ECONOMIC EVALUATION AND OPTIMIZATION

OF SOLAR HEATING SYSTEMS

BY

MICHAEL JOHN BRANDEMUEHL

A thesis submitted in partial fulfillment of the

requirements for the degree of

MASTER OF SCIENCE

(Engineering)

at the

UNIVERSITY OF WISCONSIN-MADISON

(1977)

ACKNOWLEDGEMENTS

I would like to express great thanks for the time, guidance, and support extended by my advisor, Professor William A. Beckman, and the other faculty members of the Solar Energy Laboratory, Professors John A. Duffie, Sanford A. Klein, and John W. Mitchell. The support and camaraderie of the staff and my fellow graduate students has been appreciated. I thank my wife, Leane, for her unlimited moral support and understanding, and my parents for their stimulus and direction. The financial support of the Energy Research and Development Administration (now the Department of Energy) through contract E(11-1)-2588 has been appreciated.

TABLE OF CONTENTS

LIST OF FIGURES

NOMENCLATURE

A	collector area
C	non-commercial/commercial flag (0 or 1)
C_A	investment cost which is proportional to collector area
C_E	Investment cost which is independent of collector area
C_F	cost of delivered energy when $C_{FS} = C_{FC}$
C_{FC}	cost of delivered energy for conventional system
C_{FS}	cost fo delivered energy for solar energy auxiliary system
d	annual market discount
D	ratio of down payment to initial investment
e	annual market rate of fuel price inflation
f	monthly load fraction supplied by solar energy
f_s	monthly space heating load fraction supplied by solar energy
f_w	monthly water heating load fraction supplied by solar energy
FUELC	life cycle cost of fuel for conventional system
FUELS	life cycle cost of fuel for solar auxiliary system
F_R	collector heat removal factor
F_R'	collector heat exchanger efficiency factor
$\mathcal{F}$	annual load fraction supplied by solar energy
g	annual rate of general inflation
G	ratio of salvage or resale value to initial investment
i	annual market interest rate on mortgage

INCC	life cycle cost of income taxes for conventional system
INCS	life cycle cost of income taxes for solar system
I_T	rate of total insolation on collector surface (per unit area)
INV	initial solar heating system investment
j^*	years before solar heating will be competitive with conventional heating
k	annual rate of solar system performance degradation
l	monthly heating load
l_s	monthly space heating load
l_w	monthly water heating load
L	average annual heating load
LOAN	life cycle cost of loan amortization
L_j	loan principal remaining in year j
$(\dot{m}c_p)_c$	collector fluid capacitance rate
$(\dot{m}c_p)_m$	minimum capacitance rate of collector fluid and storage fluid through collector-storage heat exchanger
M	ratio of first year miscellaneous costs to initial investment
MISC	life cycle cost of solar system miscellaneous costs such as maintenance, insurance, and parasitic power
N_D	depreciation lifetime
N_E	duration of life cycle cost analysis
N_L	loan amortization period
N_1	number of annual loan payments which contribute to life cycle cost analysis
PMT	constant annual mortgage payment

PROP — life cycle cost of property taxes for solar system

P_1 — factor relating life cycle cost of fuel savings associated expenses to fuel savings in first year

P_2 — factor relating life cycly cost of investment associated expenses to initial investment

S — monthly total radiation incident on collector surface per unit area

SAV — life cycle savings of solar heating system over conventional hèating system

SR — life cycle cost of salvage or resale value of solar system

t — property tax rate based on assessed value

$\bar{t}$ — effective state and federal income tax rate

T_A — monthly average ambient temperature

T_i — collector fluid inlet temperature

T_m — delivered hot water temperature

T_w — water main supply temperature

U — ratio of life cycle depreciation cost to initial investment

UA — overall house energy loss coefficient-area product

U_L — collector overall energy loss coefficient

V — ratio of assessed value in first year to initial investment

X — related to ratio of reference collector loss to monthly load for space heating

X_w — related to ratio of reference collector loss to monthly load for water heating

Y — related to ratio of absorbed energy to monthly load

Z^* — value of P_2C_A/P_1C_F at critical economic condition

ε defined by $\varepsilon = e-k-ek$

ε_c collector-storage heat exchanger effectiveness

$(\tau\alpha)_n$ transmittance-absorptance product of collector for radiation at normal incidence

$(\overline{\tau\alpha})$ monthly average transmittance-absorptance product of collector

CHAPTER 1 INTRODUCTION

1.1 Background

Within recent years, the world has become increasingly aware of the delicate balance existing between the supply of fossil fuel energy and its rate of consumption. In the United States, this awareness has been prompted by events such as the 1973 oil embargo and the natural gas curtailments in the winter of 1976-1977. In light of this dilemma, government and industry are searching for alternative sources of the energy we consume, such as nuclear fusion, solar, wind, and geothermal. The future of these alternatives rests on overcoming the technical, environmental, social, and economic barriers associated with any young technology. When the task is to provide a nation with a long-term renewable supply of energy, these barriers are very imposing. It has been variously estimated that the combined contributions of nuclear fusion and solar energy will probably not exceed 10% of the U.S. energy requirement until 2000-2010.

In the near future, though, low-temperature applications of solar energy could significantly reduce the national energy demand for conventional fuels [1]. In particular, water heating, space heating, and air conditioning comprise about 25% of the national energy demand. The technology for applying solar energy to space and water heating is well known and has been successfully demonstrated throughout most the U.S. Air conditioning with solar energy, though not as developed as space heating, has also been demonstrated.

The major barrier facing low temperature applications of solar energy today is one of economic viability. There are undoubtedly instances in which economic considerations are unimportant. There are individuals who have very uneconomical solar heated residences on the basis of ecological or social appeal. However, in a free enterprise society, solar energy will gain widespread acceptance only if it can economically compete with conventional energy sources.

There are many factors which influence this economic comparison. One of the most important is the cost of conventional fuel energy. This cost, and its inflation in the future, is in turn determined by a host of other factors. General inflation, fossil fuel supply and supply predictions, governmental directives, and the policies of oil producing nations combine to make fuel price predictions difficult. Since the economic viability of solar energy depends so strongly on future fuel prices, the accuracy of an economic analysis is limited by the accuracy of these predictions.

Another important factor in an economic comparison is the cost of solar energy system components. These costs will be determined largely by the success of the currently struggling solar energy industry. Improvements in component design, manufacturing techniques, and installation methods could reduce current price levels and enhance the economic attractiveness of solar energy systems.

The federal and state governments can also influence the economics of solar energy, e.g. [2,3]. As of 1976 almost half of the states have enacted legislation providing tax incentives for solar energy users, including sales and property tax exemptions. Federal programs, such as income tax credits and low interest loans can make an otherwise uneconomical solar heating system decidedly economical.

Widespread utilization of solar energy will be realized only if the overall economic scenario facilitates its effective competition with conventional fuels. Since the decisions governing this economic scenario are largely political, there is little that the individual citizen can do to affect the economic viability of solar energy. However several groups have recently conducted analyses which show that, in many circumstances, residential solar heating systems can be economically competitive with conventional fuels, depending on the location and economic situation of the individual user, e.g. [4,5]. The overall development of solar energy will be hampered unless there is a straightforward method by which an individual can evaluate the suitability of solar energy to his particular situation. This thesis is meant to provide such a method.

1.2 Objectives

The main objectives of this thesis are:

1) To present a method by which the architect, engineer, or homeowner can evaluate the economic viability of a solar space and/or domestic water heating system in terms of the life cycle savings of the solar heating system over the conventional heating system.

2) To develop a simple and straightforward procedure for determining the optimum solar system collector area and corresponding load fraction supplied by solar energy.

Any type of economic analysis for solar heating systems requires knowledge of the long-term solar system performance. The f-chart method as developed by Klein, et al. [6,7,8,9,10] is capsulated in Chapter 2. This design procedure uses monthly meteorological data, monthly heating loads, and collector characteristics to estimate the annual fraction of the heating load supplied by solar energy. The general methods described in this thesis do not rely specifically on the f-chart design method. However, the method is used to generate the tables of Appendix C.

Life cycle cost analysis has grown to become a standard method for evaluation solar heating system economics. The federal government has fostered and used many life cycle cost analysis studies, and most solar energy system computer design programs use it for economic assessment. The explicit model presented in Chapter 3 is based on that of Ruegg [11], and includes costs for system components, loan amortization, operating expenses, and taxes. The result of this analysis is the life cycle savings of a solar heating system over a conventional system. The explicit model is then collapsed into a compact and flexible form by introducing two economic parameters, P_1 and P_2.

Using the parameterized life cycle savings equation, and the f-chart design method, Chapter 4 develops a non-iterative tabular method for optimizing the solar system design to yield the maximum life cycle savings. This optimum system design is a function of location, collector type, and one economic parameter. As an extension of this optimization procedure, the critical economic condition is defined as the economic condition at which the life cycle cost of the optimum solar system design equals that of the conventional system. A similar tabular method for determining this critical condition is developed. Appendix C presents a set of tables for domestic water heating systems and combined space and domestic water heating systems with both air and liquid transfer mediums. Tables for each of these systems corresponding to a typical one cover selective surface collector, two cover non-selective surface collector, and two cover selective surface collector are given for 19 U.S. locations. A more complete set of tables can be obtained by contacting the University of Wisconsin Solar Energy Laboratory.

CHAPTER 2 Thermal Analysis

2.1 General Considerations

The purpose of any type of heating system is to meet a heating load. The design of a conventional heating system is determined by the maximum heating load that could be encountered in a heating season. This heating load depends only on the building envelope and the design weather condition. Theoretically, a solar heating system could be designed in the same manner by sizing the solar system to meet the largest anticipated heating load. Unfortunately, this would in general require a very large collector area and introduce prohibitive costs. As a result, a solar heating system will usually be designed to meet only a fraction of the total load, with a backup heating system providing the remainder.

The sizing of a solar heating system is then a matter of economics. The economic analysis, in turn, will be strongly influenced by the long-term thermal performance of the solar heating system, which is a complicated function of the specific system design, meteorological conditions, and the heating load (also dependent on meteorological conditions). In general there are three sources of the necessary system performance information: 1) experimental data, 2) computer simulations, 3) simplified design procedures. This section deals with the applicability of each of these sources of performance information for economic analyses.

Accurate data from carefully monitored system installations is the best source of performance information. This data reflects system response to actual weather and load conditions without the assumptions, approximations, or generalizations of simulations or design procedures. Unfortunately, there is very little data available, and only a fraction of it is meaningful. There are currently thousands of solar heating systems operating in the United States, but most have been in operation a relatively short period of time, and only a handful have produced useful data. Of the data that has been collected, much of it is either obtained from non-standard, experimental systems or riddled with errors, inconsistencies, and information gaps.

With the striking lack of experimental data, performance information can be obtained from computer simulations. This method is based on mathematical modelling of the solar heating system, the heating load, and the interaction with meteorological conditions. Using actual hourly weather data, the algebraic and differential equations can be solved as a function of time. There are currently several such programs in general use, e.g. TRNSYS [12].

The method of computer simulation is a powerful research tool that facilitates detailed study of both short- and long-term system performance. However, simulations are impractical as a general design tool for several reasons. First, the expertise required for modelling solar heating systems exceeds that of the typical architect or engineer in the building industry. Second, simulation programs generally require large computing facilities not common to the building industry. Third, repeated simulations necessary for optimal system design can be expensive.

Simplified design procedures, on the other hand, are quick methods of estimating solar system performance. These methods have been developed using the general knowledge gained from computer simulations. The investigation of many simulations often reveals that the long-term performance of a solar system is insensitive to some parameters, or simply related to others. By observing the trends which result from extensive simulations, simple methods can be developed by taking advantage of rule-of-thumb generalizations and semi-empirical correlations. The results can often be presented as nomograms, or simple algrebraic relations which can be solved on a hand-held calculator. These simplified methods are the obvious choice for solar heating system economic analysis.

2.2 The f-chart Method

Klein, et al. have developed a simple method for estimating the long-term thermal performance of standard solar heating systems [6,7,8,9,10]. This procedure, referred to as the f-chart method, is applicable to the standard domestic water heating system configuration shown in Figure 2-1, and the standard combined space and domestic water heating systems shown in Figures 2-2 and 2-3. The method uses monthly average meteorological data, average monthly heating loads, and collector characteristics to estimate the monthly load fraction supplied by solar energy, f. The annual load fraction, F, can be calculated from the monthly fractions and corresponding monthly loads.

The f-chart method is based on hundreds of hour by hour computer simulations of the standard system configurations over a wide range of design variables. The results of these simulations were correlated to two dimensionless parameters, X and Y, related to the ratio of a reference collector loss to the load and the ratio of the absorbed energy to the load, respectively. The specific correlation, as well as the form of the dimensionless parameters, depends on the type of solar system.

FIGURE 2-1 DOMESTIC WATER HEATING SYSTEM CONFIGURATION.

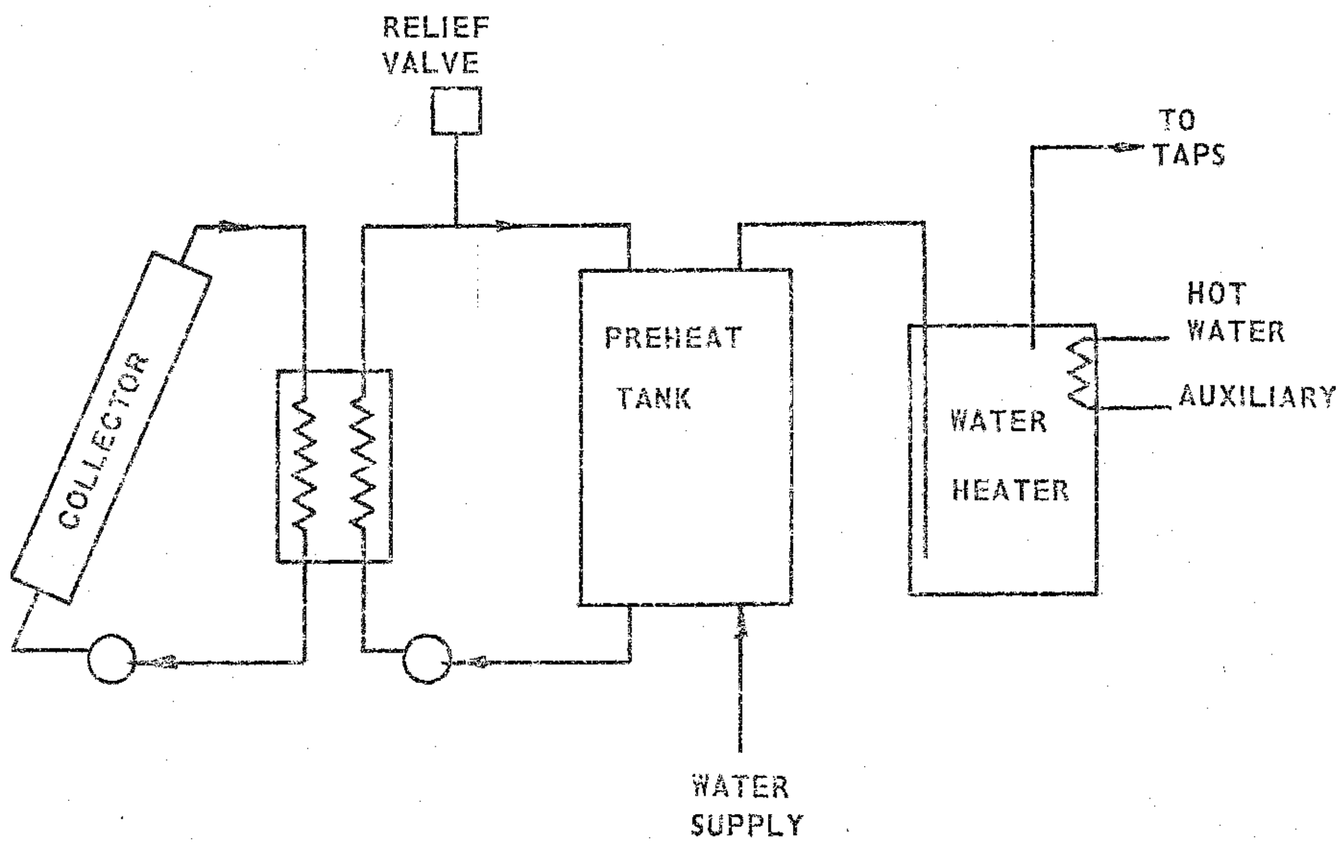


FIGURE 2-2 AIR-BASE COMBINED SPACE AND DOMESTIC WATER HEATING SYSTEM CONFIGURATION.

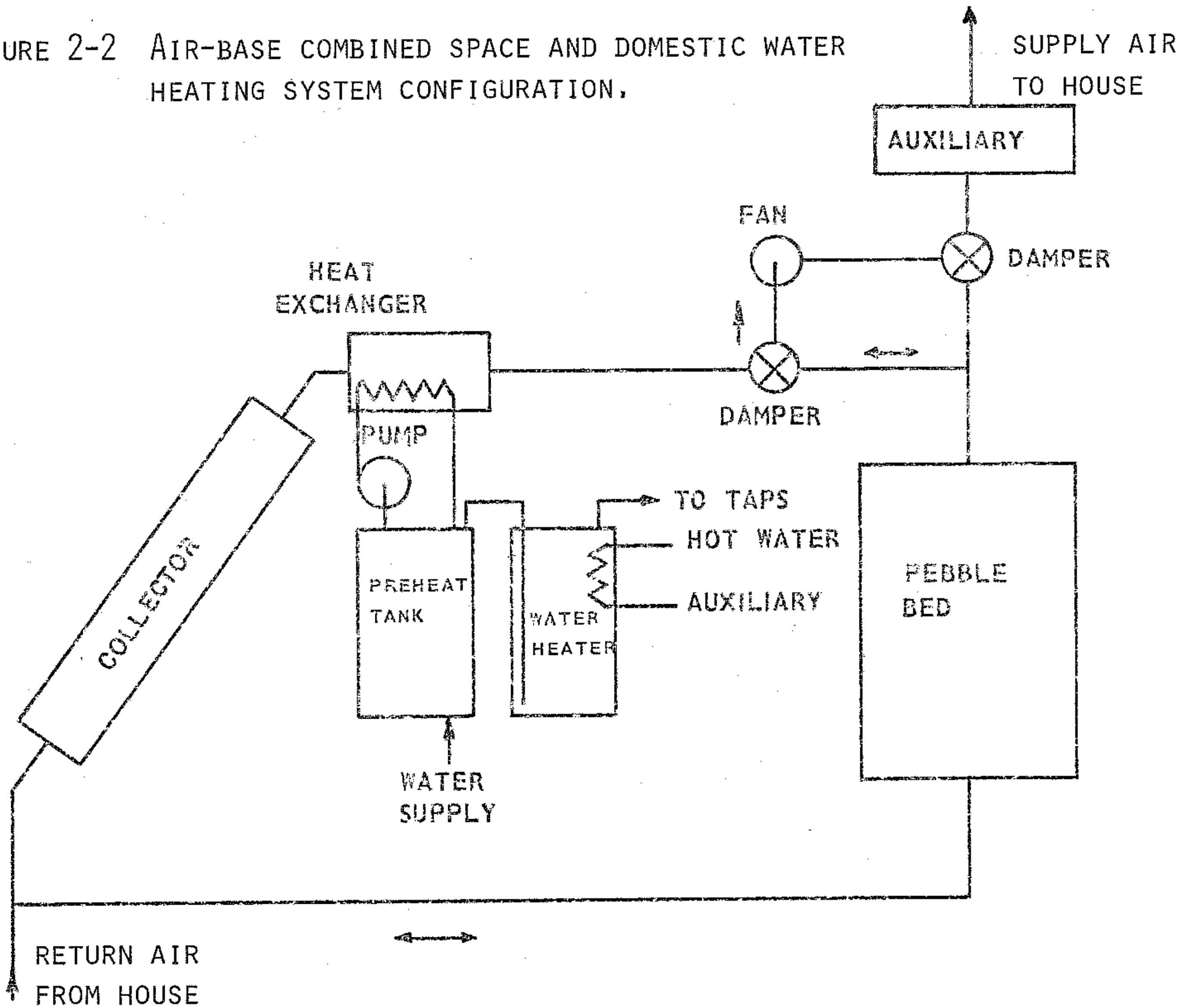
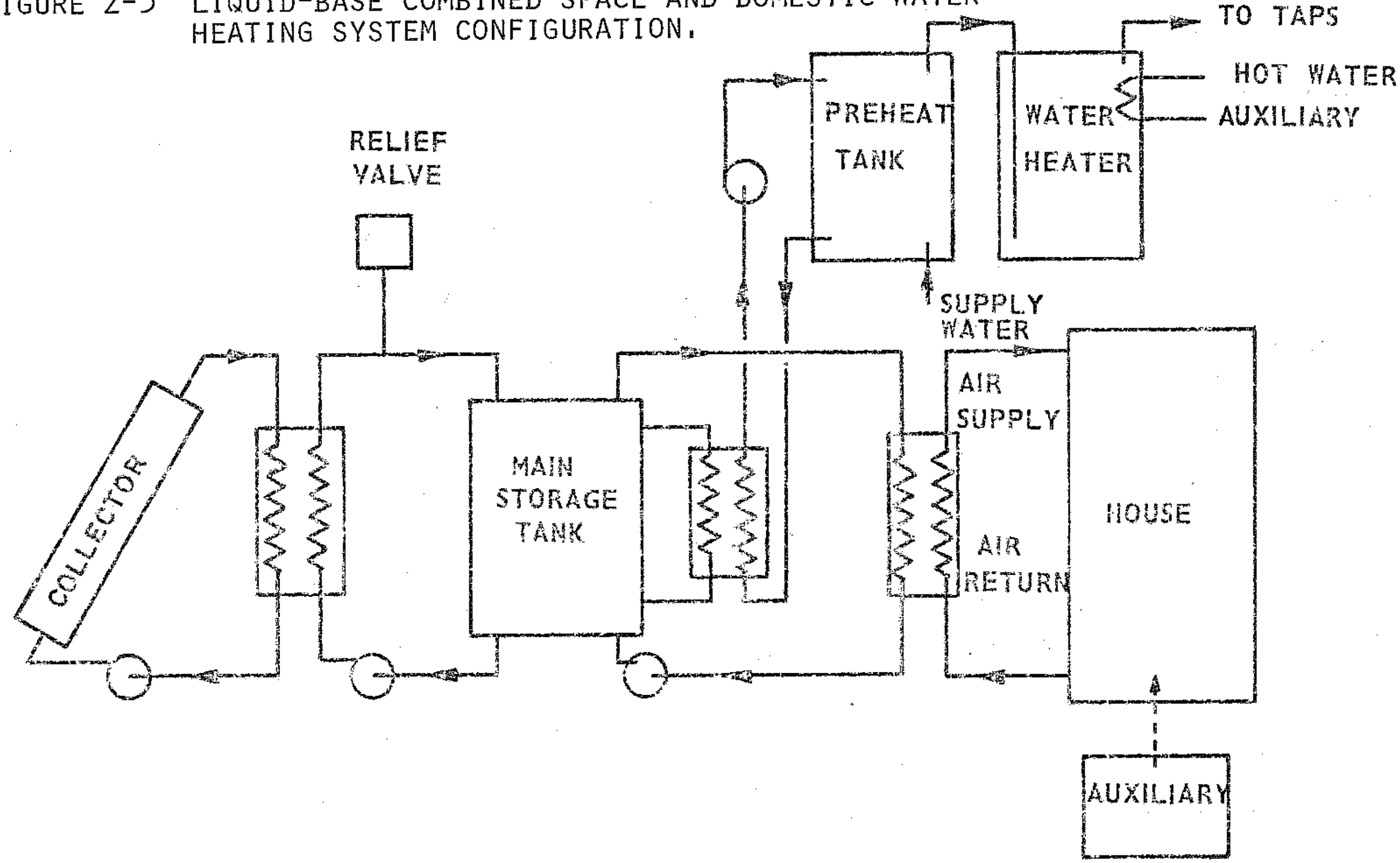


FIGURE 2-3 LIQUID-BASE COMBINED SPACE AND DOMESTIC WATER HEATING SYSTEM CONFIGURATION.



For space heating systems, the dimensionless parameters take the following form.

$$X = \frac{F_R' U_L A (T_{ref} - T_A) \Delta t}{l} \tag{2-1}$$

$$Y = \frac{F_R' (\tau\alpha)_n A S}{l} \frac{(\overline{\tau\alpha})}{(\tau\alpha)_n} \tag{2-2}$$

where

A is the collector area

F_R' is the collector-heat exchanger efficiency factor

l is the monthly heating load

S is the monthly total radiation incident on the collector surface per unit area

T_A is the monthly average ambient temperature

T_{ref} is a reference temperature chosen to be 100°C

Δt is the length of the month in appropriate units (usually seconds or hours)

U_L is the collector overall energy loss coefficient

$(\tau\alpha)_n$ is the transmittance-absorptance product of the collector for radiation at normal incidence

$\frac{(\overline{\tau\alpha})}{(\tau\alpha)_n}$ is the ratio of the monthly average transmittance-absorptance product to the transmittance-absorptance product at normal incidence

The collector-heat exchanger efficiency factor, F_R', accounts for the performance penalty resulting from a heat exchanger between the collector and the storage medium. It is related to the collector heat removal factor, F_R, [9,12].

$$F_R'/F_R = 1 + \frac{F_R A}{(\dot{m}c_p)_c}\left[\frac{(\dot{m}c_p)_c}{\varepsilon_c(\dot{m}c_p)_m} - 1\right]^{-1} \tag{2-3}$$

where

$(\dot{m}c_p)_c$ is the collector fluid capacitance rate

$(\dot{m}c_p)_m$ is the minimum of the collector fluid capacitance rate and the storage fluid capacitance rate

ε_c is the heat exchanger effectiveness

For systems in which a heat exchanger is not required, such as air based systems or drain down liquid based systems, $F_R'/F_R = 1$.

The groupings $F_R(\tau\alpha)_n$ and F_RU_L are often called the collector characteristics. The values of these parameters can be obtained from standard collector performance tests in which collector efficiency is plotted against $(T_i-T_A)/I_T$. (T_i is the collector fluid inlet temperature and I_T is the rate of total insolation on the collector surface.) On these graphs, an example of which is shown

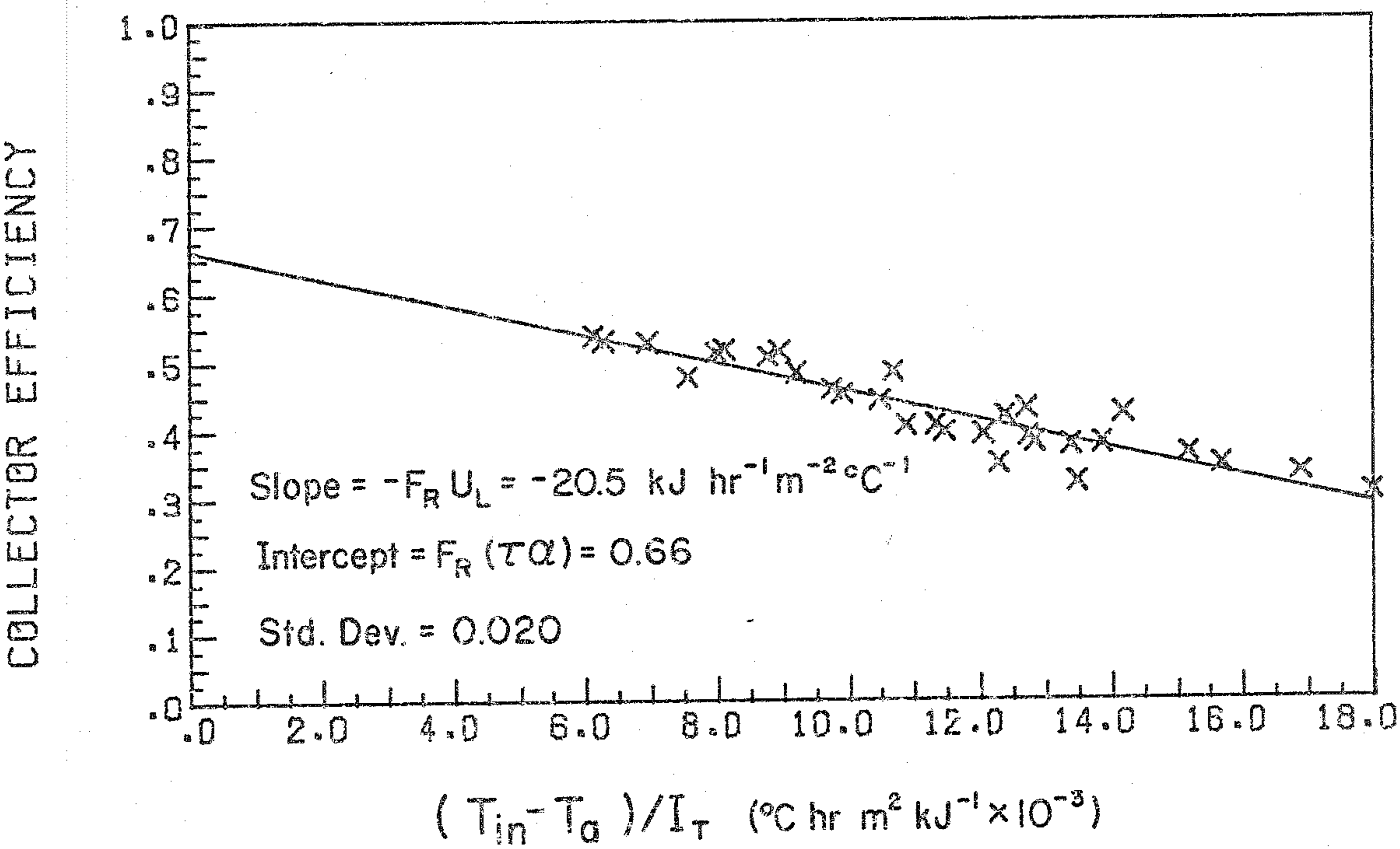


FIGURE 2-4 COLLECTOR PERFORMANCE TEST RESULTS.

in Figure 2-4, $F_R(\tau\alpha)_n$ is the value of the collector efficiency when $(T_i - T_A) = 0$, and $F_R U_L$ is the absolute value of the slope. While the data often exhibits some curvature, it can usually be closely approximated by a straight line.

The ratio $(\overline{\tau\alpha})/(\tau\alpha)_n$ can be estimated by the methods described in ref. [9,10]. In most cases though, for months during the heating season, and for collectors oriented directly towards the equator tilted at an angle within 15° of the latitude, this ratio is approximately 0.94 for collectors with two covers, and 0.96 for one cover.

The monthly load fraction, f, has been correlated to X and Y by the following algebraic expression.

$$f = C_1 Y + C_2 X + C_3 Y^2 + C_4 X^2 + C_5 Y^3 \qquad (2\text{-}4)$$

$$0 \le X \le 18$$

$$0 \le Y \le 3.3$$

The constants for liquid and air based systems are

	Liquid	Air
C_1	1.029	1.04
C_2	-0.065	-0.065
C_3	-0.245	-0.159
C_4	0.0018	0.00187
C_5	0.0215	-0.0095

These correlations are shown graphically in Figures 2-5 and 2-6.

For domestic water heating systems, the collector loss will be influenced by the water mains temperature, T_w, and the delivered hot water temperature, T_m. This effect is accomodated by redefining the abscissa coordinate as follows, with T_m and T_w in °C.

$$X_w = \frac{F_R' U_L A(11.6+1.18T_w+3.86T_m-2.32T_A)\Delta t}{l} \tag{2-5}$$

The correlation of f to X_w and Y for domestic water heating systems is the same as that used for liquid based space heating systems.

When using (2-4), care must be taken to respect the range of the correlation. The upper bound on X corresponds to the approximate value at which the slope of the curves of constant f changes sign. The upper bound on Y corresponds to the approximate value above which $f > 1$. Both of these bounds are approximate and further safeguards should be taken when using the algebraic correlation to ensure meaningful results.

There are many solar heating system design variables which affect the long-term system performance. The f-chart correlations presented here account for the influ-

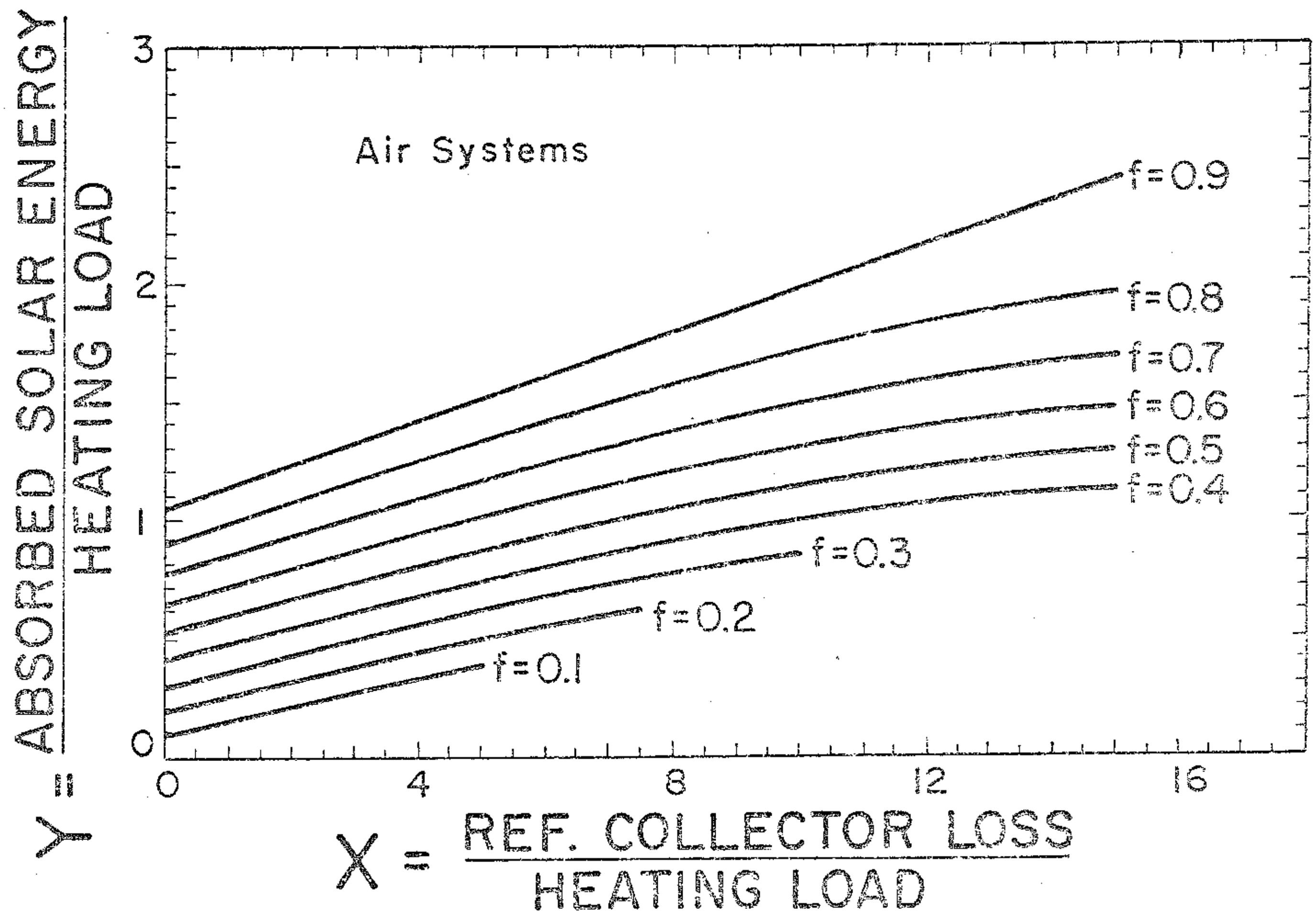


FIGURE 2-5 F-CHART FOR AIR-BASED HEATING SYSTEMS.

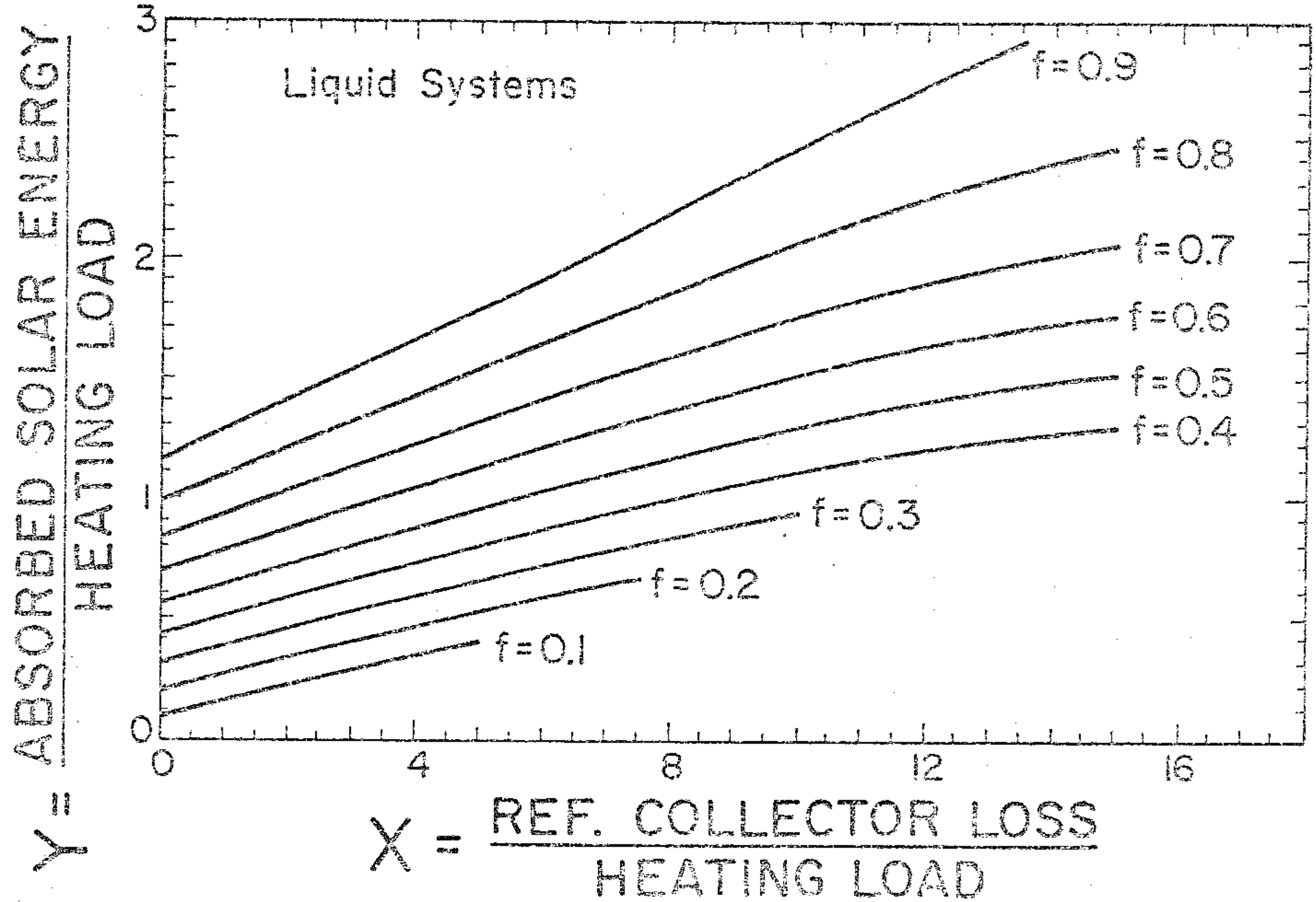


FIGURE 2-6 F-CHART FOR LIQUID BASED HEATING SYSTEMS.

ence of collector area and collector characteristics, $F_R(\tau\alpha)_n$ and F_RU_L. The general f-chart method presented in ref. [9,10] also accounts for the effects of storage capacity, load heat exchanger size, and air collector flow rate, which will not be investigated here. The correlations used in this analysis are valid for a storage capacity of $Mc_p/F_R'A = 350$ kJ-°C^{-1}m^{-2} for liquid based systems and $Mc_p/F_RA = 400$ kJ-°C^{-1}m^{-2} for air based systems, for the ratio of load heat exchanger size to load size $\varepsilon_L C_{min}/UA = 2$, and for an air collector flow rate of $(\dot{m}c_p)_c/F_RA = 58.7$ kJ-hr^{-1}m^{-2}°C^{-1}. These have been shown to be the approximate optimum values of the variables; modest variations have little effect on the solar load fraction.

It is also assumed throughout this thesis that the collector azimuth angle is zero (i.e. facing due south), and the collector slope is equal to the latitude for water heating systems and the latitude plus 15° for combined space and domestic water heating systems. (The annual load fraction is not very sensitive to angles within 15° of those cited.)

2.3 The Annual Solar Load Fraction

The economic analysis of Chapter 3 is performed on an annual basis, and is strongly influenced by the annual solar load fraction. Similarly, the economic optimization of a solar heating system is strongly influenced by its partial derivatives with respect to the optimized variables. In this light, it is beneficial to examine the functional form of the annual solar fraction.

For domestic water heating systems, the annual load fraction supplied by solar energy, F, is calculated by the following summation.

$$F = \frac{1}{L}\sum f_{wi}\, l_i \tag{2-6}$$

For space heating systems, and combined space and domestic water heating systems for which the water heating load is small compared to the total load,

$$F = \frac{1}{L}\sum f_{si}\, l_i \tag{2-7}$$

When considering combined systems for which the water heating load is not small compared to the total load, the annual solar load fraction will depend on both the water heating and space heating correlations. It seems reasonable to combine these two correlations using the following method.

$$F = \frac{1}{L}\sum(f_{wi}l_{wi} + f_{si}l_{si}) \tag{2-8}$$

When using the f-chart correlations for f_{wi} and f_{si}, the parameters X, X_w, and Y are calculated using the combined monthly heating load, $l_i = l_{wi}+l_{si}$. This combination is adequate due to the strong similarity between the domestic water heating and space heating correlations.

The annual solar load fraction is a complicated function of meteorological conditions, collector characteristics ($F_R(\tau\alpha)_n$ and F_RU_L), collector orientation, collector area, building latitude, annual load, and annual load distribution. It is often desirable to reduce this functional dependence for easy manipulation or compact presentation. Both manipulation and presentation could be simplified if a simple expression for the annual solar load fraction could be found. Unfortunately, this could only be done by beginning again with extensive computer simulation. The functional dependence can be greatly reduced, though, by noting that meteorological conditions, building location, and annual load distribution can be expressed as a general location dependence. Examination of the f-chart correlations show that, for a fixed load distribution, F can be expressed as a function of A/L rather than A and L separately. The annual load fraction can then be presented in graphical or tabular form as a function of location, collector characteristics, and the ratio of collector area to annual load. Figure 2-7 shows F versus A/L for a typical solar heating system in Madison, WI.

The assumption that the annual load distribution depends only on location, and not on the magnitude of the annual load, is built into almost all methods for calculating domestic water heating loads or space heating loads. Domestic water heating loads are usually calculated assuming a constant average daily hot water demand, and the difference between the delivered hot water temperature and the water main supply temperature (location dependent). The annual water heating load is then distributed according to the number of days in a month. Studies have also shown that the average annual space heating load is proportional to the long-term average degree days [14]. The annual space heating load is then distributed according to the long term average monthly degree days. The annual domestic water and space heating load distributions for Madison, WI are shown in Figure 2-8.

When considering combined space and domestic water heating systems, the combined load distribution is affected by the individual load distributions according to the relative size of the individual loads. For example, the load distribution when the space heating load is twice as large as the water heating load will differ from the distribution when the water heating load is twice as large as

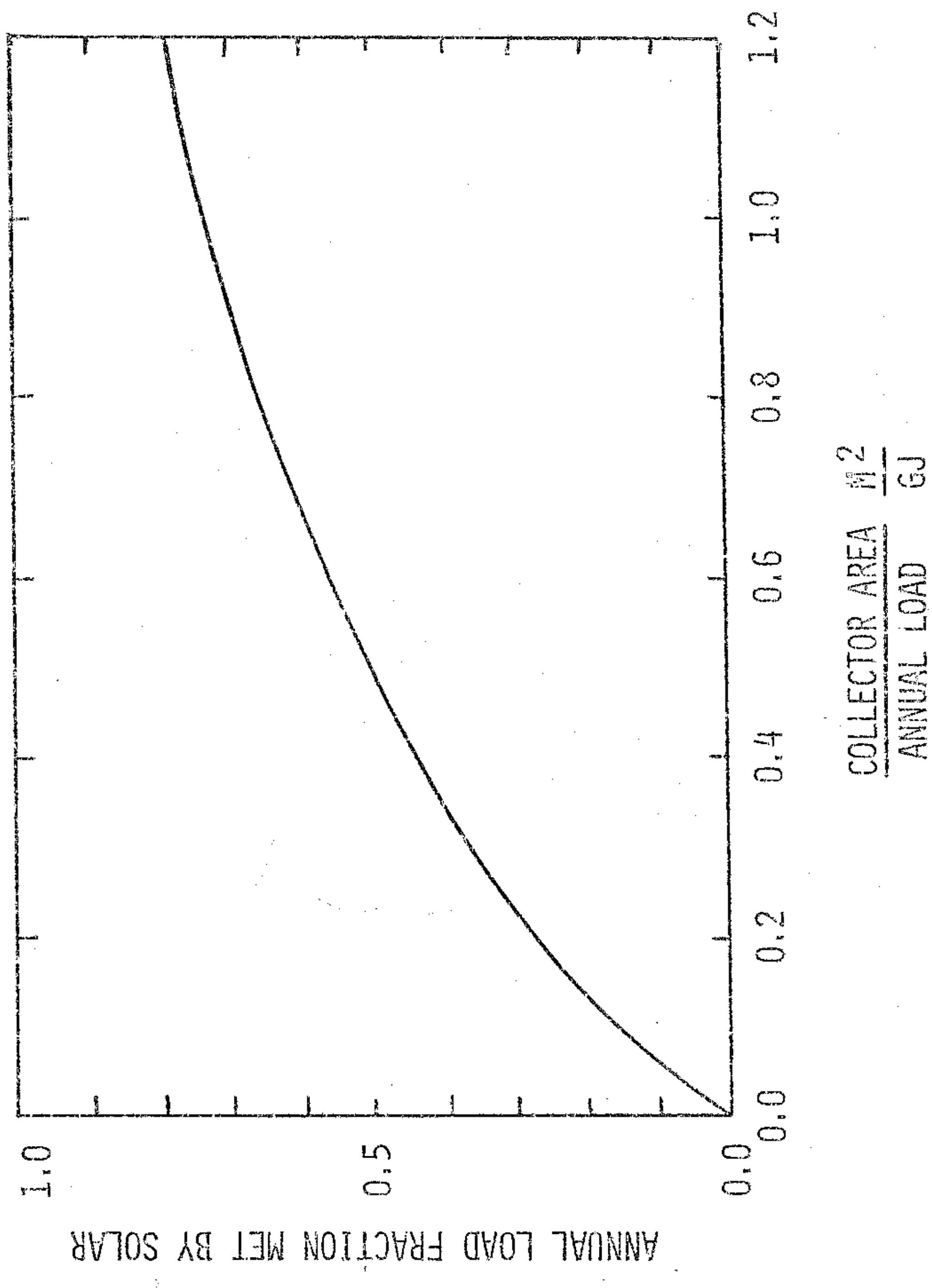
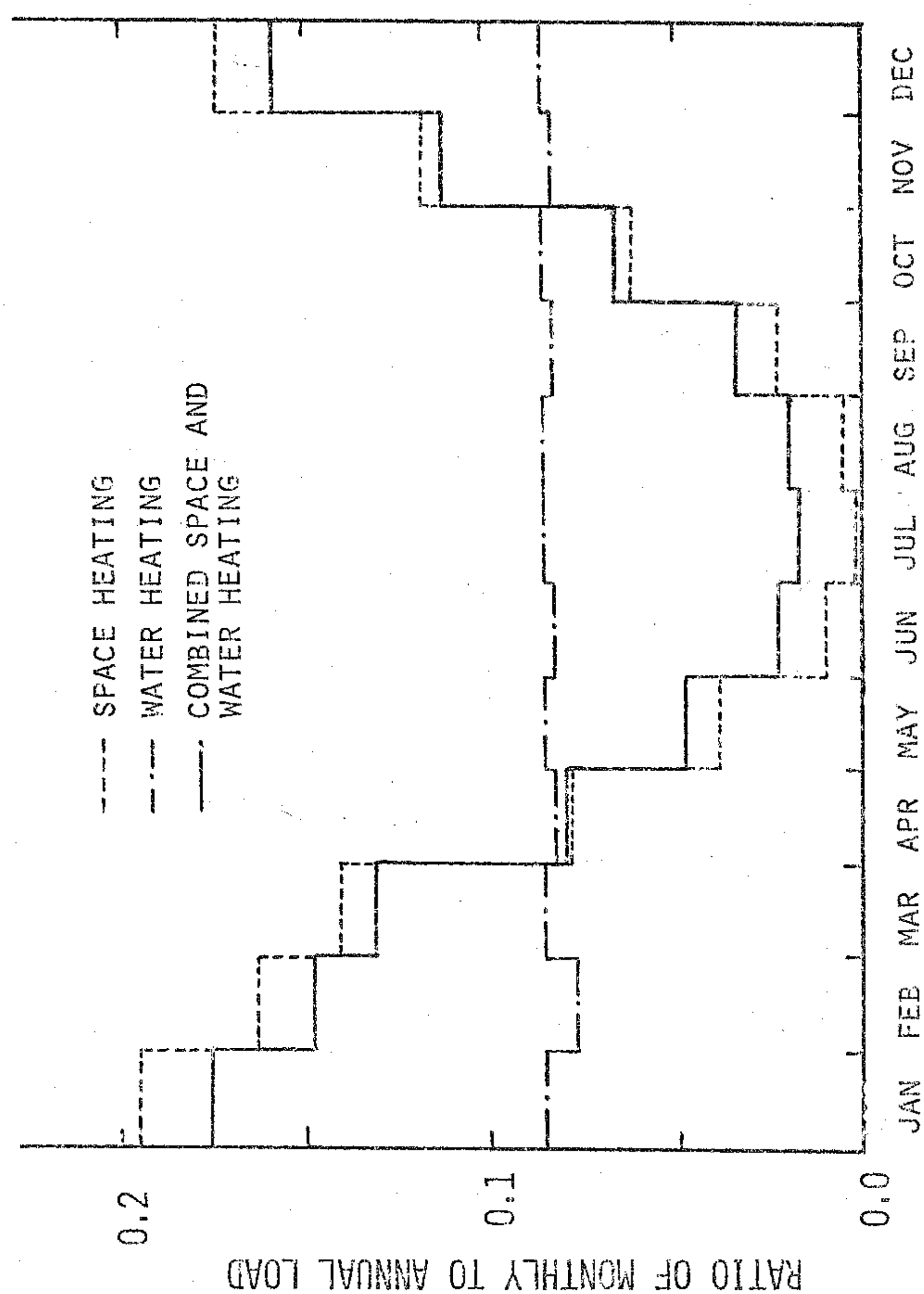


FIGURE 2-7 ANNUAL LOAD FRACTION MET BY SOLAR ENERGY AS A FUNCTION OF THE COLLECTOR AREA TO ANNUAL LOAD RATIO.

FIGURE 2-8 SPACE HEATING, WATER HEATING, AND COMBINED SPACE AND WATER HEATING ANNUAL LOAD DISTRIBUTIONS FOR MADISON, WI.



the space heating load. The assumption that the combined load distribution is a function only of location implies that, in a given location, the ratio of the space heating load to the domestic water heating load is a constant. This assumption seems reasonable over a modest range of home sizes. Most new home construction will conform to building codes which specify minimum insulation requirements. The space heating load in a particular location will then depend mainly on the size of the house. Most larger residences will house more people with a correspondingly larger hot water usage.

In order to obtain a representative load division between space and domestic hot water heating, loads were estimated for each location using the degree day method [15] for a residence insulated to ASHRAE 90-75 standards [16]. This typical residence is a single family, one story house with 150 m^2 floor area, 120 m^2 wall area, and an infiltration rate of one air change per hour. This results, for example, in an overall house UA of 294 $W\text{-}°C^{-1}$ in Great Falls, MT, and 347 $W\text{-}°C^{-1}$ in Dallas, TX. The water heating loads were calculated for a constant daily average hot water demand of 300 liters per day heated from the water main temperature to 60°C. The annual average water main temperatures, which vary from city to city, were obtained from ref. [17]. The information used to estimate the load distributions are given in Appendix C, Table C-1.

Strictly speaking, the annual solar load fraction which is generated for a given load distribution will not be valid for a different load distribution. Fortunately, the annual solar load fraction is not very sensitive to the time distribution of the load [6,18]. Since location has the strongest influence on the load distribution, this analysis will give acceptable results for almost any residential heating load.

CHAPTER 3 ECONOMIC ANALYSIS

3.1 Life Cycle Cost Analysis

Life cycle cost analysis is a method of economic evaluation which accounts for all pertinent and quantifiable costs incurred during a given period of time. Since the various costs will be realized at different times during the analysis, all costs are discounted to current dollar values, reflecting the changing value of money with time. In a comparative analysis between all possible alternatives, the alternative with the least life cycle cost is most economical.

When considering energy related systems, life cycle cost analysis is especially applicable. Most other methods of economic evaluation focus on short term expenses and benefits. However, with ever-increasing energy costs, the operating expenses of energy consuming processes become very significant. Proper accounting of these future costs is important in making effective economic decisions.

There are two variables which characterize the life cycle cost method: the duration of the analysis, N_E, and the discount rate, d. The duration of the life cycle cost analysis for solar heating systems is often chosen to be the expected operating life of the system. While this variable has a strong influence on the life cycle cost evaluation, its accurate estimation is virtually impossible in the face of so little long-term solar energy system data. Because of this uncertainty, the duration of the analysis is usually understood to be no more than some "reasonable" period of time over which the economic analysis is made.

When considering a variety of costs incurred at different times in an analysis (as in the life cycle cost method), it is important to recognize the changing value of money with time. This is accommodated by the discount rate, defined as the rate of return which can be obtained from the best alternative investment. As an example, consider a man who has the choice of repaying a $1000 debt either now or one year from now. The time value of money dictates that it would be in his best interest to delay it for a year. If he can, for example, obtain a rate of return of 8% from his best investment (i.e. $d = 0.08$), he need only have $925.93 today to have $1000 one year from now. In this case, $1000 one year in the future has a "present value" of $925.93. It is clear, though, that the present value of money in the future depends on the investment opportunity of the individual. If the man could obtain a 15% rate of return on investment, the $1000 one year in the future would have a present value of $869.57. In life cycle cost analysis, the discount rate is used to reflect this time value of money, depending on the investment opportunity of the individual or business.

The discount rate can be expressed in either real or nominal terms. A real discount rate is that rate of return above and beyond the general rate of inflation; a nominal, or market, discount rate includes inflation. Business investors tend to think in terms of real rates, while the typical home owner is more likely to think in terms of nominal rates. A more complete discussion of discount rates can be found in engineering economics texts, such as [19].

The life cycle cost model used in this solar heating system analysis is based on that of Ruegg [11]. It will be developed for a comparison between two alternatives: a solar heating system and a conventional heating system. The general procedure consists of four steps: 1) identify pertinent expenses which contribute to the life cycle cost of both a solar heating system and an alternative conventional heating system, 2) evaluate each of these costs in each year of the analysis, 3) discount all costs to current dollar values, i.e. obtain the present worth of each cost 4) subtract the solar energy system present worth from the conventional system present worth to obtain the life cycle savings of the solar heating system over the conventional heating system.

3.2 Solar Energy System Life Cycle Costs

3.2.1 Initial investment

The most obvious solar energy system cost is that required to purchase and install system components. If this initial investment, INV, is a cash payment at the beginning of the analysis, it is already expressed in present dollar value. On the other hand, the investment may be paid with a loan, with payments to be made in the future. For a loan amortized over N_L years at an annual interest rate, i, the annual payment, PMT, is determined by multiplying the loan principle by the capital recovery factor. (See, for example, [19].)

$$PMT = (1-D)(INV)\frac{i(1+i)^{N_L}}{(1+i)^{N_L}-1} \quad (3\text{-}1)$$

The contribution of loan payments to the solar heating system life cycle cost will depend on the relationship between the term of the loan, N_L, and the duration of the analysis, N_E. If $N_L \leq N_E$, all N_L loan payments will contribute. However, if $N_L > N_E$, only N_E loan payments will be made during the analysis. The procedure used to account for the remaining loan payments depends on the rationale for choosing the value of N_E. If N_E is chosen merely as a period over which to consider the discounted cash flow, with no concern for costs outside the period, only N_E payments should be discounted. If N_E is the expected operating life of the system, and loan payments will continue to be made as scheduled, all N_L loan payments will be made and the problem is identical to that for which $N_L \leq N_E$. If N_E is chosen as the period of building ownership, with the building to be sold after N_E years and the remaining loan principal paid in full at that time, the life cycle loan cost will consist of N_E loan payments plus the principal remaining in year N_{E+1}. It can easily be shown that the loan principal remaining in year j is calculated by the following.

$$L_j = (1-D)(INV)(1+i)^{j-1} - PMT\sum_{k=0}^{j-2}(1+i)^k \qquad (3-2)$$

For the purposes of this analysis, define N_1 as the number of loan payments which contribute to the life cycle cost analysis. The life cycle cost of these N_1 payments plus the loan down payment is

$$LOAN = (INV)\left[D+(1-D)\frac{i(1+i)^{N_L}}{(1+i)^{N_L}-1}\sum_{j=1}^{N_1}\frac{1}{(1+d)^j}\right] \qquad (3-3)$$

The contribution of any remaining loan principal will be recognized as a resale value and considered in Section 3.2.4.

3.2.2 Operating Costs

The mechanical nature of a solar heating system will usually require payments for maintenance at various times during the system lifetime. In most cases, the home or business owner will also wish to insure his investment against potential damages. This analysis will combine maintenance, insurance, and parasitic power (power for pumps, fans, etc.) expenses in the form of annual miscellaneous costs, which are assumed to inflate at the general inflation rate, g. The miscellaneous cost in the first year of the analysis is assumed proportional to the initial investment by the factor M. In year j, the miscellaneous cost can be expressed by the following.

$$MISC_j = M(INV)(1+g)^{j-1}$$

The life cycle miscellaneous cost over a period of N_E years is

$$MISC = \sum_{j=1}^{N_E} M(INV)\frac{(1+g)^{j-1}}{(1+d)^j}$$

Solar heating system costs include expenses for auxiliary (backup) system equipment, maintenance, and fuel. The auxiliary system fuel expenses in any one year depend on the solar system performance, the heating/hot water load, and the unit cost of delivered conventional fuel energy. (This unit delivered fuel cost must account for furnace efficiency.) For a given average annual load, L, a first year unit auxiliary energy cost, C_{FS}, an annual load fraction supplied by solar energy, F, and assuming a fixed annual fuel price inflation rate, e; the auxiliary fuel expense in year j is

$$FUELS_j = C_{FS}L(1-F)(1+e)^{j-1}$$

If the solar system performance degrades at an annual rate, k, the fuel expense in year j is

$$FUELS_j = C_{FS}L[1-F(1-k)^{j-1}](1+e)^{j-1}$$

The life cycle auxiliary fuel cost for N_E years is then

$$\text{FUELS} = \sum_{j=1}^{N_E} C_{FS} L[1-F(1-k)^{j-1}]\frac{(1+e)^{j-1}}{(1+d)^j} \quad (3\text{-}5)$$

The cost of auxiliary system components and their maintenance is not quantified in this analysis. It is assumed that these auxiliary system costs are identical to those of the alternative conventional system components and maintenance. While they contribute to the life cycle cost of both solar and conventional heating systems, respectively, the terms will negate each other in the final comparative analysis.

3.2.3 Taxes

State and federal tax laws influence the life cycle cost of solar heating systems. The impact of taxes varies with local assessment practices, property and income tax laws, the income tax bracket of the owner, and building usage. If the building is sold, the owner may also be subject to capital gains taxes. Depending on the owner's situation, these tax influences could either increase or decrease the solar energy system life cycle cost.

Property taxes are determined as a percentage of a share of the market value of a building. Since a building with a solar energy system would cost more to build than an otherwise identical building with a conventional heating system, the market value of the solar heated building is increased. Although over one third of the states have recently revised property tax laws regarding solar energy systems, most require assessment based on the full market value. Local variations are reflected in different assessment rates and tax rates per assessed dollar value. Sample census data for single family residences in a number of U.S. cities show assessment values from 8% to 93% of the market value, and nominal tax rates from 1% to 25% per assessed dollar. While these individual quantities vary tremendously, the data also shows that the effective property tax rate (based on market value) ranges from only 1% to 4% [11].

The property tax model used in this analysis assumes that the real value of the solar heating system remains fixed throughout the system lifetime, i.e. the market value increases at the general inflation rate, g. The market value in the first year of the analysis is assumed to be equal to the original investment. For a local assessment rate V, and property tax rate t, the property tax paid in year j is

$$PROP_j = tV(INV)(1+g)^{j-1}$$

The life cycle property tax cost over N_E years is determined from the following.

$$PROP = \sum_{j=1}^{N_E} tV(INV)\frac{(1+g)^{j-1}}{(1+d)^{j}} \tag{3-6}$$

Income tax deductions which alter solar energy system life cycle costs depend on the owner's federal and state tax rates, state tax laws, and whether the system is installed in an owner-occupied residence or a commercial building. For either type of building, loan interest and property tax payments are deductible from federal, and most state, income taxes. If the building is not an owner occupied residence, most state and federal income tax laws allow commercial deductions of operating expenses (fuel and miscellaneous costs) and depreciation.

If the building is sold during, or at the end, of the analysis the owner will probably be subject to tax on capital gains, which will in turn affect the life cycle cost of the solar system. The effect of capital gains depends on the type of building, individual state tax laws, the specifics of the sale, and whether the gain is reinvested. Since the exact form of this contribution depends strongly on the specific situation of the sale and owner, capital gain taxes are not quantified in this analysis. However, if capital gain tax is encountered, it may strongly influence the economics of the solar heating system and should not be ignored.

This analysis accommodates income tax deductions through an effective income tax rate, $\bar{t}$, which accounts for both state and federal tax deductions. For example, if both state and federal laws allow tax deductions and state taxes are deductible from federal taxes; if the federal rate is 30% and the state rate is 10%, then the effective income tax rate is $\bar{t} = .30+.10-(.10)(.30) = .37$.

The symbol C used in this analysis is a commercial/non-commercial flag. For an owner-occupied residence, C = 0; for commercial buildings, C = 1. The symbol U_j denotes the amount depreciated in j, and L_j is the remaining loan principal in year j as given by (3-2). The income tax contribution to the solar system cost in year j is then

$$INCS_j = -\bar{t}(iL_j + PROP_j) - C\bar{t}(MISC_j + FUELS_j + U_j)$$

The present value of the income tax contribution to the solar heating system life cycle cost is

$$INCS = -\sum_{j=1}^{N_E} \frac{\bar{t}}{(1+d)^j}[iL_j+PROP_j+C(MISC_j+FUELS_j+U_j)] \tag{3-7}$$

This term is negative because income tax deductions reduce the solar system life cycle cost.

The amount depreciated in year j, U_j, will depend on the choice of depreciation schedule. The three most commonly used schedules are straight line, double declining balance, and sum of digits. Straight line depreciation calls for an equal amount to be depreciated annually over the depreciation lifetime, N_D. This annual amount is simply

$$U_j = (INV)/N_D$$

The double declining balance method allows a certain fraction of the remaining undepreciated balance to be depreciated each year until year N_D, with the remaining balance deducted in the last year. Except for the year N_D, the amount depreciated in year j is

$$U_j = \frac{2}{N_D}(INV)[1-2/N_D]^{j-1}$$

The sum of digits depreciation method allows a different fraction of the original investment to be deducted each year, such that the total depreciable amount is deducted by the year N_D. In year j, the amount depreciated is

$$U_j = (INV)\frac{2(N_D - j + 1)}{N_D(N_D + 1)} \qquad (3\text{-}10)$$

3.2.4 Salvage and Resale Value

Depending on the rationale behind the user's choice of the period of economic analysis, the solar system will probably have a salvage or resale value at the end of the analysis. If the period of analysis, N_E, is chosen as the expected operating life of the system, the value of the scrap metal and inoperable equipment will correspond to the salvage value. If N_E is chosen as the duration of building ownership, the resale value will correspond to the market value of the solar heating system at that time. As mentioned in Section 3.2.1, there may be some outstanding loan principal remaining at the time of sale, as per (3-2). This amount must be subtracted from any anticipated resale value.

If the net salvage or resale value at the end of the analysis is proportional to the initial investment by the factor G, the contribution to the solar energy system life cycle cost is

$$SR = -\frac{G(INV)}{(1+d)^{N_E}} \tag{3-11}$$

Care must be taken to be consistent between the effects of property taxes and salvage value. If the market value of the solar heating system (and hence the property tax) are assumed to inflate at the general inflation rate, the salvage value will equal the market value in the final year of the analysis.

3.3 Conventional System Costs

The life cycle costs of a conventional heating system consist of conventional system components, maintenance, and fuel expense. Since the objective of this development is to quantify costs which contribute to a comparison between solar heating systems and conventional heating systems, conventional system components and maintenance will not be evaluated here. These costs are present in both alternatives and cancel each other in a comparative analysis.

The conventional system fuel cost in year j is evaluated in the same manner as solar system fuel costs, except the entire load must be met. Also, the cost of fuel for the conventional heating system may differ from that of the solar energy auxiliary system. If the unit cost of delivered energy for the conventional heating system in the first year of the analysis is C_{FC}, the life cycle conventional fuel cost is

$$FUELC = \sum_{j=1}^{N_E} C_{FC} L \frac{(1+e)^{j-1}}{(1+d)^j} \tag{3-12}$$

Commercial tax deductions could influence conventional system life cycle costs, as described in the previous section. The following term can account for this contribution.

$$INCC = -\sum_{j=1}^{N_E} C\bar{t}C_{FC} L \frac{(1+e)^{j-1}}{(1+d)^j} \tag{3-13}$$

3.4 Life Cycle Cost Comparison

An economic comparison between a solar heating system and a conventional heating system can be made by subtracting the solar energy system life cycle cost from the conventional system life cycle cost. If this life cycle savings is positive, the solar energy system can be deemed more economical.

$$SAV = (FUELC+INCC) - (FUELA+LOAN+MISC+PROP+INCS+SAL)$$

Substituting from Sections 3.2 and 3.3, and rearranging, the life cycle savings are

$$\begin{aligned} SAV = {} & (1-C\bar{t})C_{FC}L\sum_{j=1}^{N_E}\frac{(1+e)^{j-1}}{(1+d)^j} - (1-C\bar{t})C_{FS}L\sum_{j=1}^{N_E}\frac{(1+e)^{j-1}}{(1+d)^j} \\ & + (1-C\bar{t})C_{FS}L\mathcal{F}\sum_{j=1}^{N_E}\frac{(1+e-k-ek)^{j-1}}{(1+d)^j} - D(INV) \\ & - (1-D)(INV)\frac{i(1+i)^{N_L}}{(1+i)^{N_L}-1}\sum_{j=1}^{N_1}\frac{1}{(1+d)^j} + \bar{t}i\sum_{j=1}^{N_1}\frac{L_j}{(1+d)^j} \\ & - t(1-\bar{t})V(INV)\sum_{j=1}^{N_E}\frac{(1+g)^{j-1}}{(1+d)^j} - (1-C\bar{t})M(INV)\sum_{j=1}^{N_E}\frac{(1+g)^{j-1}}{(1+d)^j} \\ & + \frac{G(INV)}{(1+d)^{N_E}} + C\bar{t}\sum_{j=1}^{N_D}\frac{U_j}{(1+d)^j} \end{aligned} \qquad (3\text{-}14)$$

Equation (3-14) expresses the life cycle savings in terms of several geometric series. These series can be collapsed into a closed form by noting that

$$(x+x^2+x^3+\cdots+x^n) = \frac{x(1-x^n)}{(1-x)}$$

For the purpose of this analysis, this equation can be applied by the following definition.

$$f(a,b,c) = \sum_{j=1}^{a} \frac{(1+b)^{j-1}}{(1+c)^j} = \frac{1}{c-b}\left[1-\left(\frac{1+b}{1+c}\right)^a\right] \qquad (3\text{-}15)$$

The factor $f(a,b,c)$ is defined as a discount-inflation factor. When multiplied by a first period cost (which is inflated at a rate b and discounted at a rate c over a periods), the resulting value is the life cycle cost. When the inflation rate is zero, $f(a,0,c)$ is the familiar series-payment present-worth factor, and $[f(a,0,c)]^{-1}$ is the capital recovery factor. The discount-inflation factor is tabulated in Table B-1 of Appendix B.

As a further extension of the discount-inflation factor, it can be shown that the present worth of the loan interest can be expressed as

$$\sum_{j=1}^{N_1} \frac{iL_j}{(1+d)^j} = \frac{f(N_1,0,d)}{f(N_L,0,i)} + f(N_1,i,d)\left[i-\frac{1}{f(N_L,0,i)}\right] \quad (3\text{-}16)$$

The present worth of depreciation deductions can also be expressed in terms of the discount-inflation factor. For straight line depreciation, the ratio of the life cycle depreciation costs to the initial investment is

$$U = \frac{1}{N_D} f(N_D,0,d) \quad (3\text{-}17)$$

For double declining balance depreciation,

$$U = C + \frac{2C}{N_D}\left[f(N_D-1,-\frac{2}{N_D},d)-\frac{f(N_D-1,-2/N_D,0)}{(1+d)^{N_D}}\right] \quad (3\text{-}18)$$

For sum of digits depreciation,

$$U = \frac{2}{N_D(N_D+1)}\left[f(N_D,0,d) + \frac{N_D-1-f(N_D-1,0,d)}{d}\right] \quad (3\text{-}19)$$

Tables of these functions are given in Appendix B.

Using these relationships, and the definition ε = e-k-ek, in (3-14), the savings equation in closed form is

$$SAV = (1-C\bar{t})(C_{FC}-C_{FS})\,L\,f(N_E,e,d)$$

$$+\ (1-C\bar{t})C_{FS}\,L\,\mathcal{F}\ f(N_E,\varepsilon,d)\ -\ D(INV)$$

$$-(1-D)(INV)\frac{f(N_1,0,d)}{f(N_L,0,i)}\ -\ (1-C\bar{t})M(INV)\ f(N_E,g,d)$$

$$+\ (1-D)(INV)\bar{t}\left[\frac{f(N_1,0,d)}{f(N_L,0,i)}\ +\ f(N_1,i,d)\left(i-\frac{1}{f(N_L,0,i)}\right)\right]$$

$$-\ t(1-\bar{t})\ V\ (INV)\ f(N_E,g,d)$$

$$+\left[C\bar{t}\ U\ +\ \frac{G}{(1+d)^{N_E}}\right](INV) \qquad (3\text{-}20)$$

If it is assumed that the cost of energy for the conventional system is the same as that for the auxiliary system, $C_{FS} = C_{FC} = C_F$, it is seen that all costs are proportional to either the initial investment or the fuel savings in the first year. The savings equation can then be collapsed to the following simple form.

$$SAV = P_1 C_F L \mathcal{F} - P_2(INV) \qquad (3\text{-}21)$$

Equation (3-21) involves only one assumption: All costs which contribute to the life cycle savings of a solar heating system over a conventional heating system are proportional to either the first year fuel savings or the initial solar system investment. If this requirement is satisfied, P_1 and P_2 can be of any form. The formulation of the life cycle cost model in the preceding sections is merely one example consistent with one set of possible economic assumptions. The multiplying factors, P_1 and P_2, facilitate the use of life cycle cost analysis in a compact and flexible form. Depending on the desired economic complexity, they can include any or all of the terms presented in Sections 3.2 and 3.3, with future costs varied arbitrarily. The explicit functional form of P_1 and P_2 depend on the significance and applicability of each of these costs, and the economic assumptions of the individual analyst.

3.5 Economic Sensitivity

The life cycle cost comparison between a solar heating system and a conventional heating system will be a function of many economic variables. Since some economic senario can be formulated which will make solar heating look economical for almost any location or system design, the validity of the economic evaluation depends on the values assigned to these variables. Judicious choice of values will result in a powerful and reliable method for assessing the economic viability of solar energy. Poor selection of the economic variable values can yield distorted and misleading results.

Unfortunately, there is an inherent uncertainty in predicting future expenses and savings, especially for energy related processes. Even the most sophisticated economic assumptions can not yield absolutely accurate results. If the results of any life cycle cost analysis are to be viewed objectively, the possible uncertainty of the economic variables and their effects on the analysis results must be recognized.

This section sketches the sensitivity of the life cycle cost analysis to the constituent economic variables. In particular, the economic model of (3-20) is used to estimate the effects of economic variables on P_1 and P_2. Since the sensitivity to any one variable will be strongly influenced by other economic variable values, the discussion will be qualitative in nature. A procedure for quantitatively evaluating economic sensitivities is given in Appendix A.

The values of some economic variables will be fixed by the conditions surrounding the individual owner's situation. There is little uncertainty surrounding loan cost variables, since the down payment, interest rate, and term of the mortgage are determined by the loan contract for the entire building. However, knowledge of their effects can still be useful.

The down payment has little effect on the value of P_2. The effect it does have is determined by the interest rate, discount rate, mortgage term, and income tax rate. If the discount rate is greater than the interest rate, an increase in the down payment will increase the value of P_2. If the discount rate is less than the interest rate, the effect on P_2 will depend on the income tax rate. Since the interest is deductible from income taxes, it has the effect of lowering the interest rate. In most cases, it remains beneficial to have a small down payment and long mortgage term.

The loan interest rate can have a significant effect on the value of P_2. Clearly, a 5% loan interest rate will make a noticable change in P_2 over a 12% interest rate. The effect of a small change in the interest rate, though, is generally not very significant. This is especially true if the discount rate is approximately equal to the loan interest rate. If the difference between the interest and discount rates is large, and the income tax rate is large, the sensitivity to the interest rate increases.

The operating costs of both a solar and conventional heating system are among the most difficult costs to evaluate. Unfortunately, the results of a life cycle cost analysis are strongly influenced by, and highly sensitive to, the economic variables which determine these costs. The rate of inflation of conventional and auxiliary heating system fuel costs will depend on a host of uncontrollable and often unpredictable factors. An error in estimating this fuel cost inflation rate as small as 1% can easily result in an 8-10% error in the value of P_1 for a 20 year analysis. This error can in turn cause errors of over a thousand dollars in the life cycle savings. If the results of a life cycle cost analysis are to viewed objectively, this uncertainty must not be overlooked. It must also be realized that the error caused by this uncertainty will overshadow those caused by many other economic variables. This situation often makes results from very sophisticated models no more accurate than those from very simple models.

Miscellaneous costs are often deemed insignificant when considering the error introduced by fuel inflation uncertainty. This assumption can be very misleading, since the errors caused by this neglect can be as large as those caused by fuel inflation uncertainties. For example, consider a 20 year analysis with a 6% general inflation rate, an 8% discount rate, and an \$8000 solar system investment. The difference in life cycle savings between an analysis which neglects miscellaneous costs and one which assumes a first year miscellaneous cost of 1% of the initial investment is \$1248. This sensitivity is further complicated by the uncomfortable ignorance about the magnitude and future variations of the miscellaneous costs. Clearly, these uncertainties must not be overlooked.

Property tax life cycle cost contributions and their uncertainty can also be significant, especially if the property tax rate is high and the income tax rate is low. If the building is located in a state which does tax solar systems, the significance of this contribution will depend largely on the specific situation. One of the main sources of uncertainty is that many states that now tax solar systems may exempt them from property taxes in the near future.

It has been assumed that miscellaneous costs and property taxes will increase with the general inflation rate. This assumption is based more on its convenience than its validity. Maintenance costs will probably by irregular, and property taxes will eventually decrease near the end of the system lifetime. In light of these uncertainties, increases at the general inflation rate seen the most reasonable choice. If this basic assumption is accepted, the general inflation rate becomes a rather uncontroversial variable, since there is little uncertainty in its value and little sensitivity to small errors in its estimation.

Like property taxes, income taxes can also be significant, depending on the loan agreement. Both the sensitivity and the significance of income taxes will greater for loans with low down payments, high interest rates, and long mortgage terms. However the main uncertainty is the possible variation of the income tax rate, especially if the owner is young and the duration of the analysis is long.

In an effort to simplify the life cycle cost model, miscellaneous costs and taxes are often omitted on grounds that they will cancel each in the final analysis. In many cases, this can be a very good assumption. Depending on the individual situation, though, it is often better to assume that either property taxes or miscellaneous costs will cancel income taxes, since it is usually unlikely that income taxes will offset both.

When considering commercial buildings, the effect of income taxes can be tremendous. This increased effect is accompanied by a dramatic increase in life cycle cost sensitivity to the value of the income tax rate (an increase in sensitivity by a factor of 5 is not uncommon). Fortunately, it is usually known at the beginning of the analysis whether the building will qualify for commercial tax deductions. However, if this eligibility is changed at some time in the future, it can strongly influence the analysis results.

The salvage value is another variable that is difficult to quantify. For short analyses, its value and sensitivity can be significant and must be recognized. For longer analyses, its value and sensitivity will probably be small, due to discounting to present value. The salvage value is often omitted from longer analyses to yield a slightly conservative life cycle savings.

The discount rate has a major effect in tempering the uncertainty of many economic variables. While costs incurred far in the future are the most uncertain, the discounting of future costs will reduce the significance of these uncertainties. However, the broad influence of the discount rate makes the life cycle cost analysis results rather sensitive to its value. Similarly, the duration of the analysis has a strong and obvious influence on the life cycle cost analysis. Like the discount rate, the magnitude of the sensitivity is determined by the value of almost every economic variable. The best way to recognize these sensitivities is to use several values of the discount rate and period of analysis.

In general, then, a life cycle cost analysis can not be viewed objectively without recognizing the uncertainty of the results.

CHAPTER 4 ECONOMIC OPTIMIZATION

For a particular location and set of economic conditions, the economic analysis presented in Chapter 3 can be used to evaluate the economic viability of a particular solar heating system design in terms of the life cycle savings. A different system design will generally yield a different life cycle savings through the influence of design on the annual solar load fraction or the solar system investment cost, or both. It is clearly advantageous to choose the system design which yields the greatest life cycle savings.

The optimum design can be characterized in terms of system design variables, such as collector area, storage size, collector characteristics, and collector orientation. Previous thermal and economic analyses have identified the optimum values of many design variables, which have been incorporated into the f-chart design method as cited in Section 2.2. Collector characteristics ($F_R(\tau\alpha)_n$ and F_RU_L) are then the only design variables which explicitly appear in the f-chart correlations. The following analysis focuses on optimization with respect to collector area to yield the maximum life cycle savings for a given location and pair of collector characteristics.

The most obvious method of determining the optimum collector area for given location and collector type is by iteration. A value of the collector area is chosen as an initial guess, for which the annual solar load fraction is calculated. Using this load fraction and the solar system investment corresponding to the chosen area in a life cycle cost analysis, the life cycle savings can be determined. The collector area is then varied and the process repeated until the optimum area is found. While this procedure may be simple for a computer, it can be very painstaking when done by hand.

There have recently been several methods proposed for simplifying this procedure. Ward [20] has developed a method for direct computation of the optimum collector from knowledge of the average radiation in the month of January. However, this method is somewhat location dependent. Barley and Winn [21] have also developed a non-iterative method based on a location dependent curve-fit of the annual load fraction to collector area. While the correlation accounts for the influence of collector characteristics on the load fraction, this influence is not present in the calculation of the optimum collector area. The errors introduced by ignoring this effect can be large.

4.1 Collector Area Optimization

In order to economically optimize a solar heating system with respect to collector area using (3-21), the solar heating system investment cost must be expressed as a function of collector area. While various factors such as economies of scale will generally result in a non-linear relationship, this functional dependence can be approximated as a linear relationship by the following.

$$INV = C_A A + C_E \tag{4-1}$$

where

C_A is the solar energy system investment cost which is directly proportional to collector area

C_E is the solar energy system investment cost which is independent of collector area

The area dependent cost, C_A, will consist mainly of the cost of installed collectors, but can also include a portion of the storage cost. The area independent cost, C_E, can include the cost of controls, ducting, etc., and the remaining storage cost. (Generally, the storage volume will increase linearly with collector area, while the cost of a storage container generally will not.) Rather than evaluating the individual costs of solar system components, the values of C_A and C_E can be obtained from solar heating system equipment distributors in the form of total system costs for different collector areas.

Using (4-1) to relate the investment to collector area, Figure 4-1 shows an example of the life cycle savings versus collector area for four different economic conditions. Curve A corresponds to an economic senario in which solar energy obviously can not compete. Clearly, the conventional heating system is the economic choice. Curve B exhibits a non-zero optimum area, but the conventional system is still the economic choice. Curve C corresponds to the "critical" condition, i.e. the optimum solar system design can just compete with the conventional system. Curve D corresponds to an economic senario which is favorable to solar energy. In this case, the solar heating system is the economic choice.

Each curve of Figure 4-1 begins with a negative savings for zero collector area. The magnitude of this loss is equal to P_2C_E, and reflects the presence of solar energy system fixed costs in the absence of any fuel savings. As collector area increases, all curves except curve A show increased savings until reaching a maximum at some optimum collector area. As the collector area is further increased, the fuel savings continue to increase, but the excessive system costs force the net savings to decrease.

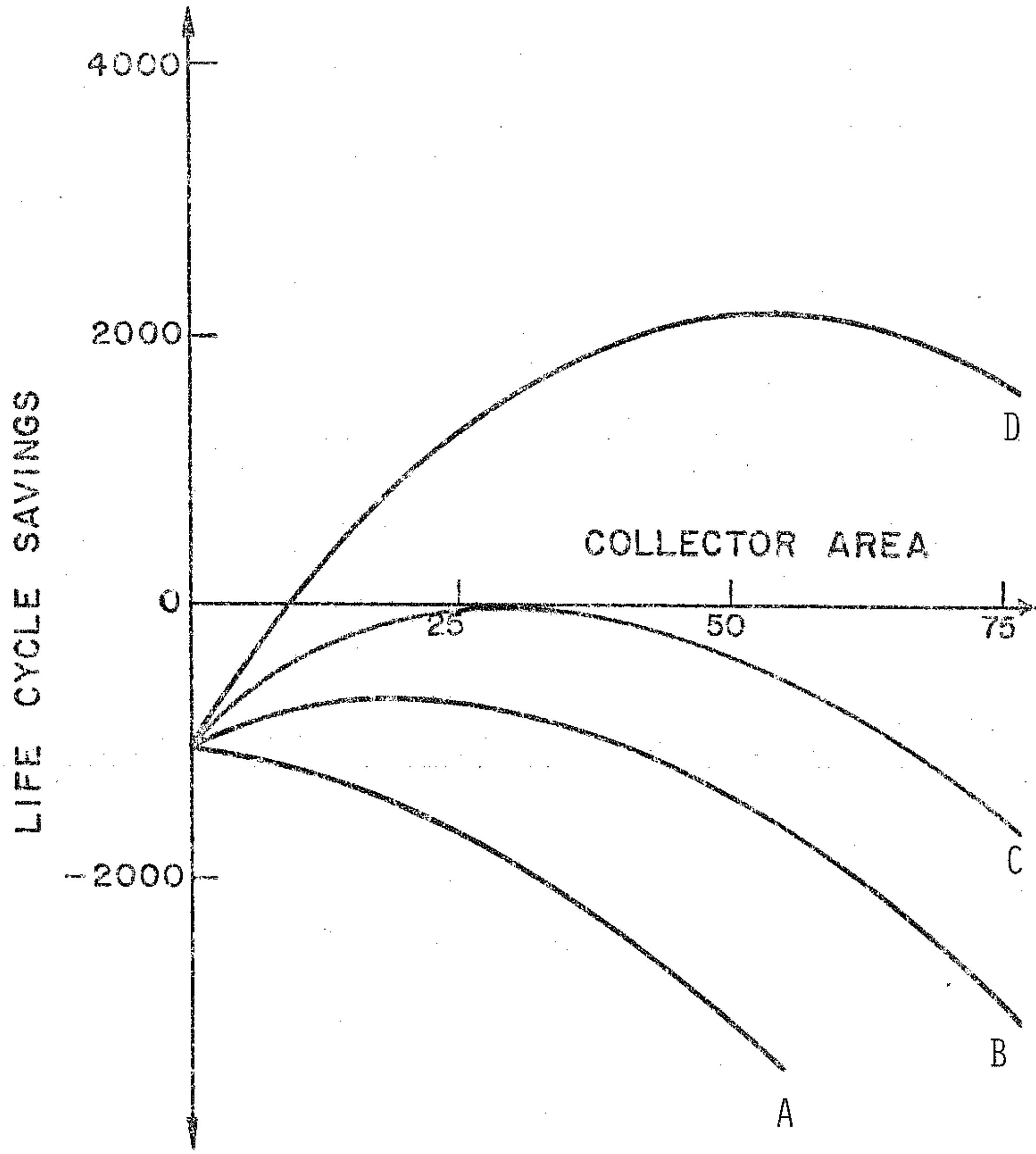


FIGURE 4-1 RELATIONSHIP BETWEEN LIFE CYCLE SAVINGS AND COLLECTOR AREA.

The maximum life cycle savings, and hence the optimum collector area, is characterized by the point at which the derivative of the life cycle savings with respect to collector area is zero.

$$\frac{\partial(\mathrm{SAV})}{\partial \mathrm{A}} = 0 = P_1 C_F L \frac{\partial F}{\partial A} - P_2 C_A \tag{4-2}$$

Rearranging, the maximum savings are realized when the relationship between collector area and solar load fraction satisfies the following.

$$L \frac{\partial F}{\partial A} = \frac{P_2 C_A}{P_1 C_F} \tag{4-3}$$

(This relationship is shown in Figure 4-2.) Since the load is constant throughout the optimization, it can be incorporated into the derivative to give at the optimum:

$$\frac{\partial F}{\partial (A/L)} = \frac{P_2 C_A}{P_1 C_F} \tag{4-4}$$

At this point, we employ the load distribution assumptions developed in Section 2.3. Recall that if the annual load distribution is independent of the magnitude

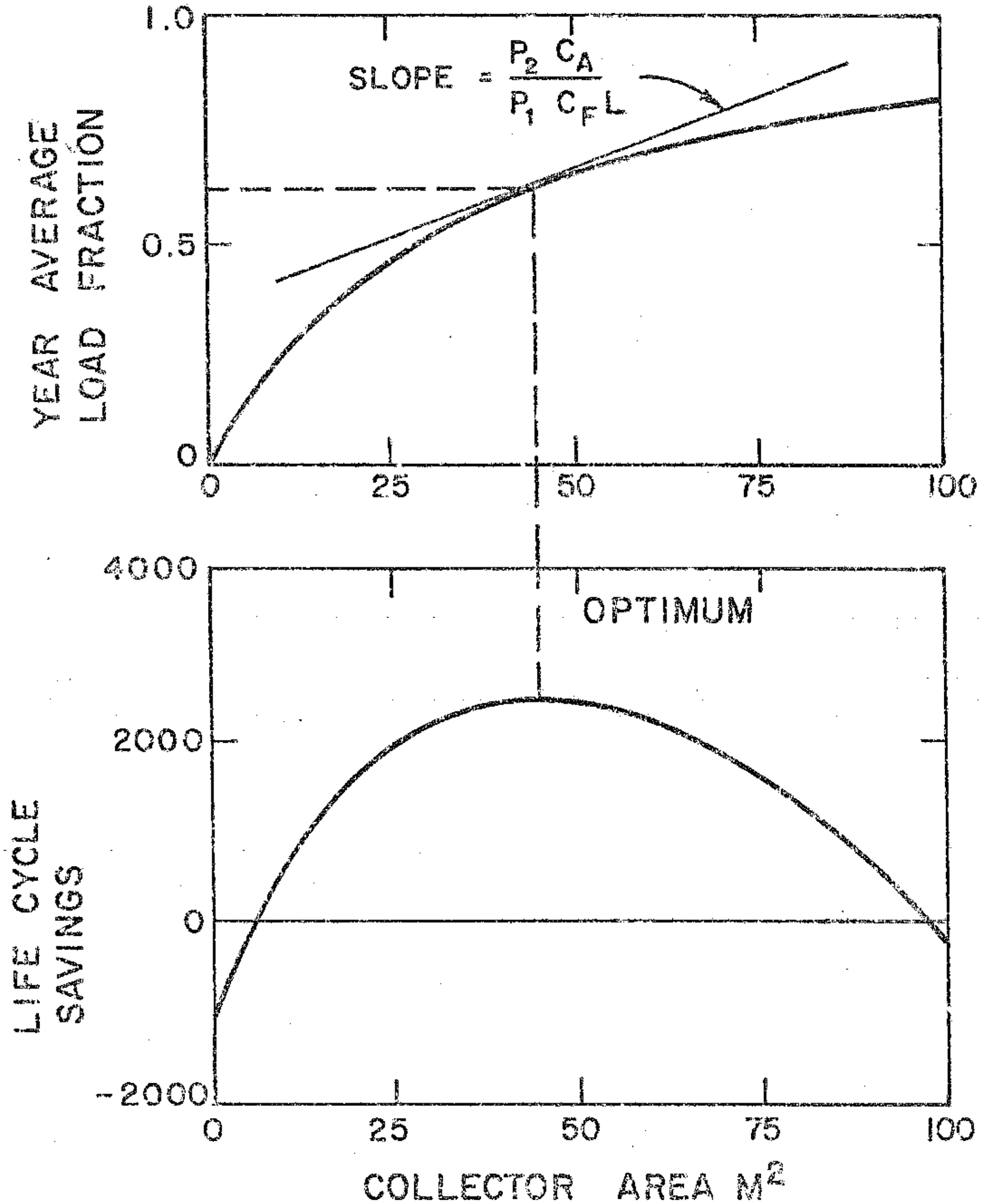


Figure 4-2 Relationship among life cycle savings, annual solar load fraction, and collector area.

of the load, can be expressed as a function only of location, collector characteristics, and the ratio A/L. Equation (4-4) then implies that, for a given location and collector type, the area to load ratio at which maximum savings can be achieved is a unique function of one economic parameter, P_2C_A/P_1C_F.

The results of this analysis can be presented in a tabular form for a given set of collector characteristics and type of system (domestic water heating, combined space and water heating with an air transfer medium, and combined space and water heating with a liquid transfer medium). These tables, as presented in Appendix C, provide a non-iterative method for determining the optimum system design. The parameter P_2C_A/P_1C_F and the optimum load fraction supplied by solar energy have been tabulated for 19 U.S. locations and different collector types as a function of the optimum collector area to annual load ratio. A more complete set of tables can be obtained by contacting the University of Wisconsin Solar Energy Laboratory. Section 4.3 describes the use of these tables and presents three illustrative examples.

It should be noted that the optimum collector area is independent of any life cycle cost contribution that is not a function of collector area. In particular, the optimum collector area obtained from this method does not depend on the cost of energy for the conventional heating system, but it does depend on the cost of energy for the solar system auxiliary heater.

4.2 Critical Economic Condition

The critical economic condition, sometimes called the break-even condition, is defined by curve C of Figure 4-1 for a given location and collector type. It is the condition at which the life cycle cost of the optimized solar heating system just equals that of the alternative conventional system. This critical condition is defined analytically by the simultaneous solution of (4-2) and (4-5).

$$SAV = 0 = P_1 C_F L \mathcal{F} - P_2 (C_A A + C_E) \tag{4-5}$$

Rearranging, it is seen that

$$\frac{P_2 C_A}{P_1 C_F} = \frac{\mathcal{F}}{A/L + C_E/C_A L} \tag{4-6}$$

Equating (4-4) and (4-6) the critical condition (denoted by an asterisk) is realized when the area to load ratio satisfies the following.

$$\left.\frac{\partial F}{\partial (A/L)}\right|_{(A/L)^*} = \frac{F^*}{(A/L)^* + (C_E/C_A L)} = Z^* \qquad (4\text{-}7)$$

For a given location and collector type, (4-7) shows the critical condition to be a function of one parameter, $C_E/C_A L$. Equating (4-6) and (4-7),

$$\left(\frac{P_2 C_A}{P_1 C_F}\right)^* = Z^*(\text{location}, \text{collector}, C_E/C_A L) \qquad (4\text{-}8)$$

Equation (4-8) can be presented in tabular form as in Appendix C. In these tables, the value of the parameter P_2C_A/P_1C_F at the critical condition is given as a function of location and C_E/C_AL for different values of collector characteristics. The use of these tables are demonstrated in Section 4.3.

The critical economic condition serves as a basis for the a priori decision of whether solar heating is economically viable. For a particular location and collector type, tables of the critical condition specify the maximum value of P_2C_A/P_1C_F for which solar energy can be economical. If P_2C_A/P_1C_F, as calculated by the individual for a particular economic senario, is less than or equal to the critical value, then the optimization method cited in the previous section can be used to determined the optimum system design for the particular economic senario.

For cases in which the value of P_2C_A/P_1C_F is greater than the critical value, the economic conditions prohibit the economic feasibility of any system design. However, economic conditions will probably evolve over time to permit economical solar heating in the future. If P_2C_A/P_1C_F could be expressed as a function of time, it would be possible to predict when the critical condition will occur. In particular, if P_1 and P_2 are assumed to be constant, if the cost of fuel is assumed to increase at an annual rate e, and the cost of solar system components (both C_A and C_E) are assumed to inflate at an annual rate g, then the value of P_2C_A/P_1C_F at a time j years in the future can be calculated.

$$C_{Fj} = C_F(1+e)^j$$
$$C_{Aj} = C_A(1+g)^j$$

$$\frac{P_2C_{Aj}}{P_1C_{Fj}} = \frac{P_2C_A}{P_1C_F}\,\frac{(1+g)^j}{(1+d)^j}$$

If $e > g$, there will be some point in the future when P_2C_{Aj}/P_1C_{Fj} will equal the critical value. Denoting this time as the year j^*,

$$j^* = \frac{\ln\left(Z^* \frac{P_1 C_F}{P_2 C_A}\right)}{\ln\left(\frac{1+g}{1+e}\right)} \qquad (4\text{-}9)$$

This analysis indicates that the solar heating system will be just competitive with the conventional system j^* years in the future. If the year j^* is large, the assumption that P_1 and P_2 are constant may not be valid. However, this value will serve as a first approximation of the time required for the critical value to occur.

4.3 Use of Optimization Tables

Appendix C contains a set of tables based on the optimization methods of Sections 4.1 and 4.2. These tables allow the individual user to assess the economic viability of solar space and/or domestic water heating, and to determine the solar system design which yields the greatest life cycle saving. The tables of Appendix C have been limited to 19 locations and selected collector types. A more complete set of tables can be obtained by contacting the University of Wisconsin Solar Energy Laboratory.

The tables have been generated by differentiating the algebraic form of the f-chart correlations. Special care has been taken to respect the ranges of these correlations by requiring that $f \leq 1$ and $\partial f/\partial A \geq 0$ for each month. If these conditions are not satisfied, $f = 1$ and $\partial f/\partial A = 0$. Solar insolation, long term average ambient temperatures, and long term average degree days have been abstracted from the FCHART computer program data. The domestic water heating tables employ monthly heating loads annually distributed according to the days per month. Combined space and domestic water heating loads are distributed according to the load combinations of Table C-1.

To use the tables, it is first necessary to evaluate P_1 and P_2 from economic conditions. By consulting with a solar system equipment supplier, the installed system costs can be determined. Knowing the present cost of delivered fuel energy and the annual heating load, the values of P_2C_A/P_1C_F and C_E/C_AL can be calculated. Entering the tables at the appropriate collector type and location, the critical value of $(P_2C_A/P_1C_F)^* = Z^*$ can be determined corresponding to the value of C_E/C_AL. If $P_2C_A/P_1C_F \leq Z^*$, the solar heating system is economically competitive with the conventional heating system. Using the same line of the table, the user can interpolate to determine the optimum area to load ratio and the corresponding load fraction met by the optimized solar system. If $P_2C_A/P_1C_F > Z^*$, the solar heating system is not currently economical. However, (4-9) can be used to determine the year in which it will become competitive with the conventional system.

To illustrate this procedure, consider a home owner in Miami, FL, who is installing a solar domestic hot water system with two cover non-selective collectors ($F_R'(\tau\alpha)_n = 0.6$, $F_R'U_L = 4.0$ $W\text{-}m^{-2}\text{-}°C^{-1}$) having area dependent costs of \$150 m^{-2} and fixed costs of \$500. It is estimated that the annual household hot water load is 16.3 GJ (300 liters per day heated from 25°C to 60°C). Assume that water could be heated using conventional fuel for \$7.00 GJ^{-1}. The home owner has decided to pay cash for the system (D = 1) and wants to consider only fuel costs and this cash initial payment in the economic analysis. Assuming that the discount rate is 8%, the energy inflation rate is 10%, and the economic period is 20 years, we have:

$$P_1 = f(N_E, e, d) = \frac{1}{0.08-0.10}\left[1-\left(\frac{1.10}{1.08}\right)^{20}\right] = 22.169$$

$$P_2 = 1$$

$$\frac{P_2 C_A}{P_1 C_F} = 0.967$$

From Table C-2 it is seen that the critical value of P_2C_A/P_1C_F for $C_E/C_AL = 0.204$ is $Z^* = 1.320$. Since $P_2C_A/P_1C_F \leq Z^*$, the solar water heating system is the economic choice. To determine the optimum value of A/L, it is seen that for $P_2C_A/P_1C_F = 0.967$, $A/L = 0.40$ and $F = 0.79$. With the assumed load of 16.3 GJ, the optimum collector area is 6.5 m^2. Using (3-21), the life cycle savings are

$$SAV = (22.169)(7.00)(16.3)(0.79) - (150)(6.5) - 500 = \$523$$

As another example, consider a home owner in Madison, WI who is installing a solar air system with combined space and domestic water heating capabilities. The system has two cover non-selective collectors ($F_R'(\tau\alpha)_n = 0.5$, $F_R'U_L = 3.0$ W-m^{-2}-°C^{-1}) with area dependent costs of \$200 m^{-2} and fixed costs of \$1000. The water heating load is estimated to be 23.4 GJ. The long term average annual degree days for Madison is 4294 °C-days. Using the typical house described in Section 2.3, the building UA from Table C-1 is 294 W-°C^{-1} resulting in an annual space heating load of 109.1 GJ. The total load is then 132.5 GJ. The present cost of conventional energy is assumed to be \$9.90 GJ^{-1} (corresponding to electric resistance heating). The home owner in this case has decided to include loan costs, miscellaneous costs, and taxes, while neglecting salvage value. Assuming $d = 0.08$, $e = 0.10$, $i = 0.09$, $g = 0.06$, $N_E = 20$, $N_L = 20$, $\bar{t} = 0.30$, $t = 0.02$, $D = 0.10$, $M = 0.01$,

$V = 0.7$, $C = 0$ (residence), and $G = 0$, P_1 and P_2 are calculated from (3-20).

$$\begin{aligned} f(N_E,e,d) &= f(20,0.10,0.08) = 22.169 \\ f(N_1,0,d) &= f(20,0.00,0.08) = 9.818 \\ f(N_L,0,i) &= f(20,0.00,0.09) = 9.129 \\ f(N_1,i,d) &= f(20,0.09,0.08) = 20.242 \\ f(N_E,g,d) &= f(20,0.06,0.08) = 15.596 \end{aligned}$$

$$P_1 = 22.169$$

$$\begin{aligned} P_2 &= .1 + (.9)(9.818)/(9.129) - .27\{20.242[.09-1/(9.129)] \\ &\quad + (9.818)/(9.129)\} + (.01)(15.596) + (.0098)(15.596) \\ &= 1.193 \end{aligned}$$

In this case, $P_2C_A/P_1C_F = 1.087$. Using Table C-3 with $C_E/C_AL = 0.038$, it is seen that $Z^* = 1.395$. Since $P_2C_A/P_1C_F < Z^*$, the combined solar heating system is economically viable. To determine the optimum ratio of A/L, the user must interpolate from the values in the table. For $P_2C_A/P_1C_F = 1.087$, the interpolation yields the values $A/L = 0.244$, and $F = 0.37$. With $L = 132.5$ GJ, $A = 32.3$ m^2, the life cycle savings from (3-21) are \$1860.

The home owner may also wish to compare the solar heating system with an oil furnace for which the delivered cost of energy (including efficiency) is \$5.40 GJ^{-1}. In this case, the value of P_2C_A/P_1C_F is calculated to be

$$\frac{P_2C_A}{P_1C_F} = 1.993$$

Comparing this value with the critical value of $Z^* = 1.395$, the solar heating system is not competitive with the conventional oil furnace. However the year at which the solar system will become competitive can be calculated from (4-9) by assuming that the collector component costs will increase at the general inflation rate, $g = 0.06$.

$$j^* = \frac{\ln(1.395/1.993)}{\ln(1.06/1.10)} = 9.6$$

That is, it will be 10 years before any system design with these collector characteristics will be competitive with oil heating in Madison, WI.

CHAPTER 5 SUMMARY AND DISCUSSION

The technical and environmental feasibility of solar energy for domestic water heating and space heating has been well established. The question of economic viability is the major barrier restricting its widespread usage. On a general level, the current economic conditions suggest that solar heating may be economically competitive with conventional fuels, depending on the individual circumstances. This guarded position dictates the need for a simple procedure by which the individual engineer, architect, or home owner can realistically evaluate the applicability of solar heating for a particular situation. The fact that, for any economic condition, there is an optimum system design requires that there also be a simple procedure for determining this design. The methods set forth in this thesis provide such procedures.

Life cycle cost analysis has been shown to be the most realistic method of economic evaluation for this purpose. It accounts for all pertinent costs incurred throughout the period of analysis and recognizes the time value of money. The identification of these pertinent costs and estimation of their future variations is, to a large extent, determined by the situation of the solar energy user and the assumptions of the economic analyst. An explicit life cycle cost model has been developed based on one such set of contributing costs and economic assumptions. Admittedly, the model in this form is somewhat restricted by these assumptions and may not apply in all cases. However, the procedure for developing this particular model is generally applicable.

To eliminate the bulk of assumptions which restrict the detailed model, two economic parameters have been defined, P_1 and P_2, which relate all life cycle cost present values to either the first year fuel savings or the initial solar heating system investment. The use of these parameters requires one economic assumption: all costs which contribute to the life cycle costs of the solar heating system or the conventional heating system are directly proportional to either the first year fuel savings or the initial solar system investment. The introduction of P_1 and P_2 not only eliminates many of the assumptions of the explicit model, but presents the life cycle savings equation in a compact and managable form. This form accomodates straightforward manipulation of the savings equation to determine the optimum system design. The optimization method presented here uses the parameterized savings equation and the f-chart design correlations to develop a tabular method for estimating the optimum collector area and evaluating its economic effectiveness.

The annual load fraction supplied by solar energy, *F*, assumes a paramount role in the optimization precedure. While it is a complicated function of many variables, the functional dependence has been reduced to three key factors: the building location, the type of collector, and the ratio of the collecotr area to annual load, A/L. This has been accomplished by making several assumptions, mostly concerning the optimum values of many other design variables as determined from previous thermal and economic analyses. The one assumption employed here which is not standard practice is that the combined annual load distribution is independent of the magnitude of the annual load. The rationale behind this assumption has been given in Section 2.3. The annual solar load fraction is then a function of the ratio A/L, rather than A and L independently.

The consequence of invoking this assumption is that, for a particular location and collector type, the optimum system design is characterized by the value of A/L at which the derivative of *F* with respect to A/L is equal to the economic parameter, P_2C_A/P_1C_F. This information, tabulated in Appendix C, provides the user with a simple method for determining the optimum system design for any set of economic conditions, in 19 U.S. locations, for several collector types.

While this optimization method is applicable for any set of economic conditions which fulfill the basic assumption of cost proportionality, it is intuitive that there should be some set of economic conditions corresponding to the critical point at which the optimized solar heating system is just competitive with the conventional system. The criteria for this critical economic condition have been identified in Section 4.2 where it is shown that the critical condition is determined by the value of $C_E/C_A L$ for a particular location and collector type. This information, also tabulated in Appendix C, allows the user to make the a priori decision of whether any solar heating system design can compete with the conventional heating system. If the solar heating system is not currently competitive, the year in which it will become competitive can be estimated by a method shown in Section 4.2.

The economic evaluation and optimization methods presented here are based on a set of assumptions: load distribution assumptions, f-chart assumptions, cost modelling assumptions, etc. However, the more subtle, and probably most important, assumption is that the user can assign realistic and reliable values to economic variables. There is an inherent uncertainty in predicting future expenses and benefits. This uncertainty is magnified by the instability of current international energy affairs. In light of this dilemma, the results of both the life cycle cost analysis and the optimization procedures must by accepted with discretion, as they may be strongly influenced by these possible errors. The reliability of the the methods developed in this thesis rest on judicious choice of economic variable values and proper recognition of the potential impact of their uncertainty.

APPENDIX A Quantitative Economic Sensitivity Analysis

The life cycle cost analysis of a solar heating system is a complicated funtion of many economic variables. Since life cycle cost analysis requires the prediction of future expenses and benefits, many of the variables can not be assign exact values. The accuracy of the resulting life cycle savings of a solar heating system over a conventional heating system may be strongly influenced by the inherent uncertainties of the individual economic variable values. In light of this uncertainty, a life cycle cost analysis can not be viewed objectively without recognizing the sensitivity of the analysis results to the constituent economic variables.

Unfortunately, the effect of any one variable is largely determined by the values of other variables. Because of the complexity, "rule of thumb" sensitivity trends are not easily recognized. The interactions dictate the need for a method to quantitatively evaluate economic sensitivities for any set of economic conditions.

The method presented in this appendix is based on partial differentiation of P_1 and P_2 with respect to the economic variables. The model of (3-20) will be used for this purpose. By repeated application of the "chain rule" of elementary calculus, the sensitivity of P_1, P_2, and ultimately the life cycle savings, to the individual economic variables can be determined.

The chain rule has been employed to derive the following expressions.

$$\frac{\partial P_1}{\partial d} = (1 - C\bar{t}) \frac{\partial}{\partial d} f(N_E, \varepsilon, d) \qquad \frac{\partial P_1}{\partial \varepsilon} = (1 - C\bar{t}) \frac{\partial}{\partial \varepsilon} f(N_E, \varepsilon, d)$$

$$\frac{\partial P_1}{\partial N_E} = (1 - C\bar{t}) \frac{\partial}{\partial N_E} f(N_E, \varepsilon, d) \qquad \frac{\partial P_1}{\partial \bar{t}} = - C f(N_E, \varepsilon, d)$$

$$\frac{\partial P_2}{\partial D} = 1 - (1 - \bar{t}) \frac{f(N_1, 0, d)}{f(N_L, 0, i)} - \bar{t} f(N_1, i, d) \left[i - \frac{1}{f(N_L, 0, i} \right]$$

$$\frac{\partial P_2}{\partial M} = (1 - C\bar{t}) f(N_E, g\ d) \qquad \frac{\partial P_2}{\partial V} = t(1 - \bar{t}) f(N_E, g, d)$$

$$\frac{\partial P_2}{\partial G} = \frac{1}{(1 + d)^{N_E}} \qquad \frac{\partial P_2}{\partial t} = (1 - \bar{t}) V f(N_E, g, d)$$

$$\frac{\partial P_2}{\partial \bar{t}} = (D - 1) \left\{ \frac{f(N_1, 0, d)}{f(N_L, 0, i)} + f(N_1, i, d) \left[i - \frac{1}{f(N_L, 0, i)} \right] \right\}$$

$$- t V f(N_E, g, d) - C \left[M f(N_E, g, d) + U \right]$$

$$\frac{\partial P_2}{\partial d} = \frac{1 - D}{f(N_L, 0, i)} \frac{\partial}{\partial d} f(N_1, 0, d) + \left[(1 - C\bar{t}) M + t(1 - \bar{t}) V \right] \frac{\partial}{\partial d} f(N_E, g, d)$$

$$- (1 - D)\bar{t} \left\{ \frac{1}{f(N_L, 0, i)} \frac{\partial}{\partial d} f(N_1, 0, d) + \left[i - \frac{1}{f(N_L, 0, i)} \right] \frac{\partial}{\partial d} f(N_1, i, d) \right\}$$

$$+ \; N_E \frac{G}{(1+d)^{N_E+1}} - C\bar{t} \frac{\partial u}{\partial d}$$

$$\frac{\partial P_2}{\partial i} = (D-1)(1-\bar{t}) \frac{f(N_1,0,d)}{[f(N_L,0,i)]^2} \frac{\partial}{\partial i} f(N_L,0,i)$$

$$- \bar{t}(1-D) \left\{ \left[i - \frac{1}{f(N_L,0,i)} \right] \frac{\partial}{\partial i} f(N_1,i,d) + f(N_1,i,d) \left[1 + \frac{1}{\{f(N_L,0,i)\}^2} \frac{\partial}{\partial i} f(N_L,0,i) \right] \right\}$$

$$\frac{\partial P_2}{\partial q} = \left[(1-C\bar{t})M + (1-\bar{t})tV \right] \frac{\partial}{\partial q} f(N_E,q,d)$$

$$\frac{\partial P_2}{\partial N_L} = -(1-D) \left\{ (1-\bar{t}) \frac{f(N_1,0,d)}{[f(N_L,0,i)]^2} - \bar{t} \frac{f(N_1,i,d)}{[f(N_L,0,i)]^2} \right\} \frac{\partial}{\partial N_L} f(N_L,0,i)$$

$$\frac{\partial P_2}{\partial N_E} = \left[(1-C\bar{t})M + t(1-\bar{t})V \right] \frac{\partial}{\partial N_E} f(N_E,q,d) + \frac{G}{(1+d)^{N_E}} \ln(1+d)$$

$$\frac{\partial P_2}{\partial N_1} = \frac{(1-D)(1-\bar{t})}{f(N_L,0,i)} \frac{\partial}{\partial N_1} f(N_1,0,d) - (1-D)\bar{t} \left[i - \frac{1}{f(N_L,0,i)} \right] \frac{\partial}{\partial N_1} f(N_1,i,d)$$

Note that since N_1 is usually equal to either N_L or N_E, the expression for the partial derivative of P_2 with respect to N_1 will usually be added to either the expression for N_L or N_E. Most of the above equations involve the following.

$$\frac{\partial}{\partial a} f(a,b,c) = -\frac{1}{c-b} \left(\frac{1+b}{1+c} \right)^a \ln\left(\frac{1+b}{1+c} \right)$$

$$\frac{\partial}{\partial b} f(a,b,c) = \frac{1}{c-b} \left[f(a,b,c) - \frac{a}{1+b} \left(\frac{1+b}{1+c} \right)^a \right]$$

$$\frac{\partial}{\partial c} f(a,b,c) = \frac{1}{c-b} \left[\frac{a}{1+c} \left(\frac{1+b}{1+c} \right)^a - f(a,b,c) \right]$$

To illustrate the use of these relationships in a quantitative economic sensitivity analysis, consider the example of Table A-1. For a given set of economic variables, the values of P_1, P_2, and the life cycle savings, SAV, have been calculated. From the above equations, the partial derivatives of P_1, P_2, and SAV with respect to each variable have been evaluated. The table shows, for example, that an increase in the discount rate (variable 6) from 8% to 9% yields a decrease in the life cycle savings of approximately \$587.

The information of Table A-1 can also be used to estimate the maximum error in the life cycle savings due to uncertainties in the constituent economic variable values. For example, consider the following estimated uncertainties: $\Delta M = 0.005$, $\Delta d = 0.015$, $\Delta \varepsilon = 0.01$, $\Delta g = 0.01$. Then the maximum resulting error in the life cycle savings can be calculated by the following.

$$\Delta SAV = \left|\frac{\partial SAV}{\partial M}\Delta M\right| + \left|\frac{\partial SAV}{\partial d}\Delta d\right| + \left|\frac{\partial SAV}{\partial \varepsilon}\Delta \varepsilon\right| + \left|\frac{\partial SAV}{\partial g}\Delta g\right|$$

$$\Delta SAV = \$2903.60$$

TABLE A-1: SENSITIVITY ANALYSIS EXAMPLE

ECONOMIC VARIABLES

1.	NON-COMMERCIAL OR COMMERCIAL FLAG (0 OR 1) . . .	.000
2.	INITIAL INVESTMENT	8160.000
3.	RATIO OF DOWN PAYMENT TO INITIAL INVESTMENT . .	.100
4.	RATIO OF 1ST YEAR MISC. COSTS TO INIT. INV. . .	.010
5.	RATIO OF 1ST YEAR ASSESSED VALUE TO INIT. INV. .	.700
6.	ANNUAL MARKET DISCOUNT RATE	.080
7.	ANNUAL MARKET RATE OF FUEL PRICE INCREASE . . .	.100
8.	ANNUAL INTEREST RATE ON MORTGAGE	.090
9.	ANNUAL RATE OF GENERAL INFLATION	.060
10.	PROPERTY TAX RATE	.020
11.	EFFECTIVE INCOME TAX RATE	.300
12.	DURATION OF ECONOMIC ANALYSIS (YEARS)	20.000
13.	TERM OF MORTGAGE (YEARS)	20.000
14.	DEPRECIATION LIFETIME (YEARS)	20.000
15.	COST OF CONVENTIONAL FUEL IN 1ST YEAR ($/GJ) . .	9.900
16.	ANNUAL HEATING AND HOT WATER LOAD (GJ)	132.500
17.	ANNUAL LOAD FRACTION SUPPLIED BY SOLAR	.440
18.	RATIO OF SALVAGE VALUE TO INIT. INV.	.000

P1	22.1686
P2	1.1932
SAVINGS. .	3058.44

DERIVATIVES

	P1	P2	SAV
1	.00000	.00000	.00000
2	.00000	.00000	.11932+01
3	.00000	.12842+00	-.10479+04
4	.00000	.15596+02	-.12726+06
5	.00000	.21834+00	-.17817+04
6	-.22802+03	-.89362+01	-.58687+05
7	.20373+03	.00000	.11759+06
8	.00000	.45908+01	-.37461+05
9	.00000	.25869+01	-.21109+05
10	.00000	.76419+01	-.62358+05
11	.00000	-.83024+00	.67748+04
12	.13242+01	.12733-01	.66039+03
13	.00000	-.37943-02	.30961+02
14	.00000	.00000	.00000
15	.00000	.00000	.12925+04
16	.00000	.00000	.96568+02
17	.00000	.00000	.29080+05
18	.00000	-.21455+00	.17507+04

APPENDIX B ECONOMIC TABLES

TABLE B-1 Discount-inflation factor

NUMBER OF YEARS = 5 YEARS

MARKET DISCOUNT RATE (%)	ANNUAL INFLATION RATE (%) 0	1	2	3	4	5	6	7	8	9	10	11	12
0	5.000	5.101	5.204	5.309	5.416	5.526	5.637	5.751	5.867	5.985	6.105	6.228	6.353
1	4.853	4.950	5.049	5.150	5.253	5.358	5.466	5.575	5.686	5.799	5.915	6.033	6.153
2	4.713	4.807	4.902	4.999	5.098	5.199	5.302	5.407	5.514	5.623	5.734	5.847	5.962
3	4.580	4.669	4.761	4.854	4.950	5.047	5.146	5.246	5.349	5.454	5.561	5.669	5.780
4	4.452	4.538	4.626	4.716	4.808	4.901	4.996	5.093	5.192	5.293	5.395	5.500	5.606
5	4.329	4.413	4.497	4.584	4.672	4.762	4.853	4.947	5.042	5.139	5.238	5.338	5.441
6	4.212	4.292	4.374	4.457	4.542	4.629	4.717	4.807	4.898	4.992	5.087	5.183	5.282
7	4.100	4.177	4.256	4.336	4.418	4.501	4.586	4.673	4.761	4.851	4.942	5.036	5.131
8	3.993	4.067	4.143	4.220	4.299	4.379	4.461	4.545	4.630	4.716	4.804	4.894	4.986
9	3.890	3.961	4.035	4.109	4.185	4.263	4.342	4.422	4.504	4.587	4.672	4.759	4.847
10	3.791	3.860	3.931	4.003	4.076	4.151	4.227	4.304	4.383	4.464	4.545	4.629	4.714
11	3.696	3.763	3.831	3.900	3.971	4.043	4.117	4.191	4.268	4.345	4.424	4.505	4.586
12	3.605	3.669	3.735	3.802	3.870	3.940	4.011	4.083	4.157	4.231	4.308	4.385	4.464
13	3.517	3.580	3.643	3.708	3.774	3.841	3.909	3.979	4.050	4.122	4.196	4.271	4.347
14	3.433	3.493	3.555	3.617	3.681	3.746	3.812	3.879	3.948	4.018	4.089	4.161	4.235
15	3.352	3.410	3.470	3.530	3.592	3.655	3.719	3.784	3.850	3.917	3.986	4.056	4.127
16	3.274	3.331	3.388	3.447	3.506	3.567	3.629	3.691	3.755	3.821	3.887	3.954	4.023
17	3.199	3.254	3.309	3.366	3.424	3.482	3.542	3.603	3.665	3.728	3.792	3.857	3.924
18	3.127	3.180	3.234	3.288	3.344	3.401	3.459	3.518	3.577	3.638	3.700	3.764	3.828
19	3.058	3.109	3.161	3.214	3.268	3.323	3.379	3.436	3.493	3.552	3.612	3.673	3.736
20	2.991	3.040	3.091	3.142	3.194	3.247	3.301	3.357	3.413	3.470	3.528	3.587	3.647

TABLE B-1 Discount-inflation factor

NUMBER OF YEARS = 10 YEARS

MARKET DISCOUNT RATE (%)	ANNUAL INFLATION RATE (%) 0	1	2	3	4	5	6	7	8	9	10	11	12
0	10.000	10.462	10.950	11.464	12.006	12.578	13.181	13.816	14.487	15.193	15.937	16.722	17.549
1	9.471	9.901	10.354	10.831	11.335	11.865	12.425	13.014	13.635	14.289	14.979	15.705	16.470
2	8.983	9.383	9.804	10.248	10.716	11.209	11.728	12.275	12.851	13.458	14.097	14.770	15.479
3	8.530	8.903	9.295	9.709	10.144	10.603	11.085	11.594	12.129	12.692	13.286	13.910	14.567
4	8.111	8.459	8.825	9.210	9.615	10.042	10.492	10.965	11.462	11.986	12.537	13.117	13.727
5	7.722	8.046	8.388	8.748	9.126	9.524	9.942	10.383	10.846	11.334	11.847	12.386	12.953
6	7.360	7.664	7.983	8.319	8.672	9.043	9.434	9.845	10.277	10.731	11.208	11.710	12.238
7	7.024	7.308	7.607	7.921	8.251	8.598	8.962	9.346	9.749	10.172	10.618	11.085	11.577
8	6.710	6.976	7.256	7.550	7.859	8.184	8.525	8.883	9.259	9.655	10.070	10.507	10.965
9	6.418	6.667	6.930	7.205	7.495	7.798	8.118	8.453	8.805	9.174	9.562	9.970	10.398
10	6.145	6.379	6.625	6.884	7.155	7.440	7.739	8.053	8.382	8.728	9.091	9.472	9.872
11	5.889	6.110	6.341	6.584	6.838	7.105	7.386	7.680	7.989	8.313	8.652	9.009	9.383
12	5.650	5.858	6.075	6.303	6.543	6.793	7.057	7.333	7.622	7.926	8.244	8.578	8.929
13	5.426	5.622	5.826	6.041	6.266	6.502	6.749	7.008	7.280	7.565	7.864	8.177	8.505
14	5.216	5.400	5.593	5.795	6.007	6.229	6.462	6.705	6.961	7.228	7.509	7.803	8.111
15	5.019	5.193	5.374	5.565	5.765	5.974	6.193	6.422	6.662	6.914	7.177	7.453	7.743
16	4.833	4.997	5.169	5.349	5.537	5.734	5.940	6.156	6.383	6.619	6.867	7.127	7.399
17	4.659	4.814	4.976	5.146	5.323	5.509	5.704	5.908	6.121	6.344	6.577	6.822	7.077
18	4.494	4.641	4.794	4.955	5.123	5.298	5.482	5.674	5.875	6.085	6.305	6.536	6.776
19	4.339	4.478	4.623	4.775	4.934	5.100	5.273	5.455	5.644	5.843	6.050	6.267	6.494
20	4.192	4.324	4.462	4.606	4.756	4.913	5.077	5.248	5.428	5.615	5.811	6.016	6.230

TABLE B-1 Discount-inflation factor

NUMBER OF YEARS = 15 YEARS

MARKET DISCOUNT RATE (%)	ANNUAL INFLATION RATE (%) 0	1	2	3	4	5	6	7	8	9	10	11	12
0	15.000	16.097	17.293	18.599	20.024	21.579	23.276	25.129	27.152	29.361	31.772	34.405	37.280
1	13.865	14.851	15.926	17.098	18.375	19.767	21.285	22.942	24.748	26.718	28.867	31.212	33.770
2	12.849	13.738	14.706	15.759	16.906	18.156	19.517	21.000	22.616	24.377	26.297	28.389	30.669
3	11.938	12.741	13.614	14.563	15.596	16.719	17.942	19.273	20.722	22.300	24.017	25.888	27.925
4	11.118	11.845	12.634	13.492	14.423	15.435	16.536	17.733	19.035	20.451	21.991	23.667	25.491
5	10.380	11.039	11.754	12.530	13.372	14.286	15.279	16.357	17.529	18.802	20.187	21.691	23.327
6	9.712	10.311	10.960	11.664	12.426	13.254	14.151	15.125	16.182	17.329	18.575	19.929	21.399
7	9.108	9.654	10.244	10.883	11.575	12.325	13.138	14.019	14.974	16.010	17.134	18.354	19.677
8	8.559	9.057	9.595	10.177	10.807	11.488	12.225	13.024	13.889	14.826	15.842	16.943	18.137
9	8.061	8.516	9.007	9.538	10.111	10.731	11.402	12.127	12.912	13.761	14.681	15.678	16.757
10	7.606	8.023	8.473	8.958	9.481	10.046	10.657	11.317	12.030	12.802	13.636	14.539	15.516
11	7.191	7.574	7.986	8.430	8.909	9.425	9.982	10.584	11.233	11.935	12.694	13.514	14.400
12	6.811	7.163	7.541	7.949	8.387	8.860	9.369	9.919	10.511	11.151	11.842	12.587	13.393
13	6.462	6.786	7.135	7.509	7.912	8.345	8.812	9.314	9.856	10.440	11.070	11.749	12.483
14	6.142	6.441	6.762	7.107	7.477	7.875	8.303	8.764	9.260	9.794	10.370	10.990	11.659
15	5.847	6.124	6.420	6.738	7.079	7.445	7.839	8.262	8.717	9.206	9.733	10.300	10.911
16	5.575	5.831	6.105	6.399	6.714	7.051	7.413	7.803	8.220	8.670	9.153	9.672	10.231
17	5.324	5.561	5.815	6.087	6.378	6.689	7.024	7.382	7.767	8.180	8.623	9.100	9.612
18	5.092	5.312	5.547	5.799	6.069	6.357	6.665	6.996	7.351	7.731	8.139	8.577	9.048
19	4.876	5.081	5.300	5.533	5.783	6.050	6.336	6.641	6.969	7.320	7.696	8.099	8.532
20	4.675	4.867	5.070	5.288	5.519	5.767	6.032	6.315	6.618	6.942	7.289	7.661	8.059

TABLE B-1 Discount-inflation factor

NUMBER OF YEARS = 20 YEARS

MARKET DISCOUNT RATE (%)	ANNUAL INFLATION RATE (%)												
	0	1	2	3	4	5	6	7	8	9	10	11	12
0	20.000	22.019	24.297	26.870	29.778	33.066	36.786	40.995	45.762	51.160	57.275	64.203	72.052
1	18.046	19.802	21.780	24.009	26.524	29.362	32.568	36.190	40.284	44.913	50.150	56.074	62.778
2	16.351	17.885	19.608	21.546	23.728	26.186	28.958	32.084	35.612	39.594	44.093	49.174	54.917
3	14.877	16.221	17.727	19.417	21.317	23.453	25.857	28.564	31.613	35.050	38.926	43.299	48.232
4	13.590	14.771	16.092	17.571	19.231	21.093	23.185	25.536	28.180	31.156	34.506	38.279	42.531
5	12.462	13.503	14.665	15.965	17.419	19.048	20.874	22.922	25.222	27.806	30.710	33.977	37.651
6	11.470	12.391	13.417	14.562	15.840	17.269	18.868	20.659	22.665	24.916	27.442	30.277	33.463
7	10.594	11.411	12.320	13.332	14.459	15.717	17.122	18.692	20.448	22.414	24.617	27.086	29.856
8	9.818	10.546	11.353	12.250	13.247	14.358	15.596	16.977	18.519	20.242	22.169	24.325	26.740
9	9.129	9.779	10.498	11.296	12.181	13.164	14.258	15.476	16.834	18.349	20.039	21.928	24.040
10	8.514	9.096	9.739	10.450	11.238	12.112	13.082	14.160	15.359	16.694	18.182	19.841	21.693
11	7.963	8.487	9.063	9.700	10.403	11.182	12.044	13.001	14.063	15.243	16.556	18.018	19.647
12	7.469	7.941	8.460	9.031	9.661	10.356	11.125	11.977	12.920	13.967	15.129	16.421	17.857
13	7.025	7.451	7.919	8.433	8.998	9.622	10.310	11.070	11.910	12.841	13.872	15.017	16.287
14	6.623	7.009	7.432	7.896	8.406	8.966	9.583	10.263	11.014	11.844	12.762	13.779	14.906
15	6.259	6.610	6.994	7.414	7.874	8.379	8.934	9.545	10.217	10.959	11.779	12.685	13.687
16	5.929	6.249	6.597	6.978	7.395	7.851	8.352	8.902	9.506	10.172	10.905	11.714	12.608
17	5.628	5.920	6.238	6.585	6.963	7.376	7.829	8.325	8.870	9.468	10.126	10.851	11.650
18	5.353	5.620	5.911	6.227	6.572	6.947	7.358	7.807	8.298	8.838	9.430	10.081	10.798
19	5.101	5.347	5.613	5.902	6.216	6.558	6.932	7.339	7.784	8.272	8.806	9.392	10.036
20	4.870	5.096	5.340	5.605	5.893	6.205	6.545	6.916	7.320	7.762	8.245	8.774	9.355

TABLE B-1 Discount-inflation factor

NUMBER OF YEARS = 25 YEARS

MARKET DISCOUNT RATE (%)	ANNUAL INFLATION RATE (%) 0	1	2	3	4	5	6	7	8	9	10	11	12
0	25.000	28.243	32.030	36.459	41.646	47.727	54.864	63.249	73.106	84.701	98.347	114.413	133.334
1	22.023	24.752	27.929	31.633	35.958	41.014	46.933	53.869	62.003	71.550	82.762	95.935	111.419
2	19.523	21.832	24.510	27.622	31.245	35.470	40.401	46.164	52.906	60.800	70.051	80.897	93.621
3	17.413	19.375	21.644	24.272	27.322	30.867	34.994	39.804	45.417	51.974	59.639	68.606	79.104
4	15.622	17.298	19.229	21.459	24.038	27.028	30.498	34.531	39.224	44.693	51.071	58.516	67.213
5	14.094	15.532	17.184	19.085	21.277	23.810	26.740	30.137	34.079	38.660	43.990	50.197	57.431
6	12.783	14.024	15.444	17.072	18.943	21.098	23.585	26.458	29.784	33.639	38.112	43.308	49.350
7	11.654	12.729	13.954	15.356	16.961	18.803	20.923	23.364	26.183	29.440	33.210	37.578	42.645
8	10.675	11.611	12.674	13.885	15.269	16.851	18.666	20.750	23.148	25.912	29.103	32.791	37.058
9	9.823	10.641	11.568	12.620	13.817	15.182	16.743	18.530	20.580	22.936	25.648	28.774	32.382
10	9.077	9.796	10.607	11.525	12.566	13.749	15.097	16.636	18.396	20.412	22.727	25.388	28.452
11	8.422	9.056	9.769	10.574	11.482	12.512	13.682	15.012	16.530	18.264	20.248	22.523	25.134
12	7.843	8.405	9.035	9.743	10.540	11.440	12.459	13.615	14.929	16.425	18.133	20.086	22.321
13	7.330	7.830	8.388	9.014	9.716	10.506	11.398	12.406	13.548	14.846	16.322	18.005	19.926
14	6.873	7.320	7.817	8.372	8.993	9.689	10.473	11.356	12.353	13.483	14.764	16.220	17.878
15	6.464	6.865	7.309	7.803	8.355	8.971	9.662	10.439	11.314	12.301	13.417	14.683	16.119
16	6.097	6.457	6.856	7.298	7.790	8.338	8.950	9.636	10.406	11.272	12.249	13.353	14.602
17	5.766	6.092	6.451	6.848	7.288	7.776	8.321	8.929	9.609	10.372	11.230	12.197	13.288
18	5.467	5.762	6.086	6.444	6.839	7.277	7.763	8.304	8.907	9.582	10.339	11.189	12.146
19	5.195	5.463	5.758	6.081	6.437	6.830	7.266	7.749	8.286	8.886	9.556	10.306	11.148
20	4.948	5.192	5.460	5.753	6.075	6.430	6.822	7.255	7.735	8.269	8.864	9.529	10.272

TABLE B-1 Discount-inflation factor

NUMBER OF YEARS = 30 YEARS

MARKET DISCOUNT RATE (%)	ANNUAL INFLATION RATE (%) 0	1	2	3	4	5	6	7	8	9	10	11	12
0	30.000	34.785	40.568	47.575	56.085	66.439	79.058	94.461	113.283	136.307	164.494	199.021	241.333
1	25.808	29.703	34.389	40.042	46.878	55.164	65.225	77.462	92.367	110.545	132.735	159.843	192.981
2	22.396	25.589	29.412	34.002	39.529	46.201	54.270	64.050	75.922	90.353	107.916	129.313	155.400
3	19.600	22.235	25.374	29.126	33.624	39.029	45.541	53.404	62.914	74.435	88.413	105.392	126.034
4	17.292	19.481	22.076	25.163	28.846	33.254	38.541	44.900	52.563	61.813	73.000	86.545	102.965
5	15.372	17.203	19.363	21.919	24.955	28.571	32.891	38.065	44.276	51.746	60.748	71.613	84.744
6	13.765	15.307	17.116	19.246	21.765	24.751	28.302	32.537	37.601	43.668	50.953	59.716	70.272
7	12.409	13.716	15.241	17.028	19.131	21.612	24.549	28.037	32.190	37.147	43.076	50.182	58.715
8	11.258	12.372	13.667	15.176	16.942	19.017	21.461	24.351	27.778	31.851	36.704	42.499	49.433
9	10.274	11.230	12.335	13.618	15.111	16.856	18.904	21.313	24.157	27.523	31.518	36.271	41.937
10	9.427	10.253	11.202	12.299	13.569	15.046	16.771	18.792	21.166	23.965	27.273	31.192	35.848
11	8.694	9.411	10.232	11.175	12.262	13.520	14.982	16.687	18.681	21.022	23.776	27.027	30.873
12	8.055	8.682	9.395	10.211	11.147	12.225	13.472	14.918	16.603	18.572	20.879	23.590	26.786
13	7.496	8.046	8.670	9.379	10.190	11.119	12.188	13.423	14.855	16.520	18.464	20.738	23.407
14	7.003	7.489	8.037	8.658	9.363	10.169	11.091	12.151	13.375	14.792	16.438	18.356	20.599
15	6.566	6.997	7.482	8.028	8.646	9.347	10.147	11.063	12.115	13.327	14.729	16.356	18.250
16	6.177	6.562	6.992	7.475	8.019	8.633	9.331	10.126	11.035	12.078	13.279	14.667	16.275
17	5.829	6.174	6.558	6.987	7.468	8.009	8.621	9.315	10.104	11.007	12.041	13.231	14.605
18	5.517	5.827	6.171	6.554	6.981	7.460	7.999	8.608	9.298	10.083	10.979	12.005	13.184
19	5.235	5.515	5.825	6.168	6.550	6.976	7.453	7.990	8.596	9.282	10.061	10.951	11.968
20	4.979	5.233	5.513	5.822	6.165	6.545	6.970	7.446	7.980	8.583	9.265	10.040	10.922

TABLE B-2

SUM OF DIGITS
DEPRECIATION FACTOR

MARKET DISCOUNT RATE %	YEARS OF DEPRECIATION 5	10	15	20	25	30
0	5.0000	10.0000	15.0000	20.0000	25.0000	30.0000
1	4.8856	9.6126	14.1868	18.6138	22.8988	27.0470
2	4.7757	9.2492	13.4421	17.3741	21.0636	24.5276
3	4.6699	8.9079	12.7586	16.2620	19.4535	22.3646
4	4.5681	8.5868	12.1300	15.2611	18.0345	20.4967
5	4.4702	8.2846	11.5509	14.3577	16.7785	18.8743
6	4.3758	7.9997	11.0161	13.5398	15.6624	17.4572
7	4.2848	7.7310	10.5216	12.7973	14.6664	16.2129
8	4.1970	7.4771	10.0633	12.1213	13.7743	15.1147
9	4.1124	7.2371	9.6379	11.5042	12.9722	14.1407
10	4.0307	7.0099	9.2424	10.9395	12.2484	13.2730
11	3.9518	6.7947	8.8740	10.4214	11.5932	12.4963
12	3.8756	6.5906	8.5303	9.9449	10.9980	11.7983
13	3.8020	6.3969	8.2093	9.5057	10.4556	11.1684
14	3.7308	6.2128	7.9088	9.0999	9.9599	10.5978
15	3.6619	6.0379	7.6272	8.7242	9.5056	10.0791
16	3.5952	5.8713	7.3629	8.3757	9.0879	9.6060
17	3.5307	5.7127	7.1146	8.0517	8.7031	9.1729
18	3.4682	5.5615	6.8808	7.7499	8.3475	8.7753
19	3.4077	5.4173	6.6606	7.4682	8.0182	8.4093
20	3.3490	5.2796	6.4528	7.2050	7.7125	8.0713

TABLE B-3

DECLINING BALANCE
DEPRECIATION FACTOR

MARKET DISCOUNT RATE %	YEARS OF DEPRECIATION 5	10	15	20	25	30
0	5.0000	10.0000	15.0000	20.0000	25.0000	30.0000
1	4.8871	9.5701	14.0589	18.3630	22.4916	26.4534
2	4.7787	9.1710	13.2134	16.9394	20.3790	23.5593
3	4.6746	8.7999	12.4516	15.6953	18.5868	21.1738
4	4.5745	8.4542	11.7631	14.6028	17.0554	19.1878
5	4.4783	8.1318	11.1391	13.6388	15.7377	17.5183
6	4.3857	7.8307	10.5719	12.7843	14.5962	16.1017
7	4.2966	7.5489	10.0548	12.0234	13.6007	14.8889
8	4.2108	7.2850	9.5823	11.3430	12.7270	13.8416
9	4.1280	7.0373	9.1492	10.7318	11.9556	12.9299
10	4.0483	6.8047	8.7513	10.1807	11.2705	12.1302
11	3.9713	6.5858	8.3848	9.6818	10.6588	11.4237
12	3.8971	6.3796	8.0465	9.2284	10.1097	10.7955
13	3.8253	6.1852	7.7333	8.8149	9.6145	10.2335
14	3.7561	6.0016	7.4429	8.4365	9.1657	9.7278
15	3.6891	5.8280	7.1729	8.0891	8.7572	9.2704
16	3.6243	5.6637	6.9214	7.7692	8.3840	8.8547
17	3.5617	5.5080	6.6866	7.4737	8.0417	8.4752
18	3.5010	5.3603	6.4671	7.1999	7.7267	8.1274
19	3.4423	5.2201	6.2614	6.9457	7.4357	7.8074
20	3.3854	5.0867	6.0683	6.7089	7.1661	7.5120

APPENDIX C OPTIMIZATION TABLES

TABLE C-1 Load calculation data for combined systems.

Location	Water Mains Temperature	Annual Degree Days	Building UA	Heating Loads Water	Heating Loads Space
Albequerque, NM	13.	2384.	328.	21.6	67.6
Atlanta, GA	18.	1719.	339.	19.3	50.3
Bismark, ND	6.	5024.	281.	24.8	122.0
Boston, MA	9.	3130.	314.	23.4	84.9
Boulder, CO	9.	3078.	315.	23.4	83.8
Cleveland, OH	11.	3419.	309.	22.5	91.4
Columbia, MO	13.	2803.	320.	21.6	77.5
Dallas, TX	19.	1272.	347.	18.8	38.1
Great Falls, MT	7.	4306.	294.	24.3	109.4
Lander, WY	6.	4372.	293.	24.8	110.7
Lincoln, NB	12.	3259.	312.	22.0	87.9
Los Angeles, CA	19.	1011.	352.	18.8	30.7
Madison, WI	9.	4294.	294.	23.4	109.1
Miami, FL	25.	119.	368.	16.3	3.8
Nashville, TN	16.	2053.	333.	20.2	59.1
New York, NY	11.	2673.	323.	22.5	74.6
Phoenix, AZ	17.	862.	354.	19.7	26.4
Reno, NV	11.	3345.	311.	22.5	89.9
Washington, DC	14.	2347.	328.	21.1	66.5

Temperatures in C

Degree days in C-days

UAs in W- C^{-1}

Heating loads in GJ

TABLE C-2: WATER HEATING OPTIMIZATION TABLES, FR'[TAU*ALPHA] = .70   FR'UL = 3.00 W/M**2 C

TABULATED: (P2*CA)/(P1*CF), CRITICAL

CE/(CA*L) [M**2/GJ]

LOCATION		.00	.02	.05	.10	.30	.50	.80
ALBUQUERQUE	NM	5.585	3.689	3.230	2.675	1.585	1.127	.785
ATLANTA	GA	3.691	2.650	2.320	1.921	1.139	.985	.734
BISMARCK	ND	4.092	2.910	2.547	2.110	1.250	.888	.619
BOSTON	MA	2.982	2.231	1.954	1.618	1.126	.873	.667
BOULDER	CO	3.855	2.782	2.435	2.017	1.195	.849	.775
CLEVELAND	OH	3.102	2.276	1.993	1.650	.978	.695	.649
COLUMBIA	MO	3.753	2.694	2.359	1.953	1.158	.823	.735
DALLAS	TX	3.883	2.765	2.421	2.005	1.188	.844	.752
GREAT FALLS	MT	4.048	2.878	2.520	2.087	1.237	.879	.613
LANDER	WY	5.112	3.481	3.048	2.524	1.496	1.063	.741
LINCOLN	NE	3.759	2.708	2.371	1.963	1.164	.827	.750
LOS ANGELES	CA	4.429	3.063	2.682	2.221	1.316	.935	.652
MADISON	WI	3.437	2.509	2.196	1.819	1.078	.766	.709
MIAMI	FL	4.193	2.932	2.567	2.126	1.260	.895	.795
NASHVILLE	TN	3.430	2.485	2.176	1.802	1.068	.759	.700
NEW YORK	NY	2.732	2.064	1.807	1.496	1.060	.834	.638
PHOENIX	AZ	5.458	3.629	3.177	2.631	1.559	1.108	.773
RENO	NV	5.200	3.487	3.053	2.529	1.499	1.065	.742
WASHINGTON	DC	3.223	2.372	2.077	1.720	1.019	.912	.689

TABULATED: (PC*CA)/(P1*CF) [M**2/GJ]
AND LOAD FRACTION

A/L [M**2/GJ]

LOCATION		.00	.10	.20	.30	.40	.60	.80
ALBUQUERQUE	NM	5.585 .00	3.965 .48	2.635 .80	.511 .97	.079 1.00	.000 1.00	.000 1.00
ATLANTA	GA	3.691 .00	2.877 .33	2.169 .58	1.566 .76	.975 .89	.167 .96	.057 .98
BISMARCK	ND	4.092 .00	3.155 .36	2.347 .64	1.508 .83	.530 .93	.136 .97	.038 .99
BOSTON	MA	2.982 .00	2.432 .27	1.942 .49	1.512 .66	1.032 .79	.310 .90	.131 .94
BOULDER	CO	3.855 .00	3.022 .34	2.294 .61	1.672 .81	.676 .94	.088 .99	.000 1.00
CLEVELAND	OH	3.102 .00	2.476 .28	1.926 .50	1.453 .67	.664 .78	.242 .87	.144 .90
COLUMBIA	MO	3.753 .00	2.924 .33	2.205 .59	1.594 .78	.620 .89	.180 .95	.067 .98
DALLAS	TX	3.883 .00	2.999 .34	2.235 .60	1.590 .79	.704 .91	.161 .97	.039 .99
GREAT FALLS	MT	4.048 .00	3.121 .36	2.321 .63	1.494 .82	.547 .92	.150 .97	.050 .99
LANDER	WY	5.112 .00	3.756 .44	2.620 .76	.681 .96	.177 .99	.000 1.00	.000 1.00
LINCOLN	NE	3.759 .00	2.941 .33	2.227 .59	1.618 .78	.629 .90	.166 .97	.037 .99
LOS ANGELES	CA	4.429 .00	3.312 .39	2.363 .67	1.584 .86	.431 .96	.072 1.00	.000 1.00
MADISON	WI	3.437 .00	2.728 .31	2.105 .55	1.570 .73	.720 .85	.196 .93	.104 .96
MIAMI	FL	4.193 .00	3.174 .37	2.301 .64	1.575 .83	.845 .96	.000 1.00	.000 1.00
NASHVILLE	TN	3.430 .00	2.700 .31	2.063 .54	1.519 .72	.690 .84	.235 .92	.110 .95
NEW YORK	NY	2.732 .00	2.252 .25	1.821 .45	1.440 .61	1.109 .74	.305 .87	.139 .92
PHOENIX	AZ	5.458 .00	3.903 .47	2.622 .79	.521 .96	.171 .99	.000 1.00	.000 1.00
RENO	NV	5.200 .00	3.755 .45	2.558 .76	.651 .94	.179 .98	.000 1.00	.000 1.00
WASHINGTON	DC	3.223 .00	2.582 .29	2.015 .52	1.524 .70	.912 .82	.235 .93	.106 .96

TABLE C-2: WATER HEATING OPTIMIZATION TABLES, FR′[TAU*ALPHA] = .60 FR′UL = 4.00 W/M**2 C

		TABULATED: (P2*CA)/(P1*CF), CRITICAL							TABULATED: (PC*CA)/(P1*CF) [M**2/GJ] AND LOAD FRACTION						
		CE/(CA*L) [M**2/GJ]							A/L [M**2/GJ]						
LOCATION		.00	.02	.05	.10	.30	.50	.80	.00	.10	.20	.30	.40	.60	.80
ALBUQUERQUE	NM	4.459	3.058	2.677	2.217	1.314	.934	.651	4.459	3.302	2.325	1.528	.367	.053	.000
									.00	.39	.67	.86	.96	.99	1.00
ATLANTA	GA	2.801	2.068	1.811	1.500	1.031	.810	.619	2.801	2.251	1.767	1.347	.993	.480	.186
									.00	.25	.45	.61	.72	.87	.92
BISMARCK	ND	3.210	2.349	2.057	1.703	1.009	.900	.681	3.210	2.555	1.980	1.483	.964	.249	.111
									.00	.29	.51	.69	.81	.92	.95
BOSTON	MA	2.265	1.734	1.518	1.257	.907	.720	.550	2.265	1.894	1.559	1.262	1.001	.538	.231
									.00	.21	.38	.52	.63	.79	.85
BOULDER	CO	3.010	2.230	1.953	1.617	1.116	.877	.670	3.010	2.429	1.915	1.465	1.081	.330	.092
									.00	.27	.49	.66	.78	.93	.96
CLEVELAND	OH	2.340	1.759	1.540	1.276	.900	.711	.544	2.340	1.918	1.544	1.217	.937	.402	.236
									.00	.21	.39	.52	.63	.76	.82
COLUMBIA	MO	2.888	2.131	1.866	1.545	1.061	.833	.637	2.888	2.319	1.818	1.385	1.018	.350	.170
									.00	.26	.47	.63	.75	.87	.92
DALLAS	TX	2.975	2.180	1.909	1.581	.937	.844	.645	2.975	2.372	1.842	1.385	1.003	.333	.165
									.00	.27	.48	.64	.76	.88	.93
GREAT FALLS	MT	3.179	2.324	2.035	1.685	.999	.888	.672	3.179	2.528	1.957	1.465	.854	.292	.114
									.00	.28	.51	.68	.80	.91	.94
LANDER	WY	4.098	2.887	2.528	2.093	1.241	.882	.615	4.098	3.128	2.294	1.596	.656	.098	.014
									.00	.36	.63	.82	.94	.99	1.00
LINCOLN	NE	2.895	2.143	1.876	1.554	1.072	.842	.644	2.895	2.334	1.837	1.406	1.039	.344	.155
									.00	.26	.47	.63	.75	.89	.93
LOS ANGELES	CA	3.423	2.443	2.139	1.772	1.050	.912	.692	3.423	2.651	1.983	1.419	.960	.291	.088
									.00	.30	.53	.70	.82	.94	.97
MADISON	WI	2.631	1.970	1.725	1.428	1.001	.790	.603	2.631	2.147	1.718	1.342	1.019	.372	.204
									.00	.24	.43	.58	.70	.84	.89
MIAMI	FL	3.202	2.309	2.022	1.675	.992	.874	.665	3.202	2.508	1.904	1.391	.967	.390	.122
									.00	.28	.50	.67	.79	.92	.97
NASHVILLE	TN	2.595	1.931	1.690	1.400	.974	.767	.586	2.595	2.103	1.669	1.291	.971	.392	.213
									.00	.23	.42	.57	.68	.82	.88
NEW YORK	NY	2.039	1.576	1.379	1.142	.835	.664	.508	2.039	1.722	1.435	1.179	.953	.592	.285
									.00	.19	.35	.48	.58	.73	.82
PHOENIX	AZ	4.361	3.007	2.632	2.180	1.292	.918	.640	4.361	3.249	2.308	1.536	.382	.067	.028
									.00	.38	.66	.85	.95	.99	.99
RENO	NV	4.129	2.868	2.511	2.080	1.233	.876	.758	4.129	3.102	2.229	1.508	.624	.133	.039
									.00	.36	.63	.81	.92	.97	.99
WASHINGTON	DC	2.432	1.835	1.607	1.331	.943	.745	.569	2.432	2.002	1.618	1.281	.990	.467	.253
									.00	.22	.40	.55	.66	.81	.87

TABLE C-2: WATER HEATING OPTIMIZATION TABLES, FR′[TAU*ALPHA] = .60   FR′UL = 2.00 W/M**2 C

TABULATED: (P2*CA)/(P1*CF), CRITICAL

CE/(CA*L) [M**2/GJ]

LOCATION		.00	.02	.05	.10	.30	.50	.80
ALBUQUERQUE	NM	4.883	3.400	2.977	2.465	1.461	1.038	.724
ATLANTA	GA	3.272	2.444	2.140	1.772	1.235	.955	.730
BISMARCK	ND	3.595	2.662	2.330	1.930	1.144	.813	.759
BOSTON	MA	2.641	2.042	1.788	1.480	1.081	.856	.654
BOULDER	CO	3.389	2.541	2.224	1.842	1.282	.986	.754
CLEVELAND	OH	2.755	2.096	1.835	1.519	1.077	.834	.637
COLUMBIA	MO	3.314	2.475	2.167	1.794	1.063	.959	.719
DALLAS	TX	3.432	2.547	2.230	1.846	1.094	.981	.737
GREAT FALLS	MT	3.555	2.632	2.305	1.909	1.131	.804	.747
LANDER	WY	4.462	3.186	2.789	2.310	1.369	.973	.678
LINCOLN	NE	3.319	2.486	2.176	1.802	1.068	.978	.734
LOS ANGELES	CA	3.908	2.831	2.478	2.052	1.216	.864	.786
MADISON	WI	3.040	2.302	2.015	1.669	1.174	.910	.696
MIAMI	FL	3.711	2.713	2.375	1.967	1.166	1.037	.778
NASHVILLE	TN	3.043	2.291	2.006	1.661	1.164	.900	.687
NEW YORK	NY	2.431	1.895	1.659	1.374	1.014	.807	.617
PHOENIX	AZ	4.770	3.339	2.923	2.421	1.435	1.019	.711
RENO	NV	4.554	3.212	2.812	2.329	1.380	.981	.684
WASHINGTON	DC	2.861	2.182	1.910	1.582	1.133	.886	.677

TABULATED: (PC*CA)/(P1*CF) [M**2/GJ]
AND LOAD FRACTION

A/L [M**2/GJ]

LOCATION		.00	.10	.20	.30	.40	.60	.80
ALBUQUERQUE	NM	4.883	3.678	2.654	1.041	.100	.000	.000
		.00	.43	.74	.96	1.00	1.00	1.00
ATLANTA	GA	3.272	2.664	2.122	1.645	1.233	.219	.063
		.00	.30	.54	.72	.87	.97	.99
BISMARCK	ND	3.595	2.899	2.282	1.745	.711	.175	.035
		.00	.32	.58	.78	.91	.97	.99
BOSTON	MA	2.641	2.232	1.860	1.524	1.225	.425	.161
		.00	.24	.45	.62	.75	.90	.94
BOULDER	CO	3.389	2.770	2.217	1.729	1.307	.119	.000
		.00	.31	.56	.75	.90	.99	1.00
CLEVELAND	OH	2.755	2.288	1.868	1.495	1.054	.275	.160
		.00	.25	.46	.63	.76	.87	.91
COLUMBIA	MO	3.314	2.697	2.148	1.665	.915	.227	.070
		.00	.30	.54	.73	.87	.96	.98
DALLAS	TX	3.432	2.774	2.189	1.678	.806	.216	.059
		.00	.31	.56	.75	.89	.97	1.00
GREAT FALLS	MT	3.555	2.867	2.257	1.728	.721	.184	.072
		.00	.32	.58	.77	.90	.97	.99
LANDER	WY	4.462	3.456	2.586	1.853	.313	.000	.000
		.00	.39	.70	.92	.99	1.00	1.00
LINCOLN	NE	3.319	2.710	2.165	1.685	1.154	.220	.058
		.00	.30	.54	.74	.88	.97	1.00
LOS ANGELES	CA	3.908	3.076	2.347	1.723	.707	.053	.000
		.00	.35	.62	.82	.95	1.00	1.00
MADISON	WI	3.040	2.512	2.037	1.616	1.131	.233	.139
		.00	.28	.50	.69	.83	.93	.97
MIAMI	FL	3.711	2.951	2.280	1.700	1.016	.000	.000
		.00	.33	.59	.79	.94	1.00	1.00
NASHVILLE	TN	3.043	2.499	2.011	1.581	1.208	.284	.140
		.00	.28	.50	.68	.82	.92	.96
NEW YORK	NY	2.431	2.073	1.745	1.448	1.181	.411	.160
		.00	.22	.42	.57	.71	.87	.92
PHOENIX	AZ	4.770	3.614	2.629	1.202	.207	.000	.000
		.00	.42	.73	.94	.99	1.00	1.00
RENO	NV	4.554	3.480	2.559	1.319	.400	.000	.000
		.00	.40	.70	.91	.97	1.00	1.00
WASHINGTON	DC	2.861	2.383	1.950	1.564	1.225	.287	.110
		.00	.26	.48	.65	.79	.93	.96

TABLE C-3: COMBINED HEATING (AIR) OPTIMIZATION TABLES, FR′[TAU*ALPHA] = .60 FR′UL = 2.50 W/M**2 C

		TABULATED: (P2*CA)/(P1*CF), CRITICAL							TABULATED: (PC*CA)/(P1*CF) [M**2/GJ] AND LOAD FRACTION						
		CE/(CA*L) [M**2/GJ]							A/L [M**2/GJ]						
LOCATION		.00	.02	.05	.10	.30	.50	.80	.00	.10	.20	.30	.40	.60	.80
ALBUQUERQUE	NM	4.682	2.731	2.391	1.980	1.174	.965	.729	4.682	2.596	1.854	1.362	.964	.386	.022
									.00	.37	.59	.75	.87	.99	1.00
ATLANTA	GA	3.106	2.026	1.774	1.469	.871	.730	.561	3.106	2.200	1.407	1.024	.864	.584	.386
									.00	.26	.44	.56	.65	.81	.90
BISMARCK	ND	3.360	2.184	1.912	1.583	.938	.804	.615	3.360	2.238	1.652	1.166	.903	.554	.244
									.00	.28	.48	.62	.72	.87	.95
BOSTON	MA	2.430	1.627	1.425	1.180	.792	.627	.501	2.430	1.792	1.184	.980	.788	.626	.457
									.00	.21	.35	.46	.56	.70	.80
BOULDER	CO	3.214	2.248	1.969	1.630	1.088	.854	.652	3.214	2.474	1.762	1.417	1.044	.449	.219
									.00	.28	.49	.65	.76	.92	.99
CLEVELAND	OH	2.523	1.590	1.392	1.153	.683	.589	.453	2.523	1.733	1.126	.905	.713	.509	.340
									.00	.21	.35	.45	.53	.65	.74
COLUMBIA	MO	3.121	1.932	1.692	1.401	.830	.709	.545	3.121	2.033	1.363	1.059	.840	.655	.363
									.00	.26	.42	.54	.64	.78	.88
DALLAS	TX	3.248	2.087	1.827	1.513	.897	.738	.567	3.248	2.268	1.325	1.016	.879	.509	.403
									.00	.27	.45	.57	.66	.81	.90
GREAT FALLS	MT	3.307	2.218	1.942	1.608	.953	.824	.629	3.307	2.318	1.670	1.265	.950	.510	.219
									.00	.28	.48	.63	.74	.88	.95
LANDER	WY	4.240	2.743	2.402	1.989	1.179	.979	.741	4.240	2.873	2.063	1.406	.860	.183	.000
									.00	.36	.60	.77	.88	.99	1.00
LINCOLN	NE	3.152	2.018	1.767	1.464	.957	.755	.577	3.152	2.133	1.516	1.129	.894	.685	.372
									.00	.27	.44	.57	.67	.83	.93
LOS ANGELES	CA	3.731	2.660	2.329	1.928	1.143	.960	.724	3.731	2.901	1.994	1.434	.944	.231	.032
									.00	.33	.58	.75	.87	.98	1.00
MADISON	WI	2.833	1.852	1.622	1.343	.893	.706	.539	2.833	1.941	1.366	1.112	.880	.621	.402
									.00	.24	.40	.53	.63	.77	.87
MIAMI	FL	3.506	2.568	2.248	1.862	1.104	.984	.742	3.506	2.794	2.165	1.620	1.076	.145	.000
									.00	.31	.56	.75	.89	.98	1.00
NASHVILLE	TN	2.847	1.768	1.548	1.282	.760	.634	.505	2.847	1.932	1.205	.848	.742	.601	.341
									.00	.24	.38	.49	.57	.70	.80
NEW YORK	NY	2.240	1.506	1.319	1.092	.725	.575	.459	2.240	1.645	1.075	.899	.730	.527	.440
									.00	.19	.33	.43	.51	.64	.73
PHOENIX	AZ	4.543	2.840	2.486	2.059	1.220	.867	.705	4.543	3.137	1.548	1.226	.796	.404	.042
									.00	.38	.61	.75	.85	.95	.99
RENO	NV	4.323	2.781	2.435	2.017	1.195	.972	.729	4.323	2.868	2.013	1.334	.974	.310	.018
									.00	.37	.61	.77	.88	.98	1.00
WASHINGTON	DC	2.679	1.746	1.529	1.266	.837	.660	.504	2.679	1.908	1.263	1.057	.849	.610	.337
									.00	.23	.38	.49	.59	.73	.82

TABLE C-3: COMBINED HEATING (AIR) OPTIMIZATION TABLES, FR′[TAU*ALPHA] = .50 FR′UL = 3.00 W/M**2 C

TABULATED: (P2*CA)/(P1*CF), CRITICAL

CE/(CA*L) [M**2/GJ]

LOCATION		.00	.02	.05	.10	.30	.50	.80
ALBUQUERQUE	NM	3.707	2.331	2.041	1.690	1.002	.854	.655
ATLANTA	GA	2.384	1.653	1.447	1.199	.710	.625	.501
BISMARCK	ND	2.609	1.790	1.567	1.298	.879	.694	.531
BOSTON	MA	1.842	1.309	1.146	.949	.652	.518	.422
BOULDER	CO	2.493	1.832	1.604	1.328	.928	.735	.561
CLEVELAND	OH	1.912	1.288	1.128	.934	.622	.493	.393
COLUMBIA	MO	2.409	1.598	1.399	1.159	.767	.608	.484
DALLAS	TX	2.505	1.717	1.503	1.245	.738	.633	.505
GREAT FALLS	MT	2.567	1.807	1.582	1.310	.900	.709	.542
LANDER	WY	3.346	2.296	2.010	1.665	.987	.870	.664
LINCOLN	NE	2.435	1.661	1.454	1.204	.811	.643	.513
LOS ANGELES	CA	2.899	2.154	1.886	1.562	1.068	.840	.642
MADISON	WI	2.169	1.507	1.320	1.093	.748	.593	.453
MIAMI	FL	2.677	2.026	1.774	1.469	1.041	.822	.628
NASHVILLE	TN	2.175	1.457	1.275	1.056	.626	.538	.428
NEW YORK	NY	1.684	1.195	1.046	.866	.594	.472	.384
PHOENIX	AZ	3.595	2.434	2.131	1.765	1.046	.846	.641
RENO	NV	3.406	2.334	2.044	1.692	1.003	.866	.659
WASHINGTON	DC	2.040	1.418	1.242	1.028	.698	.554	.444

TABULATED: (PC*CA)/(P1*CF) [M**2/GJ]
AND LOAD FRACTION

A/L [M**2/GJ]

LOCATION		.00	.10	.20	.30	.40	.60	.80
ALBUQUERQUE	NM	3.707	2.525	1.710	1.290	1.028	.513	.255
		.00	.31	.51	.65	.76	.92	.99
ATLANTA	GA	2.384	1.781	1.318	.984	.755	.595	.466
		.00	.21	.36	.47	.56	.69	.80
BISMARCK	ND	2.609	1.949	1.443	1.126	.848	.597	.378
		.00	.23	.39	.52	.62	.76	.86
BOSTON	MA	1.842	1.417	1.027	.825	.730	.529	.436
		.00	.16	.29	.38	.45	.58	.68
BOULDER	CO	2.493	1.997	1.532	1.210	1.021	.614	.370
		.00	.22	.40	.54	.65	.81	.91
CLEVELAND	OH	1.912	1.396	.938	.769	.635	.469	.382
		.00	.16	.28	.37	.44	.54	.63
COLUMBIA	MO	2.409	1.731	1.148	.951	.778	.569	.456
		.00	.21	.35	.45	.54	.67	.77
DALLAS	TX	2.505	1.847	1.344	.917	.761	.601	.410
		.00	.22	.37	.48	.57	.70	.81
GREAT FALLS	MT	2.567	1.986	1.494	1.143	.959	.609	.352
		.00	.23	.39	.53	.63	.78	.87
LANDER	WY	3.346	2.421	1.798	1.441	1.038	.539	.116
		.00	.29	.50	.66	.78	.93	.99
LINCOLN	NE	2.435	1.806	1.288	1.053	.836	.604	.475
		.00	.21	.36	.48	.57	.71	.82
LOS ANGELES	CA	2.899	2.347	1.819	1.389	1.022	.439	.161
		.00	.26	.47	.63	.75	.90	.97
MADISON	WI	2.169	1.643	1.191	.955	.828	.584	.429
		.00	.19	.33	.43	.52	.66	.76
MIAMI	FL	2.677	2.211	1.791	1.419	1.094	.533	.175
		.00	.24	.44	.60	.73	.89	.96
NASHVILLE	TN	2.175	1.561	1.115	.825	.661	.509	.419
		.00	.19	.32	.41	.48	.59	.69
NEW YORK	NY	1.684	1.291	.985	.739	.634	.490	.403
		.00	.15	.26	.35	.41	.53	.62
PHOENIX	AZ	3.595	2.629	1.777	1.102	.897	.495	.278
		.00	.31	.53	.66	.76	.89	.96
RENO	NV	3.406	2.551	1.814	1.398	1.001	.407	.211
		.00	.30	.51	.67	.78	.92	.97
WASHINGTON	DC	2.040	1.526	1.132	.875	.754	.556	.425
		.00	.18	.31	.41	.49	.62	.71

TABLE C-3: COMBINED HEATING (AIR) OPTIMIZATION TABLES, FR′[TAU*ALPHA] = .50 FR′UL = 1.75 W/M**2 C

TABULATED: (P2*CA)/(P1*CF), CRITICAL

CE/(CA*L) [M**2/GJ]

LOCATION		.00	.02	.05	.10	.30	.50	.80
ALBUQUERQUE	NM	3.941	2.458	2.152	1.782	1.056	.897	.687
ATLANTA	GA	2.632	1.832	1.604	1.328	.787	.673	.517
BISMARCK	ND	2.842	1.953	1.710	1.416	.950	.748	.571
BOSTON	MA	2.065	1.466	1.284	1.063	.728	.578	.470
BOULDER	CO	2.717	1.995	1.747	1.447	.999	.791	.604
CLEVELAND	OH	2.145	1.441	1.261	1.045	.696	.550	.438
COLUMBIA	MO	2.641	1.738	1.522	1.260	.834	.659	.526
DALLAS	TX	2.749	1.896	1.660	1.375	.815	.679	.522
GREAT FALLS	MT	2.797	1.975	1.729	1.432	.849	.762	.586
LANDER	WY	3.572	2.446	2.141	1.773	1.051	.916	.698
LINCOLN	NE	2.668	1.809	1.584	1.312	.879	.697	.557
LOS ANGELES	CA	3.154	2.356	2.063	1.708	1.012	.897	.684
MADISON	WI	2.404	1.665	1.458	1.207	.823	.651	.519
MIAMI	FL	2.978	2.270	1.987	1.646	1.175	.925	.707
NASHVILLE	TN	2.415	1.607	1.407	1.165	.690	.587	.472
NEW YORK	NY	1.907	1.364	1.195	.989	.669	.530	.430
PHOENIX	AZ	3.823	2.594	2.271	1.881	1.115	.879	.666
RENO	NV	3.643	2.486	2.177	1.803	1.068	.911	.695
WASHINGTON	DC	2.274	1.574	1.378	1.141	.769	.610	.488

TABULATED: (PC*CA)/(P1*CF) [M**2/GJ]
AND LOAD FRACTION

A/L [M**2/GJ]

LOCATION		.00	.10	.20	.30	.40	.60	.80
ALBUQUERQUE	NM	3.941 .00	2.619 .33	1.669 .53	1.287 .68	1.094 .80	.414 .95	.026 1.00
ATLANTA	GA	2.632 .00	1.991 .23	1.280 .40	1.003 .51	.817 .60	.662 .75	.387 .85
BISMARCK	ND	2.842 .00	2.056 .25	1.500 .43	1.203 .56	.953 .67	.667 .82	.332 .92
BOSTON	MA	2.065 .00	1.615 .18	1.106 .32	.954 .42	.812 .51	.603 .65	.458 .75
BOULDER	CO	2.717 .00	2.198 .25	1.633 .44	1.320 .58	1.015 .70	.681 .87	.338 .96
CLEVELAND	OH	2.145 .00	1.600 .19	1.059 .31	.835 .41	.730 .49	.546 .61	.417 .70
COLUMBIA	MO	2.641 .00	1.934 .23	1.288 .38	.966 .49	.849 .58	.635 .73	.361 .84
DALLAS	TX	2.749 .00	2.057 .24	1.328 .41	.920 .52	.807 .61	.668 .76	.408 .85
GREAT FALLS	MT	2.797 .00	2.174 .25	1.663 .43	1.257 .57	.938 .68	.516 .83	.389 .92
LANDER	WY	3.572 .00	2.529 .31	1.871 .53	1.368 .70	1.015 .82	.352 .97	.019 1.00
LINCOLN	NE	2.668 .00	2.004 .23	1.372 .39	1.055 .52	.908 .61	.683 .77	.381 .88
LOS ANGELES	CA	3.154 .00	2.574 .29	1.993 .52	1.415 .68	1.045 .80	.446 .95	.055 .99
MADISON	WI	2.404 .00	1.841 .21	1.265 .36	1.089 .48	.836 .58	.601 .72	.404 .83
MIAMI	FL	2.978 .00	2.479 .27	2.027 .50	1.623 .68	1.238 .82	.280 .97	.076 1.00
NASHVILLE	TN	2.415 .00	1.766 .21	1.097 .35	.921 .45	.680 .53	.571 .65	.484 .76
NEW YORK	NY	1.907 .00	1.488 .17	1.014 .30	.866 .39	.725 .47	.561 .59	.433 .69
PHOENIX	AZ	3.823 .00	2.833 .33	1.567 .56	1.138 .69	.865 .79	.400 .92	.175 .98
RENO	NV	3.643 .00	2.718 .32	1.874 .54	1.307 .71	.948 .82	.347 .95	.021 1.00
WASHINGTON	DC	2.274 .00	1.729 .20	1.187 .34	.979 .45	.792 .54	.588 .67	.412 .78

TABLE C-4: COMBINED HEATING (LIQUID) OPTIMIZATION TABLES, FR′[TAU*ALPHA] = .70 FR′UL = 3.00 W/M**2 C

TABULATED: (P2*CA)/(P1*CF), CRITICAL

CE/(CA*L) [M**2/GJ]

LOCATION		.00	.02	.05	.10	.30	.50	.80
ALBUQUERQUE	NM	5.413	2.870	2.513	2.081	1.233	.976	.733
ATLANTA	GA	3.586	2.146	1.879	1.556	.922	.750	.573
BISMARCK	ND	3.872	2.301	2.015	1.669	.989	.816	.621
BOSTON	MA	2.800	1.728	1.513	1.253	.816	.643	.492
BOULDER	CO	3.706	2.388	2.090	1.731	1.026	.864	.659
CLEVELAND	OH	2.907	1.680	1.471	1.218	.722	.605	.463
COLUMBIA	MO	3.602	2.044	1.790	1.482	.878	.728	.557
DALLAS	TX	3.756	2.211	1.936	1.603	.950	.759	.580
GREAT FALLS	MT	3.810	2.344	2.052	1.699	1.007	.833	.634
LANDER	WY	4.892	2.876	2.518	2.085	1.236	.878	.743
LINCOLN	NE	3.635	2.135	1.869	1.548	.918	.768	.587
LOS ANGELES	CA	4.308	2.829	2.477	2.051	1.216	.864	.735
MADISON	WI	3.264	1.960	1.716	1.421	.842	.715	.547
MIAMI	FL	4.063	2.839	2.486	2.059	1.220	.867	.767
NASHVILLE	TN	3.287	1.873	1.640	1.358	.805	.654	.500
NEW YORK	NY	2.581	1.600	1.401	1.160	.750	.590	.451
PHOENIX	AZ	5.263	2.979	2.608	2.160	1.280	.910	.634
RENO	NV	4.987	2.911	2.549	2.111	1.251	.889	.739
WASHINGTON	DC	3.090	1.854	1.623	1.344	.796	.675	.517

TABULATED: (PC*CA)/(P1*CF) [M**2/GJ]
AND LOAD FRACTION

A/L [M**2/GJ]

LOCATION		.00	.10	.20	.30	.40	.60	.80
ALBUQUERQUE	NM	5.413	2.702	1.765	1.263	.852	.304	.000
		.00	.40	.62	.77	.88	.99	1.00
ATLANTA	GA	3.586	2.300	1.369	.975	.815	.587	.321
		.00	.29	.47	.58	.67	.81	.90
BISMARCK	ND	3.872	2.328	1.585	1.183	.870	.545	.237
		.00	.31	.50	.64	.74	.87	.94
BOSTON	MA	2.800	1.877	1.201	.997	.795	.551	.375
		.00	.23	.38	.49	.57	.70	.80
BOULDER	CO	3.706	2.584	1.661	1.317	.918	.450	.232
		.00	.31	.52	.67	.78	.91	.98
CLEVELAND	OH	2.907	1.636	1.129	.854	.675	.497	.350
		.00	.23	.36	.47	.54	.66	.74
COLUMBIA	MO	3.602	2.024	1.408	1.001	.844	.566	.302
		.00	.28	.44	.56	.65	.79	.88
DALLAS	TX	3.756	2.387	1.296	.999	.828	.520	.327
		.00	.30	.48	.59	.68	.82	.90
GREAT FALLS	MT	3.810	2.337	1.572	1.242	.851	.417	.217
		.00	.31	.51	.65	.75	.88	.94
LANDER	WY	4.892	2.965	1.978	1.276	.881	.247	.000
		.00	.38	.62	.79	.89	.99	1.00
LINCOLN	NE	3.635	2.140	1.503	1.065	.893	.581	.300
		.00	.29	.46	.59	.69	.83	.92
LOS ANGELES	CA	4.308	3.056	1.888	1.284	.834	.234	.014
		.00	.37	.61	.77	.88	.98	1.00
MADISON	WI	3.264	1.969	1.364	1.104	.800	.527	.326
		.00	.26	.43	.55	.64	.78	.86
MIAMI	FL	4.063	3.073	2.231	1.540	.718	.089	.000
		.00	.36	.62	.81	.92	.99	1.00
NASHVILLE	TN	3.287	2.005	1.172	.900	.697	.535	.400
		.00	.26	.41	.51	.59	.71	.80
NEW YORK	NY	2.581	1.733	1.093	.880	.732	.522	.369
		.00	.21	.35	.45	.53	.65	.73
PHOENIX	AZ	5.263	3.286	1.547	1.153	.653	.323	.038
		.00	.42	.64	.77	.86	.96	1.00
RENO	NV	4.987	2.953	1.928	1.213	.824	.243	.007
		.00	.39	.63	.79	.88	.97	1.00
WASHINGTON	DC	3.090	1.996	1.273	1.001	.779	.530	.348
		.00	.25	.40	.52	.60	.73	.82

TABLE C-4: COMBINED HEATING (LIQUID) OPTIMIZATION TABLES, FR′[TAU*ALPHA] = .60 FR′UL = 4.00 W/M**2 C

TABULATED: (P2*CA)/(P1*CF), CRITICAL

LOCATION		CE/(CA*L) [M**2/GJ] .00	.02	.05	.10	.30	.50	.80
ALBUQUERQUE	NM	4.345	2.472	2.165	1.793	1.062	.865	.658
ATLANTA	GA	2.763	1.764	1.544	1.279	.758	.642	.491
BISMARCK	ND	3.029	1.889	1.653	1.369	.812	.696	.533
BOSTON	MA	2.123	1.383	1.211	1.003	.661	.523	.399
BOULDER	CO	2.895	1.952	1.709	1.415	.839	.734	.564
CLEVELAND	OH	2.202	1.348	1.180	.977	.579	.491	.377
COLUMBIA	MO	2.796	1.689	1.478	1.224	.726	.614	.471
DALLAS	TX	2.912	1.841	1.612	1.335	.791	.652	.498
GREAT FALLS	MT	2.979	1.911	1.673	1.385	.821	.709	.543
LANDER	WY	3.909	2.431	2.128	1.762	1.045	.872	.663
LINCOLN	NE	2.825	1.762	1.543	1.277	.757	.649	.498
LOS ANGELES	CA	3.373	2.316	2.027	1.679	.995	.848	.646
MADISON	WI	2.505	1.588	1.390	1.151	.754	.595	.455
MIAMI	FL	3.111	2.244	1.965	1.627	.964	.854	.650
NASHVILLE	TN	2.518	1.541	1.349	1.117	.662	.551	.422
NEW YORK	NY	1.934	1.267	1.109	.918	.602	.476	.379
PHOENIX	AZ	4.222	2.616	2.290	1.897	1.124	.799	.652
RENO	NV	3.976	2.465	2.158	1.787	1.059	.869	.659
WASHINGTON	DC	2.357	1.505	1.318	1.091	.709	.561	.429

TABULATED: (PC*CA)/(P1*CF) [M**2/GJ] AND LOAD FRACTION

LOCATION		A/L [M**2/GJ] .00	.10	.20	.30	.40	.60	.80
ALBUQUERQUE	NM	4.345 .00	2.515 .34	1.637 .54	1.195 .68	.886 .78	.411 .92	.189 .98
ATLANTA	GA	2.763 .00	1.869 .23	1.268 .38	.924 .49	.728 .58	.513 .70	.373 .79
BISMARCK	ND	3.029 .00	1.952 .25	1.387 .41	1.036 .53	.817 .62	.512 .75	.333 .84
BOSTON	MA	2.123 .00	1.477 .18	1.011 .30	.792 .39	.667 .46	.487 .58	.364 .66
BOULDER	CO	2.895 .00	2.101 .25	1.501 .43	1.133 .56	.895 .66	.543 .80	.337 .89
CLEVELAND	OH	2.202 .00	1.423 .18	.940 .29	.719 .37	.597 .44	.421 .54	.316 .61
COLUMBIA	MO	2.796 .00	1.784 .23	1.138 .37	.916 .47	.712 .55	.508 .67	.363 .76
DALLAS	TX	2.912 .00	1.952 .24	1.291 .40	.882 .50	.725 .59	.514 .71	.374 .80
GREAT FALLS	MT	2.979 .00	2.053 .25	1.444 .42	1.054 .54	.834 .63	.492 .77	.322 .85
LANDER	WY	3.909 .00	2.455 .32	1.705 .53	1.289 .68	.898 .79	.415 .92	.185 .98
LINCOLN	NE	2.825 .00	1.867 .23	1.242 .38	.986 .49	.771 .58	.536 .71	.377 .80
LOS ANGELES	CA	3.373 .00	2.495 .29	1.799 .51	1.257 .66	.903 .76	.455 .90	.197 .96
MADISON	WI	2.505 .00	1.689 .21	1.169 .35	.895 .45	.740 .53	.503 .65	.357 .74
MIAMI	FL	3.111 .00	2.437 .28	1.855 .49	1.365 .65	.968 .77	.443 .90	.083 .97
NASHVILLE	TN	2.518 .00	1.618 .20	1.021 .33	.780 .42	.639 .49	.445 .60	.347 .68
NEW YORK	NY	1.934 .00	1.349 .16	.948 .28	.706 .36	.598 .42	.438 .53	.335 .60
PHOENIX	AZ	4.222 .00	2.791 .35	1.572 .57	1.041 .69	.803 .78	.412 .89	.217 .95
RENO	NV	3.976 .00	2.632 .33	1.705 .54	1.247 .68	.837 .78	.444 .91	.150 .96
WASHINGTON	DC	2.357 .00	1.593 .20	1.064 .33	.840 .42	.694 .50	.490 .62	.356 .70

TABLE C-4: COMBINED HEATING (LIQUID) OPTIMIZATION TABLES, FR′[TAU*ALPHA] = .60 FR′UL = 2.00 W/M**2 C

TABULATED: (P2*CA)/(P1*CF), CRITICAL CE/(CA*L) [M**2/GJ]

LOCATION		.00	.02	.05	.10	.30	.50	.80
ALBUQUERQUE	NM	4.719	2.667	2.335	1.934	1.146	.933	.709
ATLANTA	GA	3.161	2.008	1.758	1.456	.863	.714	.547
BISMARCK	ND	3.401	2.142	1.875	1.553	.920	.782	.597
BOSTON	MA	2.479	1.621	1.419	1.175	.779	.617	.488
BOULDER	CO	3.254	2.208	1.933	1.601	.949	.824	.631
CLEVELAND	OH	2.575	1.583	1.386	1.148	.743	.585	.447
COLUMBIA	MO	3.168	1.907	1.669	1.383	.819	.695	.532
DALLAS	TX	3.303	2.077	1.819	1.506	.893	.721	.552
GREAT FALLS	MT	3.347	2.174	1.904	1.577	.934	.799	.610
LANDER	WY	4.270	2.660	2.329	1.929	1.143	.947	.716
LINCOLN	NE	3.197	1.986	1.739	1.440	.854	.731	.562
LOS ANGELES	CA	3.782	2.624	2.298	1.903	1.128	.938	.710
MADISON	WI	2.881	1.832	1.604	1.329	.873	.687	.525
MIAMI	FL	3.591	2.622	2.296	1.901	1.127	.993	.745
NASHVILLE	TN	2.901	1.761	1.542	1.277	.757	.625	.479
NEW YORK	NY	2.292	1.506	1.319	1.092	.719	.567	.434
PHOENIX	AZ	4.587	2.799	2.450	2.029	1.203	.855	.692
RENO	NV	4.357	2.701	2.364	1.958	1.161	.825	.711
WASHINGTON	DC	2.731	1.735	1.519	1.258	.746	.644	.495

TABULATED: (PC*CA)/(P1*CF) [M**2/GJ] AND LOAD FRACTION A/L [M**2/GJ]

LOCATION		.00	.10	.20	.30	.40	.60	.80
ALBUQUERQUE	NM	4.719 .00	2.488 .37	1.669 .58	1.233 .73	1.015 .84	.348 .97	.000 1.00
ATLANTA	GA	3.161 .00	2.190 .27	1.301 .44	1.030 .55	.797 .64	.616 .78	.338 .88
BISMARCK	ND	3.401 .00	2.163 .28	1.520 .47	1.171 .60	.899 .70	.593 .84	.355 .93
BOSTON	MA	2.479 .00	1.784 .21	1.143 .35	.982 .46	.794 .55	.573 .68	.405 .78
BOULDER	CO	3.254 .00	2.414 .28	1.652 .48	1.306 .63	.929 .74	.588 .89	.274 .97
CLEVELAND	OH	2.575 .00	1.739 .21	1.086 .34	.864 .44	.729 .52	.520 .64	.376 .73
COLUMBIA	MO	3.168 .00	2.000 .26	1.353 .41	.966 .53	.842 .62	.591 .76	.327 .86
DALLAS	TX	3.303 .00	2.273 .28	1.275 .45	.961 .56	.813 .65	.621 .79	.341 .88
GREAT FALLS	MT	3.347 .00	2.190 .28	1.661 .47	1.239 .61	.863 .72	.458 .86	.326 .93
LANDER	WY	4.270 .00	2.741 .35	1.901 .58	1.268 .74	.915 .85	.279 .98	.000 1.00
LINCOLN	NE	3.197 .00	2.088 .27	1.439 .43	1.027 .56	.892 .65	.614 .80	.334 .90
LOS ANGELES	CA	3.782 .00	2.846 .33	1.898 .57	1.335 .73	.946 .85	.345 .96	.018 1.00
MADISON	WI	2.881 .00	1.924 .24	1.301 .40	1.096 .52	.796 .61	.551 .75	.358 .85
MIAMI	FL	3.591 .00	2.852 .32	2.205 .57	1.650 .77	.959 .90	.110 .99	.000 1.00
NASHVILLE	TN	2.901 .00	1.932 .24	1.166 .38	.942 .48	.677 .56	.549 .68	.437 .78
NEW YORK	NY	2.292 .00	1.653 .20	1.046 .33	.895 .42	.734 .50	.541 .63	.392 .72
PHOENIX	AZ	4.587 .00	3.108 .38	1.532 .60	1.134 .73	.691 .83	.340 .94	.152 .99
RENO	NV	4.357 .00	2.735 .36	1.858 .59	1.208 .75	.856 .85	.279 .96	.019 1.00
WASHINGTON	DC	2.731 .00	1.905 .23	1.217 .38	.978 .49	.772 .58	.547 .71	.376 .81

REFERENCES

1. *Project Independence Blueprint Task Force Report on Solar Energy*. Federal Energy Administration, U.S.Government Printing Office, 4118-00012 (1974)
2. Peterson, H. C., *The Impact of Tax Incentives and Auxiliary Fuel Prices on the Utilization Rate of Solar Energy Space Conditioning*. Prepared for the NSF/RANN Program at Utah State University, Logan, Utah (1976)
3. Ruegg, R.T., *Evaluating Incentives for Solar Heating*. National Bureau of Standards IR 76-1127 (1976)
4. MITRE Corporation, *An Economic Analysis of Solar Water and Space Heating*. Prepared for Energy Research and Development Administration. Available from Government Printing Office, 060-000-00038-7 (1976)
5. Schulze, W.D., Ben-David, S., and Balcomb, J.D., *The Economics of Solar Home Heating*. Prepared for the Joint Economic Committee of the United States Congress (1977)
6. Klein, S.A., Beckman, W.A., Duffie, J.A., "A Design Procedure for Solar Heating Systems," *Solar Energy*, Vol.18, 113 (1976)

7. Klein, S.A., Beckman, W.A., Duffie, J.A., "A Design Procedure for Solar Air Heating Systems," Solar Energy, Vol.19, 509 (1977)

8. Klein, S.A., "Estimation of the Performance of Solar Domestic Water Heating Systems," to be presented ISES Conference in New Dehli, India (1978)

9. Beckman, W.A., Klein, S.A., Duffie, J.A., Solar Heating Design, Wiley Interscience, New York (1977)

10. Klein, S.A., "A Design Precedure for Solar Heating Systems," Ph.D. Thesis, University of Wisconsin - Madison (1976)

11. Ruegg, R.T., Solar Heating and Cooling in Buildings: Methods of Economic Evaluation. National Bureau of Standards IR 75-712 (1975)

12. Klein, S.A. et al., TRNSYS - A Transient System Simulation Program, User Manual. Report #38, Engineering Experiment Station, University of Wisconsin - Madison (1973)

13. de Winter, F., "Heat Exchanger Penalties in the Double Loop Solar Water Heating Systems," Solar Energy, Vol.17, 335 (1975)

14. House Heating, American Gas Association, Industrial Gas Series, 3rd ed.

15. ASHRAE Handbook and Product Directory, Systems, American Society of Heating, Refrigerating, and Air Conditioning Engineers, New York (1973)

16. ASHRAE Standards 90-75, "Energy Conservation in New Building Design," American Society of Heating, Refrigerating, and Air Conditioning Engineers, New York (1975)

17. Collins, W.D., "Temperature of Water Available for Industrial Use in the United States," U.S. Geological Survey Water Supply Paper, No. 520 F (1925)

18. Gutierrez, G., Hincapie, F., Duffie, J.A., Beckman, W.A., "Simulation of Forced Circulation Water Heaters; Effect of Auxiliary Energy Supply, Load Type, and Storage Capacity," Solar Energy, Vol. 15, 287 (1974)

19. Smith, G.W., Engineering Economy: Analysis of Capital Expenditures, 2nd ed., Iowa State University Press (1973)

20. Ward, J.C., "Minimum Cost Sizing of Solar Heating Systems." Presented at the Sharing the Sun Conference, Winnepeg, Alberta, Canada (1976)

21. Barley, C.D. and Winn, C.B., "Optimal Sizing of Solar Collectors by the Method of Relative Areas." M.S.Thesis, Department of Mechanical Engineering, Colorado State University, Fort Collins, Colorado (1977)