Fraction
By Heat Pump Work

FPUR DESIGN PROCEDURE

FPUR SIMULATION

9 U.S. Locations

Std. Dev. = 0.0089

Seasonal Fraction
By Purchased Energy

0    0.10    0.20    0.30    0.40    0.50    0.60    0.70

0    0.10    0.20    0.30    0.40    0.50    0.60    0.70

CAPACITY , kW

COP

5 ton

3 ton

Capacity
5 ton

3 ton
5 ton

3 ton

C O P

$T_{source}$ , °C