

**MODELING OF HEAT TRANSFER IN BUILDINGS  
USING COMPREHENSIVE ROOM TRANSFER FUNCTIONS**

by

**STEVEN W. CARLSON**

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## ABSTRACT

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Seem (1987) developed a method to analytically reduce the system of equations representing heat transfer in a building zone, to a single comprehensive room transfer function. The comprehensive room transfer function requires significantly less computational effort than solving a system of equations, and produces results that are just as accurate. Seem presented a method to combine pairs of wall transfer functions, a "star" heat transfer network which provides a common point for all radiation and convection heat transfer to pass through, and model reduction techniques to reduce the number of past time steps that are required in a transfer function.

This thesis applies the methods presented by Seem in developing a building zone model, which uses the comprehensive room transfer function formulation to represent the structural heat transfer. The building zone model is formulated to include solar radiation, ambient temperature effects, internal radiation gains, ventilation loads, infiltration loads, gains from people, effects of adjacent zones, and direct internal gains on the room air. The comprehensive room transfer function formulation is extended to include the formulation of the heat transfer star network from the mean radiant temperature network of Carroll (1980). The mean radiant temperature network allows a simpler description of the building geometry and makes it easy to include room furnishings in the radiation heat transfer within the room. The implementation of model reduction techniques is discussed to include limits on the

application of the techniques, along with practical considerations of using model reduction. A formulation of a multiple zone building is given, which allows a multiple zone building to be solved simultaneously as opposed to sequentially calculating the energy flows on each zone.

The simulation run time of the building zone model is shown to be significantly faster (5 to 10 times faster) than another zone model which is based on solving a system of transfer function equations. The loss of information such as wall surface temperatures and individual wall heat fluxes is discussed as it concerns possible minor limitations on the capabilities of the CRTF zone model. Methods to overcome some of the limitations are presented. Specific methods are developed for: determining a mean radiant temperature, used in estimating thermal comfort, which is derived from the wall surface temperatures that are no longer available; modeling a change in the building parameters that are not available in the CRTF, as encountered with the addition of night insulation.

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## NOMENCLATURE

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### Symbol

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$A$	area
$a$	transfer function coefficient
$B$	back shift operator
$b$	transfer function coefficient
$C$	thermal capacitance
$C_p$	specific heat of air
$c$	transfer function coefficient
$d$	CRTF coefficient for ambient temperature
$e$	error between resistance networks, CRTF coefficient for room temperature
$f$	CRTF coefficient for solar inputs
$f_a, f_b$	area fractions
$F_i$	mean radiant temperature network radiation exchange factor
$F_{ij}$	radiation view factor: fraction of energy leaving surface $i$ that directly reaches surface $j$
$\hat{F}_{ij}$	total radiation exchange factor: radiation leaving surface $i$ that is incident on surface $j$ by all possible paths, divided by the total radiation leaving surface $i$
$g$	CRTF coefficient for internal radiation gains
$h$	CRTF output coefficient
$\Delta h_{\text{vap}}$	heat of vaporization of water
$I$	incident solar radiation
$\dot{m}$	mass flow



## Symbol

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$N$	number of surfaces in a room
$N_s$	number of solar orientations
$N_z$	number of adjacent zones
$\dot{Q}$	instantaneous heat flow
$\bar{\dot{Q}}$	average heat flow
$q''$	heat flux
$R$	resistance
$s$	complex variable resulting from Laplace transformation
$T$	temperature
$\bar{T}$	average temperature
$T_{j,i}$	indoor surface temperature $j$ time steps prior to current time
$T_{j,o}$	outdoor surface temperature $j$ time steps prior to current time
$U$	thermal conductance
$V$	volume
$w$	complex variable resulting from the bilinear transformation
$\mathbf{X}$	square matrix
$x$	one dimensional distance
$x(i,j)$	element of matrix $\mathbf{X}$ at row $i$ , column $j$
$x^{-1}(i,j)$	element of the inverse of matrix $\mathbf{X}$ at row $i$ , column $j$
$\mathbf{Y}$	vector
$y(i)$	element of vector $\mathbf{Y}$ at row $i$
$\mathbf{Z}$	vector
$z(i)$	element of vector $\mathbf{Z}$ at row $i$

## Symbol

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$z$	complex variable resulting from z-transformation
$\alpha$	thermal diffusivity, solar absorptance
$\delta$	time step
$\varepsilon$	surface emittance
$\phi_k$	fraction of radiation passing through window that is absorbed by wall k
$\sigma$	Stefan-Boltzmann constant
$\tau$	time
$\omega$	humidity ratio
$\psi$	error function

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# **CHAPTER 1**

## **Introduction**

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### **1.1 Objective**

Modeling heat transfer in buildings is important for sizing heating, ventilating, and air conditioning equipment; determining the effect of design changes; and developing control strategies. These objectives often require accurate calculation of transient heat transfer processes, which involve significant computational effort. The goal of this thesis is to describe the implementation of a modeling method which greatly reduces the computation time in determining transient heating and cooling loads of buildings.

The Comprehensive Room Transfer Function (CRTF) method presented by Seem (1987) analytically determines a single equation that calculates a building zones's heating or cooling load and temperature. A computer subroutine to model the heat transfer in a building zone or room based on the CRTF method was developed for use in TRNSYS [Klein (1988)], a transient simulation program. This thesis presents the building room model and investigates the effect of assumptions and limitations on the model's performance.

### **1.2 Building Energy Simulation Methods**

Methods for determining energy loads on buildings vary widely, depending on the effects that are included in the calculations. The procedures can be classified into

three categories: single measure methods, multiple measure methods, and detailed simulation methods. The simple single measure methods include annual degree day calculations [ASHRAE Fundamentals (1981)]. Multiple measure methods such as the Bin method ASHRAE (1983) attempt to account for time and temperature dependent loads by calculating the building performance at a number of different operating conditions. The detailed simulation methods perform energy balance calculations over a small time step, one hour or smaller, for the period of analysis, typically one year.

The detailed simulation methods try to account for the constantly changing internal and external factors effecting the thermal loads on the building. The purpose of these simulations often includes more than simply finding annual energy use. Peak loads, the response of equipment and the building environment to control strategies, and human comfort are just a few of the concerns that generate a need for these complex and detailed dynamic simulations.

The simulation methods vary in the representation of different effects. Some methods require detailed information about the room geometry so that thermal radiation can be accurately modeled, while other methods apply simplifications which make the simulation easier to set up, but may limit the accuracy in some instances. Heat transfer convection coefficients from walls are allowed to vary in some methods while they are constant in other methods. Depending on the accuracy and the effects that need to be included in a simulation, there are appropriate simplifications and assumptions that can be made to eliminate unnecessary detail. Including great detail in a simulation usually results in a corresponding increase in computation effort.

The CRTF model described in this thesis is a detailed simulation method that,

with little approximation or limitation, can provide the same level of accuracy as other detailed simulation methods. The main advantage of the CRTF model over other models is the reduction in computation requirements.

### **1.3 Organization**

The formulation of the comprehensive room transfer function as developed by Seem (1987) is presented in chapter 2. The formulation is extended to include another thermal radiation exchange network which simplifies the geometry information required to describe the room.

Chapter 3 describes two methods presented by Seem (1987) which allow a simplification of the CRTF resulting in the computational time savings when using CRTFs in a simulation program. The chapter also includes a discussion of implementing model reduction.

Chapter 4 presents the building zone model: the formulation of equations based on an energy balance of the zone, the possible modes of operation, the performance of the model in terms of accuracy and computation time, and the capabilities of the model.

Chapter 5 describes methods to overcome limitations when modeling with CRTFs. A method to include night insulation is examined, along with ways to extract information from the CRTF model that can be used in predicting thermal comfort of a human in the room. Methods of running simulations with smaller than one hour time steps are also presented.

The conclusions and recommendations are included in chapter 6.

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## CHAPTER 2

### Comprehensive Room Transfer Functions

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Modeling of heat transfer and temperatures in buildings requires the solution of the transient conduction equation for the walls, roof and floor of a room. Each individual element in a room is usually modeled separately, allowing the entire room to be modeled as a system of equations. Planar walls and slabs can be individually modeled by the classical heat conduction equation.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial \tau} \quad (2.0.1)$$

The solution of equation 2.0.1 for constant properties can be formulated in terms of: finite differences, finite elements, response factors, or transfer functions. The transfer function representation has a computational advantage over the other methods. In this case, the transient heat transfer through the wall is calculated from a times series of past temperatures on either side of the wall, and a time series of past heat fluxes through the wall. No intermediate calculations are made for nodes within the wall itself, which leads to the computational savings over the finite difference and the finite element methods. The response factor method is similar to the transfer function method except that no time series of past heat fluxes is used, resulting in a much larger time series of temperatures to produce the same accuracy as obtained

from a transfer function.

Stephenson and Mitalas (1972) first presented a method for determining transfer functions for one-dimensional heat transfer through multi-layered slabs by solving the conduction equation with Laplace and z-transform theory. Ceylan and Myers (1980) and Seem (1987) present methods for calculating transfer functions for multi-dimensional heat transfer. Regardless of the method used to obtain the transfer function, relating the heat flux through a single wall to the temperatures on either side, the form is the same.

$$q''_0 = \sum_{j=0} [(a_j T_{j,o}) + (b_j T_{j,i})] - \sum_{j=1} (c_j q''_j) \quad (2.0.2)$$

where

$a_j$  = transfer function coefficient for  $j$  time steps prior to the current time

$b_j$  = transfer function coefficient for  $j$  time steps prior to the current time

$c_j$  = transfer function coefficient for  $j$  time steps prior to the current time

$q''_j$  = heat flux  $j$  time steps prior to the current time

$T_{j,i}$  = inside surface temperature  $j$  time steps prior to the current time

$T_{j,o}$  = outside surface temperature  $j$  time steps prior to the current time

The summation in equation 2.0.2 sums over the time steps in the equation. The upper limit on the summation is not specified since the number of past time steps required to represent the heat flow through a wall varies with the construction of the wall. The current time is represented by a subscript of  $j=0$ , the value of a variable one time step back in time is represented by a subscript of  $j=1$ . A subscript of  $t$  used later in this thesis denotes the value of the variable at time  $t$ . This subscript is used when the equation containing the variable does not involve past time steps.

Subscripts after the  $j$  subscript are used to identify the variable. In equation 2.0.2 the  $o$  subscript in  $T_{j,o}$  is used to indicate the temperature on the outside surface. Equation 2.0.2 will be extended in section 2.3 to include other energy inputs to the wall, such as solar radiation and internal radiation gains from people and equipment.

A transfer function can be written for each wall, interior partition, roof and floor of a building zone. The interior surface temperatures are coupled together by infrared radiation exchange with other surfaces and with convection to the room air. Energy balances on each surface and the room air allow the formulation of a system of equations that can be solved for the surface and the air temperatures. This system energy balance approach is commonly used in computer simulations of buildings such as the TRNSYS type 19 building zone model, and will be referred to as the transfer function (TF) method through out this thesis.

Madsen (1982) introduced the comprehensive room transfer function (CRTF) method based on linear regression from an energy balance simulation. The CRTF method relates the zone load to a time series of past inputs and outputs, which results in significantly reduced computational effort as compared to the system energy balance approach (TF). The time savings is gained because what was once represented by a system of equations with the energy balance method is now represented by a single equation, the comprehensive room transfer function. Seem (1987) introduced a method to analytically combine the separate wall transfer functions to produce a single transfer function which relates the zone temperature, ambient temperature, solar gains, and internal radiation gains to the zone energy load. Seem (1987) presented the combination of transfer functions for parallel path heat



transfer, the formulation of a star radiation network from the exact radiation network, and a reformulation of wall transfer functions which includes the inputs to the entire zone, rather than the inputs to the wall alone.

This chapter extends the formulation of the star radiation network, so it can be determined from the Carroll (1980) mean radiant temperature network (MRT). The MRT network is an approximate thermal radiation network which does not require detailed geometry information to describe the room. Besides making the description of the room easier, this simplification allows room furnishings to be included in the radiation network as a wall. The reformulation of the wall transfer functions to include room energy inputs rather than just wall inputs is also presented in more detail, with explanations given on distributing room energy inputs through out the room.

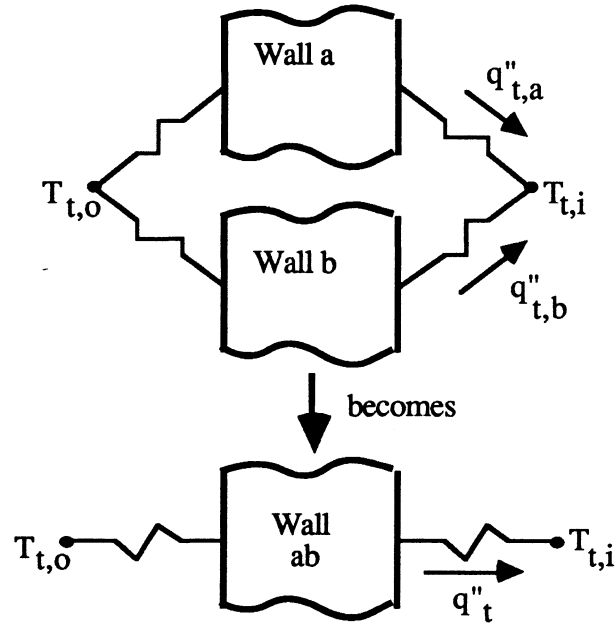
## 2.1 Transfer Function Combination

As shown by Seem (1987), individual wall transfer functions can be combined by adding the individual heat fluxes through the walls together in parallel. This section details the algebraic combination procedure for two transfer functions. The process is pictured in figure 2.1 and in equation 2.1.1.

$$\begin{aligned} q''_t &= \frac{A_a}{A_a + A_b} q''_{t,a} + \frac{A_b}{A_a + A_b} q''_{t,b} \\ &= f_a q''_{t,a} + f_b q''_{t,b} \end{aligned} \quad (2.1.1)$$

where the area fractions  $f_a$  and  $f_b$  are defined as follows:

$$f_a = \frac{A_a}{A_a + A_b} \quad f_b = \frac{A_b}{A_a + A_b}$$



**Figure 2.1** Combination of walls with parallel heat transfer paths.

The heat flux through each wall is given by the transfer function introduced earlier as equation 2.0.2. The equation is presented here specifically for wall a.

$$q''_{0,a} = \sum_{j=0} \left[ (a_{j,a} T_{j,o}) + (b_{j,a} T_{j,i}) \right] - \sum_{j=1} (c_{j,a} q''_{j,a})$$

The function can be rewritten to have the heat flux term in a single time series, by bringing the  $q''_{0,a}$  term inside the summation and resetting the lower limit to  $j=0$ .

$$\sum_{j=0} (c_{j,a} q''_{j,a}) = \sum_{j=0} \left[ (a_{j,a} T_{j,o}) + (b_{j,a} T_{j,i}) \right] \quad (2.1.2)$$

where  $c_{0,a} = 1$ .

The back shift operator<sup>1</sup>,  $B$ , allows the heat flux and temperatures to be factored out of the time series. The transfer function equation now becomes:

$$q''_{t,a} \sum_{j=0} (c_{j,a} B^j) = T_{t,o} \sum_{j=0} (a_{j,a} B^j) + T_{t,i} \sum_{j=0} (b_{j,a} B^j) \quad (2.1.3)$$

Solving for the heat flux,  $q''_{t,a}$  yields,

$$q''_{t,a} = \frac{T_{t,o} \sum_{j=0} (a_{j,a} B^j) + T_{t,i} \sum_{j=0} (b_{j,a} B^j)}{\sum_{j=0} (c_{j,a} B^j)} \quad (2.1.4)$$

Substituting the equation for the heat flux through wall-a and the similar equation for the heat flux through wall-b, into the equation for the total heat flux results in the following equation.

$$q''_t = f_a \left[ \frac{T_{t,o} \sum_{j=0} (a_{j,a} B^j) + T_{t,i} \sum_{j=0} (b_{j,a} B^j)}{\sum_{j=0} (c_{j,a} B^j)} \right] + f_b \left[ \frac{T_{t,o} \sum_{j=0} (a_{j,b} B^j) + T_{t,i} \sum_{j=0} (b_{j,b} B^j)}{\sum_{j=0} (c_{j,b} B^j)} \right] \quad (2.1.5)$$

---

<sup>1</sup>The  $B$  operator, which is the same as the inverse of the  $z$ -transform ( $B=z^{-1}$ ), shifts the value of a state or variable back one time step. A linear response at time  $j=0$  can be represented as the response one time step back at  $j=1$  plus a constant.

$$y_0 = y_1 + C$$

Using the  $B$  operator, the above equation can be represented as:

$$y_0 = B^1 y_0 + C$$

and the definition of the  $B$  operator becomes apparent.

$$B^i y_j = y_{j+i}$$

Multiplying through by both denominators,

$$q''_t \sum_{j=0} (c_{j,a} B^j) \sum_{j=0} (c_{j,b} B^j) = f_a \sum_{j=0} (c_{j,b} B^j) \left[ T_{t,o} \sum_{j=0} (a_{j,a} B^j) + T_{t,i} \sum_{j=0} (b_{j,a} B^j) \right] \\ + f_b \sum_{j=0} (c_{j,a} B^j) \left[ T_{t,o} \sum_{j=0} (a_{j,b} B^j) + T_{t,i} \sum_{j=0} (b_{j,b} B^j) \right] \quad (2.1.6)$$

and combining common temperature coefficients gives the following combined transfer function equation,

$$q''_t \left[ \sum_{j=0} (c_{j,a} B^j) \sum_{j=0} (c_{j,b} B^j) \right] = \left[ f_a \sum_{j=0} (a_{j,a} B^j) \sum_{j=0} (c_{j,b} B^j) + f_b \sum_{j=0} (a_{j,b} B^j) \sum_{j=0} (c_{j,a} B^j) \right] T_{t,o} \\ + \left[ f_a \sum_{j=0} (b_{j,a} B^j) \sum_{j=0} (c_{j,b} B^j) + f_b \sum_{j=0} (b_{j,b} B^j) \sum_{j=0} (c_{j,a} B^j) \right] T_{t,i} \quad (2.1.7)$$

When multiplying two time series together, the B operator allows the time series to be treated as polynomials, whose order is the time step. Multiplying two polynomials produces a polynomial of higher order.

$$\sum_{j=0}^n (c_{j,a} B^j) \sum_{j=0}^n (c_{j,b} B^j) = \sum_{j=0}^{2n} (c_{j,c} B^j) \quad (2.1.8)$$

Polynomial multiplication increases the number of past time steps of the combined transfer function equation.

Multiplying the time series and combining common powers of the backshift operator gives the following formulation as presented by Seem (1987).

$$q''_{t,c} \sum_{j=0} (c_{j,c} B^j) = T_{t,o} \sum_{j=0} (a_{j,c} B^j) + T_{t,i} \sum_{j=0} (b_{j,c} B^j) \quad (2.1.9)$$

Removing the back shift operator and defining the new coefficients in terms of the old coefficients completes the combination process.

$$q''_{0,c} = \sum_{j=0} [a_{j,c} T_{j,o} + b_{j,c} T_{j,i}] - \sum_{j=1} (c_{j,c} q''_{j,c}) \quad (2.1.10)$$

$$a_{j,c} = \sum_{k=0}^j (f_a a_{k,a} c_{j-k,b} + f_b a_{k,b} c_{j-k,a}) \quad (2.1.11)$$

$$b_{j,c} = \sum_{k=0}^j (f_a b_{k,a} c_{j-k,b} + f_b b_{k,b} c_{j-k,a}) \quad (2.1.12)$$

$$c_{j,c} = \sum_{k=0}^j (c_{k,a} c_{k,b}) \quad (2.1.13)$$

The combined transfer function of equation 2.1.10 has the same form as a single wall transfer function. Other walls can be combined with this result until all of the walls in a zone have been combined into a single equation. As seen above, the computations to produce this combined equation are straight forward and simple, they only involve calculating the new combined coefficients in equations 2.1.11, 2.1.12, and 2.1.13. No assumptions were made in the combination process, so the results are as accurate as the original system of equations. However, since the number of time steps has increased in the equation, no computational speed has been gained. Model reduction techniques, which will reduce the number of past time steps, will be discussed later.

## 2.2 Long-Wave Radiation Networks

The combination method presented above is based upon parallel heat transfer

paths from the outside to the inside of a zone. Actually, the interior surface temperatures are coupled to each other by thermal radiation exchange. Seem (1987) introduced a "star" radiation network that preserves the parallel heat transfer path between the indoor and outdoor temperature. This section discusses the exact and mean radiant temperature network, and converts both networks into the star network.

A three wall room is represented by the radiation and convection network shown in figure 2.2. Each surface is connected to all the other surfaces by thermal radiation, and to the air through convection resistances. The resistance to long-wave radiation exchange between surfaces is

$$R_{i-j,rad} = \frac{1}{\epsilon_i \epsilon_j \hat{F}_{i-j} A_i \sigma 4 \bar{T}^3} \quad (2.2.1)$$

where

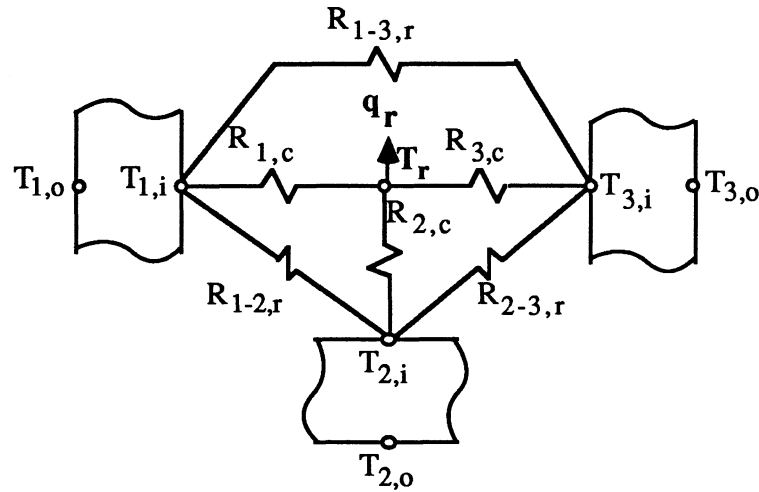
$\epsilon_i$  = emittance of gray surface i

$\epsilon_j$  = emittance of gray surface j

$\hat{F}_{i-j}$  = total exchange factor Beckman (1971). The energy leaving surface i that strikes surface j divided by the energy leaving surface i

$\sigma$  = Stefan-Boltzmann constant

$\bar{T}$  = area weighted average temperature of surfaces i and j



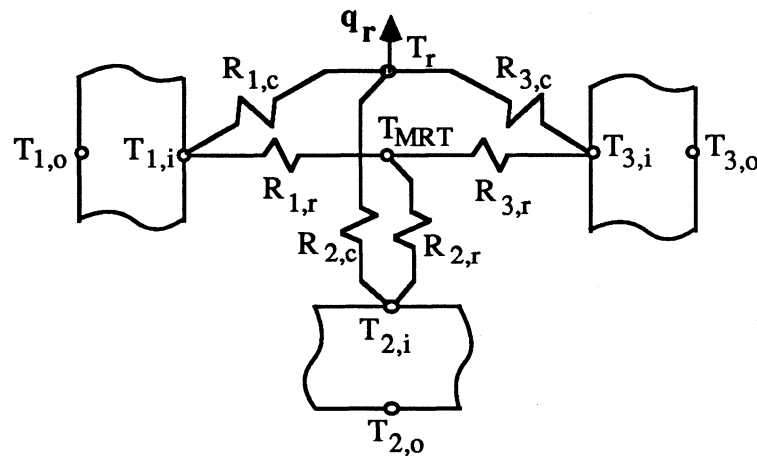
**Figure 2.2** Exact network for three wall room

An alternate form of the radiation network presented by Carroll (1980), the mean radiant temperature network (MRT), approximates the geometric configuration factors with an adjusted area ratio. The network couples each surface in a room to an MRT node, through which all radiation heat transfer occurs. This network has the advantage of requiring less information about the geometry of the zone, and makes it easier to include furnishings in the zone model, since their location is not required. The network only needs the area of each wall in a zone, rather than requiring information to generate the actual configuration factors. Some error is introduced with this network, but it is small for typical zone geometries as shown by Walton (1980). The MRT "exchange factors" are formulated by the following equation, which compensates for the self-weighting of surface  $i$  in the MRT network as noted by Carroll (1980):

$$F_i = \frac{1}{\left(1 - \frac{A_i F_i}{\sum_{j=1}^n A_j F_j}\right)} \quad (2.2.2)$$

The resistances in the radiation network then become:

$$R_{i,r} = \frac{\left(\frac{1}{F_i} + \frac{(1 - \epsilon_i)}{\epsilon_i}\right)}{4 \sigma \bar{T}^3} \quad (2.2.3)$$

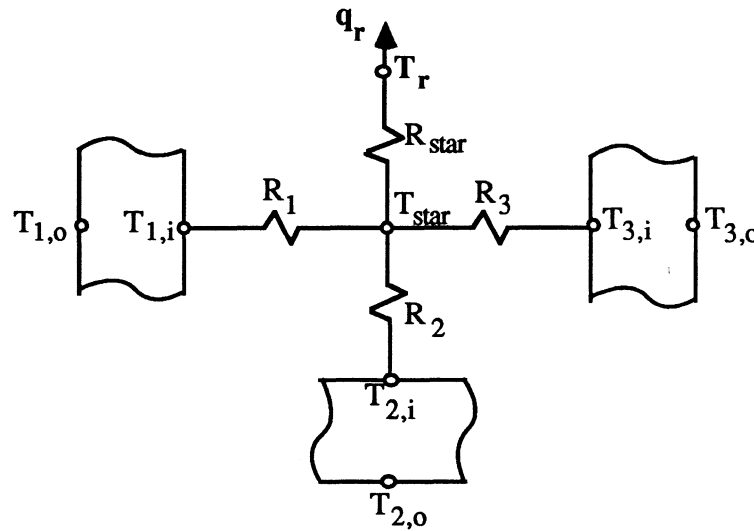


**Figure 2.3** Mean radiant temperature network for three wall zone.

A star network used to approximate a three wall zone consists of a resistance from each wall to the star point and a resistance from the star point to the room air. A star network as shown in figure 2.4 can be determined from either the exact network or the MRT network. Seem (1987) shows that the star network is only exact for a

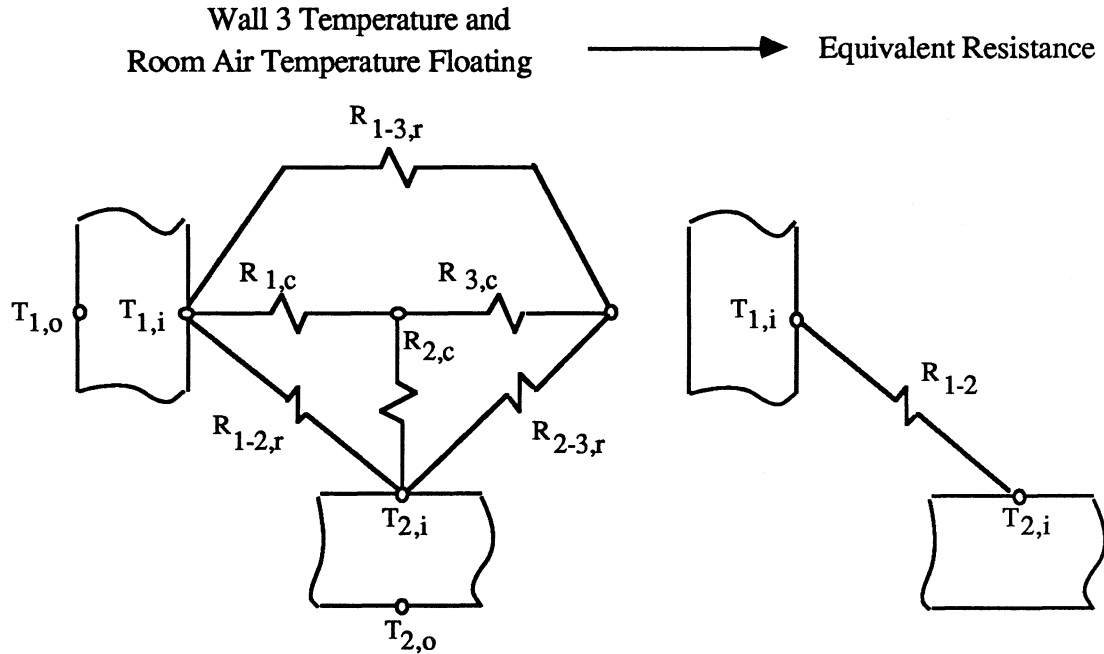


two surface room, or a room having all resistances in the network the same. In formulating the star network, energy balances on each wall and the room air give  $N$  equations, where  $N$  is the number of walls in the room. The star network contains  $N+1$  unknowns: the  $N$  wall resistances and the resistance from the star point to the room air. Section 2.2.3 will present the  $N+1$  equation. First the energy balances in both the exact and the MRT networks will be formulated.



**Figure 2.4** The star network for a three wall zone.

The first step in determining the star resistances is to determine the resistances between each pair of walls, in the exact or the MRT network, when all other temperatures are floating. In other words, a single resistance is found between two walls when heat is being transferred only between those two walls. This procedure is illustrated in figure 2.5 for the exact network.



**Figure 2.5** Reducing the exact network to an equivalent resistance between the only two walls exchanging heat.

The equivalent resistance between two walls can be found by setting up a system of equations representing energy balances on each wall and the room air. By realizing that heat flows only between the two walls of interest (no heat inputs from any other wall or the air), the system of equations can be solved for the equivalent resistance.

### 2.2.1 Energy balances in the exact network

For the exact network, the energy balance on wall  $i$  of a room with  $N$  walls is

$$q_i + \frac{(T_r - T_i)}{R_{i,c}} + \sum_{j=1}^N \frac{(T_j - T_i)}{R_{j-i,r}} = 0 \quad (2.2.4)$$

and the energy balance on the room air is

$$\sum_{j=1}^N \frac{(T_j - T_r)}{R_{j,c}} = q_r \quad (2.2.5)$$

Seem (1987) formulates a system of equations from the above energy balances which can be solved for the temperature differences between each wall, the room air temperature and the room load.

$$X Y = Z \quad (2.2.6)$$

$$X = \begin{bmatrix} x(1,1) & \frac{1}{R_{1-2,r}} & \frac{1}{R_{1-3,r}} & \cdots & \frac{1}{R_{1-N,r}} & 0 \\ \frac{1}{R_{2-1,r}} & x(2,2) & \frac{1}{R_{2-3,r}} & \cdots & \frac{1}{R_{2-N,r}} & 0 \\ \frac{1}{R_{3-1,r}} & \frac{1}{R_{3-2,r}} & x(3,3) & \cdots & \frac{1}{R_{3-N,r}} & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & 0 \\ \frac{1}{R_{N-1,r}} & \frac{1}{R_{N-2,r}} & \frac{1}{R_{N-3,r}} & \cdots & x(N,N) & 0 \\ \frac{1}{R_{1,c}} & \frac{1}{R_{2,c}} & \frac{1}{R_{3,c}} & \cdots & \frac{1}{R_{N,c}} & -1 \end{bmatrix}$$

$$Y = \begin{bmatrix} T_1 - T_r \\ T_2 - T_r \\ \vdots \\ T_N - T_r \\ q_r \end{bmatrix} \quad Z = \begin{bmatrix} -q_1 \\ -q_2 \\ \vdots \\ -q_N \\ 0 \end{bmatrix}$$

where

$$x(i,i) = - \left( \sum_{\substack{j=1 \\ j \neq i}}^N \frac{1}{R_{i-j,r}} \right) - \frac{1}{R_{i,c}}$$

### 2.2.2 Energy Balances in the Mean Radiant Temperature Network

Starting with the MRT network leads to a different formulation of the energy balance. The energy balance on wall  $i$  becomes

$$\frac{(T_r - T_i)}{R_{i,c}} + \frac{(T_m - T_i)}{R_{i,r}} + q_i = 0 \quad (2.2.7)$$

while the energy balance on the room air remains the same as equation 2.2.5.

$$\sum_{j=1}^N \frac{(T_j - T_r)}{R_{j,c}} = q_r$$

An additional equation results from an energy balance on the MRT node at temperature  $T_m$ .

$$\sum_{j=1}^N \frac{(T_j - T_m)}{R_{j,r}} = 0 \quad (2.2.8)$$

The system of equations, which can be solved for the temperature differences between any two nodes in the MRT network, are formulated again as a matrix equation,

$$\mathbf{X} \mathbf{Y} = \mathbf{Z} \quad (2.2.9)$$

The  $\mathbf{X}$ ,  $\mathbf{Y}$ , and  $\mathbf{Z}$  matrices are defined below.

where

$$x(i,i) = -\frac{1}{R_{i,c}} - \frac{1}{R_{i,r}}$$

$$\mathbf{X} = \begin{bmatrix} x(1,1) & 0 & \dots & 0 & \frac{1}{R_{1,r}} & 0 \\ 0 & x(2,2) & \dots & 0 & \frac{1}{R_{2,r}} & 0 \\ \vdots & \vdots & \dots & 0 & \vdots & 0 \\ 0 & 0 & 0 & x(N,N) & \frac{1}{R_{N,r}} & 0 \\ \frac{1}{R_{1,r}} & \frac{1}{R_{2,r}} & \dots & \frac{1}{R_{N,r}} & \sum_{i=1}^N \frac{1}{R_{i,r}} & 0 \\ \frac{1}{R_{1,c}} & \frac{1}{R_{2,c}} & \dots & \frac{1}{R_{N,c}} & \frac{1}{R_{N,c}} & -1 \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} T_1 - T_r \\ T_2 - T_r \\ \vdots \\ T_N - T_r \\ T_m - T_r \\ q_r \end{bmatrix} \quad \mathbf{Z} = \begin{bmatrix} -q_1 \\ -q_2 \\ \vdots \\ -q_N \\ 0 \\ 0 \end{bmatrix}$$

### 2.2.3 Formulating the Star Resistances from the Energy Balances

The equivalent resistance in figure 2.5 between two walls, is found by using the energy balance matrix equation 2.2.6 or 2.2.9, and setting all the heat flows except those between the two walls of interest to zero. Since heat is only flowing between the two walls, the heat flux leaving one wall must be the heat flux entering the other wall. Setting this heat flux arbitrarily to one results in the following equation:

$$q_i = -q_j = 1.0 = \frac{T_i - T_j}{R_{ij}} \quad (2.2.10)$$

Solving for the equivalent resistance  $R_{ij}$ ,

$$R_{ij} = T_i - T_j = (T_i - T_r) - (T_j - T_r) \quad (2.2.11)$$

Substituting the definition of  $\mathbf{Y}$  from the system of equations gives,

$$\begin{aligned} R_{ij} &= y(i) - y(j) \\ &= x^{-1}(i,j) - x^{-1}(i,i) + x^{-1}(j,i) - x^{-1}(j,j) \end{aligned} \quad (2.2.12)$$

The terminology  $x^{-1}(i,j)$  refers to the element at row  $i$  and column  $j$  of the inverse of the  $\mathbf{X}$  matrix. Similarly, the equivalent resistance between surface  $i$  and the room air  $R_{i-r}$  can be defined as

$$\begin{aligned} q_i = q_r = 1.0 &= \frac{T_i - T_r}{R_{i-r}} \\ R_{i-r} &= T_i - T_r \\ &= -x^{-1}(i,i) \end{aligned} \quad (2.2.13)$$

The star network and the exact or the MRT network can be equated by setting the equivalent resistances between each wall and the room air equal to the resistance from the wall to the star point plus the resistance from the star point to the room air. The following  $N$  equations result.

$$R_1 + R = R_{1-r}$$

•  
•  
•

$$R_N + R = R_{N-r} \quad (2.2.14)$$

An additional equation is needed to find the  $N+1$  unknown, the star resistance  $R$ . Seem (1987) suggests that the additional equation can be obtained by minimizing the error between the star network and the exact or MRT network. The equivalent resistance  $R_{ij}$  would be equal to the resistance between wall  $i$  and wall  $j$  in the star network, if the two networks were identical. The error,  $e$ , can then be defined.

$$e = R_i + R_j - R_{ij} \quad (2.2.15)$$

Noting that  $R_i + R = R_{i-r}$  in the star network, the error can be rewritten as

$$e = R_{i-r} + R_{j-r} - R_{ij} - 2R \quad (2.2.16)$$

Squaring and non-dimensionalizing the error term produces the following error function.

$$\psi = \sum_{i=2}^N \sum_{j=1}^{i-1} \frac{(R_{i-r} - R_{j-r} - R_{ij} - 2R)^2}{R_{ij}^2} \quad (2.2.17)$$

Seem (1987) shows that the star network becomes more accurate by placing more weight on lower resistances in the error function, since they will have higher heat transfer for the same temperature difference. This weighting is accomplished by cubing the  $R_{ij}$  term in equation 2.2.17. Setting the derivative of  $\psi$  with respect to  $R$  equal to zero and solving for  $R$  yields:

$$R = \frac{\sum_{j=2}^N \sum_{i=1}^{j-1} (R_{i-r} + R_{j-r} - R_{i-j})}{2 \sum_{j=2}^N \sum_{i=1}^{j-1} \frac{1}{R_{ij}^3}} \quad (2.2.18)$$

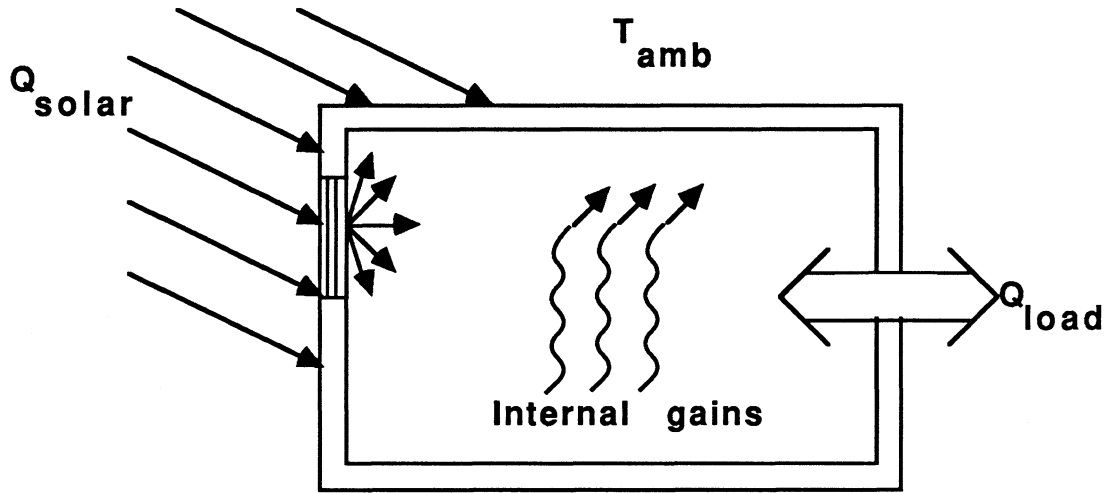
Once the star resistance is found, the wall resistances can be calculated from equation 2.2.14.

Appendix A shows comparisons between the exact network, the MRT network, and the star network, constructed from both the MRT and the exact network, for five different geometries. The star network produced heat flows that are usually within 2% of the exact network heat flows.

### 2.3 Transfer Functions Based on Zone Inputs

The CRTF is a single equation which relates the heat flows into or out of a zone to the energy load on the zone. The energy inputs to the zone are included in the equation when the transfer function for an individual element, wall or roof, is formed. Unlike the element transfer function which relates only the surface temperatures to the heat flux through the element, a CRTF must relate all energy inputs on the zone and the star temperature to the zone heat flux. The possible energy inputs to a building zone are shown in figure 2.6. Instantaneous room air gains, such as ventilation air, are not included in the room transfer functions, since the transfer function represents the time dependent heat flows. The instantaneous gains will be included later when an energy balance is formulated on the zone air.





**Figure 2.6** Energy inputs and outputs to a zone.

Different solar intensities are incident on exterior walls of different orientations, and solar radiation transmitted through the windows is incident on the interior surfaces. The solar energy transmitted through the windows is treated as being totally diffuse. Each wall orientation will have an input in the CRTF for solar radiation. Exterior walls will all see the same ambient temperature, but interior walls will see the temperature of adjacent zones. This consideration leads to the need for a unique ambient input temperature input for each wall adjacent to other zones. The above considerations lead to the following form of a CRTF which relates the zone inputs to the star temperature.

$$q_0 = \sum_{m=0}^{N_z} \sum_{j=0} (d_{j,m} T_{j,m}) + \sum_{j=0} (e_j T_{j,star}) + \sum_{n=1}^{N_s} \sum_{j=0} (f_{j,n} I_{j,n}) + \sum_{j=0} (g_j q_{j,rad}) - \sum_{j=1} (h_j q_j) \quad (2.3.1)$$

where

$N_z$  = number of adjacent zones

$N_s$  = number of solar orientations

$T_{j,m}$  = ambient temperature if  $m=0$ , adjacent zone temperature if  $m>0$

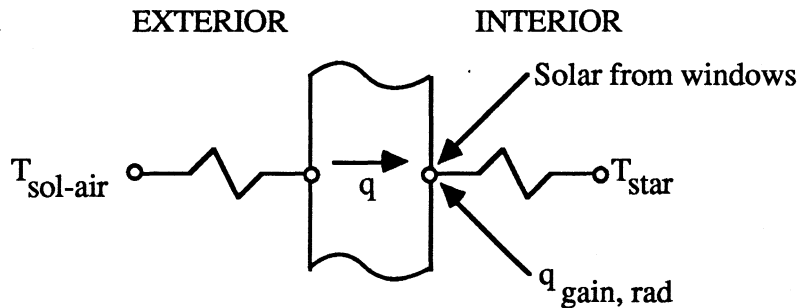
$I_{j,n}$  = incident solar radiation on orientation  $n$

$q_{j,rad}$  = internal radiation gains

The transfer functions for each type of wall (exterior, interior, or window) will be reformulated in terms of the zone inputs, so that they can be combined together using the technique of section 2.1.

### 2.3.1 Exterior wall

A typical exterior wall will have energy flows shown in figure 2.7.



**Figure 2.7** Energy flows for an exterior wall.

The form of a wall transfer function given in ASHRAE (1985) relates the sol-air temperature and the interior wall surface temperature to the heat flow through the wall. This is also the form of the wall transfer function generated by Stephenson and

Mitalas (1971). Equation 2.3.2 gives the heat flux through wall k.

$$q_{0,k} = \sum_{j=0} (a_{j,k} T_{j,k,\text{sol-air}} + b_{j,k} T_{j,k,\text{int}}) - \sum_{j=1} (c_{j,k} q_{j,k,\text{int}}) \quad (2.3.2)$$

The definition of the sol-air temperature can be substituted to give separate inputs for ambient temperature and solar radiation.

$$T_{j,k,\text{sol-air}} = T_{j,k,\text{amb}} + I_{j,k} \alpha_k R_{k,\text{out}} A_k \quad (2.3.3)$$

An energy balance on the interior surface allows radiation gains from people and equipment, and solar gains through windows to be included in the transfer function.

$$q_{j,k,\text{int}} = q_{j,k} - q_{j,k,\text{rad}} - q_{j,k,\text{sol}} \quad (2.3.4)$$

The radiation gains from people and equipment are specified on a zone basis. The energy reaching wall k from the internal radiation sources can be defined as a fraction of the total radiation gain to the zone.

$$q_{j,k,\text{rad}} = \phi_k q_{j,\text{rad}} \quad (2.3.5)$$

where  $\phi_k$ , the fraction of radiation gain absorbed on wall k, can be approximated by the area fraction of wall k in the zone, so as not to require the use of radiation exchange factors in the calculations. The solar gain on the interior surface includes solar radiation transmitted through each window in the zone. Treating each window as a diffuse source of solar radiation, gives the following form of solar radiation transmitted through all windows which is absorbed by the inner surface of wall k.

$$\begin{aligned} q_{j,k,\text{solar}} &= \sum_{n=1}^{N_s} \sum_{w=1}^{N_p} I_n A_{w,n} \tau_w \alpha_k \hat{F}_{w-k} \\ &= \sum_{n=0}^{N_s} I_n \phi_n \end{aligned} \quad (2.3.6)$$

$\phi_n$  = fraction of radiation passing through all windows of orientation  $n$  that is absorbed by wall  $k$ .

$\hat{F}_{w-k}$  = the total exchange factor Beckman (1971). The radiation leaving window  $w$  which strikes surface  $k$  divided by the radiation leaving window  $w$ .

The interior surface temperature is related to the star temperature and the heat flow to the star node by

$$T_{j,k,int} = R_k q_{j,k} + T_{j,star} \quad (2.3.7)$$

Substituting the definition of the sol-air temperature, the energy balance at the interior surface, and the relationship between the star temperature and the interior surface temperature results in the formulation of a transfer function for an exterior wall based on zone inputs.

$$\begin{aligned} q_{0,k} &= \sum_{j=0} \left( d_{j,k} T_{j,amb} + e_{j,k} T_{j,star} + \sum_{n=1}^{N_s} (f_{j,k,n} I_{j,n}) + g_j q_{j,rad} \right) - \sum_{j=0} (h_{j,k} q_{j,k}) \\ d_{j,k} &= \frac{a_{j,k} A_k}{1 - b_{0,k} A_k R_k} \\ e_{j,k} &= \frac{b_{j,k} A_k}{1 - b_{0,k} A_k R_k} \\ f_{j,k,n} &= \sum_{w=0}^{N_n} \frac{c_{j,k} \phi_{w,n} - a_{j,k} \alpha_k R_{out,k} A_k^2}{1 - b_{0,k} A_k R_k} \\ g_{j,k} &= \frac{c_{j,k} \phi_k}{1 - b_{0,k} A_k R_k} \\ h_{j,k} &= \frac{c_{j,k} - b_{j,k} R_k A_k}{1 - b_{0,k} A_k R_k} \end{aligned} \quad (2.3.8)$$

### 2.3.2 Interior Partition

The transfer function for an interior partition would be identical to the exterior wall except that the ambient air coefficient,  $d$ , would be zero and the solar coefficients,  $f$ , would not include incident exterior solar radiation.

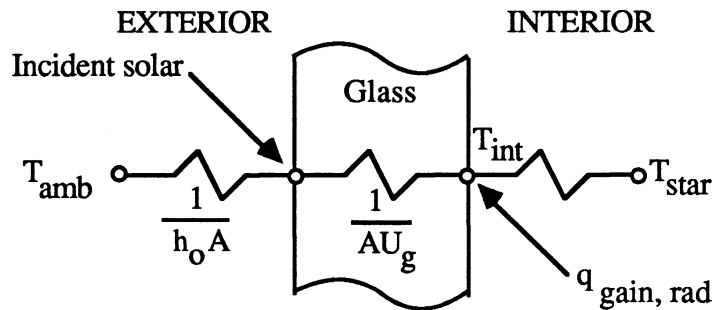
$$f_{j,k,n} = \sum_{w=0}^{N_n} \frac{c_{j,k} \phi_{w,n}}{1 - b_{0,k} A_k R_k} ; \quad d_{j,k} = 0 \quad (2.3.9)$$

### 2.3.3 Interior Wall Separating Two Zones

A wall between two zones also would not have the exterior solar radiation term, and the ambient air temperature input would be replaced by the adjacent zone temperature. The coefficients would be the same as the interior partition, except that the ambient temperature coefficients,  $d$ , would not be zero. The  $d$  coefficients are the same as the  $d$  coefficients of the exterior wall.

### 2.3.4 Windows

Windows are considered as having no thermal capacitance, but since they respond to zone inputs, they must be formulated as a wall to be included in the combination procedure. No capacitance means that there are no past time steps in the transfer function.



**Figure 2.8** Energy flows on a window.

The formulation of transfer function coefficients will be the same as a wall if the window transfer function coefficients are derived as given below. The single window transfer function desired is of the following form.

$$q = a_0 A_w T_{\text{sol-air}} - b_0 A_w T_{\text{int}} - c_0 q \quad (2.3.10)$$

using the sol-air definition, the function becomes,

$$q = a_0 A_w T_{\text{amb}} - a_0 A_w \alpha \frac{I_t}{h_o} + b_0 A_w T_{\text{int}} - c_0 q \quad (2.3.11)$$

A portion of the absorbed incident radiation is conducted to the interior, so the heat flux through a window can be written as,

$$q = \frac{1}{\frac{1}{h_o A_w} + \frac{1}{U_g A_w}} (T_{\text{amb}} - T_{\text{int}}) + I_t \alpha \frac{U_g A_w}{h_o} \quad (2.3.12)$$

Equating the transfer function formulation and the form of the heat flux given above results in the definition of the transfer function coefficients for a window.

$$a_0 A_w T_{amb} + b_0 A_w T_{int} = \frac{1}{\frac{1}{h_o A_w} + \frac{1}{U_g A_w}} (T_{amb} - T_{int})$$

$$a_0 = -b_0 = \frac{1}{\frac{1}{h_o} + \frac{1}{U_g}}$$

The resistance to the ambient air becomes:

$$R_{out} = \frac{1}{h_o A_w} + \frac{1}{U_g A_w} \quad (2.3.13)$$

Now the transfer function of a window is of identical form to the transfer function of an exterior wall, so the function can be rewritten in terms of the zone inputs by treating it as an exterior wall.

### 2.3.5 Formulating the CRTF in Terms of the Zone Air Temperature

Once all of the separate element transfer functions for walls, roof, floor and windows have been reformulated in terms of the same zone inputs, they can be combined two at a time as described in section 2.1. The combined coefficients become:

$$d_{j,c} = \sum_{k=0}^i (d_{j-k,1} h_{k,2} + d_{k,2} h_{j-k,1})$$

$$e_{j,c} = \sum_{k=0}^i (e_{j-k,1} h_{k,2} + e_{k,2} h_{j-k,1})$$

$$\begin{aligned}
f_{j,c} &= \sum_{k=0}^i (f_{j-k,1} h_{k,2} + f_{k,2} h_{j-k,1}) \\
g_{j,c} &= \sum_{k=0}^i (g_{j-k,1} h_{k,2} + g_{k,2} h_{j-k,1}) \\
h_{j,c} &= \sum_{k=0}^i (h_{k,1} h_{j-k,2})
\end{aligned} \tag{2.3.14}$$

The star temperature can be replaced with the room temperature in the combined transfer function equation by using the heat transfer equation through the star resistance.

$$q_r = \frac{T_{\text{star}} - T_{\text{room}}}{R} \tag{2.3.15}$$

Solving the above equation for the star temperature, and substituting it into equation 2.3.14, the CRTF, results in the following:

$$q_{0,r} = \sum_{j=0} \left( \sum_{m=0}^{N_z} (d_{j,m} T_{j,m}) + e_{j,k} T_{j,r} + \sum_{n=1}^{N_s} (f_{j,n} I_{j,n}) + g_j q_{j,\text{rad}} \right) - \sum_{j=0} (h_{j,k} q_{j,r}) \tag{2.3.16}$$

with the coefficients redefined as:

$$d_{j,m} = \frac{d_{j,m,c}}{1 - R e_{0,c}}$$

$$e_j = \frac{d_{j,c}}{1 - R e_{0,c}}$$

$$f_{j,n} = \frac{f_{j,n,c}}{1 - R e_{0,c}}$$



$$\begin{aligned}
 g_j &= \frac{g_{j,c}}{1 - R e_{0,c}} \\
 h_j &= \frac{h_{j,c} - R e_{j,c}}{1 - R e_{0,c}}
 \end{aligned}
 \tag{2.3.17}$$

Equation 2.3.16 is a comprehensive room transfer function which relates ambient temperature, room temperature, solar radiation, and internal radiative gains to the energy load of the zone. The equation, however, has many more past time steps than any of the individual transfer functions, due to the multiplying of time series in the combination algorithm. Model reduction techniques will be used to reduce the number of past time steps so that the CRTF is more computationally efficient.

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## CHAPTER 3

### Model Reduction

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The procedure which combines the transfer function equations representing walls, windows, roofs, and floors into a single CRTF equation, does not reduce the computational effort of modeling the room. The single equation that results from the combination procedure has significantly more past time steps than any of the individual equations used to produce it. This increase in time steps was explained in chapter 2 as resulting from the multiplication of time series together, which is similar to the product of two polynomials having a higher order than the original polynomials. Model reduction techniques are used to reduce the number of past time steps in the combined transfer function equation. The effect is analogous to representing a high order polynomial curve with a polynomial of lower order.

Physical reasons exist to suggest that model reduction should be possible. Consider two identical walls with the heat flow through each wall represented by a transfer function with three past time steps. The sum of the heat flows through the walls can be found by first combining the transfer functions of each wall into a single function. This process would produce a single function with six past time steps, due to the polynomial multiplying effect of the combination process. The total heat flow could also have been found by using one of the single wall transfer functions with twice the wall area, since the walls are identical. This method gives the same total

heat flow as the combined equation, but with fewer past time steps. In a sense, the single transfer function is a reduced form of the combined function since it gives the same results with fewer time step calculations. This example shows that the combination process produces extraneous information which is manifested in having more time steps than actually required. The example also illustrates that it is the model reduction step which gives comprehensive room transfer functions computational efficiency.

Seem (1987) presented two methods of reducing the number of past time steps in a transfer function. The Padé approximation with the bilinear transformation equates power series expansions of the transfer function with a power series expansion of the reduced transfer function. The method produces an output coefficient for each input coefficient, which means it is a method for reducing single input single output transfer functions. The second method, dominant root model reduction, determines the most significant roots of the transfer function. It directly calculates the output coefficients for the reduced transfer function from the dominant roots, and then determines the input coefficients by equating the power series of the pre-reduced transfer function with the power series of the reduced transfer function, much the same way as in the Padé approximation method. The dominant root reduction method only produces a single set of output transfer function coefficients, so it will reduce multiple input single output transfer functions. Stability of the reduced function is guaranteed since the roots are taken from the original set of transfer function roots, which are stable.

This chapter gives an over view of both the Padé approximation and the dominant root model reduction. It does not go into the mathematical detail presented

by Seem (1987), rather it presents the concepts. A final section gives results from the practical application of these methods, including limitations and implementation requirements.

### 3.1 Overview of Padé Approximation

The basis for the Padé approximation is to equate the power series expansion of a continuous domain transfer function with the power series expansion of an unknown reduced transfer function. First the coefficients of the power series expansion are determined from the original transfer function. The power series expansion coefficients are then used to determine the coefficients of a transfer function having a reduced number of time steps.

The starting point of the reduction process is the Laplace transfer function in the form of output divided by input.

$$G(s) = \frac{\sum_{j=0}^n a_j s^j}{\sum_{j=0}^n b_j s^j} \quad (3.1.1)$$

The goal of the reduction process is to approximate the transfer function equation 3.1.1 with the following Laplace transfer function of smaller order, where  $m < n$ .

$$G_r(s) = \frac{\sum_{j=0}^m d_j s^j}{\sum_{j=0}^m e_j s^j} \quad (3.1.2)$$

The original transfer function equation 3.1.1 can be written as a power series

expansion. The limit on the summation of  $2m$  will become apparent when solving for the reduced coefficients.

$$G(s) = \sum_{j=0}^{2m} c_j s^j \quad (3.1.3)$$

Equating the two forms of  $G(s)$ , equation 3.1.1 and equation 3.1.3, results in:

$$\frac{\sum_{j=0}^n a_j s^j}{\sum_{j=0}^n b_j s^j} = \sum_{j=0}^{2m} c_j s^j$$

$$\sum_{j=0}^n a_j s^j = \left( \sum_{j=0}^{2m} c_j s^j \right) \left( \sum_{j=0}^n b_j s^j \right) \quad (3.1.4)$$

The input,  $a$ , and output,  $b$ , coefficients from the original transfer function are known, so the power series expansion coefficients,  $c$ , can be found by equating equal powers of the Laplace variable  $s$  in equation 3.1.4.

The power series expansion of  $G(s)$  is equated to the reduced transfer function  $G_r(s)$  through the power series expansion (equation 3.1.2 is set equal to equation 3.1.3).

$$\frac{\sum_{j=0}^m d_j s^j}{\sum_{j=0}^m e_j s^j} = \sum_{j=0}^{2m} c_j s^j$$

$$\sum_{j=0}^m d_j s^j = \left( \sum_{j=0}^{2m} c_j s^j \right) \left( \sum_{j=0}^m e_j s^j \right) \quad (3.1.5)$$

The right hand side of equation 3.1.5 contains higher order powers of  $s$  than the left hand side, due to the choice of  $2m$  for the upper limit of the power series expansion. These higher order coefficients on the left side of the equation must be equal to zero, which allows the reduced output coefficients,  $e$ , to be determined. This procedure is more evident with a short algebraic example given below.

Equation 3.1.5 is expanded with  $m=2$ .

$$\begin{aligned}
 d_0 s^0 + d_1 s^1 + d_2 s^2 &= (c_0 s^0 + c_1 s^1 + c_2 s^2 + c_3 s^3 + c_4 s^4)(e_0 s^0 + e_1 s^1 + e_2 s^2) \\
 &= (c_0 e_0) s^0 \\
 &\quad + (c_0 e_1 + c_1 e_0) s^1 \\
 &\quad + (c_0 e_2 + c_1 e_1 + c_2 e_0) s^2 \\
 &\quad \dots\dots\dots \\
 &\quad + (c_1 e_2 + c_2 e_1 + c_3 e_0) s^3 \text{ (These } s \text{ coefficients must be zero)} \\
 &\quad + (c_2 e_2 + c_3 e_1 + c_4 e_0) s^4
 \end{aligned} \tag{3.1.6}$$

Since the order of the left hand side of equation 3.1.6 is 2 and the order of the right hand side of the equation is 4, the higher order coefficients of the  $s$  variable must equal zero.

$$\begin{aligned}
 0 &= c_1 e_2 + c_2 e_1 + c_3 e_0 \\
 0 &= c_2 e_2 + c_3 e_1 + c_4 e_0
 \end{aligned} \tag{3.1.7}$$

The  $e_0$  coefficient by definition is equal to one. It is the first output coefficient: the multiplier of the current output (heat flux). The two equations of 3.1.7 are now solved for the remaining output,  $e$ , coefficients. The input coefficients,  $d$ , are found in a similar manner by equating the lower order terms of equation 3.1.6.

$$\begin{aligned}
 d_0 &= c_0 e_0 \\
 d_1 &= c_0 e_1 + c_1 e_0 \\
 d_2 &= c_0 e_2 + c_1 e_1 + c_2 e_0
 \end{aligned} \tag{3.1.8}$$

The preceding discussion has applied to continuous transfer functions in the  $s$  domain, but the transfer functions used to represent heat transfer through walls are in the discrete  $z$  domain. The bilinear transformation is used to convert the discrete  $z$ -transfer functions of the CRTF into continuous transfer functions of the form required by the Padé approximation. Seem (1987) presented the details of the conversion with the bilinear transformation<sup>1</sup> defined as:

$$z = \frac{1 + w}{1 - w} \quad (3.1.9)$$

where  $w$  is a complex variable in the continuous domain having the same stability criteria as the Laplace  $s$  domain. Basically, equation 3.1.9 is substituted into the discrete transfer function at every occurrence of the variable  $z$ . The transfer function is now in the continuous domain and can be reduced with the Padé approximation method.

---

<sup>1</sup>Unit steps are often encountered in simulating discrete inputs in discretely modelled systems. The Laplace transform of a sampled unit step function contains the term  $e^{st}$ , since this term is often found when dealing with discrete systems, a new complex variable was defined.

$$z = e^{st}$$

Solving this equation for  $s$  develops a transformation from the  $s$  to the  $z$  domain.

$$s = \frac{1}{T} \ln(z)$$

Expanding the natural logarithm function into a series

$$s = \frac{2}{T} \frac{z - 1}{z + 1} + \frac{2}{3T} \frac{(z - 1)^3}{z + 1} + \frac{2}{5T} \frac{(z - 1)^5}{z + 1} + \dots$$

and keeping only the first term, results in the following approximation which is referred to as a bilinear transformation.  $T$  is the discrete time step.

$$s = \frac{2}{T} \frac{z - 1}{z + 1}$$

Since the Padé approximation works only with functions having a single input and a single output, the multiple input CRTF must be reformulated as a superposition of single input, single output equations. The CRTF with one solar input and no adjacent zones is given in equation 3.1.10.

$$Q_{0,load} = \sum_{j=0} (d_j T_{j,amb} + e_j T_{j,r} + f_j I_j + g_j q_{j,rad}) - \sum_{j=1} h_j Q_{j,load} \quad (3.1.10)$$

becomes:

$$Q_{0,load,amb} = \sum_{j=0} (d_j T_{j,amb}) - \sum_{j=1} (h_j Q_{j,load,amb})$$

$$Q_{0,load,r} = \sum_{j=0} (e_j T_{j,r}) - \sum_{j=1} (h_j Q_{j,load,r})$$

$$Q_{0,load,solar} = \sum_{j=0} (f_j I_j) - \sum_{j=1} (h_j Q_{j,load,solar})$$

$$Q_{0,load,rad} = \sum_{j=0} (g_j q_{j,rad}) - \sum_{j=1} (h_j Q_{j,load,rad})$$

$$Q_{0,load} = Q_{0,load,amb} + Q_{0,load,r} + Q_{0,load,solar} + Q_{0,load,rad} \quad (3.1.11)$$

This formulation increases the number of terms in the equations since four output time series must be kept rather than just one time series.

The Padé approximation with the bilinear transformation works well in representing a transfer function with one having fewer past time steps as shown by Seem (1987). The main drawbacks of the method are that it requires an output time series for each input term and the stability of the reduced transfer function is not guaranteed.

### 3.2 Overview of Dominant Root Model Reduction

The dominant root model reduction method introduced by Seem (1987) has an



advantage over the Padé approximation method of being able to reduce the CRTF without introducing additional output time series terms. This method also preserves the stability of the transfer function through the reduction process.

The basis of this method is to determine and retain only the dominant roots of the transfer function. The dominant roots are found by analytically determining the response to a step input. This is accomplished by multiplying the z-transform of the transfer function by a z-transform representation of a step input. The roots of the resulting product with the largest effect on the transient response are the dominant roots.

The output coefficients (e from section 3.1) of the reduced transfer function can be calculated directly from the dominant roots. The input coefficients (d from section 3.1) are found by equating the power series expansion of the original transfer function with the reduced transfer function as presented earlier in the Padé approximation.

The dominant root model reduction method is formulated by Seem (1987) to work only with a transfer function having unique roots. Buildings which have identical walls will be represented by a combined transfer function having multiple roots. These multiple roots must be eliminated so that the dominant root method can be applied. A possible method described by Seem (1987) to eliminate multiple roots determines the roots of the input transfer function and cancels common roots appearing in the numerator and the denominator.

Numerical problems can exist when reducing a transfer function. First, the roots need to be accurately determined if the dominant root model is to be used. The combined transfer function from a 15 wall zone may have as many as 90 past time steps, which corresponds to trying to find the roots of a 90th order polynomial.

Finding accurate roots of large order polynomials is difficult if not impossible for polynomials of this size. The roots can, however, be found from the individual transfer functions before they are combined, since the combination process does not alter the roots of the equations. A single wall transfer function usually has less than 6 past time steps, making the root finding easier and more reliable.

The dominant root model reduction preserves the original form of the transfer function, in that only one output time series term is needed regardless of the number of input time series terms. This continuity of form allows each pair of wall transfer functions to be combined and reduced before they are combined with another transfer function. The alternative combines all of the transfer functions together to form a single function with many past time steps before carrying out the reduction process. This large intermediate equation can be avoided by reducing each combined pair of transfer functions in the combination process when using the dominant root model reduction method.

The dominant root model reduction method will maintain the stability of the transfer functions, preserve the form of the transfer functions and allow a combination method that does not generate an intermediate combined transfer function with many past time steps. The method requires, however, that multiple roots of the transfer function be eliminated before the reduction can take place. Seem (1987) shows that this method produces reduced transfer functions whose response compares well with the original transfer function.

### **3.3 Applications of Model Reduction Techniques**

Seem (1987) presented a detailed mathematical representation of the model

reduction techniques. This section outlines difficulties and concerns that appear when trying to implement model reduction.

The TRNSYS CRTF model developed for this thesis uses the Padé approximation as its model reduction technique, because the dominant root model method was not fully developed at the time the CRTF model was being formulated.

The CRTF zone model combines all of the wall elements into a CRTF before any reduction takes place. The combination process produces a CRTF with a large number of past time steps. This large transfer function is converted into the continuous domain using the bilinear approximation. A power series expansion is generated to represent the transfer function. The expansion consists of  $2m + 1$  terms. Typically most CRTFs can be reduced to two past time steps ( $m=2$ ). The expansion coefficients are then used in equation 3.1.5 to determine both the input and output coefficients of the reduced transfer function.

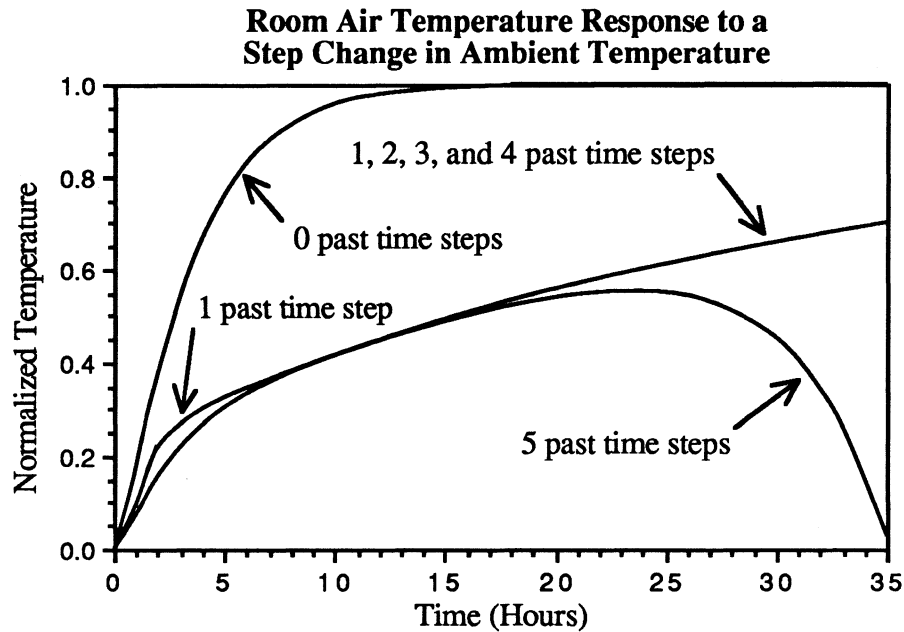
The Padé method does not require any complicated mathematics: it does not need to determine roots of equations. The approximation in the method comes from representing the original transfer function with a power expansion series of finite length. The expansion requires double precision coefficients. Table 3.1 shows the power expansion coefficients that are calculated from a 4 wall zone. The powers series expansion is a diverging series, meaning that the power series coefficients will continuously increase. The  $a$  and  $b$  coefficients are from the continuous domain transfer function representing the heat flows due to changes in ambient temperatures. The number of coefficients in the expansion series that are required is determined by the number of past time steps,  $m$ , in the reduced transfer function. The last coefficient used for different values of  $m$  is shown in table 3.1.

Laplace Transfer Function Ambient Temperature Coefficients

Time	$a_j$	$b_j$	$c_j$	
0	66.95	1.00	66.95	
1	2763.52	44.81	-236.72	
2	37430.10	647.25	4703.93	
3	265381.19	4850.35	-116932.60	
4	1157930.52	22204.32	3014896.19	m=2
5	3344421.43	66833.53	-78109464.29	
6	6604686.93	136691.83	2024862840.90	m=3
7	8979062.44	191382.23	-52495145244.11	
8	8269460.94	180628.77	1360963364982.90	m=4
9	4927279.64	109829.91	-35283704063656.00	
10	1713083.88	38828.89	914749031918900.00	m=5
11	263751.10	6061.22		

**Table 3.1** Power series expansion coefficients from the ambient temperature Laplace transfer function coefficients.

Figure 3.1 illustrates the response of a four wall room to a step change in ambient temperature. The results are shown for CRTFs having from zero to five past time steps. The CRTF without past time steps can not include any wall capacitance. The results show only the capacitance of the room furnishings. The CRTF with just one past time step coefficient has an error representing the initial transient. The 2, 3, and 4 past time step CRTF produce consistent results. When more past time steps are included in the CRTF, the transfer function becomes unstable as illustrated by the response of the CRTF with 5 past time steps, which goes to negative infinity. This instability limits the maximum number of coefficients that can be used in a reduced transfer function.



**Figure 3.1** Response of room temperature to a step change in ambient temperature using transfer functions having different past time steps.

The coefficients of the CRTFs used in figure 3.1 are presented in tables 3.2 and 3.3. The tables show the original set of coefficients before any reduction takes place, along with the coefficients that are generated for the reduced CRTF having from zero to five past time steps. A quick comparison of the first input coefficients, which represent the first response to a transient, should be the same as the pre-reduced coefficients. Only the CRTF with 2, 3, or 4 past time steps match the first coefficients well.

Ambient Temperature Input Coefficients							
Time	Full set	Reduced Sets					
0	46.920	66.951	55.609	46.401	46.921	46.920	44.044
1	-157.343		-49.194	-62.159	-79.665	-86.427	-131.453
2	222.631			17.952	41.654	53.183	133.391
3	-174.293				-7.405	-13.468	-50.969
4	82.586					1.082	2.895
5	-24.173						1.742
6	4.242						
7	-0.407						
8	0.017						
9 - 11	*						
Sum	0.181	66.951	6.416	2.194	1.505	1.290	-0.348

\* smaller than 0.001

**Table 3.2** Ambient temperature output coefficients for CRTF representing a four wall zone.

Ambient Temperature Output Coefficients							
Time	Full set	Reduced Sets					
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	-3.478		-0.904	-1.484	-1.822	-1.966	-3.237
2	5.093			0.517	1.029	1.293	3.793
3	-4.106				-0.184	-0.334	-1.979
4	1.991					0.027	0.453
5	-0.593						-0.035
6	0.105						
7	-0.010						
8 - 11	*						
Sum	0.003	1.000	0.096	0.033	0.022	0.019	-0.005

\* smaller than 0.001

**Table 3.3** Ambient temperature output coefficients for CRTF representing a four wall zone.

The number of past time steps in a reduced CRTF is limited on both the high and the low end. It is desired to have as few past time steps as possible to reduce the computation required, but there must be enough past time steps to accurately model the initial transients. On the other side, there appears to be some numerical stability problems in attempting to include too many past time steps. This may not pose too great of a problem since fewer time steps is always desirable. But this limit does impose an upper bound on the number of past time steps, which may become a problem in representing thermally massive rooms. Heavy rooms respond slowly to changes in their environment. This slow change translates into requiring more past time steps. It might happen that the minimum number of time steps needed to accurately represent the initial transient is greater than the maximum limit imposed by numerical concerns of the Padé reduction method. This scenario, however, has not yet been encountered by any room that has been modeled with the CRTF model based on the Padé approximation with the bilinear transformation.

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## CHAPTER 4

### Building Zone Model

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The preceding chapters have dealt with the transient heat transfer through the structural elements of a building. The transient effects arise from the thermal capacitance of the walls. This has the effect of delaying heat flows through the walls due to ambient conditions or absorbing internal radiation gains from lights or people. Other energy flows act directly and instantaneously on the room air. These gains include: ventilation air, equipment, people, and infiltration. The two types of loads, transient and instantaneous, will be combined together in this chapter to form a model useable for predicting the total energy requirements of an entire zone.

#### 4.1 Energy Balance on a Zone

All of the energy flows into and out of a zone can be related to each other through an energy balance on the zone air. The change in energy of the zone is equal to the sum of the energy flows entering and leaving the zone.

$$C \frac{dT_r}{dt} = \dot{Q}_{\text{room}} \quad (4.1.1)$$

The energy flow to the room,  $\dot{Q}_{\text{room}}$ , includes heat transferred through the walls, heat generated within the zone, and energy associated with convective mass transfer between the zone, ventilation air, adjacent zone air, and ambient air. Equation 4.1.1



can be approximated by assuming that temperatures vary linearly over a discrete time step,  $\delta$ . This assumption facilitates the use of the comprehensive room transfer function, which is based on discrete time steps, and the formulation of the equation for use in a simulation model with discontinuous inputs (such as people gains). Further definition of the average zone energy flows over the time step results in equation 4.1.2.

$$C \left( \frac{T_{0,r} - T_{1,r}}{\delta} \right) = \dot{Q}_{\text{vent}} + \dot{Q}_{\text{infl}} + \dot{Q}_{\text{people}} + \dot{Q}_{\text{inst}} + \dot{Q}_{\text{zones}} + \dot{Q}_{\text{CRTF}} \quad (4.1.2)$$

where

$C$  = lumped capacitance of the room air (including furnishings).

$\dot{Q}_{\text{vent}}$  = controlled ventilation load from air conditioning equipment.

$\dot{Q}_{\text{infl}}$  = uncontrolled ventilation load due to infiltration.

$\dot{Q}_{\text{people}}$  = sensible load due to people.

$\dot{Q}_{\text{inst}}$  = miscellaneous instantaneous sensible loads.

$\dot{Q}_{\text{zones}}$  = load due to convection with adjacent zones.

$\dot{Q}_{\text{CRTF}}$  = structural loads represented by a transfer function.

$T_{0,r}$  = zone temperature at time  $t$ .

$T_{1,r}$  = zone temperature one time step  $\delta$  prior to time  $t$ .

The ventilation, infiltration, and zone convection energy gains depend on a difference between the room air temperature and the input temperature. The temperature difference driving the heat transfer during a time step is approximated as the average temperature difference over the time period, resulting in the following formulations:

$$\dot{Q}_{\text{vent}} = \dot{m}_v C_p (\bar{T}_v - \bar{T}_r) \quad (4.1.3)$$

$$\dot{Q}_{\text{infl}} = \dot{m}_f C_p (\bar{T}_{\text{amb}} - \bar{T}_r) \quad (4.1.4)$$

$$\bar{T} = \frac{T_0 + T_1}{2} \quad (4.1.5)$$

Convected air through hallways and doors makes up the instantaneous energy transfer between zones. The conducted heat flows through walls separating adjacent zones are included in the zone transfer function. Again, the temperatures are averaged over the time step for the convection heat transfer between zones.

$$\dot{Q}_{\text{zones}} = \sum_{i=1}^{N_z} UA (\bar{T}_{z,i} - \bar{T}_r) \quad (4.1.6)$$

The sensible load due to people is found from tabular values in the ASHRAE Handbook of Fundamentals (1985) for people at different activity levels. Of the total sensible people load, 30% is assumed to be an instantaneous sensible load convected directly to the room air, while the remaining 70% is treated as a radiative gain to the walls.

The CRTF Equation 2.3.16 represents the loads due to the ambient temperature, solar radiation, and radiative internal gains. The transfer function of equation 2.3.16 is based on inputs and outputs at discrete times. The energy balance equation 4.1.2 is based on averaged temperatures and heat flows. Fortunately, the same transfer function can be used in the energy balance with averaged inputs and outputs. The following equations show that the same transfer function that produces instantaneous outputs from instantaneous inputs will give averaged outputs from averaged inputs.

The average heat flux over one time step can be written as:

$$\dot{Q}_{t,\text{room}} = \frac{\dot{Q}_{t,\text{room}} + \dot{Q}_{t-\delta,\text{room}}}{2} \quad (4.1.7)$$

A single input single output transfer function based on the room temperature gives the instantaneous heat flux output and requires instantaneous inputs.

$$\dot{Q}_{0,room} = \sum_{j=0} (e_j T_j) - \sum_{j=1} (h_j \dot{Q}_{j,room}) \quad (4.1.8)$$

Equation 4.1.8 can also be used to find the instantaneous heat flux one time step prior to the current time.

$$\dot{Q}_{1,room} = \sum_{j=0} (e_j T_{j+1}) - \sum_{j=1} (h_j \dot{Q}_{j+1,room}) \quad (4.1.9)$$

Substituting equation 4.1.8 and 4.1.9 into the average heat flux equation 4.1.7 and combining the terms having common summation intervals yields:

$$\begin{aligned} \dot{\bar{Q}}_{0,room} &= \frac{\sum_{j=0} (e_j T_j + e_j T_{j+1}) - \sum_{j=1} (h_j \dot{Q}_{j,room} + h_j \dot{Q}_{j+1,room})}{2} \\ \dot{\bar{Q}}_{0,room} &= \sum_{j=0} \left[ e_j \left( \frac{T_j + T_{j+1}}{2} \right) \right] - \sum_{j=1} \left[ h_j \left( \frac{\dot{Q}_{j,room} + \dot{Q}_{j+1,room}}{2} \right) \right] \\ \dot{\bar{Q}}_{0,room} &= \sum_{j=0} (e_j \bar{T}_j) - \sum_{j=1} (h_j \bar{\dot{Q}}_{j,room}) \end{aligned} \quad (4.1.10)$$

Equation 4.1.10 is the same transfer function as equation 4.1.8, except with averaged inputs and outputs. Therefore, the same transfer function based on instantaneous inputs and outputs can be used to produce averaged outputs from averaged inputs, as expected since this is a linear system.

The instantaneous room temperature, which is unknown, is contained in the lumped zone capacitance term in the energy balance, and in the average room temperature found in the transfer function equation and the convected air loads. The individual load definitions can be substituted into the zone energy balance equation, and the average room temperature at time  $t$  can be replaced with its definition, so that

the energy balance can be solved for the instantaneous room temperature  $T_r$ .

$$T_{r,t} = \frac{\left[ T_{r,t-\delta} \left( \frac{C}{\delta} - S \right) + \dot{m}_v C_p \bar{T}_v + \dot{m}_f C_p \bar{T}_{amb} + \sum_{i=1}^{N_z} (UA)_i \bar{T}_{z,i} + 0.3 \dot{Q}_{people} + \dot{Q}_{inst} + \dot{Q}_{TF} \right]}{\left( \frac{C}{\delta} + S \right)} \quad (4.1.11)$$

where,

$$S = \frac{\dot{m}_v C_p}{2} + \frac{\dot{m}_f C_p}{2} + \frac{\sum_{i=1}^{N_z} (UA)_i}{2} - \frac{e_0}{2}$$

and  $\dot{Q}_{TF}$  is the CRTF with the current time zone temperature term,  $e_0 \bar{T}_{0,r}$ , removed from the CRTF so that  $T_r$  can be solved for.

$$\dot{Q}_{TF} = \sum_{j=0}^{N_z} \left( \sum_{m=1}^{N_z} (d_{j,m} \bar{T}_{j,m}) + \sum_{n=1}^{N_s} (f_{j,m} \dot{I}_{j,n}) + g_j \dot{q}_{j,rad} \right) + \sum_{j=1}^{N_z} (e_j \bar{T}_{j,r} - h_j \dot{Q}_{j,TF})$$

Once the instantaneous room temperature is found, the average room temperature can be calculated.

$$\bar{T}_{0,r} = \frac{T_{0,r} + T_{1,r}}{2} \quad (4.1.12)$$

An energy balance on the room air has been used in this section to solve for the room air temperature. The energy balance consists of a single equation with terms to account for structural loads, instantaneous air loads, and internal gains. The model assumes linear behavior over the time step period, so that average values of energy gains and temperatures can be used to represent the average energy flows through a room over the simulation time step period.

## 4.2 Model Operation

A simulation based on the above model can be run in two modes of operation; a temperature control mode, and an energy rate control mode. Once the temperature of the zone is calculated with respect to all of the energy transfers, the model can do nothing, or take an action to control the room temperature. This section outlines the two modes, detailing the calculation of sensible and latent room loads, that can be used when running building simulations with the model.

In temperature level control mode, the zone temperature reflects the ambient conditions and the heating or cooling equipment inputs. No limits are set on the zone temperature. An independent controller is required to command the heating and cooling equipment. Heat is added or removed through ventilation flow streams or instantaneous inputs as directed by an external controller. The model simply reacts to all of the inputs. This mode is useful in studying control strategies, since the control functions are independent of the model.

With energy rate control, the zone temperature is checked against a set temperature range. The control of the zone is handled within the model. If the calculated zone temperature is outside the range, then the zone temperature is held at the limit and the energy required to maintain the zone temperature at the limit is calculated.

$$\dot{Q}_{\text{load}} = C \left( \frac{T_{\text{set}} - T_{1,r}}{\delta} \right) + \dot{Q}_{\text{vent}} + \dot{Q}_{\text{infl}} + \dot{Q}_{\text{people}} + \dot{Q}_{\text{inst}} + \dot{Q}_{\text{zones}} + \dot{Q}_{\text{CRTF}} \quad (4.2.1)$$

If the limits are not exceeded, then no energy load exists. Energy rate control assumes that the load is exactly met at each time step; the model gives the energy required by the heating or cooling equipment to maintain the zone temperature. This mode of operation is suited for predicting energy load under given conditions.

The formulation of latent loads is identical to the TRNSYS type 19 zone model. A moisture balance on the room air is solved for the zone humidity ratio. In temperature level control mode, the humidity ratio reflects the ambient conditions and the heating or cooling equipment moisture inputs. No limits are set on the zone humidity ratio. It is free to respond to the moisture inputs and outputs. Moisture is added or removed through ventilation flow streams or instantaneous inputs as directed by an external controller. In energy rate control mode, the latent load is the energy required to bring the zone humidity ratio within a specified range. When the humidity ratio would fall outside the limits, the humidity ratio is set equal to the limit and a latent load is calculated from equation 4.2.2., as presented in the TRNSYS user's manual, otherwise the latent load is zero.

$$\dot{Q}_{\text{lat}} = \Delta h_{\text{vap}} \left[ \dot{m}_f (\omega_{\text{amb}} - \omega_r) + \dot{m}_v (\omega_v - \omega_r) + \omega_i \right] \quad (4.2.2)$$

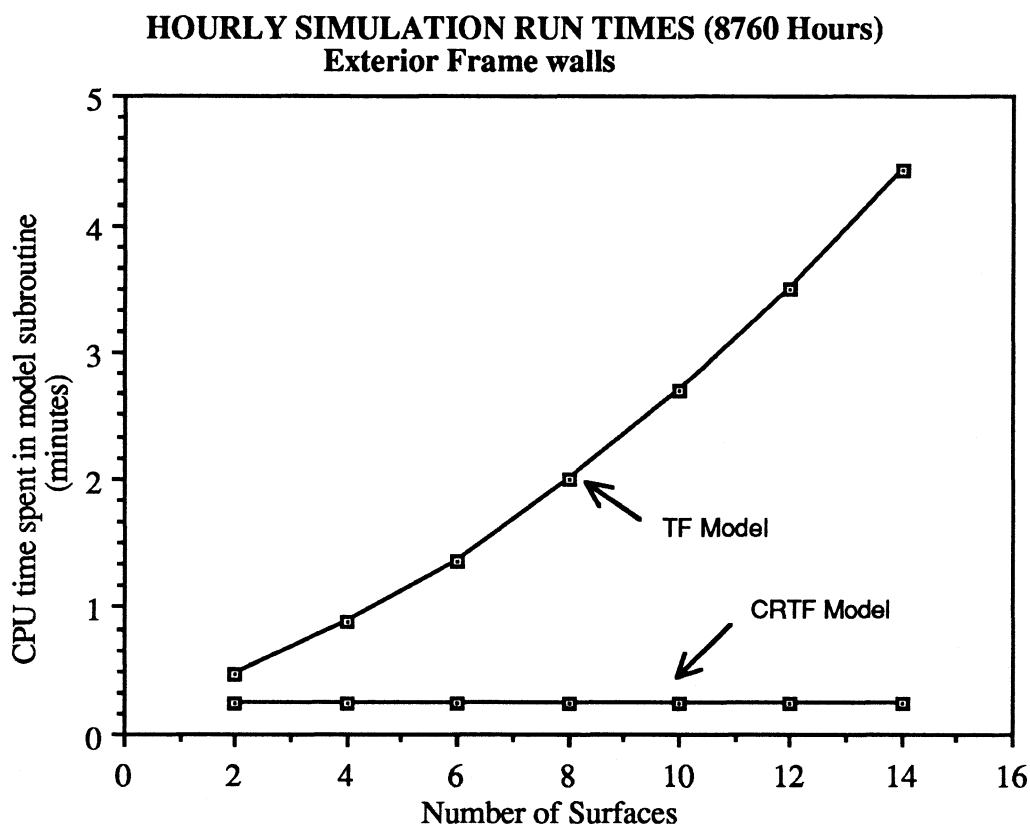
### 4.3 Model Performance

The most significant difference between the comprehensive room transfer function method and the heat balance transfer function (TF) method of modeling building energy loads, is that the energy balance in the CRTF model consists of only a single equation while the TF method uses an energy balance on each element in the zone formulating a system of equations. The single equation energy balance used in the CRTF model gives the zone temperature immediately, without having to solve a system of equations for wall surface temperatures and then calculate heat flows through each element in a zone. This intermediate temperature and heat flow information has been eliminated resulting in a decrease in computational effort. The following subsections show examples of the CRTF model's accuracy and calculation

time when compared to the TF method.

#### 4.3.1 Computational speed

Single zone simulations were run to show the calculation time differences between the CRTF model and the TRNSYS type 19 zone model which is based on solving a system of transfer functions. Figure 4.1 shows the CPU time spent within the zone routine on a Digital Equipment MicroVAX II computer.



**Figure 4.1** Hourly simulation of a single zone building for a period of one year. The CRTF model and the TF model of TRNSYS type 19. CPU time spent within the zone subroutine.

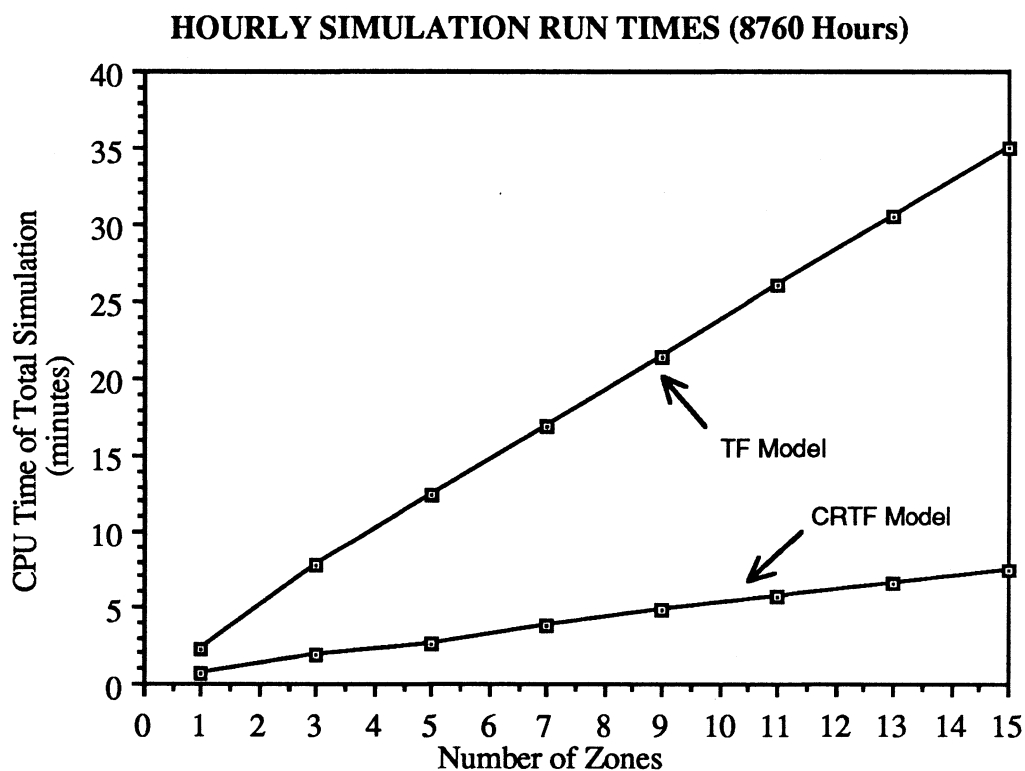
The simulations were run with hourly time steps for a period of one year. The CRTF model consumes 15 seconds of CPU time regardless of the number of walls comprising the zone. This result is expected since the CRTF model reduces the system of equations into a single equation, which has the same form (the same number of past time steps) regardless of the number of walls in a zone. The TF model is eighteen times slower for a fourteen surface room. Typical rooms will have between six and eight surfaces where the CRTF model is between five and eight times faster. These timings, however, include no overhead associated with the simulation, since they only measure the time spent in the zone model subroutine. The actual time savings realized for an entire simulation will depend on the portion of time spent in the zone model. A complex simulation with many other subroutines modeling equipment and loads will have less of an overall computational time savings than a simple simulation including only the zone model.

The difference in computation time between the two models is further increased when running multiple zone building simulations. The CRTF model has one equation per zone, while the type 19 model will have a system of equations per zone. Figure 4.2 shows the simulation time for a simple model including only the building zones. Each zone consists of six walls.

A more complex simulation with a larger temperature difference between zones would have caused TRNSYS to perform its successive substitution iteration scheme, since the zones are coupled together. TRNSYS calculates each zone temperature successively since each zone is represented by a separate component subroutine. With the zones coupled, the first zone energy balances calculated will use old values of the adjacent zone temperatures. TRNSYS checks the difference between the zone temperature used and the actual updated temperature. If the difference is greater than



a specified tolerance, TRNSYS recalculates the zone temperatures. This recalculation has the same effect of adding more zones to the simulation. A five zone building with three iterations per time steps will require as much computation time as a fifteen zone building without iterations.



**Figure 4.2** Hourly simulation of multi-zone buildings for a period of one year. Comparison of the CRTF model and the TF model of TRNSYS type 19. Six walls per zone.

A multiple zone model which solves the equations of the entire building simultaneously would eliminate the need for the iterations. The CRTF model can be formulated as a system of equations that will solve a multi-zone building simultaneously. The energy balance equation which will be solved for the room temperature is shown in equation 4.3.1

$$\mathbf{A} \mathbf{T}_{0,\text{zone}} = \mathbf{B} \mathbf{T}_{1,\text{zone}} + \mathbf{C} \quad (4.3.1)$$

$$\mathbf{A} = \left[ \left( \frac{C}{\delta} + \frac{\dot{m}_{v,i} C_p}{2} + \frac{\dot{m}_{f,i} C_p}{2} - \frac{e_{0,i}}{2} + \sum_{j=1}^{N_z} \frac{UA_{i-j}}{2} \right) \delta_{i,j} + \left( -\frac{UA_{i-j}}{2} - \frac{d_{0,i-j}}{2} \right) (1 - \delta_{i,j}) \right]$$

$$\mathbf{B} = \left[ \left( \frac{C}{\delta} - \frac{\dot{m}_{v,i} C_p}{2} - \frac{\dot{m}_{f,i} C_p}{2} + \frac{e_{0,i}}{2} - \sum_{j=1}^{N_z} \frac{UA_{i-j}}{2} \right) \delta_{i,j} + \left( \frac{UA_{i-j}}{2} + \frac{d_{0,i-j}}{2} \right) (1 - \delta_{i,j}) \right]$$

$$\mathbf{C} = \left[ \dot{m}_v C_p \bar{T}_{v,i} + \dot{m}_f C_p \bar{T}_{amb} + \dot{Q}_{TF,i} + \dot{Q}_{inst,i} + \dot{Q}_{people,i} \right]$$

$$\mathbf{T}_{0,\text{zone}} = [T_{0,\text{zone},i}]$$

$$\mathbf{T}_{1,\text{zone}} = [T_{1,\text{zone},i}]$$

where  $\dot{Q}_{TF,i}$  is the CRTF with the current time,  $t$ , zone temperatures removed.

Also,  $\delta_{i,j} = 1$  when  $i=j$  and  $\delta_{i,j} = 0$  when  $i \neq j$ .

The matrix  $\mathbf{A}$  can be made constant by assuming constant air flow between zones, constant infiltration air flow rate, and constant ventilation air flow rate. These assumptions may be acceptable for some limited cases where these items would normally be constant or have little effect. Another possible simplifying assumption to make the  $\mathbf{A}$  matrix constant involves approximating the temperature difference used in calculating the energy flow from infiltration, ventilation, and zone air exchange with the temperature difference at the prior time step. By replacing the current room temperature with the last room temperature, the  $0.5 \dot{m} C_p T_{0,\text{zone}}$  terms in the energy balance would be replaced with a  $0.5 \dot{m} C_p T_{1,\text{zone}}$  terms. Since the current room temperature is no longer in the infiltration, ventilation, or zone convection heat flow equations, the  $\dot{m}_f$ ,  $\dot{m}_v$ , and zone  $UA$  terms would not appear in matrix  $\mathbf{A}$ ,

making it time invariant.

The CRTF model is considerably faster than a model based on a system of equations representing heat flows through each individual element in a room. The speed difference becomes greater with zones having more walls, due to the form of the CRTF equation remaining unchanged, whereas the TF method adds an equation for each wall added to a zone. The CRTF model is also well suited for formulation as a multi-zone model, in which an entire building can be represented by a system of equations made up of a single equation for each zone.

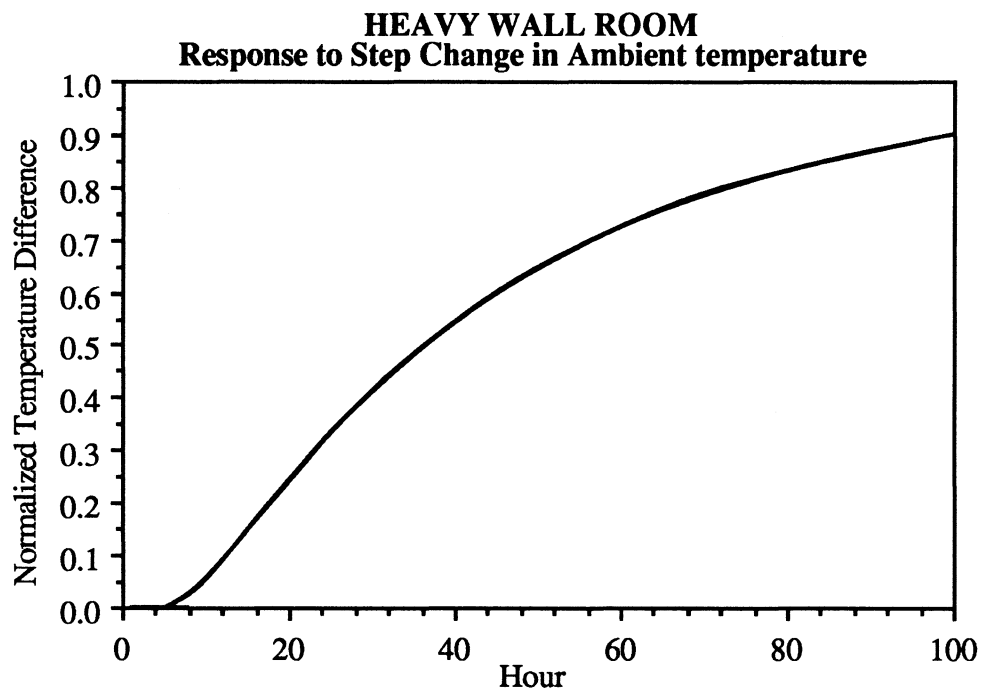
#### **4.3.2 Model Accuracy**

The accuracy of the CRTF model was judged against the TRNSYS type 19 model, which is based on the TF method. Simulations on different geometries of buildings made of different building materials were run while varying the possible energy load inputs. The following figure shows typical results of simulations run with both models. The differences between the two models are nearly indistinguishable. The only approximation in the CRTF method is from the star network, which, as shown in Appendix A, produced results within 2% of the exact network results. The results of a simulation on a two wall room are shown in table 4.1 and figure 4.3.

**Heavy Wall Room**  
**Response to Step Change in Ambient Temperature**

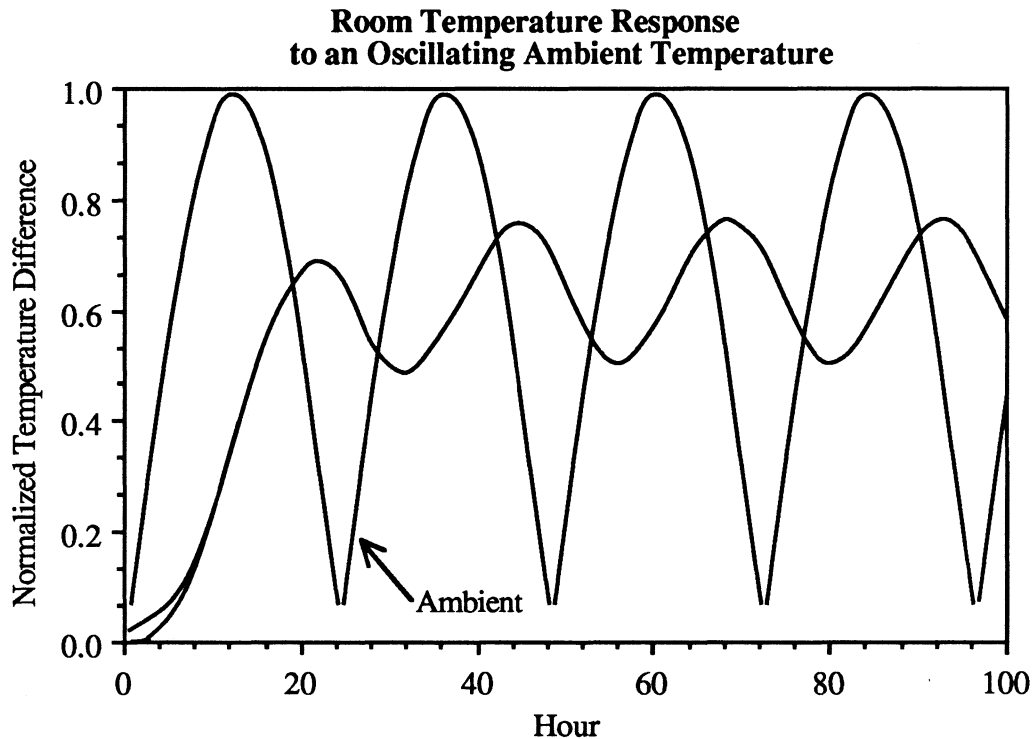
Time Hour	Room Temperatures		Heat Flows	
	TF	CRTF	TF	CRTF
1	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.89
3	0.00	0.00	0.02	-0.91
4	0.00	0.00	0.29	-0.07
5	0.00	0.00	0.96	1.04
6	0.01	0.01	1.77	1.97
7	0.02	0.02	2.51	2.67
8	0.04	0.04	3.08	3.17
9	0.05	0.05	3.48	3.52
10	0.07	0.07	3.75	3.76
11	0.09	0.09	3.92	3.91
12	0.11	0.11	4.01	3.99
13	0.13	0.13	4.05	4.02
14	0.15	0.15	4.04	4.02
15	0.17	0.17	4.01	3.99
16	0.19	0.19	3.97	3.94
17	0.21	0.21	3.90	3.88
18	0.23	0.23	3.83	3.82
19	0.25	0.25	3.76	3.74
20	0.27	0.27	3.68	3.66

**Table 4.1** Response of a room consisting of two heavy walls to a step change in ambient temperature. Comparison of TF and CRTF models.



**Figure 4.3** Comparison of CRTF model and TF model for a two wall zone consisting of heavy walls.

Figure 4.4 shows the response of a four surface room to an oscillating ambient temperature. Again the results of the two models are nearly the same. The initial difference is due to the initial values of the histories of the transfer function time series. The CRTF model fills the histories with the steady state solution at the initial values of the simulation inputs. An alternate method, which is used by other programs, involves running the simulation for a period of time before the point of interest in the simulation is reached. This method attempts to fill the time series histories with more realistic values containing the dynamics of the system.



**Figure 4.4** Comparison of CRTF model and TF model for a four wall zone consisting of frame construction, with an oscillating ambient temperature.

#### 4.4 Comparison of CRTF Model and TF Model Capabilities

The preceding sections have shown that the CRTF model is much faster and as accurate as the Transfer Function (TF) method. This section addresses the trade-offs and simplifications that were made to obtain this increase in computation speed, by noting the items that were included in the TF type 19 model but could not be handled in the CRTF model. The CRTF gains its speed by not solving for intermediate results, such as interior wall surface temperatures and individual wall heat fluxes. These quantities are no longer available, even though the CRTF method requires the same amount of data as the TF method to describe the energy loads in a zone.

The CRTF approach assumes that properties are constant throughout the simulation, since the paths which are represented by transfer functions and heat transfer resistances do not retain their identity through the combination and reduction process. Exterior and interior convection coefficients must be assumed constant through out the simulation. The convection coefficients are a part of the heat transfer resistances which are combined with the radiation resistances in the star network. The star resistances become part of the transfer function coefficients in the CRTF. When the CRTF is reduced, all references to individual variables are lost, making it impossible to change the convection coefficients during the simulation. It is difficult to accurately determine these convection coefficients, so approximating them with a constant value might not introduce any more error than trying to guess their magnitudes as a function of other variables.

Conductances of windows must also remain constant. The UA of a window was allowed to change in the TF method to simulate the application of night insulation. Since the window has been treated as a wall and combined into the zone transfer function, the conductance of the window is no longer available to be modified as in the TF method. A method is presented in chapter 5 to overcome this limitation.

Solar radiation is treated as totally diffuse radiation. No beam radiation calculations are made, since geometry information has been lost. The TRNSYS single zone model allowed for beam radiation to strike separate surfaces. The user had to determine the beam radiation view factors independently, which is a difficult task in itself. Diffuse radiation probably is the most common form of solar radiation incident on windows in most instances so this limitation is not very restrictive.

The importance of these approximations must be judged by the expectations of

the model user. The CRTF method should provide accurate results for the majority of building simulation requirements with significantly less computation time than the TF method. The variables which are no longer available to be modified during the simulation should have little effect on the outcome of energy predicting simulations for mid and large size buildings, where the structural energy flows are dwarfed in comparisons to internal gains. Chapter 5 presents possible methods to overcome some of the restrictions mentioned here.



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## **CHAPTER 5**

### **Topics in Modeling with the CRTF**

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The CRTF model is faster and as accurate as the TF model for simulating energy flows in a building zone. The CRTF model is quicker because it does not calculate intermediate results such as wall surface temperatures or individual wall heat fluxes. This information, which is not available in the CRTF model, maybe needed to include certain effects in the model. For example, night insulation on windows requires the modification of the heat flow through an individual building element, the window. Comfort calculations depend on a mean radiant temperature which is a function of wall surface temperatures.

Discrete time base transfer functions carry an implicit simulation time step with them. This time step may or may not be convenient for the purpose of the simulation. The simulation may need to run at a smaller time step than the time base of the transfer function.

This chapter looks at ways of overcoming the loss of needed information and the consequences of the transfer function on the simulation time step, so that these effects can be included in the CRTF model.

#### **5.1 Night Insulation**

Night insulation refers to adding an insulated covering, such as a quilted fabric, over windows at night. Modeling night insulation is a specific example of a larger

class of problems that involve changing the building parameters during the simulation. Physical properties such as convection coefficients, conductances and transmittances could vary over the simulation period. This section describes the problem introduced by adding night insulation to windows during a simulation.

The TRNSYS type 19 zone model, which calculates the heat flows through each room element transfer function separately, referred to as the transfer function model (TF), allows the window conductance to change with time, to account for the added insulation. The thermal conduction through the window from the ambient is given as:

$$\dot{Q} = U A (T_a - T_r) \quad (5.1.1)$$

The overall loss coefficient,  $U$ , which is the reciprocal of the sum of the resistances of the window ( $1/U_g$ ), the outside air ( $1/h_o$ ), and the inside air ( $1/h_i$ ), is an input that can vary at each time step of the simulation. The window and its insulating cover are assumed to have no thermal capacitance. By changing  $U$ , the model can continue to calculate the heat flow through the window element as before, but with a different loss coefficient.

The CRTF model does not contain separate information about individual windows, since the entire system is represented by a single transfer function equation, which is simplified below, for illustration purposes by neglecting absorbed solar radiation and internal radiation gains.

$$\dot{Q}_{0,\text{total}} = \sum_{j=0} (d_j T_{j,a}) + \sum_{j=0} (e_j T_{j,r}) - \sum_{j=1} (h_j \dot{Q}_j) \quad (5.1.2)$$

Adding insulating material onto a window results in an entirely new equation with different transfer function coefficients.

$$\dot{Q}_{0,\text{total}} = \sum_{j=0} (d'_j T_{j,a}) + \sum_{j=0} (e'_j T_{j,r}) - \sum_{j=1} (h'_j \dot{Q}_j) \quad (5.1.3)$$

This representation is not limited to night insulation, since changes in any of the physical properties of the system would result in a new set of transfer function coefficients. Exterior convection coefficients could be scheduled to change on a day-night basis using a different transfer functions for each convection coefficient. The window with night insulation can now be modeled as a glass wall with insulating material on its inner surface. Modeling an insulated window as a wall, would allow insulation capacitance to be included, and would correctly exclude day time solar radiation if the insulating material were opaque and left in place during the day light hours.

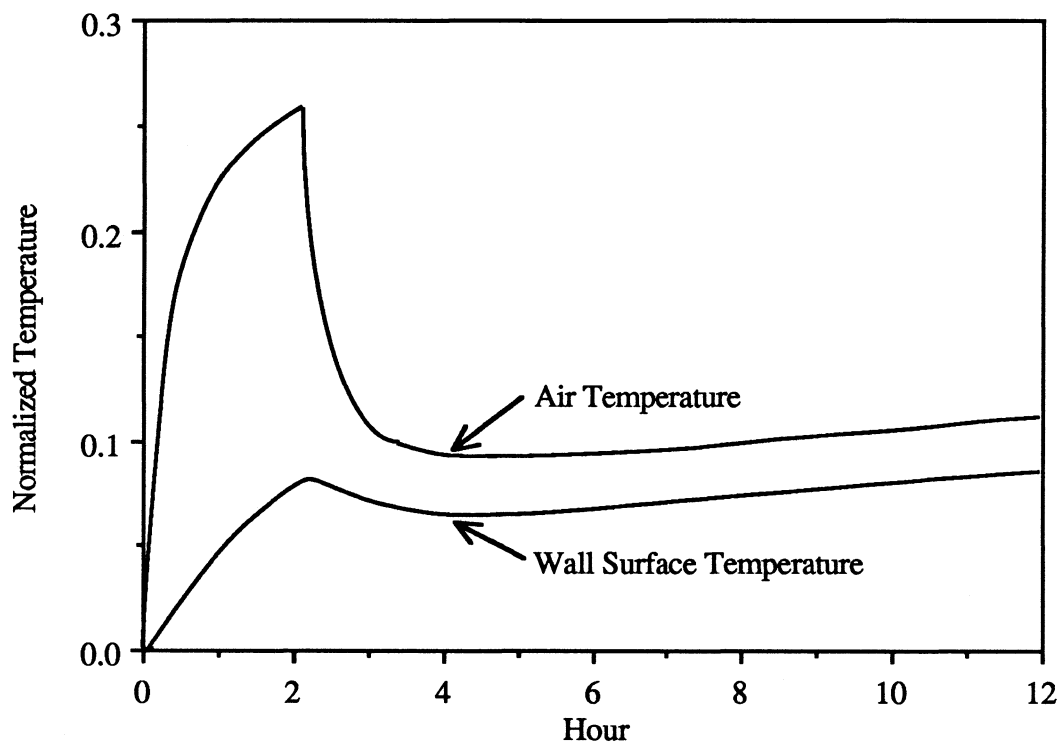
The CRTF model can account for changes in the system by switching to the new transfer function equation representing the new system. Only the transfer function coefficients are replaced. The past input and output histories remain unchanged. This method produces the same results as the TF model when used to change the overall loss coefficient of a window.

The following sub-sections compare the transfer function modeling of night insulation against a finite difference model of the same situation. Both modes of operation of the model were checked: the temperature level control model, where the air temperature of the zone is free to respond to the inputs on the zone, and the energy rate mode, where the air temperature is held at a fixed value.

### **5.1.1 Floating Room Air Temperature**

Errors occur in both the CRTF and the TF model if the window conduction is the major heat transfer path to a thermally heavy room, and the room air temperature is not constant. The errors arise from the assumption that with transfer functions there

is a uniform temperature distribution in a wall when a simulation begins. Actually, in a heavy wall, the core temperature would be lower than its surface temperature if the room air temperature is rising. To illustrate the problem, consider a finite difference simulation of a window in front of a large concrete wall, with a step change in ambient temperature at time zero, and night insulation added at hour 2.

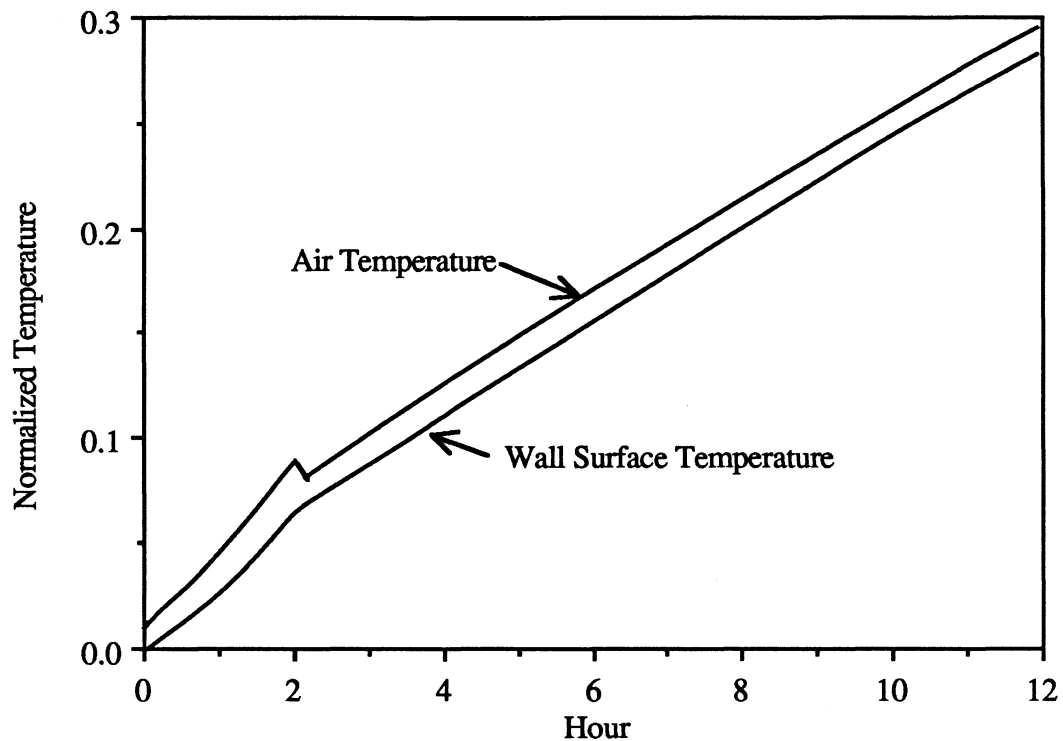


**Figure 5.1** Normalized temperature response of a concrete wall room with 100% exterior window area to a step change in ambient temperature. Finite difference simulation. Window insulation added at hour two.

As shown in figure 5.1, the air temperature rises quicker than the inner wall surface temperature. This increasing temperature differential between the room air and the wall surface indicates that the heat transfer to the wall is increasing. At hour

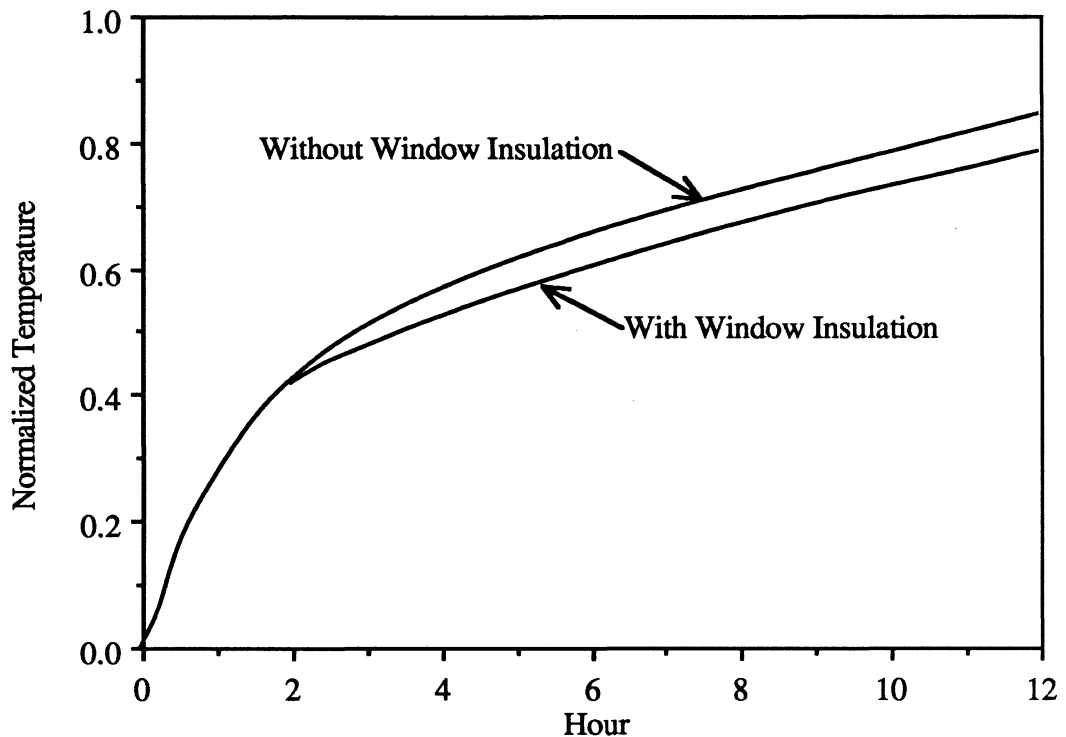
two of the simulation, an insulating material is applied to the window, reducing the heat flow into the room. With the reduced heat flow into the room, the large temperature difference between the air and the wall must decrease, which causes the room air temperature to decrease rapidly, since the air has a lower thermal mass than the wall. The wall surface temperature decreases slightly as heat flows into the wall core faster than heat flows from the air to the wall surface. The temperature difference eventually reaches a level that is supported by the incoming heat flow to the room, and the temperature begins to increase again at a slower rate as depicted by the lower slope after the insulation is added.

The situation is less severe with a lighter wall structure such as a frame wall. Figure 5.2 shows simulation results for a room having an exterior wall with only 15% window area. The walls are constructed of fiberglass with sheet rock on both sides. The lower thermal mass of a frame wall as compared to a concrete allows the wall surface temperature to increase nearly as fast as the air temperature. Additionally, the smaller exterior window area results in less of the total room heat transfer occurring through the window. Thus, when the night insulation is added at hour 2, the total heat flux is only slightly attenuated resulting in a smaller change in the difference between the room air and the wall surface temperature. Once the night insulation is applied, the total heat transfer to the room is reduced resulting in the lower slope of the temperature curves in figure 5.2.



**Figure 5.2** Normalized temperature response of a frame wall room with 15% exterior window area, to a step change in ambient temperature. Finite difference simulation. Window insulation added at hour two.

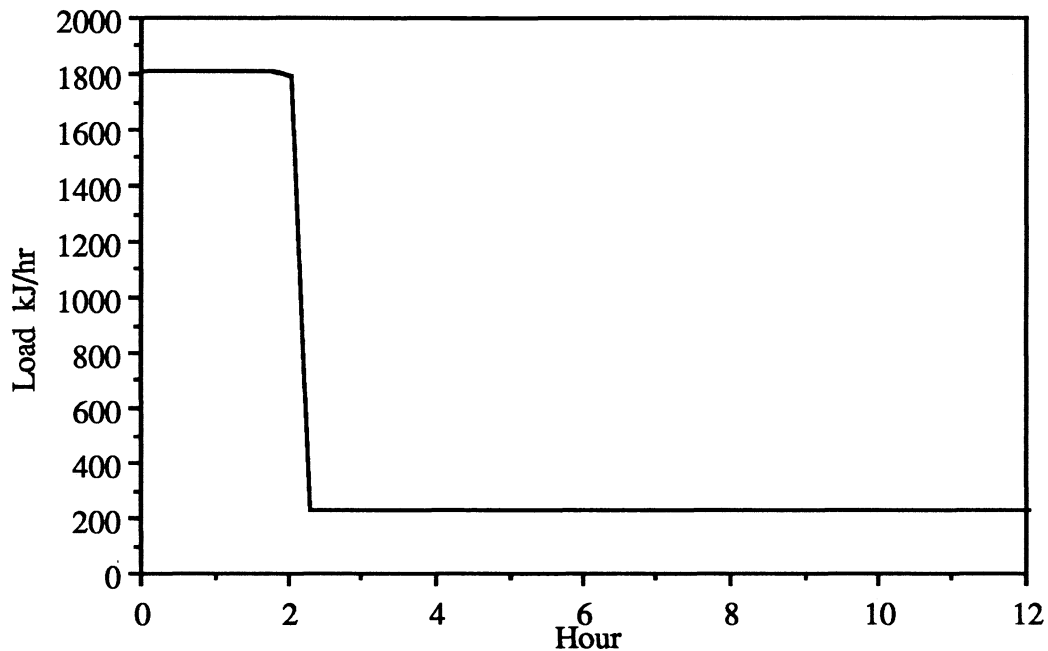
The transfer functions can not account for the temperature distribution in the wall. As far as the transfer functions are concerned, a change of conduction through the window merely represents an attenuation of the incoming heat flux to the room. When insulation is applied to the window, the room temperature continues to increase at a slower rate, as shown in figure 5.3, without the drop in room temperature indicated in the finite difference model.



**Figure 5.3** Normalized temperature response of a concrete wall room with 100% exterior window area, to a step change in ambient temperature. Transfer function simulation. Window insulation added at hour two.

### 5.1.2 Controlled Room Air Temperature

In many simulations the room air temperature is held constant at or near a set point. In this case the room air temperature and the wall surface temperature would remain constant in contrast to the fluctuating temperature differences that lead to the errors described above. The load calculated to maintain a room at the set point temperature would simply respond with a step change due to the addition of night insulation to a window, as shown in figure 5.4. Since the room temperature is held constant, transfer functions should provide a good representation of the zone energy flows when building parameters, such as the conductance of a window, are changed.



**Figure 5.4** Load required to maintain temperature of a concrete wall room with 100% exterior window area. Transfer function simulation. Window insulation added at hour two.

## 5.2 Simulation Time Step

Transfer functions are usually formulated to be used on a one hour time step, meaning the function provides the heat flow over an hour. It may be desired to run simulation at time steps smaller than one hour. This is often necessary when studying control strategies or when short duration inputs are of interest. This section presents ways to run simulations at other than the typical one hour time step.

### 5.2.1 Small Time Base Transfer Functions

The CRTF model provides two methods using smaller time steps. The first

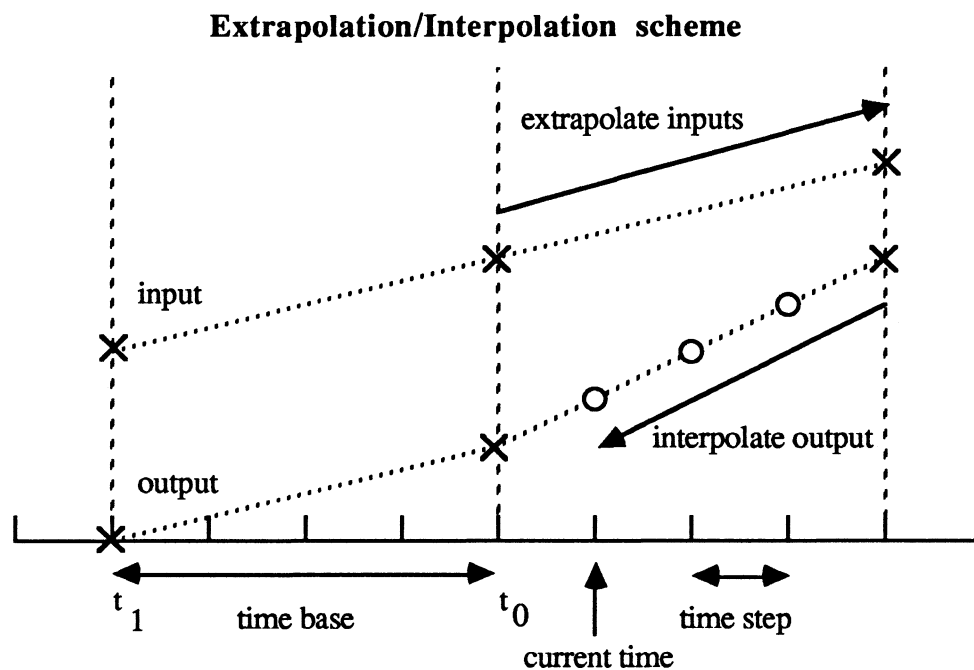


method is to use transfer functions which are based on the smaller simulation time step. In this case the transfer function for the walls must be generated using the smaller simulation time step as the transfer function time base. TRNSYS version 12.2 contains a new building preprocessor, BID (Building Input Description), which can produce transfer functions with a user specified time base. For typical building construction materials, the thermal mass limits the transfer function time base to about 10 or 15 minutes or longer. Smaller time bases cause numerical problems in the transfer function generation because of the small change in temperature over a small change in time. With thermally heavy walls, the wall temperatures do not change significantly over a small time base period, so more past time steps are required to represent heat flows over a time period of a couple of hours. This small change in output between consecutive time steps, and the increased number of time steps, contribute to the numerical problems of generating transfer functions with a small time base. Nevertheless, using small time base transfer functions is a practical method for running simulations at time steps down to 10 or 15 minutes.

### **5.2.2 Time Steps Smaller Than the Transfer Function Time Base**

An alternative method to run simulations with small time steps uses an interpolation scheme. This method can be used to further reduce the simulation time step when using small time base transfer function, or it can be used alone on large (one hour) time base transfer functions. Since the heat flows are changing slowly in the heavy wall described earlier in section 5.2.1, linear interpolation of the transfer function output within the time base of the transfer function should yield a good approximation. The output at the next time must be estimated so that there are two

outputs to interpolate between. This method assumes transfer function inputs are varying linearly over the time base period of the transfer function. Transfer function inputs for time steps less than one hour will usually have been interpolated from hourly weather data, so no new approximation is introduced with this interpolation scheme. This assumption allows the input to be extrapolated to the next step in the transfer function. The future output of the transfer function is calculated and interpolated back to the current intermediate time within the time base period. Figure 5.5 illustrates the extrapolation and interpolation required at a simulation time within the transfer function time base.



**Figure 5.5** Extrapolation of transfer function input with interpolation of transfer function output to yield the output at a time within the transfer function time base.

### **5.3 Comfort Calculations**

Predicting human comfort in an indoor environment is gaining importance in building energy use simulations. The main purpose of maintaining a building environment is usually to produce a comfortable climate for people. Issues of health, safety and productivity can all be influenced by the building environment. The emphasis of this section is on obtaining information from the building model which can be used in predicting thermal comfort.

#### **5.3.1 Environmental Parameters Effecting Comfort**

Fanger (1970) studied methods for assessing the thermal comfort of people using a comfort equation which is based on clothing, activity level, and environmental variables. The environmental variables of importance are the air velocity, air temperature, vapor pressure, and mean radiant temperature (MRT). Unlike the Carroll (1980) mean radiant temperature which was used in a simplified radiation network, the Fanger mean radiant temperature relates to comfort. Carroll's MRT is the temperature at a node through which all radiation exchange in a room occurs. Fanger defines his mean radiant temperature as follows:

"The mean radiant temperature in relation to a person in a given body posture and clothing placed at a given point in a room, is defined as that uniform temperature of black surroundings which will give the same radiant heat loss from the person as the actual case under study."

This definition of the mean radiant temperature causes the MRT to vary with body posture and position within the room. The formula for calculating the temperature is derived from a radiation balance on the person. Using the definition of the MRT, the energy balance on a person in a 'black' room having uniform wall temperatures of

$T_{MRT}$  is given as:

$$Q_p = \sigma \epsilon_p A_p (T_p^4 - T_{MRT}^4) \quad (5.3.1)$$

The energy balance on a person in an actual room, formulated using total radiation exchange factors ( $\hat{F}$ ) Beckman (1970) is,

$$Q_p = \sigma \epsilon_p A_p \sum_{i=1}^N \left[ \epsilon_i \hat{F}_{p-i} (T_p^4 - T_i^4) \right] \quad (5.3.2)$$

Equating the two energy balances, equations 5.3.1 and 5.3.2, and cancelling the common terms, results in,

$$T_p^4 - T_{MRT}^4 = T_p^4 \sum_{i=1}^N (\epsilon_i \hat{F}_{p-i}) - \sum_{i=1}^N (\epsilon_i T_i^4 \hat{F}_{p-i}) \quad (5.3.3)$$

From conservation of energy considerations,

$$\sum_{i=1}^N (\epsilon_i \hat{F}_{p-i}) = 1 \quad (5.3.4)$$

Substituting equation 5.3.4 into equation 5.3.3 results in a formula for the MRT.

$$T_{MRT}^4 = \sum_{i=1}^N (\epsilon_i T_i^4 \hat{F}_{p-i}) \quad (5.3.5)$$

Building materials often have a high emittance, so it is often acceptable to assume all surfaces in the room are black, which makes  $\hat{F}_{p-i}$  equal to the radiation view factor  $F_{p-i}$ . Equation 5.3.5 then becomes,

$$T_{MRT}^4 = \sum_{i=1}^N (T_i^4 F_{p-i}) \quad (5.3.6)$$

Assuming the surface temperatures are near the MRT allows the following linearization of the temperature difference to be made.

$$T_i^4 - T_{MRT}^4 = 4 \bar{T}^3 (T_i - T_{MRT}) \quad (5.3.7)$$

Multiplying the left hand side of equation 5.3.6 by 1, defined as the summation of the view factors from the person to the room surfaces, a final simplification to the MRT equation can be made.

$$\begin{aligned} 0 &= \sum_{i=1}^N (T_i^4 F_{p-i}) - \sum_{i=1}^N (T_{MRT}^4 F_{p-i}) \\ 0 &= \sum_{i=1}^N F_{p-i} (T_i^4 - T_{MRT}^4) \\ 0 &= \sum_{i=1}^N F_{p-i} 4 \bar{T}^3 (T_i - T_{MRT}) \\ T_{MRT} &= \sum_{i=1}^N (T_i F_{p-i}) \end{aligned} \quad (5.3.8)$$

Equation 5.3.8 shows that the mean radiant temperature can be calculated as the mean value of the surrounding temperatures weighted according to the magnitude of the respective angle factors. Fanger (1970) presented data on angle factors between persons and typical room surfaces.

### 5.3.2 Estimating the Mean Radiant Temperature

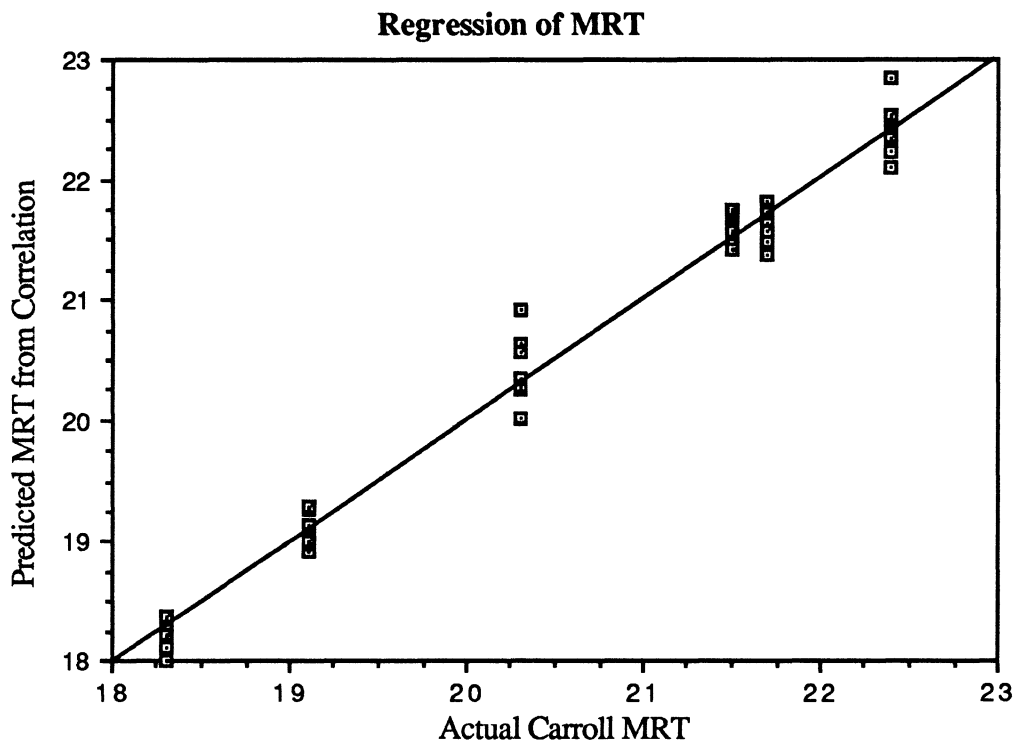
The CRTF model can not produce wall surface temperatures that are needed to calculate the mean radiant temperature. Other models based on Carroll's MRT network approximate Fanger's MRT with the Carroll's MRT. This approximation means that the comfort MRT would not vary with position in the room, nor does it specifically relate to a person in a room. Nevertheless, it has been used as an estimate

of a Fanger MRT.

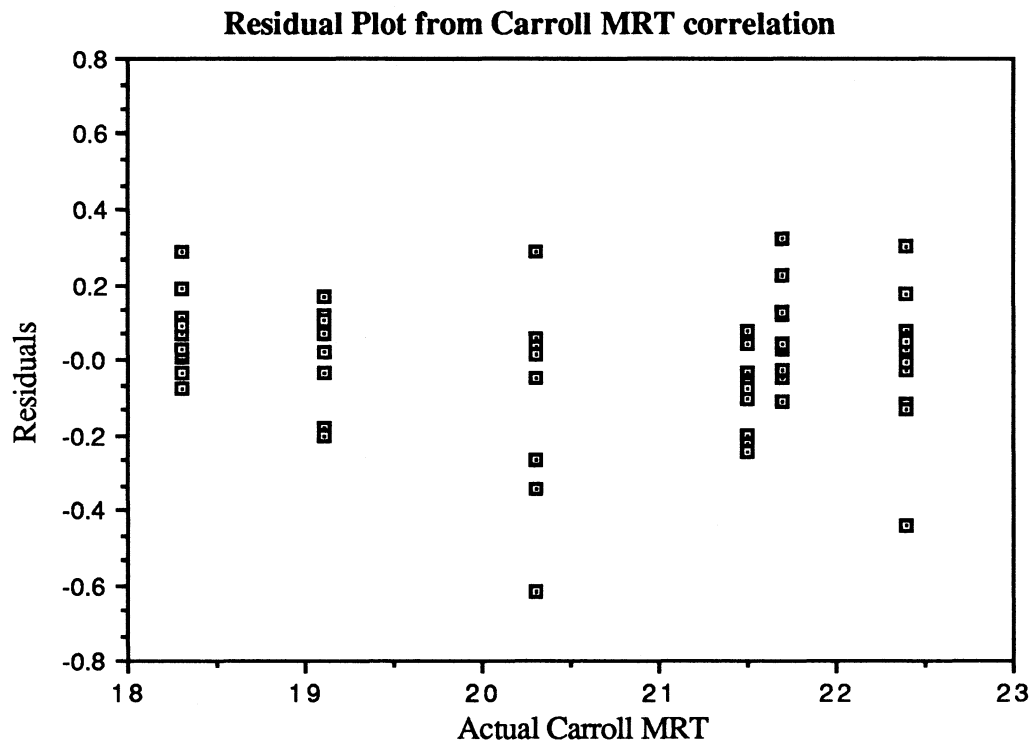
The star temperature used in the CRTF model includes convection heat transfer through the star point. The relationship between the star temperature and the Carroll MRT was determined by a regression on sixty six runs of five different room geometries with constant wall emittances of 0.9. The correlation uses room volume ( $V \text{ m}^3$ ), structural load ( $Q_{\text{star}} \text{ W}$ ), star temperature, and air temperature to estimate Carroll's MRT.

$$T_{\text{mrt}} = 1.25T_{\text{star}} - 0.25T_{\text{air}} + 0.033Q_{\text{star}}/V \quad (5.3.9)$$

Figure 5.6 shows a plot of the MRT predicted by the correlation verses the Carroll MRT. The correlation fits the MRT data to within  $\pm 0.5^\circ\text{C}$  with an  $R^2=0.987$ . Figure 5.7 shows a plot of the residuals of the correlation.

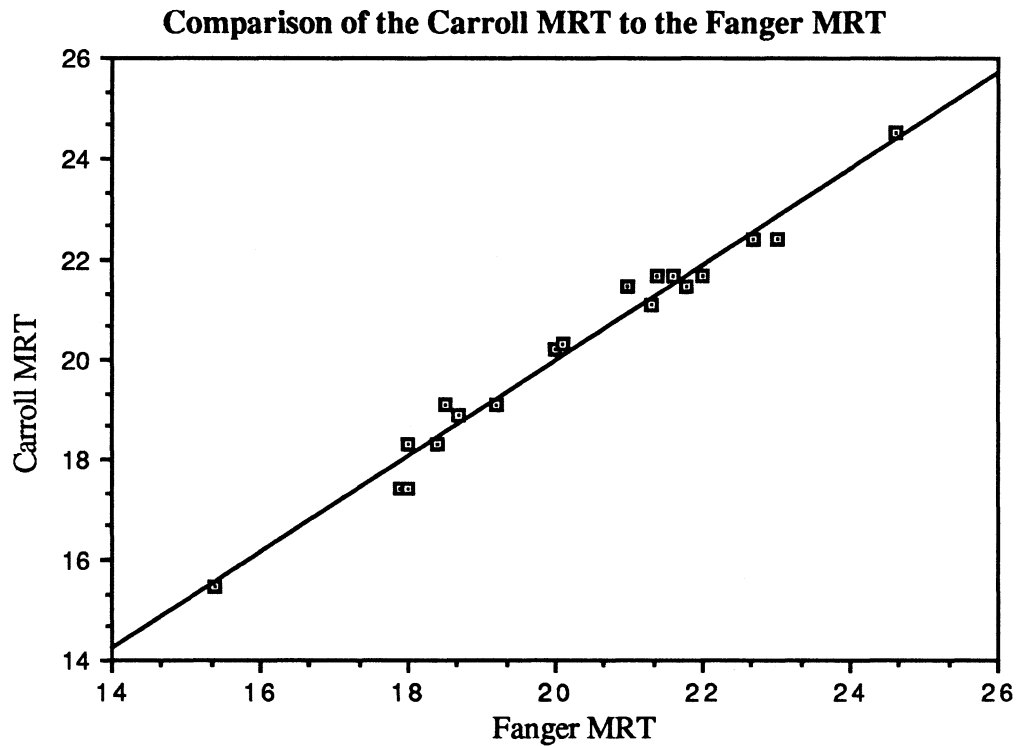


**Figure 5.6** Predicted regression correlation Carroll MRT vs. actual Carroll MRT .



**Figure 5.7** Residual plot of MRT correlation.

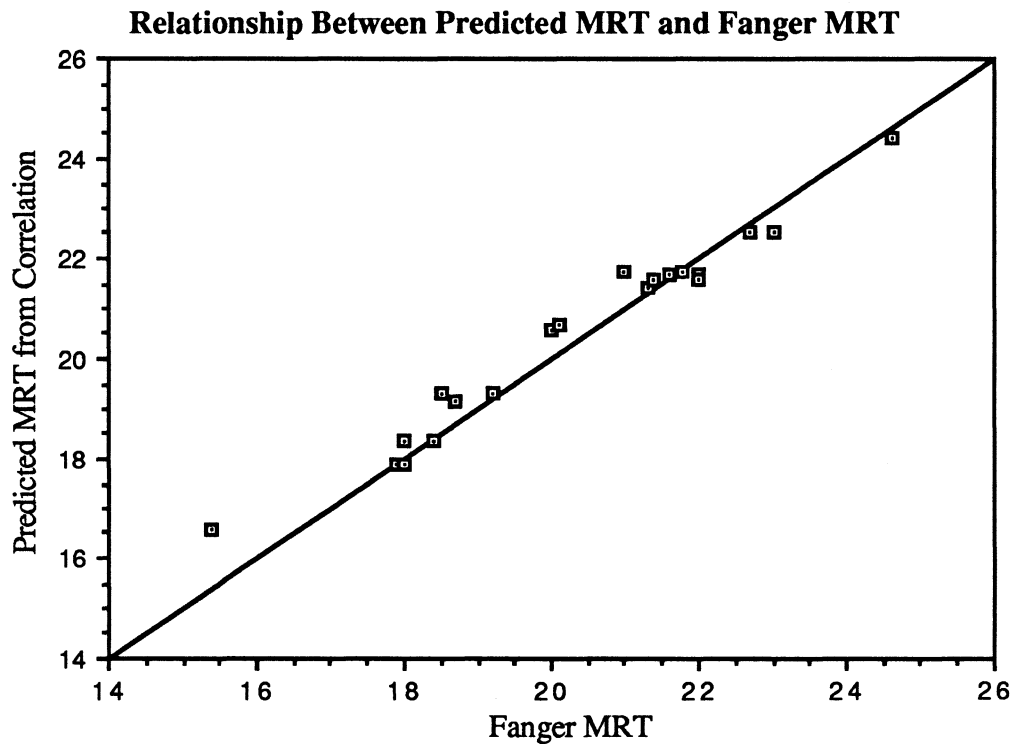
The Carroll MRT is compared to the Fanger MRT, which was calculated for a person standing in the center of each of the five test geometries. Figure 5.8 plots the Fanger MRT against the Carroll MRT. The Fanger MRT includes effects of the orientation of the person in a room, accounting for the person facing either the North or the West wall, e.g. the hot or cold wall. The largest error between the network MRT and the Fanger MRT is 3.5% with an absolute error of 0.6°C which suggests that the Carroll MRT can be used as an estimate of a Fanger MRT for a person at the center of a room.



**Figure 5.8** Carroll MRT vs Fanger MRT.

Figure 5.9 shows the relationship between the Fanger MRT and the MRT predicted from the correlation equation 5.3.9. The correlated MRT is a good approximation of a Fanger MRT used in calculating the comfort of a person at the center of a room. The differences between the two MRTs is small but must be judged for significance by its effect on a comfort indicator.





**Figure 5.9** Comparison of Fanger MRT and MRT predicted from the Correlation equation 5.3.9

### 5.3.3 Sensitivity of Comfort Indicators to Variations in the MRT

Fanger (1970) presented the predicted mean vote (PMV) as an index of thermal comfort. The PMV is a complicated function based on activity level, clothing, air temperature, mean radiant temperature, relative air velocity, and air humidity. It indicates comfort on the following scale:

- 3 cold
- 2 cool
- 1 slightly cool
- 0 neutral
- + 1 slightly warm
- + 2 warm
- + 3 hot

Fanger differentiated the PMV with respect to the MRT and presented the derivative as a function of the thermal resistance of clothing, relative air velocity, and activity level. The PMV usually changes less than 0.1 units for each degree change in MRT. The largest change is 0.25 units for a sedentary person without clothing. With this small change in PMV with a 1°C change in MRT, it is reasonable to assume that the errors introduced by approximating a Fanger MRT for a person near the center of a room with an MRT calculated by the correlation equation 5.3.9 are negligible.

Far more significant errors in predicting comfort result from assuming that the MRT is constant through out a room. Fanger (1970) shows an example of a room with large exterior double pane windows. The room is heated by a radiant panel on the ceiling for an ambient temperature of 0°C. The Fanger MRT actually varies from 22°C near the window to 26.8°C near the back wall. This difference can result in a significant change of PMV of up to 1.2 units from one location to another.

The Carroll MRT or the correlated MRT can be used in calculations of an estimate of thermal comfort. The ability to determine an MRT from a correlation of known information in the CRTF model overcomes the lack of wall surface

temperature information in the CRTF model. This correlated network MRT, can be used along with the other environmental variables in a model of the human body developed by Gagge (1986), to give indications of the comfort of a person standing in the center of a room throughout a simulation.

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## CHAPTER 6

### Conclusions

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#### 6.1 Conclusions

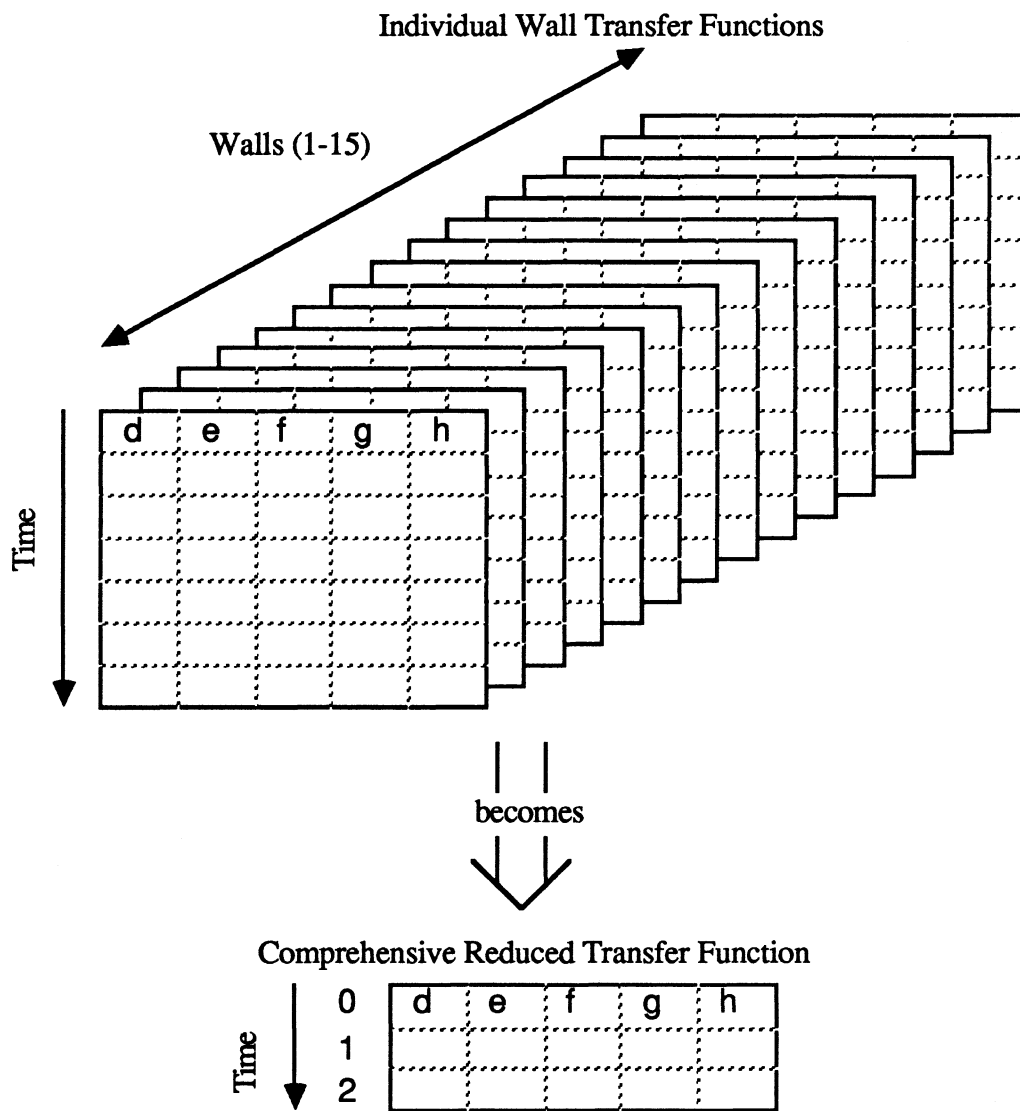
A building zone model based on the comprehensive room transfer function can significantly reduce the computation time of a simulation while maintaining accuracy. The CRTF eliminates the need to calculate intermediate heat flows or temperatures, giving instead the total zone heat transfer and the zone temperature. The combination of the equations representing individual room elements into a single equation representing the zone, reduces the computational effort involved in computing building energy loads. The reduction in computation is illustrated in figure 6.1. The large overlapping cards at the top of the figure each represent a transfer function of a single wall, roof, floor, or partition. The transfer functions each have input terms represented by the d,e,f and g columns. Each of the small dashed squares represents a multiplication and addition. In solving the system of equations, each transfer function would be calculated. The CRTF is represented in the lower portion of the figure as a single equation having only 2 past time steps.

The CRTF model includes the most common factors effecting energy loads on a zone. Conduction through walls due to the ambient temperature, incident solar radiation on different orientations, and internal radiation gains from people and lights are included in the CRTF. The direct gains on the room air such as ventilation,

infiltration and internal gains are combined with the structural loads represented by the CRTF in an energy balance on the zone air.

The mean radiant temperature network, Carroll (1980), can be used in the formulation of the CRTF in place of the exact network. The MRT network does not require the detailed geometry information used in the exact network to calculate radiation exchange factors between walls. Furnishings can easily be included in the radiation network as a wall, since only a representative surface area must be specified. The star network provides an accurate representation, within 2%, of the radiation and convection heat transfer in a room regardless of whether the star network was generated from the exact network or the MRT network.

The CRTF model performs well when compared to the TRNSYS type 19 zone model based on the TF model. The difference in accuracy between the two models is negligible. The simulation times are considerably smaller with the CRTF model. For a single zone, the CRTF model simulation time remains constant regardless of the number of walls in the room. For a multiple zone building simulation, the CRTF model only requires one equation per zone while the type 19 model requires a system of equations for each zone. In multiple zone building simulations, the CRTF model can be formulated as a system of equations representing the entire building. This formulation will efficiently account for the coupling of zones due to heat transfer between them, by solving the system of equations, rather than relying on a successive substitution iteration scheme which calculates the single zone performance independently.



**Figure 6.1** The reduction in the amount of information required to represent a building zone with the CRTF.

The combination and reduction process of the CRTF mandates that certain characteristics of the zone remain constant throughout the simulation. Night insulation added to a window in effect decreases the conductance of the window.

This change in the zone can be modeled with the CRTF method by replacing the CRTF with a new CRTF which includes insulation over the windows. Other properties could be allowed to change using this method. For example, exterior convection coefficients could change from a night value to a day value by replacing the CRTF with a new CRTF formulated with a different convection coefficient.

The ability to calculate intermediate information such as wall surface temperatures is lost in the CRTF formulation. These temperatures are of interest in the modeling of human comfort. A relationship between the star temperature, which is known, and the Carroll MRT was developed. It was shown that this approximate MRT temperature can be used in comfort calculations to give estimates of comfort indices. Even though some information is not available in the CRTF model, other means, such as the regression analysis used to relate the star temperature and the mean radiant temperature, are available.

As with any model, simplifying assumptions must be made. The relevance of the assumptions is determined with the expected results in mind. The CRTF method allows nearly as much flexibility in representing a building as any other detailed simulation method, only the CRTF method requires significantly less computation effort.

## **6.2 Recommendations**

- 1) There is a great deal of descriptive information involved in setting up a simulation of a building. Including the calculation of the CRTF in a building preprocessor such as BID in TRNSYS, would make the specification of the building geometry and materials easier. Separating the CRTF formulation from the energy balance simulation portion of the model would allow the

model to choose the CRTF which represents the room at a given time. This flexibility would facilitate the inclusion of factors such as night insulation or a convection coefficient that varied on a given schedule.

- 2) A TRNSYS multiple zone building component based on the CRTF as described in chapter 4, would eliminate internal TRNSYS iterations when significant heat transfer occurs between zones.
- 3) The dominant root model reduction should be substituted for the Padé approximation method in the CRTF model. The dominant root model reduction method guarantees stability, only produces one output time series regardless of the number of input time series, and would allow intermittent reduction within the transfer function combination process which would eliminate the generation of a CRTF with many past time steps.
- 4) The current version of the TRNSYS CRTF model relies on the user to decide on the number of past time steps to be used in the reduced transfer function. A process should be automated which checks the accuracy of the reduced CRTF. The steady state response is easily checked by comparing the sum of the coefficients of the reduced CRTF with the original CRTF. The transient error is most evident in the first time step, so these errors can be checked by comparing the first coefficient of each term in the reduced CRTF with the original CRTF.



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## APPENDIX A

### Comparison of Heat Transfer Networks

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This appendix presents a comparison of heat transfer networks for the five different geometries. The geometries were used by Walton (1980) to compare radiation exchange networks. The heat flows from each wall, and the net room heat transfer is calculated for different wall temperatures and different wall emittances. Three different radiation networks are used: the exact network<sup>1</sup>, the exact network with area ratio view factors, and the Carroll (1980) mean radiant temperature network. The star network is formulated from each of the radiation networks. The net heat transfer to a room is compared between the star network and the network from which the star network was developed.

Descriptions of the five geometries are given in table A.2. The geometries include a: cube, room, warehouse, corridor, and modified room with windows on two of the walls. All of the enclosures are simple "boxes" having no internal walls. Figure A.3 illustrates the room geometry named "windows" which contains two windows.

Tables A.3 and A.4 show the heat transfer out of each surface in watts for different emittances and temperatures of the walls. Table A.3 was generated by setting the convection coefficient to a negligibly small size ( $h_c = 0.0001 \text{ W/m}^2\text{°C}$ ).

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<sup>1</sup>Referred to as the linear network in Walton (1980). The fourth order temperature difference is approximated by the linear temperature difference multiplied by four times the area weighted average room surface temperature raised to the third power.

These results indicate the performance of the star temperature in calculating radiation heat transfer. No heat is transferred to the room since the convection resistance is extremely high. Energy is simply exchanged through radiation among the walls.

Table A.4 contains results with a convection coefficient of  $h_c = 5.0 \text{ W/m}^2\text{°C}$ . The convection is included in the star network resistances, which changes the relative errors in comparing the networks.

Cases 1 and 2, describe a cube with one hot and one cold wall respectively. Since the wall emittances and areas are the same, the star distributes the radiation properly. In cases 3 and 4, the star network distributes radiation differently than the radiation only networks. In cases 5 through 10 the room geometry becomes a factor, causing differences among the three radiation only networks as well as with the star network. Cases 11 through 15 represent extreme geometries where the use of an area weighted average wall temperature when linearizing the temperature difference between surfaces causes significant errors in the network being used as the "correct" standard. Nevertheless the comparison of the star network to the radiation only networks is still valid. The window geometry of cases 16, 17 and 18 show similar distributions of the thermal radiation within the star and radiation networks.

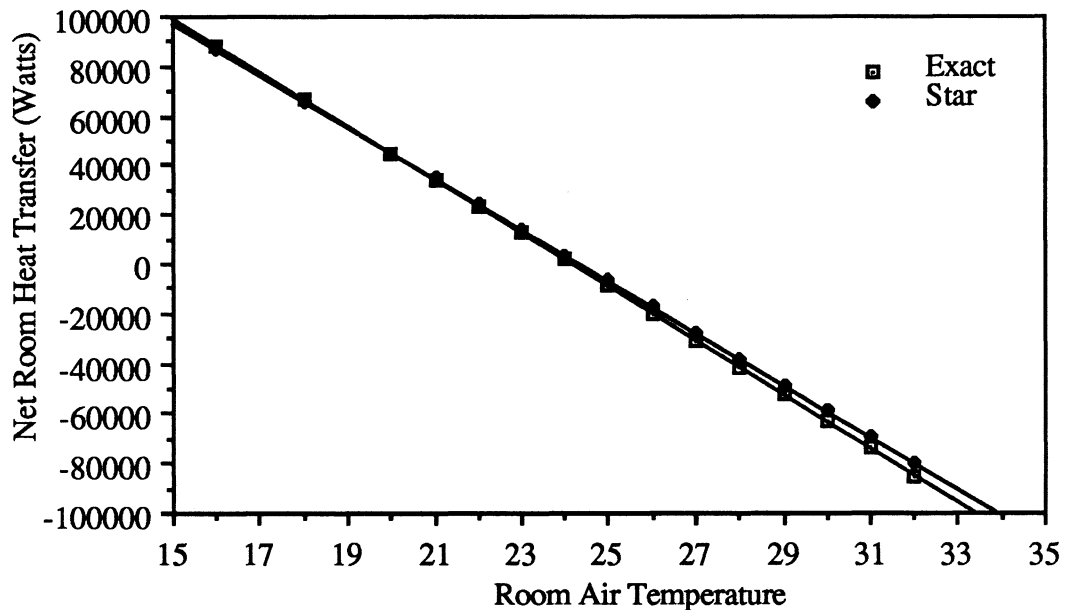
Table A.1 attempts to summarize the differences between the total heat transfer to the room air for the star network and the star network as generated from the exact network. All of the errors are within 5% except for case 12 which has an error of 23%. This large error arises from the definition of the relative error. The error is calculated as the difference divided by the actual amount. Under the conditions on case 12, the absolute heat transfer error is small (875W, about 5 light bulbs) for a  $2700 \text{ m}^3$  building. The accuracy of the star network becomes more apparent when viewed on a plot of the net room load verses the air temperature as shown in

figure A.1. From the plot it is easy to accept that the star network models the heat transfer well. The next largest error occurred in case 4 which contains a wall emittance of 0.5. Figure A.2 again shows the relevance of the error along with showing the shift of the net room load when all of the walls do not have identical emittances.

**Table A.1** Summary of relative errors between star network and exact network

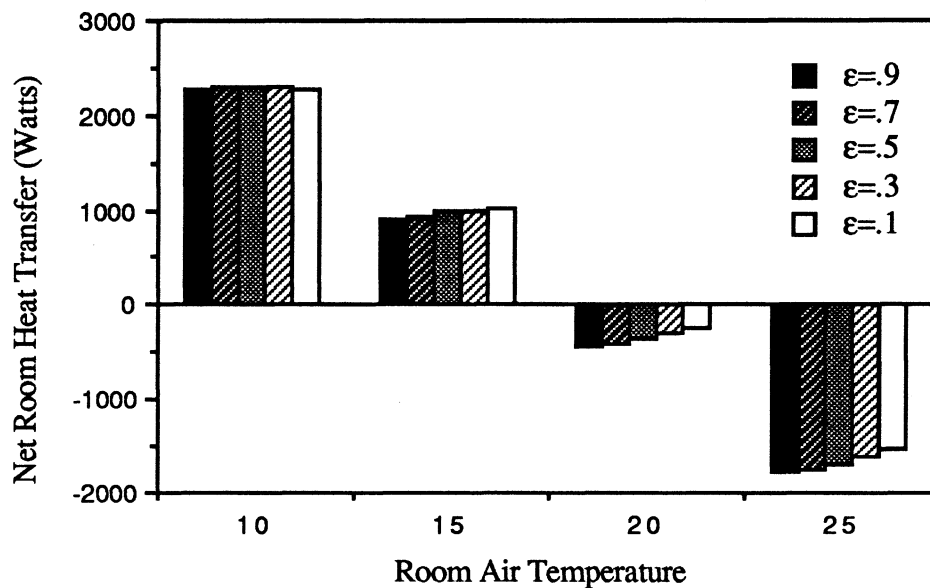
	Case	Net Zone Heat Flow (W)		Percent Difference
		Exact	Star	
Cube	1	-900	-900	0.0
	2	-1800	-1800	0.0
	3	-1800	-1781	-1.1
	4	-1800	-1703	-5.4
Room	5	-1675	-1674	-0.1
	6	-2975	-2929	-1.5
	7	-3675	-3678	0.1
	8	-1775	-1705	-3.9
	9	-1975	-1998	1.2
	10	-2975	-2890	-2.9
Warehouse	11	-49500	-48625	-1.8
	12	-9000	-6911	-23.2
	13	-99000	-97193	-1.8
Corridor	14	-4950	-4896	-1.1
	15	-9000	-8993	-0.1
	16	-1825	-1811	-0.8
	17	-3075	-2988	-2.8
Windows	18	-3075	-2949	-4.1

### Case 12 Net Room Heat Transfer Comparison



**Figure A.1** Comparison of net heat transfer to a room between the star network and the exact network representing the warehouse geometry.

### Case 4 Net Room Heat Transfer Comparison



**Figure A.2** Room heat transfer for case 4 cube as a function of wall emittance.

Table A.2 Geometry Descriptions of test cases

	area (m <sup>2</sup> )					volume	
	west	south	east	north	floor	roof	
<b>Cube</b>	9.0	9.0	9.0	9.0	9.0	9.0	(3.0 x 3.0 x 3.0)
<b>Room</b>	15.0	10.0	15.0	10.0	24.0	24.0	(4.0 x 6.0 x 2.5)
<b>Warehouse</b>	90.0	90.0	90.0	90.0	900.0	900.0	(30.0 x 30.0 x 3.0)
<b>Corridor</b>	90.0	9.0	90.0	9.0	90.0	90.0	(3.0 x 30.0 x 3.0)

	west1	west2	south1	south2	east	north	floor	roof	
<b>Windows</b>	7.5	7.5	5.0	5.0	15.0	10.0	24.0	24.0	(4.0 x 6.0 x 2.5)

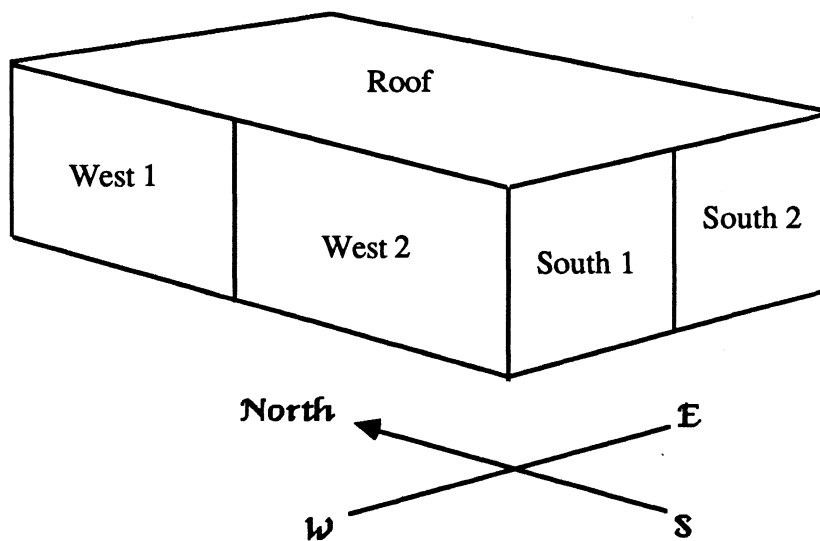


Figure A.3 Geometry of test case "Windows"

The following tables give results of heat transfer from the surfaces of the room using different radiation and convection resistance networks. In every case the room air temperature is held constant at 25°C. The wall temperatures are held constant at the values noted. The last column presents the fictitious node temperature of the network: the star temperature for the first three rows and the MRT for the sixth row.

**Table A.3 Comparison of Heat Transfer Networks (radiation heat transfer only)**

**Case 4:** Cube  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except  
west wall  $\epsilon=0.5$ ,  $T=10^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	-288.4	57.7	57.7	57.7	57.7	57.7	0.0	18.9
2	-288.4	57.7	57.7	57.7	57.7	57.7	0.0	18.9
3	-288.2	57.7	57.7	57.6	57.6	57.7	0.0	18.9
4	-249.5	49.9	49.9	49.9	49.9	49.9	0.0	-
5	-249.5	49.9	49.9	49.9	49.9	49.9	0.0	-
6	-249.5	49.9	49.9	49.9	49.9	49.9	0.0	19.1

**Case 5:** Room  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except  
west wall  $\epsilon=0.9$ ,  $T=30^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	851.3	-95.0	-148.2	-94.6	-256.7	-256.7	0.0	21.6
2	817.5	-94.0	-147.2	-94.6	-240.9	-240.9	0.0	21.6
3	836.6	-91.5	-147.4	-91.8	-252.9	-252.9	0.0	21.6
4	820.1	-99.0	-145.6	-99.0	-238.3	-238.3	0.0	-
5	818.5	-99.9	-148.9	-99.9	-234.9	-234.9	0.0	-
6	819.1	-87.4	-138.2	-87.4	-253.1	-253.1	0.0	21.5

**Case 6:** Room  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except  
south wall  $\epsilon=0.9$ ,  $T=10^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	89.9	-519.3	84.3	53.3	145.7	146.0	-0.1	19.1
2	92.5	-512.8	85.8	55.8	139.3	139.3	-0.1	19.0
3	89.2	-513.1	82.9	52.6	144.2	144.2	-0.1	19.1
4	96.4	-508.3	90.4	39.4	141.1	141.1	-0.1	-
5	97.3	-508.4	85.2	57.1	134.4	134.4	-0.1	-
6	85.1	-502.5	78.8	49.8	144.4	144.4	-0.1	19.1

**Case 7:** Room  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except  
roof  $\epsilon=0.9$ ,  $T=10^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	243.0	143.8	227.7	143.7	393.5	-1151.7	-0.1	17.5
2	229.6	137.3	213.1	138.6	345.4	-1064.0	-0.1	17.5
3	241.4	142.1	223.8	142.0	388.9	-1138.3	-0.1	17.5
4	228.6	139.0	214.4	139.0	456.5	-1177.6	-0.1	-
5	225.4	132.4	209.1	132.4	369.6	-1069.1	-0.1	-
6	242.8	142.3	225.0	142.3	412.0	-1164.4	-0.1	17.4

**Case 8:** Room  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except  
 south wall  $\epsilon=0.9$ ,  $T=10^\circ\text{C}$   
 floor  $\epsilon=0.9$ ,  $T=30^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	-160.3	-680.6	-149.7	-94.8	1345.0	-259.6	0.0	21.6
2	-143.6	-668.7	-132.8	-86.6	1247.4	-215.7	0.0	21.5
3	-159.3	-672.8	-148.0	-93.6	1330.3	-256.5	0.0	21.6
4	-139.0	-665.6	-130.3	-104.2	1369.2	-330.2	0.0	-
5	-134.8	-658.8	-130.2	-79.2	1249.6	-246.7	0.0	-
6	-165.4	-663.0	-153.2	-96.9	1358.9	-280.6	0.0	21.7

**Case 9:** Room  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except  
 south wall  $\epsilon=0.8$ ,  $T=30^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	-83.4	480.9	-78.1	-49.4	-135.0	-135.0	0.0	20.8
2	-85.9	475.2	-79.3	-51.8	-129.1	-129.1	0.0	20.9
3	-82.8	475.2	-76.9	-48.8	-133.4	-133.4	0.0	20.8
4	-87.6	461.8	-82.1	-35.8	-128.2	-128.2	0.0	-
5	-88.4	461.9	-77.4	-51.9	-122.1	-122.1	0.0	-
6	-77.3	456.5	-71.6	-45.3	-131.2	-131.2	0.0	20.8

**Case 10:** Room  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except  
 south wall  $\epsilon=0.8$ ,  $T=10^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	81.6	-471.0	76.7	48.5	132.1	132.1	-0.1	19.2
2	84.3	-466.0	78.0	50.8	126.4	126.4	-0.1	19.1
3	81.2	-465.8	75.4	47.8	130.6	130.7	-0.1	19.2
4	85.8	-452.4	80.4	35.1	125.6	125.6	-0.1	-
5	86.6	-452.6	75.8	50.9	119.6	119.6	-0.1	-
6	75.7	-447.2	70.1	44.4	128.5	128.5	-0.1	19.2

**Case 11:** Warehouse  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except  
 west wall  $\epsilon=0.9$ ,  $T=30^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	4777.7	-137.2	-137.1	-137.1	-2183.6	-2183.6	-1.0	20.3
2	4733.1	-152.0	-152.0	-152.0	-2139.1	-2139.1	-1.0	20.3
3	4770.2	-137.7	-137.7	-137.7	-2179.1	-2179.1	-1.0	20.3
4	4619.9	-248.5	-124.1	-248.5	-1999.9	-1999.9	-1.0	-
5	4609.5	-212.0	-212.0	-212.0	-1987.3	-1987.3	-1.0	-
6	4620.1	-129.1	-129.1	-129.1	-2116.9	-2116.9	-1.0	20.3



**Case 12:** Warehouse  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except

			roof $\epsilon=0.9$ , $T=30^\circ\text{C}$				
	west	south	east	north	floor	roof	Total T
1	-2273.6	-2272.5	-2273.1	-2272.5	-36184.4	45276.1	-0.1 24.5
2	-2219.6	-2219.6	-2220.1	-2220.1	-31524.4	40403.8	-0.1 24.4
3	-2262.8	-2261.7	-2262.8	-2262.8	-35804.8	44854.9	-0.1 24.4
4	-2077.1	-2077.1	-2077.1	-2077.1	-36521.0	44829.4	-0.2 -
5	-2064.0	-2064.0	-2064.0	-2064.0	-31785.4	40041.2	-0.2 -
6	-2198.7	-2198.7	-2198.7	-2198.7	-36066.7	44861.5	-0.2 24.5

**Case 13:** Warehouse  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except

			roof $\epsilon=0.9$ , $T=10^\circ\text{C}$				
	west	south	east	north	floor	roof	Total T
1	2080.9	2080.9	2080.9	2080.0	33021.2	-41345.9	-2.0 15.6
2	2040.0	2040.0	2039.1	2038.7	28688.0	-36848.0	-2.0 15.6
3	2078.4	2078.4	2078.8	2078.4	32894.2	-41210.2	-2.0 15.6
4	1907.0	1907.0	1907.0	1907.0	33530.8	-41160.9	-2.0 -
5	1895.0	1895.0	1895.0	1895.0	29182.8	-36764.6	-2.0 -
6	2018.7	2018.7	2018.7	2018.7	33113.7	-41190.3	-2.0 15.5

**Case 14:** Corridor  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except

			west wall $\epsilon=0.9$ , $T=30^\circ\text{C}$				
	west	south	east	north	floor	roof	Total T
1	4582.9	-116.8	-1448.7	-116.8	-1450.3	-1450.3	-0.1 22.4
2	4571.0	-124.7	-1441.8	-132.5	-1435.2	-1436.9	-0.1 22.4
3	4603.1	-117.0	-1458.1	-117.0	-1456.4	-1454.7	-0.1 22.4
4	4587.5	-117.3	-1731.8	-117.3	-1310.5	-1310.6	-0.1 -
5	4592.8	-147.4	-1432.7	-147.4	-1432.7	-1432.7	-0.1 -
6	4590.4	-115.8	-1453.0	-115.8	-1453.0	-1453.0	-0.1 22.4

**Case 15:** Corridor  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except

			south wall $\epsilon=0.9$ , $T=30^\circ\text{C}$				
	west	south	east	north	floor	roof	Total T
1	-114.5	466.3	-114.2	-9.2	-114.3	-114.3	-0.2 20.2
2	-122.0	496.5	-121.6	-11.2	-120.9	-120.9	-0.2 20.2
3	-114.4	466.6	-114.4	-9.2	-114.4	-114.4	-0.2 20.2
4	-114.8	462.0	-114.9	-2.5	-114.9	-114.9	-0.2 -
5	-144.2	493.4	-112.6	-11.6	-112.6	-112.6	-0.2 -
6	-113.3	462.1	-113.3	-9.0	-113.3	-113.3	-0.2 20.2

**Case 16:** Window  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except

west1 wall  $\epsilon=0.9$ ,  $T=30^\circ\text{C}$

south1 wall  $\epsilon=0.9$ ,  $T=30^\circ\text{C}$

	west1	west2	south1	south2	east	north	floor	roof	total	T
1	367.8	-48.6	237.1	-32.0	-94.8	-56.1	-186.8	-186.8	0.0	21.2
2	369.0	-51.1	240.9	-33.4	-107.4	-69.7	-173.9	-174.5	0.0	21.2
3	374.7	-48.6	243.0	-31.5	-105.2	-66.6	-182.9	-182.9	0.0	21.1
4	378.1	-61.1	250.2	-11.6	-96.7	-92.9	-183.0	-183.0	0.0	-
5	364.0	-54.1	237.4	-35.8	-106.0	-71.1	-167.2	-167.2	0.0	-
6	367.8	-46.2	239.9	-30.1	-99.9	-63.1	-184.2	-184.2	0.0	21.1

**Case 17:** Window  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except

west1 wall  $\epsilon=0.9$ ,  $T=10^\circ\text{C}$

south1 wall  $\epsilon=0.9$ ,  $T=10^\circ\text{C}$

	west1	west2	south1	south2	east	north	floor	roof	total	T
1	-358.2	47.4	-231.0	31.2	92.1	54.7	181.8	181.8	-0.1	18.8
2	-359.2	49.8	-234.7	32.5	104.4	67.9	169.7	169.7	-0.1	18.8
3	-364.5	47.3	-236.6	30.7	102.2	64.7	177.9	178.3	-0.1	18.9
4	-368.2	59.5	-243.7	11.3	94.1	90.5	178.2	178.2	-0.1	-
5	-354.5	52.7	-231.2	34.8	103.2	69.3	162.8	162.8	-0.1	-
6	-358.2	45.0	-233.6	29.3	97.3	61.4	179.4	179.4	-0.1	18.9

**Case 18:** Window  $h_c=0.0001$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except

west1 wall  $\epsilon=0.8$ ,  $T=10^\circ\text{C}$

south1 wall  $\epsilon=0.8$ ,  $T=10^\circ\text{C}$

	west1	west2	south1	south2	east	north	floor	roof	total	T
1	-325.6	43.0	-209.4	28.3	84.0	49.5	165.0	165.0	-0.1	18.9
2	-326.5	45.2	-212.4	29.5	94.8	61.8	153.8	153.8	-0.1	18.9
3	-330.8	42.8	-214.0	27.8	92.6	58.5	161.5	161.5	-0.1	19.0
4	-328.3	53.1	-217.5	10.0	84.0	80.7	159.0	159.0	-0.1	-
5	-317.3	47.2	-207.6	31.2	92.5	62.1	145.9	145.9	-0.1	-
6	-320.3	40.3	-209.5	26.3	87.1	55.0	160.6	160.6	-0.1	19.0



**Case 4:** Cube  $h_c=5.0$       all surfaces:  $\epsilon=0.9, T=20^\circ\text{C}$  except  
west wall  $\epsilon=0.5, T=10^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	-905.9	-159.5	-159.5	-159.5	-159.5	-159.5	-1703.4	21.6
2	-905.9	-159.5	-159.5	-159.5	-159.5	-159.5	-1703.4	21.6
3	-905.9	-159.5	-159.5	-159.5	-159.5	-159.5	-1703.4	21.6
4	-924.5	-175.1	-175.1	-175.1	-175.1	-175.1	-1800.0	-
5	-924.5	-175.1	-175.1	-175.1	-175.1	-175.1	-1800.0	-
6	-924.5	-175.1	-175.1	-175.1	-175.1	-175.1	-1800.0	19.1

**Case 5:** Room  $h_c=5.0$       all surfaces:  $\epsilon=0.9, T=20^\circ\text{C}$  except  
west wall  $\epsilon=0.9, T=30^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	1239.1	-334.8	-516.2	-334.8	-863.7	-863.7	-1674.1	23.1
2	1217.4	-341.1	-520.5	-342.7	-840.1	-840.1	-1667.2	23.1
3	1233.3	-334.2	-514.9	-334.2	-862.3	-862.3	-1674.6	23.1
4	1220.1	-349.0	-520.6	-349.0	-838.3	-838.3	-1675.0	-
5	1218.5	-349.9	-523.9	-349.9	-834.9	-834.9	-1675.0	-
6	1219.1	-337.4	-513.2	-337.4	-853.1	-853.1	-1675.0	21.5

**Case 6:** Room  $h_c=5.0$       all surfaces:  $\epsilon=0.9, T=20^\circ\text{C}$  except  
south wall  $\epsilon=0.9, T=10^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	-295.4	-1251.9	-276.8	-179.6	-463.0	-463.0	-2929.6	21.7
2	-306.8	-1255.6	-285.6	-188.1	-461.0	-461.0	-2958.1	21.8
3	-298.4	-1246.2	-278.3	-180.7	-465.7	-465.7	-2935.0	21.7
4	-303.6	-1258.3	-284.6	-210.6	-458.9	-458.9	-2975.0	-
5	-302.7	-1258.4	-289.8	-192.8	-465.6	-465.6	-2975.0	-
6	-314.9	-1252.5	-296.2	-200.2	-455.6	-455.6	-2975.0	19.1

**Case 7:** Room  $h_c=5.0$       all surfaces:  $\epsilon=0.9, T=20^\circ\text{C}$  except  
roof  $\epsilon=0.9, T=10^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	-149.0	-90.6	-139.7	-90.6	-233.5	-2975.1	-3678.5	20.9
2	-172.7	-105.4	-160.8	-105.9	-259.4	-2869.3	-3673.4	21.0
3	-153.3	-92.9	-143.0	-92.9	-239.3	-2963.5	-3684.9	20.9
4	-171.3	-111.0	-160.6	-111.0	-143.5	-2977.6	-3675.0	-
5	-174.6	-117.6	-165.9	-117.6	-230.4	-2869.1	-3675.0	-
6	-157.2	-107.7	-150.0	-107.7	-188.0	-2964.4	-3675.0	17.4

**Case 8:** Room  $h_c=5.0$ all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  exceptsouth wall  $\epsilon=0.9$ ,  $T=10^\circ\text{C}$ floor  $\epsilon=0.9$ ,  $T=30^\circ\text{C}$ 

	west	south	east	north	floor	roof	Total	T
1	-544.2	-1417.2	-509.9	-330.7	1950.0	-853.2	-1705.2	23.0
2	-541.8	-1413.1	-504.4	-332.1	1852.1	-814.1	-1753.4	23.1
3	-544.9	-1409.4	-508.2	-329.8	1934.2	-850.9	-1708.9	23.1
4	-539.0	-1415.6	-505.3	-354.2	1969.2	-930.1	-1775.0	-
5	-534.8	-1408.8	-505.2	-329.2	1849.6	-846.7	-1775.0	-
6	-565.3	-1412.9	-528.2	-346.9	1958.9	-880.5	-1775.0	21.7

**Case 9:** Room  $h_c=5.0$ all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  exceptsouth wall  $\epsilon=0.8$ ,  $T=30^\circ\text{C}$ 

	west	south	east	north	floor	roof	Total	T
1	-483.4	747.6	-453.0	-294.0	-757.3	-757.3	-1997.6	22.7
2	-493.0	737.3	-459.1	-302.4	-740.5	-740.5	-1998.3	22.8
3	-484.8	741.6	-452.2	-293.6	-756.3	-756.3	-2001.5	22.7
4	-487.5	711.8	-457.1	-285.8	-728.2	-728.2	-1975.0	-
5	-488.4	711.9	-452.4	-301.9	-722.1	-722.1	-1975.0	-
6	-477.3	706.5	-446.6	-295.3	-731.1	-731.1	-1975.0	20.8

**Case 10:** Room  $h_c=5.0$ all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  exceptsouth wall  $\epsilon=0.8$ ,  $T=10^\circ\text{C}$ 

	west	south	east	north	floor	roof	Total	T
1	-301.3	-1189.0	-282.4	-183.3	-471.9	-471.9	-2899.8	21.7
2	-312.8	-1192.8	-291.3	-191.9	-469.8	-469.8	-2928.4	21.8
3	-304.3	-1183.9	-283.8	-184.3	-474.5	-474.5	-2905.3	21.7
4	-314.2	-1202.4	-294.6	-214.9	-474.4	-474.4	-2975.0	-
5	-313.4	-1202.5	-299.2	-199.1	-480.4	-480.4	-2975.0	-
6	-324.3	-1197.2	-304.8	-205.6	-471.5	-471.5	-2975.0	19.2

**Case 11:** Warehouse  $h_c=5.0$ all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  exceptwest wall  $\epsilon=0.9$ ,  $T=30^\circ\text{C}$ 

	west	south	east	north	floor	roof	Total	T
1	7500.0	-1932.2	-1932.2	-1932.2	-25164.6	-25164.6	-48625.8	22.0
2	7332.3	-2062.0	-2062.0	-2062.0	-24970.3	-24970.3	-48794.2	22.2
3	7487.0	-1941.3	-1941.3	-1941.3	-25213.4	-25213.4	-48763.5	22.1
4	6869.9	-2498.4	-2374.1	-2498.4	-24499.4	-24499.4	-49500.0	-
5	6859.5	-2462.0	-2462.0	-2462.0	-24486.8	-24486.8	-49500.0	-
6	6870.1	-2379.0	-2379.0	-2379.0	-24616.5	-24616.5	-49500.0	20.3

**Case 12:** Warehouse  $h_c=5.0$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except  
 roof  $\epsilon=0.9$ ,  $T=30^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	-4402.6	-4402.6	-4402.6	-4402.6	-57584.4	68284.1	-6910.8	24.6
2	-4377.0	-4377.0	-4377.0	-4377.0	-53176.3	63259.9	-7424.4	24.6
3	-4401.1	-4401.1	-4401.1	-4401.1	-57405.2	68061.3	-6948.1	24.6
4	-4327.1	-4327.1	-4327.1	-4327.1	-59020.6	67328.9	-9000.0	-
5	-4314.0	-4314.0	-4314.0	-4314.0	-54284.9	62540.7	-9000.0	-
6	-4448.7	-4448.7	-4448.7	-4448.7	-58566.3	67361.0	-9000.0	24.5

**Case 13:** Warehouse  $h_c=5.0$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except  
 roof  $\epsilon=0.9$ ,  $T=10^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	737.2	737.2	737.2	737.2	9549.4	-109691.3	-97193.2	19.2
2	463.6	463.6	463.6	463.6	5591.2	-104992.7	-97547.3	19.5
3	717.3	717.3	717.3	717.3	9266.7	-109605.3	-97469.5	19.2
4	-342.9	-342.9	-342.9	-342.9	11031.2	-108659.5	-99000.0	-
5	-355.0	-355.0	-355.0	-355.0	6683.2	-104263.3	-99000.0	-
6	-231.3	-231.3	-231.3	-231.3	10614.1	-108689.0	-99000.0	15.5

**Case 14:** Corridor  $h_c=5.0$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except  
 west wall  $\epsilon=0.9$ ,  $T=30^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	6839.9	-328.3	-3693.2	-328.3	-3693.2	-3693.3	-4896.4	23.5
2	6825.2	-340.4	-3687.7	-351.8	-3676.1	-3676.1	-4906.8	23.5
3	6844.5	-328.4	-3696.8	-328.4	-3696.8	-3696.8	-4902.8	23.5
4	6837.4	-342.3	-3981.8	-342.3	-3560.4	-3560.6	-4950.0	-
5	6842.7	-372.4	-3682.7	-372.4	-3682.7	-3682.7	-4950.0	-
6	6840.4	-340.8	-3703.0	-340.8	-3703.0	-3703.0	-4950.0	22.4

**Case 15:** Corridor  $h_c=5.0$  all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except  
 south wall  $\epsilon=0.9$ ,  $T=30^\circ\text{C}$

	west	south	east	north	floor	roof	Total	T
1	-2374.0	714.5	-2374.0	-211.2	-2374.0	-2374.1	-8992.8	22.3
2	-2389.3	737.2	-2381.9	-227.3	-2374.4	-2374.4	-9010.2	22.3
3	-2377.0	714.5	-2377.0	-211.4	-2377.0	-2377.0	-9005.0	22.3
4	-2364.8	687.0	-2364.9	-227.5	-2364.9	-2364.9	-9000.0	-
5	-2394.2	718.4	-2362.5	-236.6	-2362.5	-2362.5	-9000.0	-
6	-2363.3	687.1	-2363.3	-234.0	-2363.3	-2363.3	-9000.0	20.2

all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except

west1 wall       $\epsilon=0.9, T=30^{\circ}\text{C}$ 

**south1 wall      $\epsilon=0.9$ ,  $T=30^\circ\text{C}$**

	west1	west2	south1	south2	east	north	floor	roof	total	T
1	562.8	-230.9	369.3	-152.3	-454.8	-286.5	-809.0	-809.0	-1810.5	22.9
2	559.1	-235.9	368.7	-155.6	-483.7	-318.5	-780.4	-780.4	-1826.5	23.0
3	568.5	-229.6	373.6	-150.9	-478.7	-310.5	-802.6	-802.6	-1832.7	22.9
4	565.6	-248.6	375.2	-136.6	-471.7	-342.9	-783.0	-783.0	-1825.0	0.0
5	551.5	-241.6	362.4	-160.8	-481.0	-321.1	-767.2	-767.2	-1825.0	0.0
6	555.3	-233.7	364.9	-155.1	-474.9	-313.1	-784.2	-784.2	-1825.0	21.1

**all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except**

west1 wall       $\epsilon=0.9$ ,  $T=10^\circ\text{C}$ 

**south1 wall      $\epsilon=0.9, T=10^{\circ}\text{C}$**

	west1	west2	south1	south2	east	north	floor	roof	total	T
1	-908.6	-124.9	-596.4	-82.4	-246.1	-155.1	-437.2	-437.2	-2988.1	21.6
2	-914.2	-130.2	-603.0	-85.9	-266.9	-175.7	-430.6	-430.6	-3036.9	21.7
3	-910.4	-123.3	-598.4	-81.0	-256.9	-166.7	-430.6	-430.6	-2997.9	21.6
4	-930.7	-128.0	-618.7	-113.7	-280.9	-159.5	-421.7	-421.7	-3075.0	-
5	-917.0	-134.8	-606.2	-90.2	-271.8	-180.7	-437.2	-437.2	-3075.0	-
6	-920.7	-142.5	-608.6	-95.7	-277.7	-188.6	-420.6	-420.6	-3075.0	18.9

all surfaces:  $\epsilon=0.9$ ,  $T=20^\circ\text{C}$  except

**west1 wall**       $\epsilon=0.8, T=10^{\circ}\text{C}$

south1 wall      $\epsilon=0.8, T=10^{\circ}\text{C}$

	west1	west2	south1	south2	east	north	floor	roof	total	T
1	-863.6	-128.0	-566.4	-84.5	-252.7	-158.8	-447.5	-447.5	-2948.9	21.6
2	-868.4	-133.3	-572.1	-88.0	-273.3	-180.0	-440.7	-440.7	-2996.5	21.7
3	-864.6	-126.4	-567.8	-83.1	-263.3	-170.9	-440.9	-440.9	-2957.9	21.6
4	-890.8	-134.4	-592.5	-114.9	-291.0	-169.3	-441.0	-441.0	-3075.0	-
5	-879.8	-140.3	-582.6	-93.8	-282.5	-187.9	-454.1	-454.1	-3075.0	-
6	-882.8	-147.2	-584.4	-98.7	-287.9	-195.0	-439.4	-439.4	-3075.0	19.0

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## APPENDIX B

### Calculation of the Fanger MRT for Five Room Geometries

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The details of the calculation of the Fanger MRT are given in this appendix for the "room" test case of appendix A. The details include determining the view factors<sup>1</sup> between a person and the room walls, and applying the definition of the mean radiant temperature. The view factors of a person to room walls for the other test case geometries are given along with the Fanger MRT.

Calculation of view factors from a person facing east or west and standing at the center of the test case "room" are presented below. The view factors from a person to the shaded areas in figures B.1, B.2 , and B.3 are given by Fanger (1970). Fanger compiled charts containing view factors of a person to wall surfaces by experimentally determining the projected area of a body at different viewing angles. The data base consisted of 20 people standing, sitting; male, and female.

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<sup>1</sup>Referred to as angle factors by Fanger. Referred to as geometric configuration factors by Siegel (1981) who defines them as: the fraction of energy leaving surface 1 that is incident on surface 2.



View factor to the west wall (person facing west):  $F_{p-w}$

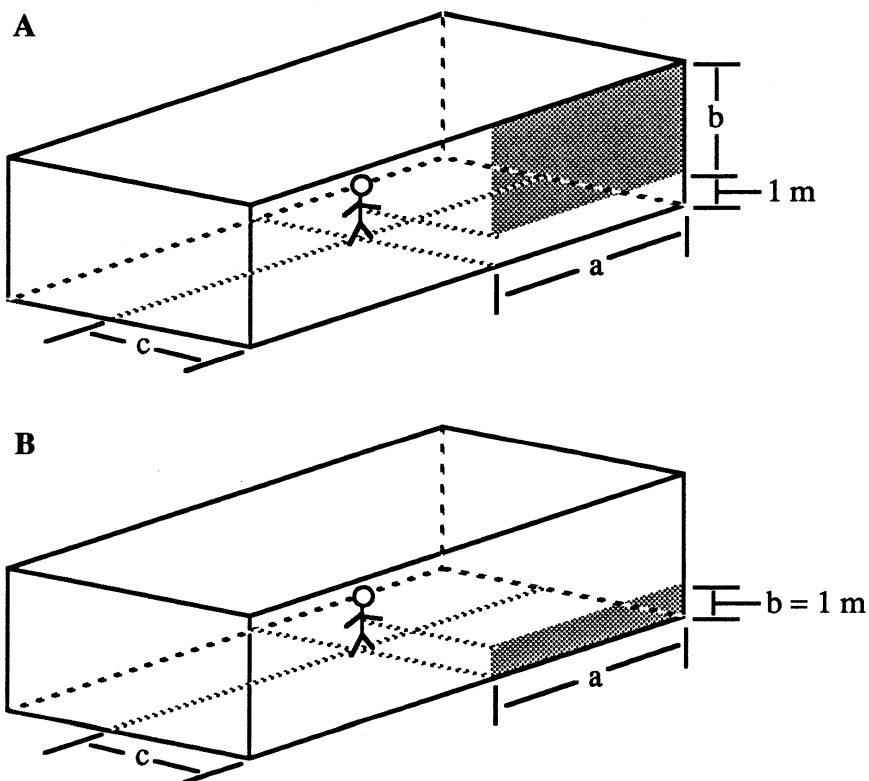
From Figure B.1A:

$$a = 6/2 = 3 \quad c = 4/2 = 2 \quad b = 2.5 - 1 = 1.5 \quad F_A = 0.050$$

From Figure B.1B:

$$a = 3 \quad b = 1 \quad c = 2 \quad F_B = 0.042$$

$$F_{p-w} = 2 F_A + 2 F_B = 0.184$$



**Figure B.1** Geometry of view factors given by Fanger for a person facing a wall

View factor to the north wall (person facing west):  $F_{p-n}$

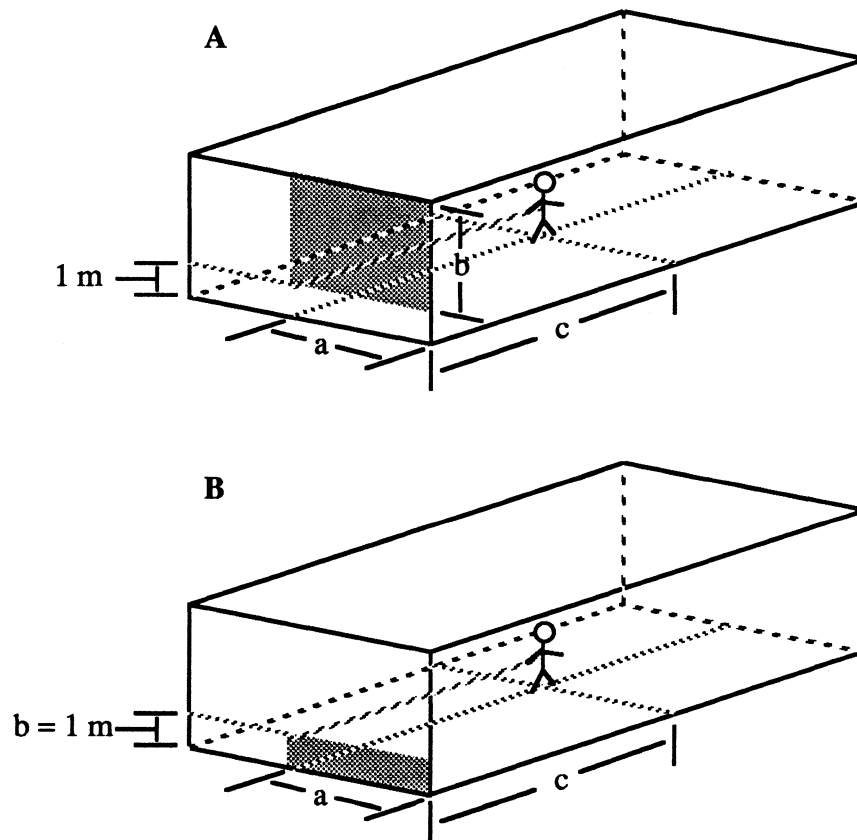
From Figure B.2A:

$$a = 2 \quad b = 1.5 \quad c = 3 \quad F_A = 0.022$$

From Figure B.2B:

$$a = 2 \quad b = 1 \quad c = 3 \quad F_B = 0.016$$

$$F_{p-n} = 2 F_A + 2 F_B = 0.076$$



**Figure B.2** Geometry of view factors given by Fanger for a person facing perpendicular to a wall.

View factor to the ceiling:  $F_{p-c}$

From Figure B.3:

$$a = 3 \quad b = 2 \quad c = 1.5 \quad F = 0.050$$

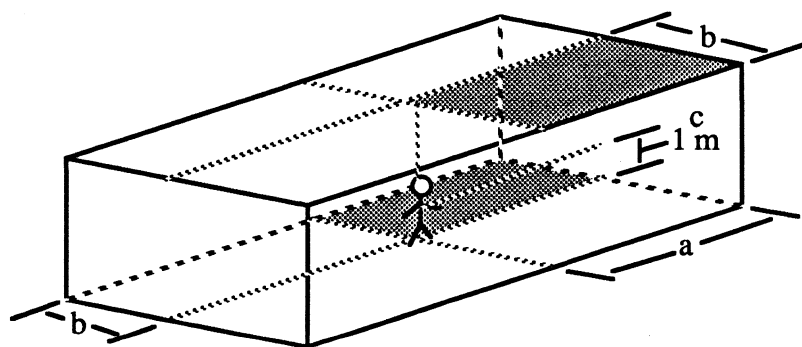
$$F_{p-c} = 4 F = 0.200$$

View factor to the floor:  $F_{p-f}$

From Figure B.3:

$$a = 3 \quad b = 2 \quad c = 1 \quad F_B = 0.070$$

$$F_{p-c} = 4 F = 0.280$$



**Figure B.3** Geometry of view factors given by Fanger for the ceiling and the floor.

The example calculations presented above were repeated with the person facing north, the short wall. Table B.1 summarizes the view factors from the person to each room surface in the test case "Room". The table shows that the orientation of the person in the room effects the view factors. Table B.2 summarizes the variation of the MRT for a person standing in the center of a room facing north and west respectively. Table B.3 gives the view factors for the other test geometries, and table B.4 gives the Fanger MRTs for the test enclosures.

The examples only considered a person standing. Fanger also gives view

factors for a person sitting. The change in posture will add more variability to the MRT. One further consideration is the location of the person in the room. The example assumes the person is at the center of the room. Fanger (1970) gives an example where the MRT is calculated through out the room. The MRT varies from 22°C to 26.8°C in the example. The reasons for the variability of the MRT are the view factors from the person to the walls.

**Table B.1** Summary of view factors for a person standing at the center of "Room"

facing west (or east)	facing north (or south)
$F_{p-w} = 0.184$	$F_{p-w} = 0.099$
$F_{p-e} = 0.184$	$F_{p-e} = 0.099$
$F_{p-n} = 0.076$	$F_{p-n} = 0.153$
$F_{p-s} = 0.076$	$F_{p-s} = 0.153$
$F_{p-c} = 0.280$	$F_{p-c} = 0.288$
$F_{p-f} = \underline{0.200}$	$F_{p-f} = \underline{0.208}$
1.000	1.000

**Table B.2** Summary of Fanger MRT calculated for a person standing at the center of "Room".

	$T_{mrt}$	
	Person facing west	Person facing east
case 5	21.8	21.0
case 6	19.2	18.5
case 7	18.0	17.9
case 8	22.0	21.4

$$T_{mrt} = \sum_{i=1}^N (T_i F_{p-i})$$

The wall temperatures are from the test cases presented in appendix A

**Table B.3** Summary of view factors for a person standing at the center of test case enclosures.

**Cube**

facing west (or east)	facing north (or south)
$F_{p-w} = 0.198$	$F_{p-w} = 0.156$
$F_{p-e} = 0.198$	$F_{p-e} = 0.156$
$F_{p-n} = 0.156$	$F_{p-n} = 0.198$
$F_{p-s} = 0.156$	$F_{p-s} = 0.198$
$F_{p-c} = 0.100$	$F_{p-c} = 0.100$
$F_{p-f} = \underline{0.192}$	$F_{p-f} = \underline{0.192}$
1.000	1.000

**Warehouse**

facing west (or east)	facing north (or south)
$F_{p-w} = 0.014$	$F_{p-w} = 0.014$
$F_{p-e} = 0.014$	$F_{p-e} = 0.014$
$F_{p-n} = 0.014$	$F_{p-n} = 0.014$
$F_{p-s} = 0.014$	$F_{p-s} = 0.014$
$F_{p-c} = 0.458$	$F_{p-c} = 0.458$
$F_{p-f} = \underline{0.486}$	$F_{p-f} = \underline{0.486}$
1.000	1.000

**Corridor**

facing west (or east)	facing north (or south)
$F_{p-w} = 0.298$	$F_{p-w} = 0.267$
$F_{p-e} = 0.298$	$F_{p-e} = 0.267$
$F_{p-n} = 0.002$	$F_{p-n} = 0.002$
$F_{p-s} = 0.002$	$F_{p-s} = 0.002$
$F_{p-c} = 0.140$	$F_{p-c} = 0.179$
$F_{p-f} = \underline{0.260}$	$F_{p-f} = \underline{0.283}$
1.000	1.000

**Windows**

facing west (or east)	facing north (or south)
$F_{p-w1} = 0.098$	$F_{p-w1} = 0.076$
$F_{p-w2} = 0.098$	$F_{p-w1} = 0.076$
$F_{p-e} = 0.184$	$F_{p-e} = 0.153$
$F_{p-n} = 0.076$	$F_{p-n} = 0.099$
$F_{p-s1} = 0.038$	$F_{p-s1} = 0.050$
$F_{p-s2} = 0.038$	$F_{p-s1} = 0.050$
$F_{p-c} = 0.200$	$F_{p-c} = 0.208$
$F_{p-f} = \underline{0.280}$	$F_{p-f} = \underline{0.288}$
1.000	1.000

**Table B.4** Summary of Fanger MRT calculated for a person standing at the center of test case geometries.

		$T_{mrt}$	
		Person facing west	Person facing east
Cube	case 1	22.0	21.6
	case 2	18.0	18.4
Room	case 5	21.8	21.0
	case 6	19.2	18.5
	case 7	18.0	17.9
	case 8	22.0	21.4
Warehouse	case 11	20.1	20.1
	case 12	24.6	24.6
	case 13	15.4	15.4
Corridor	case 14	23.0	22.7
	case 15	20.0	20.0
Windows	case 16	21.3	21.3
	case 17	18.7	18.7

---

## **APPENDIX C**

### **A Guide to Using the TRNSYS type 46 CRTF Zone Model**

---

This section describes the information needed to use the TRNSYS type 46 CRTF zone model and the format of the unit in a TRNSYS deck. Separate sets of parameters and inputs are required to describe the internal space, the walls, floor, ceiling, and windows. Walls are divided into three categories: external walls, interior partitions, and walls separating zones at different temperatures. The walls are modeled with transfer functions. The coefficients for the transfer functions may be specified from the tables in the ASHRAE Handbook of Fundamentals (1977 and 1981). For walls not described in tables, the program PREP, or BID (version 12.2), included with TRNSYS is used to generate transfer coefficients that can be included in the TRNSYS deck.

A number of options are available for describing the radiation network within a zone. View factors can be entered, or calculated internally for simple box structures, for use in the exact network. The program can also calculate area ratios to substitute for view factors. An MRT radiation network is also available. This network only requires areas of the walls to generate radiation resistances.

### ZONE PARAMETERS

<u>PARAMETER NO.</u>	<u>DESCRIPTION</u>
1	Mode: 1 - energy rate control 2 - temperature level control
2	Units: 1 - SI units 2 - English units
3	$V_a$ - zone volume of air ( $m^3$ , $ft^3$ )
4	$K_1$ - constant air change per hour
5	$K_2$ - proportionality constant for air change due to indoor / outdoor temperature difference ( $^{\circ}C^{-1}$ , $^{\circ}F^{-1}$ )
6	$K_3$ - proportionality constant for air change due to wind effects ( $(m/s)^{-1}$ , $mph^{-1}$ )
7	CAP - capacitance of room air and furnishings (KJ/ $^{\circ}C$ , BTU/ $^{\circ}F$ )
8	N - number of total surfaces comprising room description ( $1 \leq N \leq 15$ )
9	$T_o$ - initial room temperature; also used for calculation of inside radiation coefficients ( $^{\circ}C$ , $^{\circ}F$ )
10	$\omega_o$ - initial room humidity ratio
11	NUZT - number of unique adjacent zone temperature inputs
12	NUSR - number of unique solar radiation inputs (different orientations, at least 1)
13	WIND - wind speed (m/s, mph)
14	PAST - number of past time steps in transfer function (often 2)
15	BASE - time base of transfer functions
<b>Mode 1 - Additional parameters</b>	
16	$T_{min}$ - set point temperature for heating ( $^{\circ}C$ , $^{\circ}F$ )
17	$T_{max}$ - set point temperature for cooling ( $^{\circ}C$ , $^{\circ}F$ )
18	$w_{min}$ - set point humidity ratio for humidification
19	$w_{max}$ - set point humidity ratio for dehumidification



**WALLS**

<u>PARAMETER NO.</u>	<u>DESCRIPTION</u>
1	Surface number: 1 to 15
2	Surface Type: <ul style="list-style-type: none"> <li>1 - Exterior wall</li> <li>2 - Interior wall</li> <li>3 - Wall separating zones</li> <li>-1 - Duplicate last wall (enter parameters 1-4 only)</li> </ul>
3	A - area (m <sup>2</sup> , ft <sup>2</sup> )
4	ID - input ID <ul style="list-style-type: none"> <li>exterior wall - solar orientation number</li> <li>interior partition - no meaning</li> <li>wall separating zones - zone input number</li> </ul>
5	$\rho$ - reflectance of inner surface to solar
6	$\alpha$ - absorptance of exterior surface to solar
7	$I_{\text{coef}}$ <ul style="list-style-type: none"> <li>1 - standard ASHRAE roofs from Table 26, Chap. 25, 1977 ASHRAE Fundamentals and Table 28, Chap. 26, 1981 ASHRAE Fundamentals</li> <li>2 - standard ASHRAE exterior walls from Table 27, Chap. 26, 1981 ASHRAE Fundamentals</li> <li>3 - standard ASHRAE partitions, floors, and ceilings from Table 29, Chap. 25, 1977 ASHRAE Fundamentals</li> <li>4 - b, c, d coefficients entered using parameters following:</li> </ul>
( $I_{\text{coef}} < 4$ )	
8	$N_{\text{table}}$ - number of wall, roof, etc. from table specified by $I_{\text{coef}}$
( $I_{\text{coef}} = 4$ )	
8	$h_c$ - inside convection coefficient (KJ/hr-m <sup>2</sup> -°C, BTU/hr-ft <sup>2</sup> -°F)
9	$N_b$ - number of b coefficients
10	$N_c$ - number of c coefficients
11	$N_d$ - number of d coefficients
12	FLAG - 0 - no conversion of coefficients (usually 0) 1 - remove interior convection coefficient

13	$b_o$ - coefficient of current sol-air temperature (KJ/hr-m <sup>2</sup> -°C, BTU/hr-ft <sup>2</sup> -°F)
.	.
.	.
.	.
13 + N <sub>b</sub>	$c_o$ - coefficient of current equivalent zone air temperature (KJ/hr-m <sup>2</sup> -°C, BTU/hr-ft <sup>2</sup> -°F)
.	.
.	.
.	.
13 + N <sub>b</sub> + N <sub>c</sub>	$d_1$ - coefficient of previous hour's heat flux
.	.
.	.
.	.

#### NON-ASHRAE WALL - (CONDUCTION INPUT)

1	Surface number: 1 to 15
2	Surface type: Specify 4
3	A - area (m <sup>2</sup> , ft <sup>2</sup> )
4	$\rho$ - reflectance of inner surface to solar
5	$h_{c,i}$ - inside convection coefficient (KJ/hr-m <sup>2</sup> -°C, BTU/hr-ft <sup>2</sup> -°F)

#### WINDOW

1	Surface number: 1 to 15				
2	Surface Type: Specify 5				
3	A - area (m <sup>2</sup> , ft <sup>2</sup> )				
4	ID - input ID; solar orientation number				
5	Window mode: <table> <tr> <td>1</td><td>transmitted solar and thermal energy calculated internally</td></tr> <tr> <td>2</td><td>transmitted solar and thermal energy transfer provided as inputs</td></tr> </table>	1	transmitted solar and thermal energy calculated internally	2	transmitted solar and thermal energy transfer provided as inputs
1	transmitted solar and thermal energy calculated internally				
2	transmitted solar and thermal energy transfer provided as inputs				
6	$\tau_t$ - transmittance for total solar radiation				
7	$\alpha$ - absorptance of glass				
8	$h_{c,i}$ - inside convection coefficient				

(KJ/hr-m<sup>2</sup>-°C, BTU/hr-ft<sup>2</sup>-°F)

- 9 U - loss coefficient of window not including convection at the inside or outside surface (KJ/hr-m<sup>2</sup>-°C, BTU/hr-ft<sup>2</sup>-°F)

**VIEW FACTORS****Geometry mode 0: Approximate view factors with area ratios****Geometry mode 1: Rectangular Parallelepiped**

(unchanged from type 19)

**Geometry mode 2: User Defined View Factors**

- |     |   |
|-----|---|
| 1   | Geometry Mode: Specify 2                                  |
| 2   | F <sub>11</sub> - view factor from surface 1 to surface 1 |
| .   | .   |
| .   | .   |
| .   | .   |
| N+1 | F <sub>1N</sub> - view factor from surface 1 to surface N |
| N+2 | F <sub>22</sub> - view factor from surface 2 to surface 2 |
| .   | .   |
| .   | .   |
| .   | .   |
| 2N  | F <sub>2N</sub> - view factor from surface 2 to surface N |
| .   | .   |
| .   | .   |
| .   | .   |

**Geometry mode 3: Use mean radiant temperature network****INPUT NUMBER****DESCRIPTION**

- |   |  |
|---|--|
| 1 | T <sub>a</sub> - ambient temperature (°C, °F)                              |
| 2 | ω <sub>a</sub> - ambient humidity ratio                                    |
| 3 | T <sub>v</sub> - temperature of ventilation flow stream (°C, °F)           |
| 4 | m <sub>v</sub> - mass flow rate of ventilation flow stream (Kg/hr, lbm/hr) |
| 5 | ω <sub>v</sub> - humidity ratio of ventilation flow stream                 |
| 6 | ω <sub>g</sub> - rate of moisture gain (other than people) (Kg/hr, lbm/hr) |
| 7 | N <sub>pepl</sub> - number of people in zone                               |
| 8 | I <sub>act</sub> - activity level of people (see Table 2)                  |
| 9 | Q <sub>IR</sub> - radiative energy input due to lights, equipment, etc.    |

	(KJ/hr, BTU/hr)
10	$Q_{int}$ - sum of all other instantaneous gains to space (KJ/hr, BTU/hr)
11	$W$ - wind speed for residential infiltration model (m/s, mph)
<u>INPUTS AS REQUIRED</u>	
12	$T_z$ - adjacent zone temperature 1
13	$(UA)_c$ - additional conductance for heat transfer between zone 1 (KJ/hr-°C, BTU/hr-°F)
14	$T_z$ - adjacent zone temperature 2
15	$(UA)_c$ - additional conductance for heat transfer between zone 2 (KJ/hr-°C, BTU/hr-°F)
.	.
.	.
.	.
-----	
12+NUZT	$I_t$ - total incident radiation on orientation 1
13+NUZT	$I_t$ - total incident radiation on orientation 2
.	.
.	.
.	.
	$Q_{RAD}$ - total solar radiation passing through window 1 in mode 2 (KJ/hr, BTU/hr)
	$Q_{COND}$ - thermal energy gain through window 1 in mode 2 (KJ/hr, BTU/hr)
.	.
.	.
.	.
-----	
12+NUZT+NUSR	$Q_{COND}$ - energy transfer into the zone at the inside surface of the wall through conduction wall input 1
13+NUZT+NUSR	$Q_{COND}$ - energy transfer into the zone at the inside surface of the wall through conduction wall input 2
.	.
.	.
.	.

<b>OUTPUT NUMBER</b>	<b>Description (Averaged over time step)</b>
1	$T_z$ - zone Temperature
2	$w_z$ - zone humidity ratio
3	$\dot{Q}_{\text{CRTF}}$ - gains to the room due to convection from all surfaces (KJ/hr, Btu/hr)
4	$\dot{Q}_{\text{spepl}}$ - sensible energy gains due to convection from people
5	$\dot{Q}_{\text{infl}}$ - sensible infiltration gains
6	$\dot{Q}_v$ - sensible gains due to forced ventilation
Mode 1	
7	$\dot{Q}_{\text{sens}}$ - sensible load
8	$\dot{Q}_{\text{lat}}$ - latent load
9	$\dot{Q}_{\text{c,max}}$ - maximum cooling load
10	$\dot{Q}_{\text{c,max}}$ - maximum heating load
All modes	
-----	
11	$\dot{Q}_{\text{ipepl}}$ - latent people load
12	$\dot{Q}_{\text{ivent}}$ - latent ventilation load
13	$\dot{Q}_{\text{igen}}$ - latent internal generation load
14	$T_{\text{mrt}}$ - estimate of mean radiant temperature

---

**APPENDIX D**  
**TRNSYS type 46 CRTF Building Zone Component**  
**Program Listing**

---

This section lists the type 46 CRTF building zone model that is a subroutine in TRNSYS.

<u>Routine</u>	<u>Page</u>
TYPE 46	117
CRTFgen	128
FHAT	143
STAR_EXACT	144
STAR_MRT	147
EXPAND_ALL	150
TAU_ALF_AREA	152
COMBINE_ALL	153
REDUCE	156
POLY_MULT	157
PADE	158

C January 4, 1988

C SINGLE ZONE BUILDING MODEL USING A COMPREHENSIVE ROOM  
C TRANSFER FUNCTION TO MODEL THE STRUCTURAL LOADS

C Subroutine file structure

C **TYPE46** : Main  
C DIFFEQ (TRNSYS routine)  
C CRTFgen  
C TABLE (TRNSYS routine) assign ASHRAE.COF to FOR008  
C ENCL (TRNSYS routine)  
C TYPECK (TRNSYS routine)  
C FHAT  
C STAR\_EXACT  
C INVERT (TRNSYS routine)  
C STAR\_MRT  
C INVERT (TRNSYS routine)  
C SUMF (within star\_MRT file)  
C EXPAND\_ALL  
C TAU\_ALF\_AREA  
C COMBINE\_ALL  
C REDUCE  
C POLY\_MULT  
C PADE  
C INVERTD (double precision INVERT)  
C

**SUBROUTINE** TYPE46(TM,XIN,OUT,DUM1,DUM2,PAR,INFO)  
REAL MAIR,K1,K2,K3  
INTEGER WMODE2,USR,UZT,COLUMN(15),TERM,TIME,START,NCF  
DIMENSION XIN(\*),OUT(\*),PAR(\*),INFO(\*)  
REAL CFIN(15,10),CFOUT(15,10),INPUTS(15,10),OUTPUTS(15,10)  
REAL ECONV(2),CP(2),DENS(2),TCONV(2),HVAP(2),QPSEN(11)  
REAL QPLAT(11),UAZ(15),NOW,FIRST,TIME\_BASE  
COMMON /SIM/ TIME0,TIMEF,DELT  
COMMON /STORE/ NSTORE,IAV,S(5000)  
COMMON /LUNITS/ LUR,LUW,IFORM  
DATA ECONV/3.6,3.41/,CP/1.012,.24/,DENS/1.204,0.075/  
DATA TCONV/273.15,459.67/,HVAP/2468.,1061./  
DATA QPSEN/60.,65.,75.,75.,90.,100.,100.,100.,120.,165.,185./  
DATA QPLAT/40.,55.,95.,75.,95.,130.,205.,180.,255.,300.,340./

C  
C INITIALIZE TRANSFER FUNCTION COEFFICIENTS IF THIS IS THE  
C FIRST CALL IN THE SIMULATION  
C

```

      IF (INFO(7).LE.-1) THEN
        CALL CRTFgen(PAR,INFO)
      END IF
      QLOADS=0.0
      QINST=0.0
      QSPEPL=0.0
      QCOND=0.0
      FLWV=0.0
      TVENT=0.0
      FLWI=0.0
      TAMB=0.0
      CAP=0.0

C      EXTRACT POINTER INTO S(), LAST ROOM TEMP, AND THE
C      NUMBER OF TERMS IN THE EQUATION

      TR=0.0
      ISTORE=INFO(10)
      IF (INFO(7).EQ.0) S(ISTORE)=S(ISTORE+1)
      TLAST=S(ISTORE)
      WRLAST=S(ISTORE+2)
      NTERMS=INT(S(ISTORE+3))
      NCF=INT(S(ISTORE+4))
      RSTAR=S(ISTORE+5)
      WMODE2=S(ISTORE+6)
      NCOND=S(ISTORE+7)
      INDEX=ISTORE+12 !STORAGE ISTORE+8 +11 FOR INTERPOLATION

C      RECALL TRANSFER FUNCTION COEFFS. E,G,D1,D2...,F1,F2,...,H1,H2
C      EACH DIFFERENT ADJACENT ZONE HAS A 'D' COEFFICIENT, AND
C      EACH EXTERIOR ORIENTATION HAS AN 'F' COEFFICIENT.

      DO TERM=1,NTERMS
        DO TIME=1,NCF
          CFIN(TIME,TERM)=S(INDEX)
          INDEX=INDEX+1
        END DO
      END DO
      DO TERM=1,NTERMS
        DO TIME=1,NCF
          CFOUT(TIME,TERM)=S(INDEX)
          INDEX=INDEX+1
        END DO
      END DO
      IX=INDEX

```



```

C      RECALL PAST INPUT HISTORIES
C      TAMB, TROOM, TZONE1, TZONE2,..., SOLAR1, SOLAR2,..., GAINS
C      DON'T SHIFT INPUTS AND OUTPUTS IN TIME IF ITERATING

```

```

TIME_BASE = PAR(15)
NOW = AMOD(TM, TIME_BASE)
FIRST = NOW/DELT

```

```

START=2
IF (DELT.NE.TIME_BASE) START = 1
IF (FIRST.GT..99 .AND. FIRST.LT.1.01) START = 2
IF (INFO(7).GT.0) START=1
DO TERM=1, NTERMS
  DO TIME=START, NCF
    INPUTS (TIME, TERM)=S (INDEX)
    INDEX=INDEX+1
  END DO
  INDEX=INDEX+START-1
END DO

```

```

C      RECALL PAST LOADS
DO TERM=1, NTERMS
  DO TIME=START, NCF
    OUTPUTS (TIME, TERM)=S (INDEX)
    INDEX=INDEX+1
  END DO
  INDEX=INDEX+START-1
END DO

```

```

C      EXTRACT ZONE INFO FROM PAR()

```

```

MODE=INT(PAR(1))
IU=INT(PAR(2))
VOL=PAR(3)
K1=PAR(4)
K2=PAR(5)
K3=PAR(6)
CAP=PAR(7)/TIME_BASE
NS=PAR(8)
W0=PAR(10)
UZT=INT(PAR(11)+.1)
USR=INT(PAR(12)+.1)
NP=15
IN=11
IF (MODE.EQ.1) THEN
  TMIN=PAR(16)

```

```

      TMAX=PAR(17)
      WMIN=PAR(18)
      WMAX=PAR(19)
      NP=NP+4
      END IF

C      LOAD THE INPUTS

      TAMB=XIN(1)
      WAMB=XIN(2)
      TVENT=XIN(3)
      FLWV=XIN(4)
      WVENT=XIN(5)
      WGEN=XIN(6)
      NPEPL=XIN(7)
      IACT=XIN(8)
      QRAD=XIN(9)
      QINST=XIN(10)
      WIND=XIN(11)

C ----- STORE CURRENT INPUTS -----
C      DECODE INPUTS FROM DEK IN XIN ARRAY

C      AMBIENT TEMPERATURE
      IF ((FIRST.GT.0.99 .AND. FIRST.LT.1.01) .OR.
&      (NOW.GT. -0.01 .AND. NOW.LT. 0.01)) THEN
        INPUTS(1,3)=TAMB
      END IF

C      ADJACENT ZONE TEMPS

      IN=12
      DO I=4,3+UZT
        IF ((FIRST.GT.0.99 .AND. FIRST.LT.1.01) .OR.
&      (NOW.GT. -0.01 .AND. NOW.LT. 0.01)) THEN
          INPUTS(1,I)=XIN(IN)
        END IF
        UAZ(I-3)=XIN(IN+1)
        IN=IN+2
      END DO

C      SOLAR RADIATION

      QCOND=0.0
      SFACT=1./(1-CFIN(1,1)*RSTAR)

C      INTERNAL CALCULATION MODE 1 WINDOWS
      DO I=4+UZT,3+UZT+USR-WMODE2

```

```

      IF ((FIRST.GT.0.99 .AND. FIRST.LT.1.01) .OR.
&      (NOW.GT. -0.01 .AND. NOW.LT. 0.01)) THEN
        INPUTS(1,I)=XIN(IN)
      END IF
      IN=IN+1
    END DO
C      EXTERNAL CALCULATION MODE 2 WINDOWS
    DO I=4+UZI+USR-WMODE2,3+UZI+USR
      IF ((FIRST.GT.0.99 .AND. FIRST.LT.1.01) .OR.
&      (NOW.GT. -0.01 .AND. NOW.LT. 0.01)) THEN
        INPUTS(1,I)=XIN(IN)
      END IF
      QCOND=QCOND+XIN(IN+1)*SFACT
      IN=IN+2
    END DO

C      CONDUCTION INPUTS TO STAR POINT

    DO I=1,NCOND
      QCOND=XIN(IN)*SFACT+QCOND
      IN=IN+1
    END DO

C ----- CALCULATE ZONE AIR LOADS -----

C      SENSIBLE PEOPLE LOAD

    QSPEPL=ECONV(IU)*NPEPL*QPSEN(IACT)

C      RADIATION GAINS

    IF ((FIRST.GT.0.99 .AND. FIRST.LT.1.01) .OR.
&      (NOW.GT. -0.01 .AND. NOW.LT. 0.01)) THEN
      INPUTS(1,2)=QRAD+0.7*QSPEPL
    END IF

C      INFILTRATION FLOW RATE

    MAIR=VOL*DENS(IU)
    FLWI=MAIR*(K1+K2*ABS(TAMB-TLAST)+K3*WIND)

C-----

C      INITIALIZE HISTORIES IF THIS IS AN INITIALIZATION CALL.
C      THIS SECTION IS NOT IN INITIALIZATION ROUTINE BECAUSE
C      IT MAKES USE OF THE INPUTS WHICH WERE DECODED ABOVE.

```

```

IF (INFO(7).EQ.-1) THEN

    INPUTS(1,1)=PAR(9)
    DO TERM=1, NTERMS
        SUMIN=0.0
        SUMOUT=0.0
        DO TIME=1, NCF
            INPUTS (TIME, TERM)=INPUTS (1, TERM)
            SUMIN=SUMIN+CFIN (TIME, TERM)
            SUMOUT=SUMOUT+CFOUT (TIME, TERM)
        END DO
        DO TIME=1, NCF
            OUTPUTS (TIME, TERM)=SUMIN/SUMOUT*INPUTS (1, TERM)
        END DO
    END DO
    TR=PAR(9)
    QLOADS=0.0
END IF

C -----
C     DON'T CALCULATE LOADS IF THIS IS AN INTIALIZATION CALL
C     IF (INFO(7).EQ.-1) GOTO 999
C -----

C     ENVELOPE AND OTHER DELAYED LOADS (TRANSFER FUNCTIONS)

IF (DELT.EQ.TIME_BASE) THEN
C     INSTANTANEOUS ROOM AIR LOADS (TERMS INDEPENDENT OF ROOM TEMP)
    QLOADS=QINST+0.3*QSPEPL+QCOND+(FLWV*TVENT+FLWI*TAMB)*CP(IU)
    &      + (CAP+CFIN(1,1)/2)*TLAST
C     ADDITIONAL ADJACENT ZONE CONDUCTION
    UAS=0.0
    DO I=4, 3+UZT
        UAS=UAS+UAZ(I-3)
        QLOADS=QLOADS+UAZ(I-3)*INPUTS(1,I)
    END DO

    INPUTS(1,1)=0.0
    DO TERM = 1, NTERMS
        OUTPUTS(1, TERM)=0.0
        SUM=0.0
C     QNOW = SUM OF (INPUT * COEFF.) + SUM OF (QPAST * COEFF.)
        DO TIME=1, NCF
            SUM=SUM+CFIN (TIME, TERM) *INPUTS (TIME, TERM)
            &      -CFOUT (TIME, TERM) *OUTPUTS (TIME, TERM)
        END DO

```

```

        OUTPUTS (1, TERM) = SUM
        QLOADS = QLOADS + SUM
    END DO

C      ENERGY BALANCE ON ROOM AIR

        TR = QLOADS / (UAS + CAP + (FLWI + FLWV) * CP (IU) - CFIN (1, 1) / 2)

C      CALCULATE QROOM LOAD

        OUTPUTS (1, 1) = OUTPUTS (1, 1) + CFIN (1, 1) * (TR + TLAST) / 2.
        INPUTS (1, 1) = (TR + TLAST) / 2.

ELSE IF (FIRST.GT.0.99 .AND. FIRST.LT.1.01) THEN
C      FIRST STEP IN INTERPOLATION MODE
        S (ISTORE+10) = TLAST
        TLAST = S (ISTORE+11)

        QLOAD1 = QINST + 0.3 * QSPEPL + QCOND + (FLWV * TVENT + FLWI * TAMB) * CP (IU)
&          + (CAP + CFIN (1, 1) / 2) * TLAST
C      ADDITIONAL ADJACENT ZONE CONDUCTION
        UAS = 0.0
        DO I = 4, 3 + UZT
            UAS = UAS + UAZ (I)
            QLOAD1 = QLOAD1 + UAZ (I) * INPUTS (1, I)
        END DO
        INPUTS (1, 1) = 0.0
        DO TERM = 1, NTERMS

C      EXTRAPOLATE EACH INPUT TO END OF TIMEBASE

            IF (TERM.NE.1) THEN
                DEL_INPUT = ( INPUTS (1, TERM) - INPUTS (2, TERM) ) / DELT
                INPUTS (1, TERM) = INPUTS (2, TERM) + DEL_INPUT * TIME_BASE
            END IF

C      CALCULATE OUTPUTS AT END OF TIME BASE

            OUTPUTS (1, TERM) = 0.0
            SUM = 0.0
            DO TIME = 1, NCF
                SUM = SUM + CFIN (TIME, TERM) * INPUTS (TIME, TERM)
&          - CFOUT (TIME, TERM) * OUTPUTS (TIME, TERM)
            END DO
            OUTPUTS (1, TERM) = SUM
            QLOAD1 = QLOAD1 + SUM

```

```

END DO

C      ROOM TEMPERATURE AT END OF TIMEBASE

      TR=QLOAD1/ (UAS+CAP+ (FLWI+FLWV) *CP (IU) -CFIN (1,1) /2)
      S (ISTORE+11)=TR
      S (ISTORE+9) = QLOAD1
C      ROOM LOAD OUTPUT AT END OF TIME BASE

      OUTPUTS (1,1)=OUTPUTS (1,1)+CFIN (1,1) * (TR+TLAST) /2.
      INPUTS (1,1)=(TR+TLAST) /2.
END IF

IF (DELT.NE.TIME_BASE) THEN
  IF (NOW.LT.0.01) NOW = 1.0
  TR = S (ISTORE+10) + ( S (ISTORE+11) - S (ISTORE+10) ) *NOW
  QLOADS = S (ISTORE+8) + ( S (ISTORE+9) - S (ISTORE+8) ) * NOW
  IF (NOW.GT.0.99) S (ISTORE+8)=S (ISTORE+9)
END IF

C-----

IF (MODE.EQ.1) THEN

C      ENERGY RATE CONTROL
      QLOADS = 0.0
C      CHECK IF ROOM TEMPERATURE IS WITHIN BOUNDS

      IF ((TR.GT.TMAX) .OR. (TR.LT.TMIN)) THEN

C          TEMPERATURE IS OUTSIDE ALLOWED RANGE.
C          DETERMINE ENERGY TO BRING TR INTO RANGE

          IF (TR.GT.TMAX) TSET=TMAX
          IF (TR.LT.TMIN) TSET=TMIN

C      -- CALCULATE THE NEW HEAT FLOWS --
      QSVENT=FLWV*CP (IU) * (TVENT-TSET)
      QSINFL=FLWI*CP (IU) * (TAMB-TSET)
      QCAP= (TSET-TLAST) *CAP
      QZONE=0.
      DO I=4,UZT+3
        TZONE=INPUTS (1,I)
        QZONE=UAZ (I) * (TZONE-TSET) +QZONE
      END DO

```

```

C          RESET QROOM CALCULATED ABOVE

          OUTPUTS(1,1)=OUTPUTS(1,1)+CFIN(1,1)*(TSET-(INPUTS(1,1)))

C          CALCULATE ENERGY REQUIREMENT FROM ROOM AIR ENERGY BALANCE

          QLOADS=QINST+0.3*QSPEPL+QCOND+QZONE+QSVENT+QSINFL+QCAP
          DO TERM=1, NTERMS
            QLOADS=QLOADS+OUTPUTS(1,TERM)
          END DO
          TR=TSET
          S(ISTORE+11) = TSET
          INPUTS(1,1)=TR
        END IF
      END IF

C-----
C  LATENT LOADS
C
      QLPEPL=0.0
      IF (IACT.GE.1 .AND. IACT.LE.11)
        &    QLPEPL=ECONV(IU)*NPEPL*QPLAT(IACT)
      ILAT=0
      QLAT=0.
      AA=-(FLWI+FLWV)/MAIR
      BB=(FLWI*WAMB+FLWV*WVENT+WGEN+QLPEPL/HVAP(IU))/MAIR

      CALL DIFFEQ(TM,AA,BB,WRLAST,WRF,WR)

      IF (MODE.EQ.1 .AND. (WR.GE.WMAX .OR. WR.LE.WMIN)) THEN
        ILAT=1
        IF (WR.GT.WMIN) THEN
          WR=WMAX
          WRF=WR
        ELSE
          WR=WMIN
          WRF=WR
        END IF
      END IF
      QLINE=FLWI*(WAMB-WR)*HVAP(IU)
      QLVENT=FLWV*(WVENT-WR)*HVAP(IU)
      QLGEN=WGEN*HVAP(IU)
      IF (ILAT.EQ.1) QLAT=QLINE+QLVENT+QLGEN+QLPEPL
      S(ISTORE+2)=WRF
C
C-----

```

```

999  OUT(1)=(TR+TLAST)/2
      OUT(2)=WR

C      GAINS TO ROOM FROM SURFACES
C      ONLY GIVES VALUE AT TIME_BASE INTERAL

      IF (DELT.EQ.TIME_BASE) THEN
        SUM=0.0
        DO TERM=1, NTERMS
          SUM=SUM+OUTPUTS(1, TERM)
        END DO
        OUT(3)=SUM
      ELSE
        SUM = 0.0
        DO TERM = 1, NTERMS
          SUM = SUM + OUTPUTS(2, TERM)
          &      + (OUTPUTS(1, TERM) - OUTPUTS(2, TERM)) * NOW
        END DO
        OUT(3) = SUM
      END IF

C      ADDITIONAL ROOM AIR GAINS

      OUT(4)=0.3*QSPEPL
      OUT(5)=FLWI*CP(IU)*(TAMB-TR)
      OUT(6)=FLWV*CP(IU)*(TVENT-TR)
      OUT(7)=QLOADS
      OUT(8)=QLAT

C      MAXIMUM HEATING AND COOLING LOADS

      IF (QSENS.GT.0.) OUT(9)=AMAX1(QSENS+AMAX1(QLAT,0.),OUT(9))
      IF (QSENS.LT.0.) OUT(10)=AMIN1(QSENS+AMIN1(QLAT,0.),OUT(10))

      OUT(11)=QLPEPL
      OUT(12)=QLVENT
      OUT(13)=QLGEN
      Tstar=out(3)*rstar+out(1)
      OUT(14)=1.25*Tstar-0.25*out(1)+0.033*out(3)/vol

C-----

C      SAVE HISTORIES IN S()
      S(ISTORE+1)=TR
      INDEX=IX

C      PAST INPUT HISTORIES
C      TAMB, TROOM, TZONE1, TZONE2,..., SOLAR1, SOLAR2,..., GAINS

```



```
DO TERM=1, NTERMS
  DO TIME=1, NCF
    S (INDEX) = INPUTS (TIME, TERM)
    INDEX=INDEX+1
  END DO
END DO
```

C       PAST LOADS

```
DO TERM=1, NTERMS
  DO TIME=1, NCF
    S (INDEX) = OUTPUTS (TIME, TERM)
    INDEX=INDEX+1
  END DO
END DO
RETURN
END
```

C INITIALIZE INFORMATION ON FIRST CALL IN SIMULATION  
 C CREATE A CRTF BASED ON TRNSYS DECK INFORMATION  
 C 8/10/87

**SUBROUTINE CRTFgen(PAR, INFO)**

```

INTEGER WMODE, COLUMN(15), USR, UZT, WMODE2, NCOND, START
DIMENSION PAR(*), INFO(*), HCI(15), IPAR(15), SUMCF(15)
DIMENSION RHO(15), AREA(15), NCF(15, 3), LIST(15)
DIMENSION B(10, 15), C(10, 15), D(10, 15), IS(15)
DIMENSION FV(15, 15), FH(15, 15), SUMFV(15)
DIMENSION SIGMA(2), TCONV(2), ECONV(2), FHL(15, 15), RHOL(15)
DIMENSION WCONV(2), HCON(2), VLGTH(20), RTFCF(10, 30)
DIMENSION HCOEF(2), NCFCR(15), ROUT(15)
DIMENSION RS(15), TAU(15), ALPHA(15), U(15)
DIMENSION NCFE(15, 15), NCFC(15)
DOUBLE PRECISION AA(50), BB(50), DD(21), EE(21), SUM(15)
DOUBLE PRECISION SCTFCF(50, 15), ETFCF(15, 15, 50), CFR(10, 15)
DOUBLE PRECISION XCOF(30), COF(30), ROOTR(30), ROOTI(30)
DOUBLE PRECISION ROOTS(100)
CHARACTER*12 DSH$(15)
COMMON /SIM/ TIME0, TIMEF, DELT
COMMON /STORE/ NSTORE, IAV, S(5000)
COMMON /LUNITS/ LUR, LUW, IFORM
DATA IUNIT/0/, SIGMA/2.0411E-07, 1.714E-09/, TCONV/273.15, 459.67/
DATA LIST/1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15/
DATA ECONV/3.6, 3.41/, WCONV/1., 0.447/, U/15*0.0/
DATA HCOEF/9.58, 0.47/, IPAR/15*0/, TAU/15*0.0/
DATA HCON/3.6, 0.1761/
DATA NSFTM/8/, DSMIN/.125/, NSMAX/15/

```

C-----

C RESET AND INITIALIZE VARIABLES

```

DO I=1, 50
  AA(I)=0
  BB(I)=0
  DO J=1, 15
    DO K=1, 15
      ETFCF(J, K, I)=0
    END DO
  END DO
END DO

DO I=1, 15
  NCFC(I)=0
  DO J=1, 50
    SCTFCF(J, I)=0.0D0
  END DO
END DO

```

```

      END DO
    END DO

    DO I=1,15
      DSH$(I)='-----'
    END DO
  C-----
  C      INITIALIZE COUNTS
    WMODE2=0
    NCOND=0
    ATOT=0.
  C-----
  C      EXTRACT ZONE PARAMETER INFO

  C      MODE OF OPERATION ENERGY = 1; TEMPERATURE =2
    MODE=INT(PAR(1)+0.1)
  C      UNITS 1 = SI; 2 = ENGLISH
    IU=INT(PAR(2)+0.1)
  C      GENERAL ZONE PARAMETERS, SOME NOT NEEDED IN SET-UP
    NS=INT(PAR(8)+0.1)      ! number of surfaces
    T0=PAR(9)+TCONV(IU)    ! average temperature for rad. resistances
    UZT=INT(PAR(11)+.1)    ! number of adjacent zones
    USR=INT(PAR(12)+.1)    ! number of solar orientations (usr>0)
    FIT = 3.0              ! star network error exponent
    WOUT=PAR(13)*WCONV(IU) ! wind speed for convection coeff.
    MM=PAR(14)              ! past time steps in reduced CRTF
    NPAR=15                 ! counter of parameters
  C      CHECK FOR ENERGY RATE CONTROL - ADDITIONAL PARAMETERS
    IF(MODE.EQ.1) NPAR=19
    IP=NPAR
  C-----
  C      WALL AND WINDOW PARAMETERS
  C***** MAIN SURFACE LOOP START *****

    DO 50 K=1,NS
  C      SURFACE NUMBER
      I=INT(PAR(IP+1)+0.1)
  C      SURFACE TYPE
      ITYPE=INT(PAR(IP+2)+0.1)
  C-----
  C      ERROR CHECKS FOR VALID SURFACE NUMBER AND TYPE

    IF(I.LT.1 .OR. I.GT.NSMAX) GO TO 600
    IF(ITYPE.GT.5) GO TO 700
    IPAR(I)=IP
    IS(I)=ITYPE

```

```

C-----
C      SURFACE AREA

      AREA(I)=PAR(IP+3)
      ATOT=ATOT+AREA(I)
C      INPUT ID SOLAR ORIENTATION AND ADJACENT ZONE
      COLUMN(I)=PAR(IP+4)
C-----
      IF (ITYPE.LE.0) THEN
C////////////////////////////////////
C
C      DUPLICATE OF LAST SURFACE

      IS(I)=IS(I-1)
      RHO(I)=RHO(I-1)
      ALPHA(I)=ALPHA(I-1)
      HCI(I)=HCI(I-1)
      NCF(I,1)=NCF(I-1,1)
      NCF(I,2)=NCF(I-1,2)
      NCF(I,3)=NCF(I-1,3)
      DO J=1,NCF(I,1)
        B(J,I)=B(J,I-1)
      END DO
      DO J=1,NCF(I,2)
        C(J,I)=C(J,I-1)
      END DO
      DO J=1,NCF(I,3)
        D(J,I)=D(J,I-1)
      END DO
      NP=4
    END IF
    IF (ITYPE.LE.0) GOTO 45

    GO TO (20,20,20,30,40) ,ITYPE

C////////////////////////////////////
C      ASHRAE WALL OR ROOF

20    RHO(I)=PAR(IP+5)
      ALPHA(I)=PAR(IP+6)
      ICOEF=INT(PAR(IP+7)+0.1)
      IFLAG = 0
      IF (ICOEF.NE.4) THEN

C      TRANSFER FUNCTION COEFFICIENTS FROM ASHRAE TABLES

```

```

        NTABLE=INT(PAR(IP+8)+0.1)
        CALL TABLE(ICOEF,NTABLE,IU,NB,NC,ND,B(1,I),C(1,I),D(2,I))
        HCI(I)=HCOEF(IU)
        IFLAG=1
    ELSE

```

C            USER SPECIFIES TRANSFER FUNCTION COEFFICIENTS

```

        HCI(I)=PAR(IP+8)
        NB=INT(PAR(IP+9)+0.1)
        NC=INT(PAR(IP+10)+0.1)
        ND=INT(PAR(IP+11)+0.1)
        ND=ND+1
        IFLAG=INT(PAR(IP+12)+0.1)
    END IF
    NMAX=MAX0(NB,NC,ND-1)

```

C            CURRENT HOUR COEFF. OF PAST HEAT FLUX IS ALWAYS = 1

```

        D(1,I)=1.
        C0=C(1,I)

```

C-----

```

        DO 25 J=1,NMAX
            IF(ICOEF.GE.4) THEN

```

C            LOAD IN USER COEFFICIENTS

```

        IF(J.LE.NB) B(J,I)=PAR(IP+12+J)
        IF(J.LE.NC) C(J,I)=-1.0*PAR(IP+12+NB+J)
        IF (IFLAG.EQ.1) THEN
            C(J,I)=-C(J,I)
            C0=PAR(IP+12+NB+1)
        END IF

```

C            ONLY LOAD COEFF. OF PAST HOUR'S HEAT FLUXES

```

        IF(J.LE.ND-1) D(J+1,I)=PAR(IP+12+NB+NC+J)
    END IF
    IF (IFLAG.EQ.1) THEN
        B(J,I)=B(J,I)/(1.-C0/HCI(I))
        D(J,I)=(D(J,I)-C(J,I)/HCI(I))/(1.-C0/HCI(I))
        C(J,I)=-C(J,I)/(1.-C0/HCI(I))
    IF(ITYPE.EQ.2) THEN
        C(J,I)=C(J,I)+B(J,I)
        B(J,I) = 0.
    END IF

```

```

      END IF

C      CLEAR COEFFICIENTS NOT ENTERED BY USER

      IF(J.GT.NB) B(J,I)=0.
      IF(J.GT.NC) C(J,I)=0.
      IF(J.GT.ND) D(J,I)=0.

C      CLEAR AMBIENT TEMPERATURE COEFF. OF INTERIOR PARTITION TYPE

      IF(ITYPE.EQ.2) B(J,I)=0.
25    CONTINUE
C-----
C      UPDATE COUNT OF PARAMETERS AND INPUTS

      NP=8
      IF(ICOEF.EQ.4) NP=11+nb+nc+nd
      NCF(I,1)=NB
      NCF(I,2)=NC
      NCF(I,3)=ND
      GO TO 45

C////////////////////////////////////
C      EXTERIOR SURFACE WITH CONDUCTION AS INPUT
C      NEED TO SET B,C,D PROPERLY SO COMBINATION IS CORRECT
30    RHO(I)=PAR(IP+4)
      HCI(I)=PAR(IP+5)
      B(1,I)=0
      C(1,I)=0
      D(1,I)=1
      NCF(I,1)=1
      NCF(I,2)=1
      NCF(I,3)=1
      NP=5
      NCOND=NCOND+1
      GOTO 45

C////////////////////////////////////
C      WINDOW

40    WMODE=INT(PAR(IP+5)+0.1)
      IF (WMODE.EQ.2) WMODE2=WMODE2+1
      TAU(I)=PAR(IP+6)
      ALPHA(I)=PAR(IP+7)
      RHO(I)=1.-TAU(I)-ALPHA(I)
      IF (WMODE.EQ.2) THEN
        ALPHA(I)=0.0
        TAU(I)=1.0

```

```

      END IF
      HCI(I)=PAR(IP+8)
      U(I)=PAR(IP+9)

C      EXTERIOR CONVECTION COEFF. HCO SET HERE

      HCO=HCON(IU)*(5.7+3.8*WOUT)
      B(1,I)=1/(1/HCO+1/U(I))
      C(1,I)=-B(1,I)
      D(1,I)=1
      NCF(I,1)=1
      NCF(I,2)=1
      NCF(I,3)=1
      NP=9
C////////////////////////////////////
45    CONTINUE
      NPAR=NPAR+NP
50    IP=NPAR
C*****END OF MAIN SURFACE LOOP*****

      IGEOM=INT(PAR(IP+1)+0.1)
      IP=IP+1
C-----
C      CHECK FOR COMPLETE SET OF SURFACE NUMBERS

      DO I=1,NS
        IF(IPAR(I).LE.0) GO TO 1200
      END DO
C-----
C      DETERMINE VIEW FACTORS

      IF((IGEOM.EQ.0) .OR. (IGEOM.EQ.3)) THEN

C      BASE VIEW FACTORS ON AREA RATIO

      DO I=1,NS
        DO J=1,NS
          FV(I,J)=AREA(J)/(ATOT-AREA(I))
        END DO
        FV(I,I)=0.0
      END DO

      ELSE IF(IGEOM.EQ.1) THEN

C      VIEW FACTORS FOR PARALLELPiped GEOMETRY

```

```

      CALL ENCL(PAR(IP+1),PAR(IP+11),FV,INFO)
      IP=IP+10+INT(PAR(IP+10)+0.1)*6

      ELSE IF (IGEOM.EQ.2) THEN

C          VIEW FACTORS ENTERED BY USER

      DO I=1,NS
        DO J=I,NS
          IP=IP+1
          FV(I,J)=PAR(IP)
          FV(J,I)=FV(I,J)*AREA(I)/AREA(J)
        END DO
      END DO

      END IF

C-----
C          CHECK VIEW FACTORS

      IF ((IGEOM.NE.0) .AND. (IGEOM.NE.3)) THEN
        DO I=1,NS
          SUMFV(I)=0.
          DO J=1,NS
            SUMFV(I)=SUMFV(I)+FV(I,J)
          END DO
        END DO
      END IF

C-----
C          CHECK FOR INCOMPLETE ENCLOSURE

      IF ((IGEOM.NE.0) .AND. (IGEOM.NE.3)) THEN
        DO I=1,NS
          IF (SUMFV(I).LE.0.98 .OR. SUMFV(I).GE.1.02)
            &          WRITE(LUW,1400) INFO(1),INFO(2),I
          END DO
        END IF

C-----
C          NET EXCHANGE FACTORS, FHATS

      call fhat(ns,fv,rho,fh)

C-----
C          SETUP STAR RADIATION NETWORK
C          CALCULATE LONG WAVE F-HATS WITH ASSUMED EMITANCE = 0.9

      DO I=1,NSMAX

```



```

      RHOL(I)=0.1
    END DO

    CALL FHAT(NS,FV,RHOL,FHL)
    SIG=SIGMA(IU)
    IFLAG=0

    IF (IGEOM.LT.3) THEN
      CALL STAR_EXACT(NS,FHL,RHOL,AREA,HCI,T0,FIT,RSTAR,RS,IFLAG,SIG)
    ELSE
      CALL STAR_MRT(NS,FHL,RHOL,AREA,HCI,T0,FIT,RSTAR,RS,IFLAG,SIG)
    END IF

    IF(IFLAG.EQ.1) GO TO 500
C-----
C      EXPAND TRANSFER FUNCTION COEFFICIENTS TO INCLUDE
C      AMBIENT TEMP, ROOM TEMP, SOLAR RAD, INT GAINS, AND PAST HEAT
C      FLUXES. ALSO REGROUP BY STORING COEFFICIENTS TO BE
C      COMBINED IN THE SAME COLUMN OF ETFCF ARRAY.

C      EXTERIOR CONVECTION COEFF. HCO SET HERE

      HCO=HCON(IU)*(5.7+3.8*WOUT)
      IF (USR.LT.1) USR=1 ! at least one solar input is required
      NTERMS=UZT+USR+4
C      Exterior surface convection resistance
      DO I=1,NS
        IF ((IS(I).EQ.1).OR.(IS(I).EQ.5)) ROUT(I)=1./(HCO*AREA(I))
        IF (IS(I).EQ.5) ROUT(I)=ROUT(I)+1./(U(I)*AREA(I))
        IF ((IS(I).GE.2).AND.(IS(I).LE.4)) ROUT(I)=1E30
      END DO
      CALL EXPAND_ALL(B,C,D,NCF,AREA,RS,ALPHA,NS,ROUT,USR,UZT,
&                  COLUMN,IS,FH,RHO,TAU,ETFCF,NCFE)
C-----
C      COMBINE COEFFICIENTS

      CALL COMBINE_ALL(ETFCF,NCFE,NTERMS,NS,SCTFCF,NCFC,RSTAR)

C      ELIMINATE ZERO COEFFICIENTS

      NMAX=0
      DO I=1,NTERMS
        DO WHILE ((NCFC(I).GT.0).AND.(SCTFCF(NCFC(I),I).EQ.0))
          NCFC(I)=NCFC(I)-1
        END DO
      END DO
    END DO

```

```

C-----
C      REDUCE COEFFICIENTS USING DOMINANT ROOT MODEL REDUCTION
C
C      REDUCE COEFFICIENTS USING BILINEAR TRANSFORM AND PADE APPROX

DO I=1,NTERMS-1
  NMAX=MAX0 (NCFC (I) , NCFC (NTERMS) )
  DO J=1,NMAX
    AA(J)=SCTFCF (J, I)
    BB(J)=SCTFCF (J,NTERMS)
  END DO

  CALL REDUCE (AA, BB, NMAX-1, MM, DD, EE)

  DO J=1, MM+1
    RTFCF (J, I) = SNGL (DD (J) )
    RTFCF (J, NTERMS-1+I) = SNGL (EE (J) )
  END DO
END DO

C-----
C      TYPECK CALL, STORE INFORMATION IN S ARRAY
C      ADD UP TOTAL NUMBER OF STORAGE UNITS REQUIRED
C      COEFFICIENTS, PAST HISTORIES
C      ALSO CHECK THE NUMBER OF USER FURNISHED INPUTS AND PARAMETERS
C      FROM INFO ARRAY AGAINST IN AND IP.

NSTOR=7.+4*NTERMS*(MM+1)+1+4
IF (INFO(4) .NE. IP) IP=IP+1
INFO(6)=10
IN=INFO(3)
INFO(10)=NSTOR
CALL TYPECK(1, INFO, IN, IP, 0)

ISTORE=INFO(10)
S(ISTORE)=PAR(9)      ! Initial room temperature
S(ISTORE+1)=PAR(9)    ! Last room temperature
S(ISTORE+2)=PAR(10)   ! Last room humidity ratio
S(ISTORE+3)=NTERMS-1 ! Number of input terms in CRTF
S(ISTORE+4)=MM+1      ! number of past time steps in CRTF (0-mm)
S(ISTORE+5)=RSTAR     ! star resistance from star point to room air
S(ISTORE+6)=WMODE2    ! number of solar inputs from mode 2 windows
S(ISTORE+7)=NCOND     ! number of conduction walls
S(ISTORE+8)=0.        ! Storage for interpolation mode
S(ISTORE+9)=0.

```

```

S(ISTORE+10)=0.
S(ISTORE+11)=0.
INDEX=ISTORE+12

C      STORE COEFFICIENTS E, G, D's, F's

C      E  room temperature

DO J=1,MM+1
  S(INDEX)=RTFCF(J,2+UZT)
  INDEX=INDEX+1
END DO

C      G  internal radiation gains

DO J=1,MM+1
  S(INDEX)=RTFCF(J,NTERMS-1)
  INDEX=INDEX+1
END DO

C      D's  ambient, adjacent zone temperatures

DO I=1,1+UZT
  DO J=1,MM+1
    S(INDEX)=RTFCF(J,I)
    INDEX=INDEX+1
  END DO
END DO

C      F's solar radiation orientation inputs

DO I=3+UZT,3+UZT+USR-1
  DO J=1,MM+1
    S(INDEX)=RTFCF(J,I)
    INDEX=INDEX+1
  END DO
END DO

C      H ROOM (E)  output room temperature

DO J=1,MM+1
  S(INDEX)=RTFCF(J,NTERMS+1+UZT)
  INDEX=INDEX+1
END DO

C      H RAD (G)  output internal radiation gains

```

```

DO J=1,MM+1
  S(INDEX)=RTFCF(J,2*(NTERMS-1))
  INDEX=INDEX+1
END DO

C      H AMB (D's)  output ambient, adjacent zone temperatures

DO I=NTERMS,NTERMS+UZT
  DO J=1,MM+1
    S(INDEX)=RTFCF(J,I)
    INDEX=INDEX+1
  END DO
END DO

C      H SOLAR (F's) output solar radiation inputs

DO I=NTERMS+2+UZT,NTERMS+2+UZT+USR
  DO J=1,MM+1
    S(INDEX)=RTFCF(J,I)
    INDEX=INDEX+1
  END DO
END DO

C-----

C      REPORT

C-----

C      VIEW FACTORS
WRITE(LUW,1300) 'VIEW FACTOR SUMMARY FOR UNIT',INFO(1),INFO(2),
&              (LIST(J),J=1,NS)
DO I=1,NS
  WRITE(LUW,1301) I,(FV(I,J),J=1,NS)
END DO

C      SOLAR F-HATS
WRITE(LUW,1300) 'SOLAR F-HAT SUMMARY FOR UNIT',INFO(1),INFO(2),
&              (LIST(J),J=1,NS)
DO I=1,NS
  WRITE(LUW,1301) I,(FH(I,J),J=1,NS)
END DO

C      LONGWAVE F-HATS
WRITE(LUW,1300) ' I.R. F-HAT SUMMARY FOR UNIT',INFO(1),INFO(2),
&              (LIST(J),J=1,NS)
DO I=1,NS
  WRITE(LUW,1301) I,(FHL(I,J),J=1,NS)
END DO

```

```

C      STAR RADIATION NETWORK
WRITE(LUW,1325) INFO(1), INFO(2)
DO I=1, NS
    WRITE(LUW,1326) I, RS(I)
END DO
WRITE(LUW,*)
WRITE(LUW,1303) RSTAR
WRITE(LUW,*)
C      INDIVIDUAL WALL COEFFICIENTS
WRITE(LUW,*)
WRITE(LUW,*) ' INDIVIDUAL SURFACE TRANSFER FUNCTION COEFFICIENTS '
WRITE(LUW,*)
do k=1, ns
    do j=1, 3
        sumfv(j)=0
    end do
    NMAX=MAX0(Ncf(k,1), Ncf(k,2), Ncf(k,3))
    write(luw,1305) K
    do i=1, nmax
        write(luw,1306) b(i,k), c(i,k), d(i,k)
        sumfv(1)=sumfv(1)+b(i,k)
        sumfv(2)=sumfv(2)+c(i,k)
        sumfv(3)=sumfv(3)+d(i,k)
    end do
    write(luw,1307) (sumfv(j), j=1,3)
    WRITE(LUW,*)
end do
C      EXPANDED COEFF.
WRITE(LUW,*)

if (nterms.le.10) then

do k=1, ns
    write(luw,1310) K
    nmax=max0(ncf(k,1), ncf(k,2), ncf(k,3))
    do j=1, nterms
        sum(j)=0D+0
    end do
    do i=1, nmax
        write(luw,1311) (etfctf(k,j,i), j=1, nterms)
        do j=1, nterms
            sum(j)=sum(j)+etfctf(k,j,i)
        end do
    end do
    WRITE(LUW,1316) (DSH$(J), J=1, NTERMS)
    write(luw,1312) (sum(j), j=1, nterms)

```

```

end do
C      COMBINED COEFF.
NMAX=0
DO I=1, NTERMS
  IF (NCFC(I).GT.NMAX) NMAX=NCFC(I)
  sum(i)=0
END DO
WRITE(LUW,*)
WRITE(LUW,*)
write(luw,*) 'COMBINED TRANSFER FUNCTION COEFFICIENTS'
WRITE(LUW,*)
do i=1,nmax
  write(luw,1311) (sctfcf(i,j),j=1,nterms)
  do j=1,nterms
    sum(j)=sum(j)+sctfcf(i,j)
  end do
end do
WRITE(LUW,1316) (DSH$(J),J=1,NTERMS)
write(luw,1312) (sum(j),j=1,nterms)
C      REDUCED COEFF.
DO J=1,15
  SUMFV(J)=0
END DO
WRITE(LUW,*)
write(luw,*) 'COMPREHENSIVE ROOM TRANSFER FUNCTION COEFFICIENTS'
WRITE(LUW,*)
WRITE(LUW,*) '  INPUT COEFFICIENTS:'
WRITE(LUW,*)
do i=1,MM+1
  write(luw,1311) (RTFCF(i,j),j=1,nterms-1)
  do j=1,NTERMS-1
    sumfv(j)=sumfv(j)+RTFCF(i,j)
  end do
end do
WRITE(LUW,1316) (DSH$(J),J=1,NTERMS-1)
write(luw,1312) (sumfv(j),j=1,nterms-1)
write(luw,*)
WRITE(LUW,*) '  OUTPUT COEFFICIENTS'
WRITE(LUW,*)
do i=1,MM+1
  write(luw,1311) (RTFCF(i,j),j=nterms,2*(nterms-1))
  do j=NTERMS,2*(nterms-1)
    sumfv(j)=sumfv(j)+RTFCF(i,j)
  end do
end do
WRITE(LUW,1316) (DSH$(J),J=1,NTERMS-1)
write(luw,1312) (sumfv(j),j=NTERMS,2*(nterms-1))

```

```

DO J=1,15
    SUMFV(J)=0
END DO
end if

120  RETURN

C-----

C      ERROR MESSAGES

C-----

500  WRITE(LUW,501) INFO(1),INFO(2)
501  FORMAT(/2X,22H***** ERROR ***** UNIT,I3,5H TYPE,I3/4X,
.      38HMATRIX IS SINGULAR, SIMULATION STOPPED/)
STOP
600  IERR=IP+1
WRITE(LUW,601) INFO(1),INFO(2),IERR,NSMAX
601  FORMAT(/2X,22H***** ERROR ***** UNIT,I3,5H TYPE,I3,
.      12H PARAMETER #,I4,/4X,22HSURFACE # MUST BETWEEN
.      6H 1 AND,I3,21H - SIMULATION STOPPED/)
STOP
700  IERR=IP+2
WRITE(LUW,701) INFO(1),INFO(2),IERR
701  FORMAT(/2X,22H***** ERROR ***** UNIT,I3,5H TYPE,I3,
.      12H PARAMETER #,I4,/4X,25HSURFACE TYPE MUST BETWEEN
.      29H 1 AND 5 - SIMULATION STOPPED/)
STOP
800  IERR=NCK+1
WRITE(LUW,801) INFO(1),INFO(2),IERR
801  FORMAT(/2X,22H***** ERROR ***** UNIT,I3,5H TYPE,I3,
.      12H PARAMETER #,I4,/4X,24HONLY 10 OPTIONAL OUTPUTS
.      29H ALLOWED - SIMULATION STOPPED/)
STOP
1000 IERR=IL+1
WRITE(LUW,1001) INFO(1),INFO(2),IERR
1001 FORMAT(/2X,22H***** ERROR ***** UNIT,I3,5H TYPE,I3,
.      12H PARAMETER #,I4/4X,28HONLY 5 OPTIONAL OUTPUT TYPES
.      33H ARE ALLOWED - SIMULATION STOPPED/)
STOP
1100 IERR=IL+2
WRITE(LUW,1101) INFO(1),INFO(2),IERR
1101 FORMAT(/2X,22H***** ERROR ***** UNIT,I3,5H TYPE,I3,
.      12H PARAMETER #,I4/4X,24H SURFACE # NOT DEFINED -
.      19H SIMULATION STOPPED/)

```

```

      STOP
1200 WRITE(LUW,1201) INFO(1),INFO(2),I
1201 FORMAT(/2X,22H***** ERROR ***** UNIT,I3,5H TYPE,I3,/4X,
.    7HSURFACE,I3,36H IS NOT DEFINED - SIMULATION STOPPED/)
      STOP
1300 FORMAT(///6X,A29,1X,I2,
.    ' TYPE',1X,I2//2X,'I',3X,'J=',15(4X,I2)/)
1301 FORMAT(1X,I2,5X,15(1X,F5.3))
1302 FORMAT(8X,14(1X,F5.3))
1303 FORMAT(2X,'STAR POINT TO ROOM RESISTANCE: ',F10.7)
1305 FORMAT(2X,'WALL ',I2,8X,'B',12X,'C',12X,'D')
1306 FORMAT(9X,3(F13.7))
1307 FORMAT(12X,3('-----'),1X/,'SUM',6X,3(F13.7))
1310 FORMAT(/,' EXPANDED COEFFICIENTS FOR WALL',I2/)
1311 FORMAT(2X,20(F13.3))
1312 FORMAT(1X,'SUM',F11.3,20(F13.3),/)
1316 FORMAT(3X,20(A12,1X))
1325 FORMAT(///6X,'STAR NETWORK SUMMARY FOR UNIT',1X,I2,
.    ' TYPE',1X,I2/)
1326 FORMAT(1X,I2,3X,F10.7)
1400 FORMAT(/2X,24H***** WARNING ***** UNIT,I3,5H TYPE,I3,/4X,
.    47HINCOMPLETE ENCLOSURE - SUM OF VIEW FACTORS FROM,/4X,
.    7HSURFACE,I3,35H TO OTHER SURFACES IS LESS THAN ONE)
      END

```



```

subroutine fhat(ns,fv,rho,fh)

dimension fv(15,15),fh(15,15),rho(15),z(15,15)
nsmax=15

c      Calculate radiation exchange factors (F-hats)

c      Inputs:
c      fv(i,j)   View factors
c      rho(wall) Reflectance of each wall
c      ns        Number of surfaces in enclosure

c      OUTPUT:
c      fh(i,j)   F-hats

c      ROUTINES CALLED:
c      invert
c-----
do i=1,ns
  do j=1,ns
    z(i,j) = -rho(j) * fv(i,j)
  end do
  z(i,i) = 1.0 + z(i,i)
end do

call invert(nsmax,ns,z,iflag)

do i=1,ns
  do j=1,ns
    fh(i,j) = 0.0
    do k=1,ns
      fh(i,j) = fh(i,j) + z(i,k) * fv(k,j)
    end do
  end do
end do

return
end

```

```

SUBROUTINE STAR_EXACT(NS,FH,RHO,AREA,HCI,T0,FIT,
& RSTAR,RS,IFLAG,SIGMA)

```

```

DIMENSION FH(15,15),AREA(15),HCI(15),RS(15)
DIMENSION RR(15,15),RM(16,16),RA(15),RHO(15)

```

```

C      * This routine calculates the STAR network resistances      *
C      * from the information given in the TRNSYS type 19 model    *
C      * The star network is calculated from the exact network    *
C      * The exact network is a view factor radiation network    *
C      * which also includes convection resistances                *

```

```

C      REQUIRED INPUTS:

```

```

C      NS      Number of surfaces
C      FH(,)    Radiation exchange factors  F-hats
C      AREA()   Area of each surface
C      HCI()    Wall convection coefficient to room air
C      T0       Initial zone temperature
C      FIT      Weighting function exponent
C      SIGMA    Steffan-Boltzman constant
C      RHO()    Reflectance of wall used to get emitance

```

```

C      RETURNED VARIABLES:

```

```

C      RSTAR    Resistance between star point and air
C      RS()     Resistances in STAR network surface to STAR point
C      IFLAG    Flag to denote singular matrix

```

```

C      INTERMEDIATE RESULTS:

```

```

C      RR(,)    Radiation resistances between two surfaces
C      RA()     Convective resistances between surface and room air
C      RM(,)    Resistance matrix and inverse (floating nodes)
C      SUM      Sum of resistances from a surface to all others
C      RNUM     Numerator of weighting function
C      RDEN     Denominator of weighting function

```

```

C      CALLED SUBROUTINES:

```

```

C      INVERT   Invert a matrix  (from TRNSYS)

```

```

C-----
C      View factors are given from area ratios, user defined, or
C      calculated internally. Now calculate the linear
C      radiation resistance between each pair of walls, making
C      use of the symmetry  $R_{12} = R_{21}$ 
C      The initial zone temperature is used as the average temperature
C      in the linearizing of the radiation temperature difference

```

```

J2 = NS - 1
DO I=1,J2
  RR(I,I) = 0
  J1 = I
  DO J=J1,NS
    IF (FH(I,J).NE.0) THEN
      E1=1. - RHO(I)
      E2=1. - RHO(J)
      RR(I,J)=1/(4*SIGMA*FH(I,J)*E1*E2*AREA(I)*T0**3)
    ELSE
      RR(I,J)=1E+32
    END IF
    RR(J,I)=RR(I,J)
  END DO
END DO
RR(NS,NS)=0

```

```

C-----
C   Calculate the convective resistance between each wall and the
C   air.

```

```

DO I=1,NS
  RA(I) = 1/(HCI(I)*AREA(I))
END DO

```

```

C-----
C   Load the resistance matrix ala Seem, making use of symmetry
C   above and below the diagonal. The form comes from an energy
C   balance at the inside surface of each wall and an energy
C   balance on the zone air.

```

```

J1=0
DO I=1,NS
  J1=J1+1
  DO J=J1,NS
    IF (I.EQ.J) THEN
      c   Fill the diagonal
      SUM = 0.0
      DO J2=1,NS
        IF (I.NE.J2) THEN
          IF (RR(I,J2).NE.0) SUM = SUM + 1/RR(I,J2)
        END IF
      END DO
      RM(I,J) = -SUM - 1/RA(I)
    ELSE
      c   Symmetry above and below the diagonal

```

```

        RM(I,J) = 1/RR(I,J)
        RM(J,I) = RM(I,J)
    END IF
END DO
    RM(I,NS+1) = 0.0
END DO
c    The last row is all convection
DO J=1,NS
    RM(NS+1,J) = 1/RA(J)
END DO
    RM (NS+1,NS+1) = -1.0

C-----
C    Invert the resistance matrix.  If the matrix is singular
C    stop execution and pass the error flag back.

    NSMAX=15
    CALL INVERT(NSMAX+1,NS+1,RM,IFLAG)
    IF (IFLAG.GT.0) GO TO 99

C-----
C    Calculate the resistances in the STAR network.

    RNUM = 0.0
    RDEN = 0.0
    DO I=2,NS
        DO J=1,I-1
            RFLOAT=RM(I,J)+RM(J,I)-RM(I,I)-RM(J,J)
            RNUM=RNUM+(-RM(I,I)-RM(J,J)-RFLOAT)/(RFLOAT**FIT)
            RDEN=RDEN + 1/(RFLOAT**FIT)
        END DO
    END DO

    RSTAR = RNUM/(2*RDEN)

    DO I=1,NS
        RS(I) = -RM(I,I) - RSTAR
    END DO

C-----
99    RETURN

    END

```

```

SUBROUTINE STAR_MRT(NS,FH,RHO,AREA,HCI,T0,FIT,
& RSTAR,RS,IFLAG,SIGMA)

DIMENSION FH(15,15),AREA(15),HCI(15),RS(15)
DIMENSION RR(15),RM(16,16),RA(15),RHO(15),F(15)
PARAMETER (TOL=0.0001)

C      * This routine calculates the STAR network resistances      *
C      * from the information given in the TRNSYS type 19 model    *
C      * The star network is calculated from the MRT radiation    *
C      * network with convection resistances                      *

C      REQUIRED INPUTS:
C      NS      Number of surfaces
C      FH(,)   Radiation exchange factors  F-hats
C      AREA()  Area of each surface
C      HCI()   Wall convection coefficient to room air
C      T0      Initial zone temperature
C      FIT     Weighting function exponent
C      SIGMA   Steffan-Boltzman constant
C      RHO()   Reflectance of wall used to get emitance

C      RETURNED VARIABLES:
C      RSTAR   Resistance between star point and air
C      RS()    Resistances in STAR network surface to STAR point
C      IFLAG   Flag to denote singular matrix

C      INTERMEDIATE RESULTS:
C      RR(,)   Radiation resistances between two surfaces
C      RA()    Convective resistances between surface and room air
C      RM(,)   Resistance matrix and inverse (floating nodes)
C      SUM     Sum of resistances from a surface to all others
C      RNUM    Numerator of weighting function
C      RDEN    Denominator of weighting function

C      SUBROUTINES CALLED:
C      INVERT   Invert a matrix  (from TRNSYS)
C      SUMF     (Function) sum: area x view factors  (internal)

C-----
C      Calculate the "MRT view factors" iteratively
C      DO I=1,NS
C          F(I) = 1.0
C      END DO
C      ERROR=1.0
C      ITER = 0

```

```

DO WHILE (ERROR.GT.TOL)
  ITER = ITER + 1
  ERROR = 0.0
  SUM = SUMF (AREA,F,NS)
  DO I=1,NS
    FNEW = 1./ (1.-AREA(I)*F(I)/SUM)
    DIFF = ABS (FNEW-F(I))
    IF (DIFF.GT.ERROR) ERROR=DIFF
    F(I) = FNEW
  END DO
END DO

```

c      calcualte the mrt resistances

```

DO I=1,ns
  E1=1. - RHO(I)
  RR(I)=1/ ((4*SIGMA*AREA(I)*T0**3) / (1/F(I)+(1-E1)/E1))
end do

```

C-----  
C      Calculate the convective resistance between each wall and the  
C      air.

```

DO I=1,NS
  RA(I) = 1/ (HCI(I)*AREA(I))
END DO

```

C-----  
C      Load the resistance matrix.  
C      The form comes from an energy  
C      balance at the inside surface of each wall, an energy  
C      balance on the zone air AND AN ENERGY BALANCE ON THE MRT NODE.

```

DO I=1,16
DO J=1,16
  RM(I,J)=0.0
END DO
END DO
DO I=1,NS
  RM(I,I) = 1./RA(I) + 1./RR(I)
  RM(I,NS+1) = -1./RR(I)
  RM(NS+1,I) = -1./RR(I)
  RM(NS+1,NS+1) = RM(NS+1,NS+1) + 1./RR(I)
  RM(NS+2,I) = 1./RA(I)
END DO
RM(NS+2,NS+2) = 1.

```

```

C-----
C      Invert the resistance matrix.  If the matrix is singular
C      stop execution and pass the error flag back.

      IFLAG=0
      NSMAX=15
      CALL INVERT(NSMAX+1,NS+2,RM,IFLAG)
      IF (IFLAG.GT.0) GO TO 99
C-----
C      Calculate the resistances in the STAR network.

      RNUM = 0.0
      RDEN = 0.0
      DO I=2,NS
        DO J=1,I-1
          Rij=RM(I,I)-RM(I,J)+RM(J,J)-RM(J,I)
          Rir=RM(I,I)
          Rjr=RM(J,J)
          RNUM=RNUM+(Rir + Rjr - Rij)/(Rij**FIT)
          RDEN=RDEN + 1/(Rij**FIT)
        END DO
      END DO

      RSTAR = RNUM/(2*RDEN)

      DO I=1,NS
        Rir = RM(I,I)
        RS(I) = Rir - RSTAR
      END DO

C-----

99      RETURN
      END

FUNCTION SUMF(AREA,F,NS)
      DIMENSION AREA(*),F(*)

      SUM = 0.0
      DO I=1,NS
        SUM = SUM + AREA(I) * F(I)
      END DO
      SUMF = SUM
      RETURN
      END

```

```

subroutine expand_all(a,b,c,ncf,area,rs,alpha,ns,
&   rout,usr,uzt,column,itype,fh,rho,tau,etfcf,ncfe)

dimension a(10,15),b(10,15),c(10,15),ncf(15,3),area(15),rs(15)
dimension tau(15),alpha(15),rho(15),rout(15),itype(15)
DOUBLE PRECISION etfcf(15,15,50),TEMP,ATOTAL
dimension ncfe(15,15),fh(15,15)
integer column(15),UZT,USR,D,E,F,G,H,L

C      5/20/87
C      Take ASHRAE b,c,d transfer function coefficients and expand
C      to Seem's d,e,f,g,h coefficients.
C-----
C      INPUTS:
C      a, b, c      ASHRAE transfer function coeffs.
C                   a(time_step,wall)
C      ncf          Number of ASHRAE coeffs.
C                   ncf(wall,id) id is a, b, or c
C      area        Area of each wall
C                   area(wall)
C      rs          Radiation resistances from wall to star point
C                   rs(wall)
C      alpha       Radiation absorbtance of each wall to solar gains
C                   alpha(wall)
C      ns          Number of surfaces in zone
C      rout        Exterior radiation resistance 1/h*A
C                   rout(wall)
C      column      List surfaces to be combined column(wall)
C      itype       Surface identifier itype(wall)
C      usr        Number of unique solar radiation inputs
C      uzt        Number of unique zone temperature inputs

C      OUTPUTS:
C      etfcf       Expanded Transfer Function Coefficients expanded
C                   for ambient temp, star temp, solar rad, internal
C                   gains, and past heat flows
C                   etfcf(wall,term,time_step)
C      ncfe        Number of expanded coeffs.
C                   ncfe(wall,term) term is 1-15 (d,e,f,g,h)

C      ROUTINES CALLED:
C      Tau_alf_area
C-----

C      Calculate column location of the first term in each set of coeff.

```



```

d=1
e=uzt+2
g=uzt+usr+3
h=uzt+usr+4

C    Calculate total enclosure area

atotal=0.0D+0
do i=1,ns
    atotal=atotal+DBLE(area(i))
end do

C    Expand coefficients of each wall

DO K=1,NS
    TEMP = 1.0D+0 - DBLE(B(1,K)) * DBLE(AREA(K)) * DBLE(RS(K))
    nc = jmax0(ncf(k,1),ncf(k,2),ncf(k,3))
c    Adjust pointers into matrix to store coeff. with identical
C    in the same column for later combination
    d=1
    if (itype(k).eq.3) d=1+column(k)
    DO N=1,NC
        etfcf(k,d,n)=(DBLE(A(N,K))*DBLE(AREA(K)))/TEMP
        etfcf(k,e,n)=(DBLE(B(N,K))*DBLE(AREA(K)))/TEMP
        etfcf(k,g,n)=(DBLE(C(N,K))*DBLE(AREA(K))/ATOTAL)/TEMP
        etfcf(k,h,n)=(DBLE(C(N,K))-DBLE(B(N,K))*DBLE(RS(K))
&                                *DBLE(AREA(K)))/TEMP
    END DO

c    F coefficients

do L=1,usr
    taa=tau_alf_area(k,L,ns,itype,column,fh,area,rho,tau)
    f=uzt+2+L
    do n=1,nc
        if (L.eq.column(k) .AND. ITYPE(K).NE.3) then
            etfcf(k,f,n)=(dble(c(n,k))*dble(taa)
&                                +DBLE(A(N,K))*DBLE(ALPHA(K))*DBLE(ROUT(K))
&                                *DBLE(AREA(K)**2)/TEMP
        else
            etfcf(k,f,n)=dble(c(n,k))*dble(taa)/temp
        end if
    end do
    NCFE(K,f)=JMAX0(NCF(K,1),NCF(K,3))
end do

IF (ITYPE(K).EQ.4) THEN

```

```

      ETFCF(K,F,1)=0D+0
      ETFCF(K,G,1)=0D+0
    END IF

C      Calculate the number of coefficients

      NCFE(K,d)=NCF(K,1)
      NCFE(K,e)=NCF(K,2)
      NCFE(K,g)=NCF(K,3)
      NCFE(K,h)=JMAX0(NCF(K,2),NCF(K,3))
    END DO

    return
  end

  real function tau_alf_area(surface,orientation,ns,itype,
&                          orient_id,fh,area,rho,tau)

  integer itype(15),orient_id(15)
  real fh(15,15),area(15),rho(15),tau(15)
  integer surface,orientation,ns
  real result

C      Calculate tau_alpha * window area for a surface from the
c      specified orientation

C      INPUTS:
c      surface      Surface number
c      orientation  Solar input orientation
C      ns           Number of surfaces
C      itype        Type of surface 5 = window
c      orient_id    Array of orientation ID's for solar rad. input
C      fh           F-hats
C      area
C      tau          Solar transmittance of surface
C      rho          Solar reflectance of surface

C      OUTPUTS:
C      tau_alf_area  Tau * Alpha * Area of windows
C-----
    result=0.0
    if (itype(surface).ne.5) then
      do i=1,ns
        if (itype(i).eq.5 .and. orient_id(i).eq.orientation)
&          result=area(i)*fh(i,surface)*tau(i)
&          *(1-rho(surface))+result

```

```

      end do
    end if
    tau_alf_area = result
    return
  end

```

```

subroutine combine_all(etfcf,ncfe,nterms,ns,Sctfcf,ncfc,RSTAR)

```

```

  dimension ncfe(15,15),ncfc(15)
  DOUBLE PRECISION ctfcf(50,15),etfcf(15,15,50),hold(50)
  DOUBLE PRECISION TEMP,SCTFCF(50,15)

```

```

C      8/10/87
C      Combine all expanded transfer functions, each column is a
C      separate coefficient list for a unique input.
C      this allows multiple ambient temperaures (adjacent
C      zones) and multiple solar orientations.
C      Results are dnc, enc, fnc, gnc, hnc. Multiple
C      d's and f's are allowed. The routine requires no knowledge
C      of which columns represent which coefficients
C      except the last column, which MUST be past outputs!

C      INPUTS:
C      etfcf      Expanded Transfer Function Coefficients
C                  etfcf(wall,term,time_step) the last term must be
C                  the past output coefficients
C      ncfe      Number of coefficients in each column
C                  ncf(wall,term)
C      nterms    Number of terms in equation
C      ns        Number of surfaces (walls)

C      RETURNED VARIABLES:
C      ctfcf      Combined Transfer Function Coefficients
C                  cfc(time_step,term)
C      ncfc      Number of combined coefficients in each term
C                  ncfc(term)

C      ROUTINES CALLED:
C      (none)
C-----
      DO I=1,50
      DO J=1,15
        CTFCF(I,J)=0.0D0
      END DO
    END DO

```

```

C      Copy the first equation into the combined equation
do i=1,nterms
  ncfc(i) = ncfe(1,i)
  do j=1,ncfe(1,i)
    ctfcf(j,i) = etfcf(1,i,j)
  end do
end do
C-----
C      Combine each wall equation

do k=2,ns

C      Calculate the number of coefficients in the combined equation

do i=1,nterms-1
  ncfc(i)=jmax0(ncfc(i)+ncfe(k,nterms),ncfe(k,i)+ncfc(nterms))
end do
  ncfc(nterms)=ncfc(nterms)+ncfe(k,nterms)

C      Combine the coefficients

do i=1,nterms-1
  do n=1,ncfc(i)
    hold(n) = 0.0D+0
    do j=1,n
      hold(n) = hold(n) + ctfcf(n-j+1,i) * etfcf(k,nterms,j)
&      + etfcf(k,i,j) * ctfcf(n-j+1,nterms)
    end do
  end do
  do n=1,ncfc(i)
    ctfcf(n,i) = hold(n)
  end do
end do

C      Combine the past output coefficients

do n=1,ncfc(nterms)
  hold(n) = 0.0D+0
  do j=1,n
    hold(n) = hold(n) + ctfcf(j,nterms) * etfcf(k,nterms,n-j+1)
  end do
end do
do n=1,ncfc(nterms)
  ctfcf(n,nterms) = hold(n)
end do
end do

```

```
C      Combine the transfer function coeff for the the star
C      network load equation:
C      Qload = SUM(dn*Tamb+en*Troom+fn*Irads+gn*Qgain)-SUM(hn*Qpast)

      temp = 1.0D+0 - DBLE(rstar) * ctfcf(1,2)
      do i=1,NTERMS-1
        do n=1,ncfc(i)
          sctfcf(n,i) = ctfcf(n,i) / temp
        end do
      end do

      do n=1,ncfc(NTERMS)
        sctfcf(n,NTERMS)=(ctfcf(n,NTERMS)-DBLE(rstar)*ctfcf(n,2))/temp
      end do
      return
      end
```

```

SUBROUTINE REDUCE(A,B,N,M,D,E)
C
C REDUCES TRANSFER FUNCTION COEFFICIENTS BY USING THE
C BI-LINEAR TRANSFORMATION AND THE PADE' APPROXIMATION
C
C A - NUMERATOR COEFF. INPUT
C B - DENOMINATOR COEFF. INPUT
C N - NUMBER OF COEFF. INPUT
C M - NUMBER OF COEFF. IN REDUCED FUNCTION INPUT
C D - NUMERATOR COEFF. OUTPUT
C E - DENOMINATOR COEFF. OUTPUT
C
C CALLS POLY_MULT, PADE

DOUBLE PRECISION A(0:49),B(0:49),D(0:20),E(0:20),V(0:49),DEN
DOUBLE PRECISION ABAR(0:49),BBAR(0:49),DBAR(0:20),EBAR(0:20)

DO I=0,49
  ABAR(I)=0.0D0
  BBAR(I)=0.0D0
END DO
DO I=0,20
  E(I)=0.0D0
  D(I)=0.0D0
END DO
DEN=0.0D0
DO I=0,N
  CALL POLY_MULT(I,N-I,V)
  DO J=0,N
    ABAR(J)=ABAR(J)+A(I)*V(J)
    BBAR(J)=BBAR(J)+B(I)*V(J)
  END DO
  DEN=DEN+V(0)*B(I)
END DO

DO J=0,N
  ABAR(J)=ABAR(J)/DEN
  BBAR(J)=BBAR(J)/DEN
END DO

CALL PADE(ABAR,BBAR,N,M,DBAR,EBAR)

DEN=0.0D0

DO J=0,M
  CALL POLY_MULT(J,M-J,V)

```

```

      DO I=0,M
        D(I)=D(I)+DBAR(J)*V(I)
        E(I)=E(I)+EBAR(J)*V(I)
      END DO
      DEN=DEN+V(0)*EBAR(J)
    END DO

```

```

    DO I=0,M
      D(I)=D(I)/DEN
      E(I)=E(I)/DEN
    END DO

```

```

  RETURN
END

```

```

SUBROUTINE POLY_MULT(N,M,P)

```

```

C
C  MULTIPLIES (1-X)**N * (1+X)**M
C
C  MAKING USE OF PASCAL'S TRIANGLE
C
C  DOUBLE PRECISION P(0:49),WORK(0:49)

```

```

    DO I=0,49
      P(I)=0.0D0
      WORK(I)=0.0D0
    END DO

```

```

    P(0)=1.0D0

```

```

C
C    CALCULATE (1-X)**N
C

```

```

    DO I=1,N
      DO J=1,I
        WORK(J)=P(J)-P(J-1)
      END DO
      DO J=1,I
        P(J)=WORK(J)
      END DO
    END DO

```

```

C
C    MULTIPLY PRODUCT BY (1+X)**M
C

```

```

    DO I=N+1,N+M
      DO J=1,I
        WORK(J)=P(J)+P(J-1)

```

```

      END DO
      DO J=1,I
        P(J)=WORK(J)
      END DO
    END DO

```

```

RETURN
END

```

**SUBROUTINE PADE(A,B,N,M,D,E)**

```

C      USE THE PADE APPROXIMATION TO ESTIMATE THE POLYNOMIAL
C      A/B OF ORDER N WITH THE POLYNOMIAL D/E OF ORDER M

```

```

DOUBLE PRECISION A(0:49),B(0:49)
DOUBLE PRECISION C(0:49),D(0:20),E(0:20),Z(50,50),Y(50)
INTEGER M,N

```

```

DO I=1,50
  Y(I)=0.0D0
  DO J=1,50
    Z(I,J)=0.0D0
  END DO
END DO

```

```

C      CALCULATE THE POWER SERIES EXPANSION COEFFICIENTS OF A()/B()

```

```

DO I=0,2*M
  C(I) = A(I)
  DO J=0,I-1
    C(I) = C(I) - C(J)*B(I-J)
  END DO
END DO

```

```

C      CALCULATE THE DENOMINATOR COEFFICIENTS OF THE APPROXIMATION
C      BY USING THE HIGHER ORDER TERMS OF THE POWER SERIES

```

```

DO J=M,2*M-1
  DO I=1,M
    Z(J-M+1,I) = C(J-I+1)
  END DO
  Y(J-M+1)=-C(J+1)
END DO

```

```

C      SOLVE ZE=Y FOR E

```

```

CALL INVERTD(50,M,Z,IFLAG)

```



C        MULTIPLY (Z-1)\*Y

```
DO I=1,M
  E(I)=0.0D0
  DO J=1,M
    E(I)=E(I)+Z(I,J)*Y(J)
  END DO
END DO
E(0)=1.0D0
```

C        CALCULATE THE NUMERATOR COEFFS FROM THE POWER SERIES EXPANSION FORM

```
DO I=0,2*M
  D(I) = C(I)
  DO J=1,I
    D(I) = D(I) + C(I-J)*E(J)
  END DO
END DO

RETURN
END
```

---

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**APPROVED** William A Beckman 5/12/88

Prof. W. A. Beckman, Chairman Mechanical Engineering      Date