Thermodynamic Optimization of Mixed Refrigerant Joule Thomson Cycle with Heat Transfer Considerations

By

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Abstract

Joule Thomson (JT) cycles have the ability to provide low temperature cooling in a simple and compact cycle. They have many applications where a compact, low vibration cold head is needed. Mixed refrigerant (MR) working fluids can significantly increase the cooling capacity of a JT cycle. The optimization of Mixed Refrigerant Joule Thomson (MRJT) cycles has been the subject of substantial research. However, most optimization techniques do not model the recuperator in detail. For example many researchers have used a conductance model, which assumes a constant conductance in the recuperator. Work at the University of Wisconsin-Madison and elsewhere has shown that convection coefficients are a strong function of quality. When in the enhanced heat transfer region, between 15 and 85 percent quality, the convection coefficient can be up to three times higher than it is in the single phase regions [1]. This behavior causes the conductance of the recuperator to be a function of the MR quality in the recuperator. As a result, a system that optimizes a MR without also requiring that the flow be in the enhanced heat transfer region will require a large recuperator or not achieve the performance predicted by the model. To ensure optimal performance of a MRJT cycle, the MR should be selected to provide the highest cooling load while remaining in the enhanced heat transfer region.

To determine the optimal MR composition, a model of a thermodynamically ideal cycle was created and used in a parametric study. The results of the parametric study are graphically presented on a contour plot with the parameter space being the limits of quality in the recuperator (i.e., lowest and highest quality anywhere in the recuperator). The contours

show constant values of the normalized refrigeration power. This ‘map’ shows the effect of MR composition on the cycle performance and it can be used to select the MR that provides a high cooling load while also constraining the recuperator to be in the enhanced heat transfer region. The cycle map shows that the cost of operating in the enhanced heat transfer region is a reduction in the maximum possible cooling load. A design line is specified that identifies all mixtures that have the highest thermodynamic cooling load for a given quality constraint. The MRs identified along this line can be used as the initial starting point for experimental testing. The model and plotting technique is tested with different model input parameters to ensure it is valid for all cycles. Additionally, a method for determining the optimal number of components is described. The plotting technique is modified to provide the optimal mixture for a cycle operating with the maximum COP. Finally, the model is modified to consider the pressure drop in the recuperator, and the plotting method is used to determine the optimal mixture in that case as well.

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I also want to thanks the other students in the SEL for so readily providing assistance on any problems from homework to experimental testing. I'm looking forward to another 3 years with everyone.

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List of Variables

a_1	Parameter for JT effect curve fit (-)
a_2	Parameter for JT effect curve fit (-)
A_{HP}	Convection area in high pressure side of recuperator (m^2)
A_{LP}	Convection area in low pressure side of recuperator (m^2)
A_{cs}	Cross sectional area for conduction in recuperator (m^2)
C	Clearance volume fraction (-)
C_p	Specific heat capacity at constant pressure (kJ/kg-K)
C_v	Specific heat capacity at constant volume (kJ/kg-K)
$\frac{dT}{dP}$	Change in temperature with respect to pressure at constant enthalpy (K/kPa)
$h_{C,i}$	Enthalpy of cold stream at node i (kJ/kg)
h_f	Enthalpy of saturated liquid (kJ/kg)
h_g	Enthalpy of saturated vapor (kJ/kg)
$h_{H,i}$	Enthalpy of hot stream at node i (kJ/kg)
$\overline{h}_{HP}$	Convective heat transfer coefficient of high pressure stream (W/m^2-K)
h_i	Enthalpy at state i (kJ/kg)
$\overline{h}_{LP}$	Convective heat transfer coefficient of low pressure stream (W/m^2-K)
htc	Convective heat transfer coefficient (W/m^2-K)
k	Specific heat ratio (-)
k_h	Conductivity of heat exchanger (W/m-K)
L_c	Length of conduction (m)
$\dot{m}$	Mass flow rate (kg/s)
$P_{discharge}$	Discharge pressure (kPa)
P_{high}	Recuperator high pressure (kPa)
P_i	Pressure at node i (kPa)
$P_{suction}$	Suction pressure (kPa)
$\dot{Q}_{aftercool}$	Heat transfer rate in aftercooler (kW)
$\dot{Q}_{load}$	Cooling load (kW)

$\dot{Q}_{load,norm}$	Cooling load normalized by displacement rate (kW/(m^3/s))
$\dot{Q}_{load,norm,frac}$	Cooling load normalized by displacement rate and maximum cooling load (-)
$\dot{Q}_{rec}$	Heat transfer rate in recuperator (kW)
Q_{rec}	Specific heat transfer in recuperator (kJ/kg)
$\dot{Q}_{reheat}$	Heat transfer rate from reheater (kW)
RPM	Compressor speed (rpm)
ΔT_{app}	Approach temperature difference (K)
$T_{C,i}$	Temperature of cold stream at node i (K)
$T_{H,i}$	Temperature of hot stream at node i (K)
T_i	Temperature at state i (K)
UA	Conductance (W/K)
V_{clear}	Compressor clearance volume (m^3)
V_{dis}	Compressor displacement volume (m^3)
$\dot{V}_{displace}$	Compressor displacement rate (m^3/s)
v_{suc}	Specific volume at compressor suction (m^3/kg)
V_{swept}	Compressor swept volume (m^3)
$\dot{W}_{comp}$	Compressor work (kW)
x_{avg}	Average mass based quality
x_{sc}	Quality of subcooled MR
x_{sh}	Quality of superheated MR
ΔP	Differential pressure (kPa)
η_{vol}	Volumetric efficiency (-)

1.0 Introduction

A Joule Thompson (JT) cycle is one of the simplest and most reliable ways of providing cooling. It can be single staged for use at higher temperatures, or multi-staged to provide high cooling load at low temperatures. In its simplest form a JT cycle consists of a compressor, an after cooler, a recuperative heat exchanger (recuperator), an expansion valve, and a load heat exchanger as shown in Figure 1.

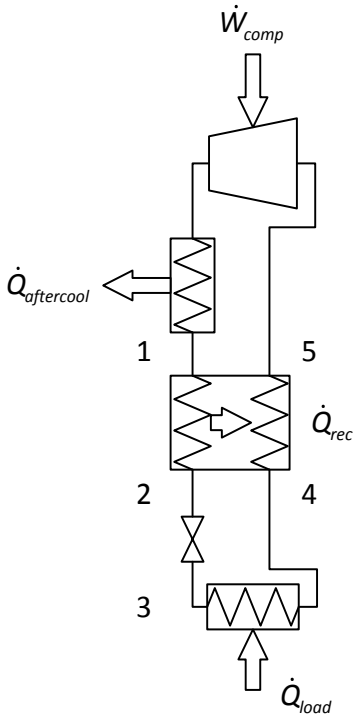


Figure 1 Diagram of simple single state JT cycle

A JT cycle can be divided into two parts, the compression unit, and the cold head. The compression unit consists of the compressor and the after cooler which returns the high pressure gas to room temperature. The cold head consists of the recuperator, expansion valve,

and load heat exchanger. This results in five distinct state points in the cold head, which are labeled in Figure 1. The recuperator allows the cycle to reach lower temperatures than a vapor compression cycle, and is the key difference between the two cycles. It is possible to see the operation of the cycle by overlaying the state points on a P-h diagram

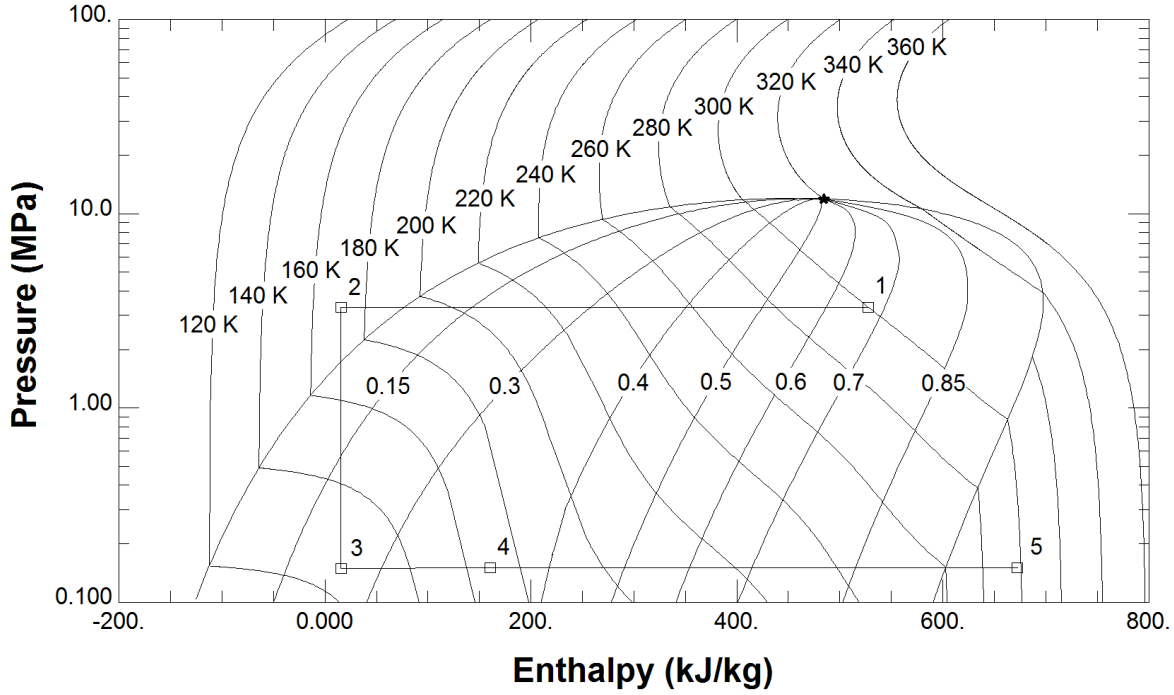


Figure 2 P-h diagram of example mixed refrigerant cycle with state points identified

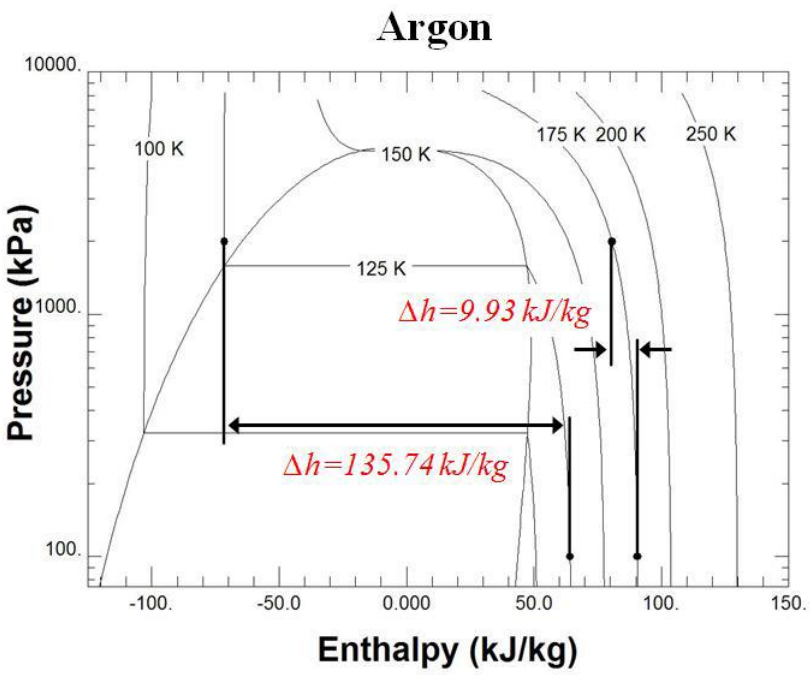
At state 1 the mixture exits the after cooler at high pressure and room temperature. From states 1 to 2 there is heat rejection from the high pressure stream to the low pressure stream. From states 2 to 3 there is isenthalpic expansion through the valve to get from high pressure to low pressure. This step is the JT effect and it causes the Mixed Refrigerant (MR) to drastically drop in temperature. The cycle then removes heat from the load between states 3 and 4. From states 4 to 5 the low pressure stream is cooling the high pressure stream. At state

5 the stream returns to the compressor. In this ideal cycle it is assumed that the entire pressure drop occurs in the expansion valve. However, in a real cycle some of the pressure drop would also occur in the recuperator. The effect of considering the pressure drop in the recuperator is analyzed later in the thesis. The cooling load is the enthalpy difference from state 3 to 4 where the MR is accepting heat from the load. The recuperator precools the hot inlet gas coming into the cycle which results in the low temperature at state 2. Then when the high pressure MR is expanded through the valve the temperature drops even further which results in the low temperature cooling load.

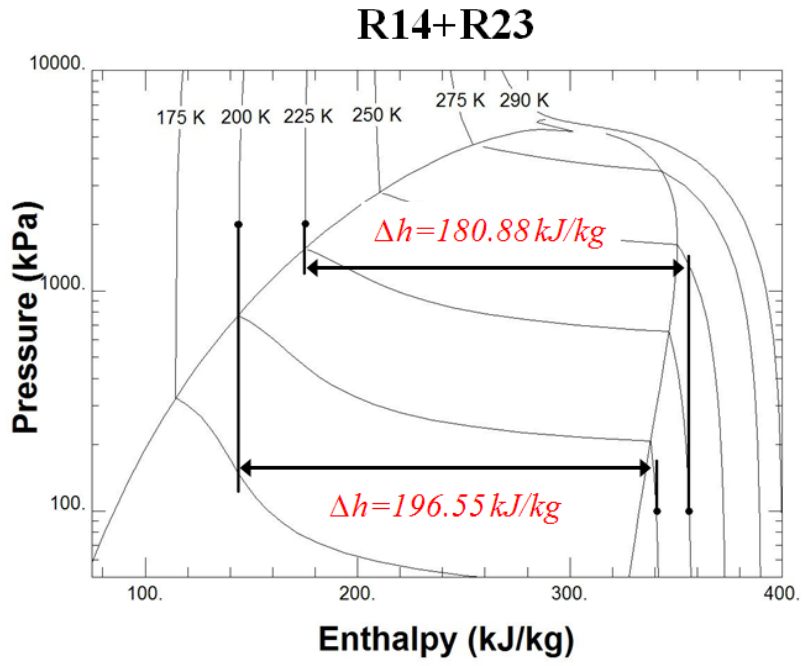
A JT cycle has many advantages over other cooling cycles because there are no moving parts in the cold head. This feature is an advantage for applications where vibrations are not acceptable. Infra-red detectors are one example where no vibration can be tolerated and low temperature cooling is needed. Using a more complex staged cycle and sorbtion pumps, JT cycles can be used for cooling sensors on telescopes that require low temperatures. Additionally, the cold head can be constructed to be very compact, making the cycle very portable and maneuverable. This makes it ideal for applications like cryosurgery where the load heat exchanger must be small enough to operate inside of a patient.

Previously, many JT cycles used an open cycle, meaning the low pressure fluid exiting the recuperator is exhausted to the atmosphere. These cycles use pure fluid refrigerants operating using high pressure cylinders. This made the cycles very simple since there were no compressors to deal with, however they were also very bulky and required large air handling systems to remove the discarded gas when operated in closed areas. Additionally the cycle needed to be periodically stopped in order to change cylinders. Mixed

refrigerants provides the solution to many of these problems as they operate at much lower pressures, in a closed cycle, and provide a higher cooling load. The advantage of refrigerant mixtures can be illustrated by comparing the P-h diagrams for a pure fluid refrigerant (argon) and a mixture of R14 and R23.


Argon
10000.
1000.
100.
Pressure (kPa)
100 K
150 K
125 K
175 K
200 K
250 K
Δh=9.93 kJ/kg
Δh=135.74 kJ/kg
-100.
-50.0
0.000
50.0
100.
150.
Enthalpy (kJ/kg)

(a)


R14+R23
10000.
1000.
100.
Pressure (kPa)
175 K
200 K
225 K
250 K
275 K
290 K
Δh=180.88 kJ/kg
Δh=196.55 kJ/kg
100.
200.
300.
400.
Enthalpy (kJ/kg)

(b)

Figure 3 P-h diagram of (a) argon (b) R14+R23 showing isothermal enthalpy difference from high pressure to low pressure from [2].

Figure 3 shows the isothermal enthalpy difference from high pressure to low pressure which is the maximum possible cooling load that a mixture can provide assuming the recuperator is perfectly effective; multiplying the mass flow rate by the enthalpy difference results in the cooling load available at a given temperature. For argon in the single phase region the enthalpy difference is rather small however, when argon is expanded through the vapor dome the enthalpy difference is drastically increased. The mixture of R14 and R23 shows a significant increase in cooling load, even when compared to argon passing through the vapor dome. By changing the mixture components and composition it is possible to match the mixture to desired cycle operating temperature and cycle operating parameters. This results in lower pressures, higher cooling, and easier operation, making Mixed Refrigerant Joule Thomson (MRJT) cycles a more attractive option.

However, if the mixture is not properly matched to the cycle, the advantages of using a MR will not be fully realized. This problem has been the area of significant study. At the University of Wisconsin Madison, several studies have been directed to optimize the mixture for a cryosurgical probe. Skye created an experimental test facility and predicted an optimal mixture using a model and experimentally investigated the behavior of the system with various mixtures [3]. Similar work was done by Pettit, Passow, Keppler, Rule, and Fredrickson [4] [5] [6] [7]. Additional modeling has also been done for optimizing a low temperature MRJT using the thermodynamics of the mixture properties [8], as well as optimizing the MR for IR cooling [9]. However none of these studies properly model the operation of the recuperator. Many break the recuperator into sub heat exchangers and use an

effectiveness-NTU model to match the conductance assigned by the user to the cycle [2] [3] [6]. However there are issues with calculating the cycle performance this way, which can be seen when analyzing the calculation for recuperator conductance. The conductance of the recuperator given by Eqn (1.1), is the inverse of the resistance of the recuperator.

$$UA = \frac{1}{\frac{1}{\bar{h}_{LP} A_{LP}} + \frac{L_c}{k_h A_{cs}} + \frac{1}{\bar{h}_{HP} A_{HP}}} \tag{1.1}$$

where $\bar{h}_{LP}$ is the convection coefficient on the low is pressure side and $\bar{h}_{HP}$ is the convection coefficient on the high pressure side of the recuperator. A_{LP} and A_{HP} are the area for convection on the low pressure and high pressure sides respectively. k is the conductivity of the material used in recuperator, A_{cs} is the cross sectional area of the material of the recuperator for conduction between the fluids, and L_c is a measure of how far the heat needs to be conducted. For most cases the conductance resistance through the recuperator is much less than the resistance of convection into and out of the heat exchanger so it can be neglected leaving Eqn (1.2).

$$UA = \frac{1}{\frac{1}{\bar{h}_{LP} A_{LP}} + \frac{1}{\bar{h}_{HP} A_{HP}}} \tag{1.2}$$

In past research the conductance for the recuperator has been assumed to be constant, meaning that the heat transfer coefficients are constant regardless of operating condition and mixture. However, recent work at the University of Wisconsin Madison has shown that the heat transfer coefficients of MRs can vary drastically with quality [1], as shown in Figure 4.

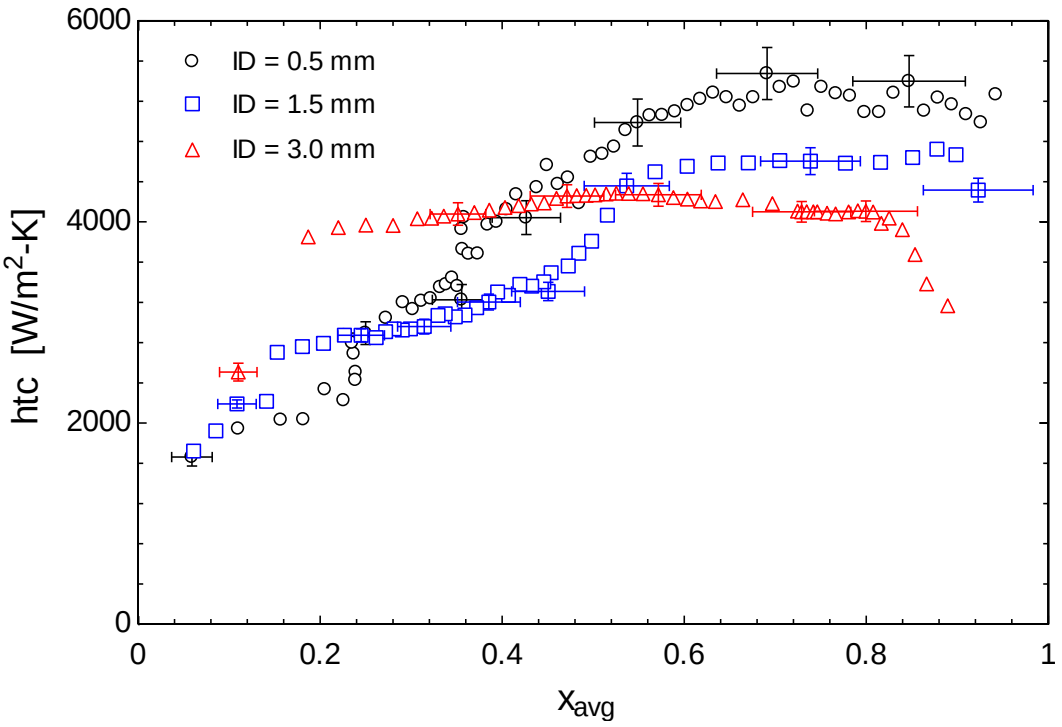


Figure 4 Heat transfer coefficient as a function of quality for a hydrocarbon mixture with different inner diameters. From [1].

Figure 4 shows that at low qualities the heat transfer coefficients drop off drastically, additionally it is apparent that the heat transfer coefficient begins to drop again when the quality approaches 1. The region between 15 and 85 percent quality will be referred to as the enhanced heat transfer region, and is the region where there is high certainty that the heat transfer coefficients will be elevated. Since the size of the recuperator is constant, and the heat transfer coefficients changes with quality, the conductance of the recuperator must also change with quality. So a cycle operating in the enhanced heat transfer region would have a higher conductance and thus a high cooling load. This behavior is not taken into account in any past research projects.

1.1 Objective

The objective of this research is to identify the mixture that will provide the largest possible cooling load while remaining in a region of high heat transfer coefficient allowing a small recuperator to be used. To keep the recuperator small, the mixture chosen needs to operate with the MR in the recuperator in the enhanced heat transfer region. This serves two purposes; first it allows for a smaller recuperator, and second, it provides a higher heat transfer effectiveness which can result in the cycle providing a higher cooling load. This objective was chosen to complement the experimental test facility for a cryosurgical probe that was previously constructed by Skye [3]. The probe needs to be small enough to be easily maneuvered, and provide the maximum cooling power possible to remain minimally invasive for large tumors and reduce the time needed for operations. Once an optimal mixture is determined it can be experimentally tested and the results of the model can be verified.

In order to accomplish this objective, a method of identifying the MR's effect on both the heat transfer and thermodynamic cooling load of the cycle is needed. This method will allow for the selection of mixture that has the optimal balance of good heat transfer and thermodynamic cooling load and results in the best real world performance.

2.0 Model

To begin the optimization process a model is needed in order to determine the thermodynamic cooling load as well as quality in a cycle with given operating parameters and mixture. In order to model multi-staged cycle, the diagram from Figure 1 has been modified to include several other components required for a precooled cycle. The modified diagram can be seen in Figure 5.

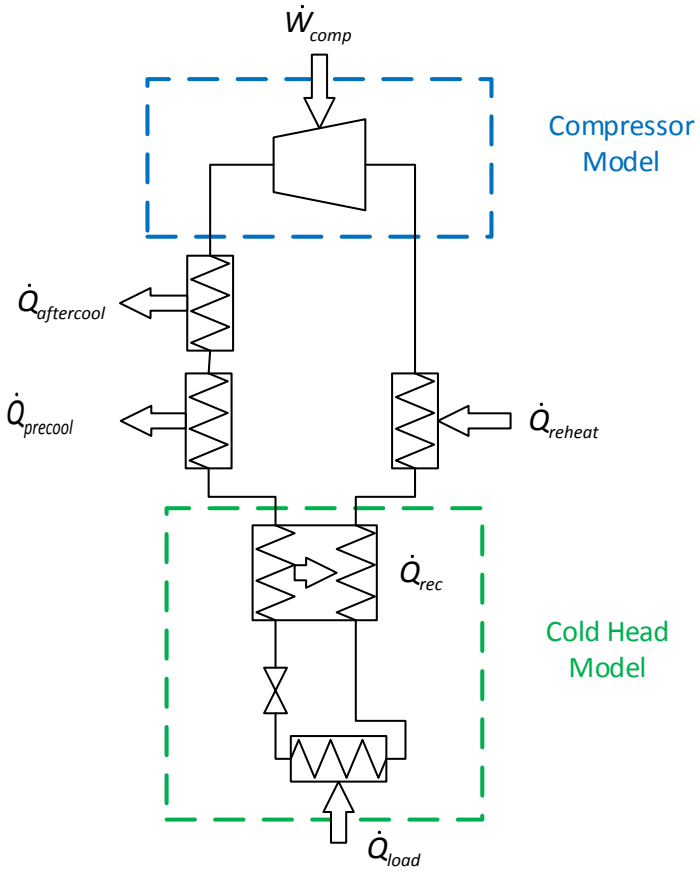


Figure 5 Total system diagram with compressor and cold head models identified

The model can be divided into two general parts, the first being the compression component, which defines the relationship between cycle pressures and mass flow rate, and the second being the cold head, which is defined as the recuperator, expansion valve, and load heat exchanger. This division allows for the flexibility to use the model to define both pre-cooled and room temperature cycles without modeling an additional cycle.

The aftercooler, precooler, and the reheater are all not explicitly modeled, and are assumed to be of sufficient size to provide the necessary heat transfer to cool or heat the MR to the proper temperature. Additionally, it is assumed that no pressure drop occurs in these

heat exchangers. These parts of the cycle are very simple, and do not need to be explicitly modeled.

2.1 Compressor

The compressor model defines the relationship between cycle operating pressures and mass flow rate, which has a large impact on the operation of the cycle. It is assumed that there is no pressure drop in the piping or valves of the compressor. The temperature of the MR at the suction of the compressor is assumed to be at ambient temperature due to the reheat heat exchanger. The compressor is modeled after work done by Jähnig which is based on an isentropic compressor model [10]. The mass flow in the cycle was determined with Eqn (2.1).

$$\dot{m} = \eta_{vol} V_{dis} \frac{1}{v_{suc}} RPM \tag{2.1}$$

where $\dot{m}$ is the mass flow rate of the MR through the system, η_{vol} is the volumetric efficiency of the compressor, V_{disp} is the volume displaced for each stroke of the compressor, and v_{suc} is the specific volume of the MR at the entrance to the compressor. The rotational speed is

assumed to be 3600 RPM which is typical for compressor operating with 60 Hz power. The volumetric efficiency is predicted by Eqn (2.2).

$$\eta_{vol} = 1 + C - C\left(\frac{P_{dis}}{P_{suc}}\right)^{\frac{1}{k}} \tag{2.2}$$

where *k* is the ratio of the specific heat at constant pressure (c_p) to the specific heat at constant volume (c_v) for the MR at the inlet to compressor, as given by Eqn (2.3).

$$k = \frac{c_p}{c_v} \tag{2.3}$$

The specific heat ratio is calculated using REFPROP [11], and is only valid when the MR is super-heated. The quality of the MR is calculated at 300 K and the suction pressure of the cycle. If the quality at the suction side of the compressor is less than one then the mixture cannot be used as the compressor will be damaged. The parameter *C* is the ratio of the clearance volume to the swept volume of the piston as shown in Eqn (2.4).

$$C = \frac{V_{clear}}{V_{swept}} \tag{2.4}$$

In the model a representative value of 0.025 was used for *C*, which is a good estimate for all reciprocating compressors [10]. With the mixture defined, as well as the suction and discharge pressures, the mass flow rate of refrigerant through the cycle can be calculated.

In order to calculate the COP, the power needed for the compressor is estimated using Eqn (2.5)which is based on [10].

$$\dot{W}_{comp} = \dot{V}_{displace}\,\eta_v\,P_{suction}\left(\frac{k}{k-1}\right)\left[\left(\frac{P_{discharge}}{P_{suction}}\right)^{\frac{k-1}{k}} - 1\right] \tag{2.5}$$

where $\dot{V}_{displace}$ is the displacement rate of the compressor. $P_{suction}$ and $P_{discharge}$ are the suction and discharge pressures respectively. It should be noted that this is only the power required for the mixed gas cycle being modeled and does not consider the power needed to power the precool cycle.

2.2 Cold Head

The cold head model consists the recuperator, expansion valve, and load heat exchanger. It is used to calculate the cooling load of the cycle as well as the quality limits of the MR in the recuperator. The diagram of the cold head is shown in Figure 6.

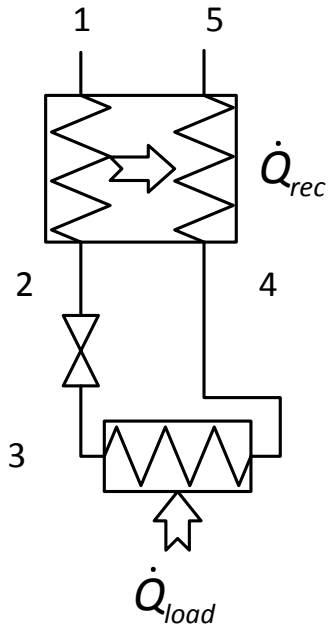


Figure 6 Cold head of MRJT cycle model with states labeled.

The states are labeled as follows; state 1: high pressure entrance to the recuperator, state 2: high pressure exit of the recuperator, state 3: low pressure entrance to the load heat exchanger, state 4: low pressure entrance to the recuperator, and state 5: low pressure exit from the recuperator. The model has been idealized so that the entire pressure drop in the cycle occurs between states 2 and 3. Thus, all of the pressures in the cycle can be specified by the compressor model. States 1 and 2 are at the compressor discharge pressure, while states 3, 4, and 5 are at the compressor suction pressure. Some temperatures in the cycle are also explicitly defined. The temperature at state 1 is specified by the user, and will be referred to as the supply temperature. Specifying the supply temperature allows the model to be applicable to both room temperature and pre-cooled MRJT cycles. The temperature at state 4 is also specified by the user, and will be referred to as the load temperature; the load temperature is assumed to be the temperature of the object to be cooled and is the highest temperature that the fluid experiences in the load heat exchanger.

To solve for the cooling load and the quality limits in the recuperator an iterative equation solver called Engineering Equation Solver, or EES, is used [12]. This program can very quickly and easily solve implicit sets of equations making it the perfect choice for modeling this cycle.

2.2.1 Recuperator Model

To solve for the cooling load first the states at 2 and 5 need to be determined. This is done using sub-heat exchanger discretization to solve for the unknown states in the recuperator [13]. This method breaks up a large heat exchanger into N smaller sub-heat

exchangers, each with equal heat transfer. Each between each section there is a node for a total of N+1 nodes, as shown in Figure 7.

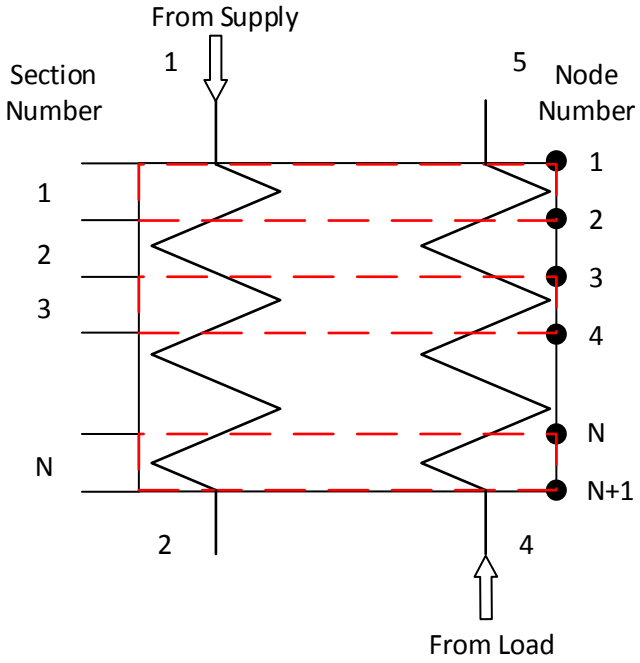


Figure 7 Sub-heat exchanger discretization

To quantify the performance of the recuperator, an approach temperature difference (ΔT_{app}) is specified depending on the effectiveness of the recuperator. The approach temperature difference is the minimum temperature difference between the two streams in the recuperator. Depending on the capacitance rates of each stream, the place location that this occurs (i.e., the pinch point) can be anywhere along the length of the recuperator. The approach temperature difference is shown for an example cycle in Figure 8.

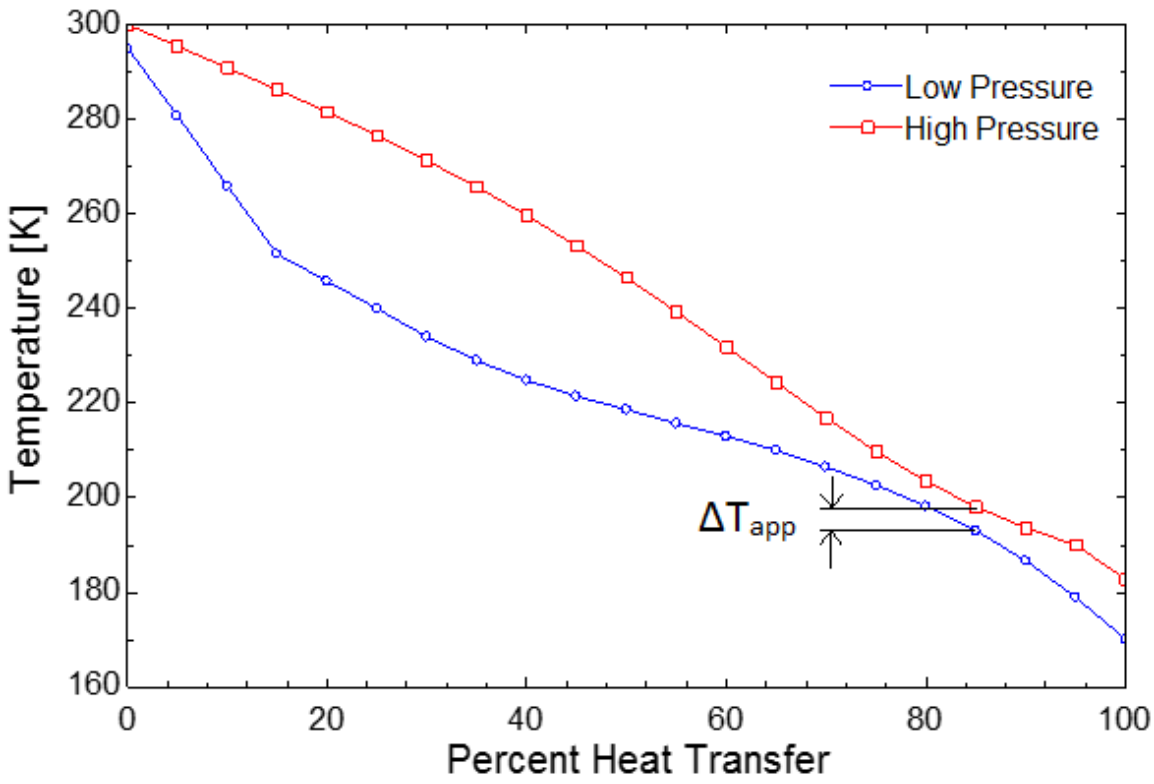


Figure 8 Temperature of high and low pressure streams in the recuperator for a representative cycle. Approach temperature difference = 5 K.

The approach temperature difference can be varied according to the performance of the recuperator. An approach temperature difference of zero means that the recuperator is operating at maximum effectiveness. As the approach temperature difference increases, the recuperator becomes less effective. The approach temperature difference can never be less than zero, as it would mean that heat is being transferred from low to high temperature. In the real world it would be possible for the streams to cross if the distributed JT effect is considered. The distributed JT effect occurs when some of the pressure drop in the cycle occurs in the recuperator. This results in cooling of the high pressure stream prematurely which reduces the effectiveness of the recuperator since the temperature difference between the streams is reduced. This effect is difficult to account for without a complicated model,

which means the computations take too long to be useful in a parametric study. The cold head model is later modified to model pressure drop in a much simpler way. The distributed JT effect is not important to consider unless the pressure drop in the recuperator is large, about 50 percent of the total pressure drop of the cycle.

The model is run with an approach temperature difference of zero Kelvin which corresponds to the thermodynamic optimal limit of the cycle. This means that a cycle operating within the enhanced heat transfer region would be able to operate with a cooling load near the predicted result, whereas a cycle that is operating outside of the enhanced heat transfer region would exhibit poorer performance.

The states of the cold head model are already partially specified, all the pressures are known, and the supply temperature, state 1, and load temperature, state 2, are specified by the user. Solving for the remaining states requires the determination of the temperature distribution that corresponds to the approach temperature difference specified by the user. This is done by first calculating the specific heat transfer rate (i.e., the ratio of the heat transfer rate to the mass flow rate) in the recuperator (Q_{rec}) using Eqn (2.6).

$$Q_{rec} = \left(h_1 - h_2\right) \tag{2.6}$$

The specific enthalpy at state 1 is calculated using the known temperature and pressure, while the specific enthalpy at state 2 is calculated using the known pressure and a guess value for the temperature. The initial guess value used for T_2 is the supply temperature, T_4. This assumes that the recuperator pinch point occurs at the cold side of the recuperator. Using an

energy balance, the specific enthalpy at the low pressure exit of the recuperator can be determined using Eqn (2.7).

$$Q_{rec} = \left(h_5 - h_4\right) \tag{2.7}$$

The recuperator is then broken into sub heat exchangers each with equal amount of heat transfer. Using an energy balance, the specific enthalpy can be determined for each stream at the entrance and exit of each sub-heat exchanger, as indicated in Eqn (2.8) for the high pressure stream and Eqn (2.9) for the low pressure stream.

$$for\, i = 1:\left(N+1\right)\, h_{H,i} = h_1 - \frac{Q_{rec}\left(i-1\right)}{N} \tag{2.8}$$

$$for\, i = 1:\left(N+1\right)\, h_{C,i} = h_4 + \frac{Q_{rec}\left(i-1\right)}{N} \tag{2.9}$$

Assuming no pressure drop in the heat exchanger, the state at each node is fully specified, and the temperature can be calculated. Once the temperature distribution in the recuperator is known, the approach temperature difference is calculated using Eqn (2.10).

$$T_{app} = \min\left(T_{H,i} - T_{C,i}\right) \tag{2.10}$$

where $T_{H,i}$ is the temperature of the high pressure stream at node i and $T_{C,i}$ is the temperature of the low pressure stream at node i. Eqn (2.10) calculates the temperature difference at each node, and returns the smallest temperature difference which is the approach temperature difference.

Since the approach temperature difference calculated using the guess value for T_2 is likely not equal to the approach temperature difference defined by the user, T_2 needs to be changed until it matches. EES [12] can be used to identify the value of T_2 that results in the correct approach temperature difference. For example if the approach temperature difference calculated is lower than the approach temperature difference defined by the user T_2 will be increased until it matches. Once the temperature that results in the proper approach temperature difference is found, the specific enthalpy at state 2 can be calculated. Then, the simple energy balance from Eqn (2.7) can be used to solve for the temperature and specific enthalpy at state 5.

After states 2 and 5 have been fully defined by the heat exchanger model, the cooling load can be calculated. The cooling load is calculated by Eqn (2.11).

$$\dot{Q}_{load} = \dot{m}\left(h_3 - h_4\right) \tag{2.11}$$

Since there is isenthalpic expansion through the valve, the specific enthalpy at state 3 is the same as state 2 and the cooling load can be rewritten as seen in Eqn (2.12).

$$\dot{Q}_{load} = \dot{m}\left(h_2 - h_4\right) \tag{2.12}$$

Assuming isenthalpic expansion and a constant suction pressure, the temperature at state 3 can be calculated from Eqn (2.13).

$$T_3 = T(h_2, P_{suction}) \tag{2.13}$$

The mass flow rate is specified by the compressor model, so the cooling load is fully defined. To make the results general to any sized cycle, the cooling load is normalized by the displacement rate of the compressor as seen in Eqn (2.14).

$$\dot{Q}_{load,norm} = \frac{\dot{Q}_{load}}{\dot{V}_{disp}} \tag{2.14}$$

2.2.2 Lookup Tables

The properties of the MR are calculated using NIST Standard Reference Database 23 also known as REFPROP [11]. Version 9.1 is the current version of the NIST Standard Reference Database and is used for all state calculations. In the past other researchers have used the NIST 4 SUPERTRAPP database which uses a simpler equation of state [14]. NIST 4 is much more robust than NIST 23, however it comes at the cost of accuracy [6]. To use REFPROP to calculate states in EES an interface program is used [12]. This interface program accepts inputs of mixture composition and two set properties and returns the remaining properties. REFPROP has issues directly calculating temperature from enthalpy and pressure which sometimes results in fatal errors in the model [12]. To solve this problem, and to increase the computation speed, lookup tables of temperature and enthalpy are prepared for the MR at the low and high pressures to solve for the temperature at each node in the recuperator. These tables allowed for the more accurate REFPROP database to be used while maintaining a robust model.

Tables of temperature versus specific enthalpy at constant pressure for both the suction and discharge pressure were prepared using REFPROP. The temperature limits are ten Kelvin above the supply temperature, to ten Kelvin below the load temperature. These temperature ranges ensure that the entire range of possible temperatures in the recuperator is covered. The area that REFPROP most often fails to converge is in the two phase regions near the bubble and dew points. If the temperatures are spaced linearly it is likely that a temperature near the saturated liquid or vapor point will be chosen causing an error in REFPROP, as shown in Table 1.

Table 1: Lookup table temperature corrections for non-converging REFPROP calls. T_{dew} = 223.39 K.

Error Prone Table		Corrected Table	
Enthalpy (kJ/kg)	Temperature (K)	Enthalpy (kJ/kg)	Temperature (K)
209.51	221	209.51	221
210.82	222	210.82	222
212.15	223	220.35	222.39
Error Prone Entry	224	221.34	224.39
224.61	225	224.61	225
231.08	226	231.08	226

The table on the left is an example of a linearly spaced temperature distribution with an entry near the dew point temperature. On the right Table 1 shows how the temperature range is modified in order to prevent calculating enthalpy at the temperature nearest the dew point temperature. Before the tables are created, the temperature of the bubble and dew

points are calculated and compared to the minimum and maximum temperatures required for the tables. When crossing a phase boundary, the tables skip 1 Kelvin on either side of either the dew or bubble point, which prevents REFPROP from trying to calculate using temperatures near the bubble or dew point temperature where errors are most likely to occur. An example of this modification is shown on the right side of Table 1. This prevents any errors from occurring in the tables while still allowing accurate calculation of properties. For more information see the code used in Appendix B.

The specific enthalpy for both the high and low pressure cases can be calculated at all the temperatures in the tables. If any errors occur when calculating specific enthalpy, the run is skipped and the program moves onto the next entry in the table. This process ensures that the entire run is not ended because of a single error in REFPROP. After all the points have been calculated the program comes back through and linearly interpolates any data points that were omitted because of errors to ensure that even if the program is missing data it can determine the temperature at a given enthalpy fairly accurately. These entries must be kept in the tables because EES can only interpolate in tables where there are no empty entries [12].

The tables can then be used to solve for the temperature at each node in the recuperator. Using the known specific enthalpy, the temperature is interpolated from the tables again using linear interpolation. The tables serve several purposes, the first catching bad data that might otherwise be fed into the program from REFPROP or the interface program (typically resulting in a discontinuity in the specific enthalpy), and second preventing EES from providing enthalpy values outside of the operational range of REFPROP, which causes a fatal error. Finally, the use of the tables greatly reduces

computation time by reducing the number of calls needed to REFPROP, which is the most time intensive part of the calculations.

2.2.3 Quality Calculations

In order to be able to assess the impact of enhanced heat transfer the quality needs to be calculated for both streams in the recuperator. Quality is calculated on a mass basis using REFPROP [11] when the MR is in the vapor dome. In some instances the cycle will operate sub-cooled or super-heated, so a way to quantify how subcooled or superheated the MR is needed. To estimate the 'quality' of a sub-cooled or super-heated MR, equations (2.15) and (2.16) were used.

$$x_{sh} = \left(\frac{h - h_g}{h_g - h_f} \right) + 1 \tag{2.15}$$

$$x_{sc} = \frac{h - h_f}{h_g - h_f} \tag{2.16}$$

In both equations (2.15) and (2.16), h_f is the specific enthalpy of the saturated liquid, h_g is the specific enthalpy of the saturated vapor, and h is the specific enthalpy of the MR at the state of interest. All the specific enthalpies are calculated assuming a constant pressure equal to the pressure used to calculate h. Using this method all superheated MRs will have a quality greater than one and all subcooled MRs will have a quality less than zero. Instead of calculating the quality of the MR at each point in the recuperator, it was observed that two points, state 2 and state 5, could be used to define the limits of quality in the recuperator.

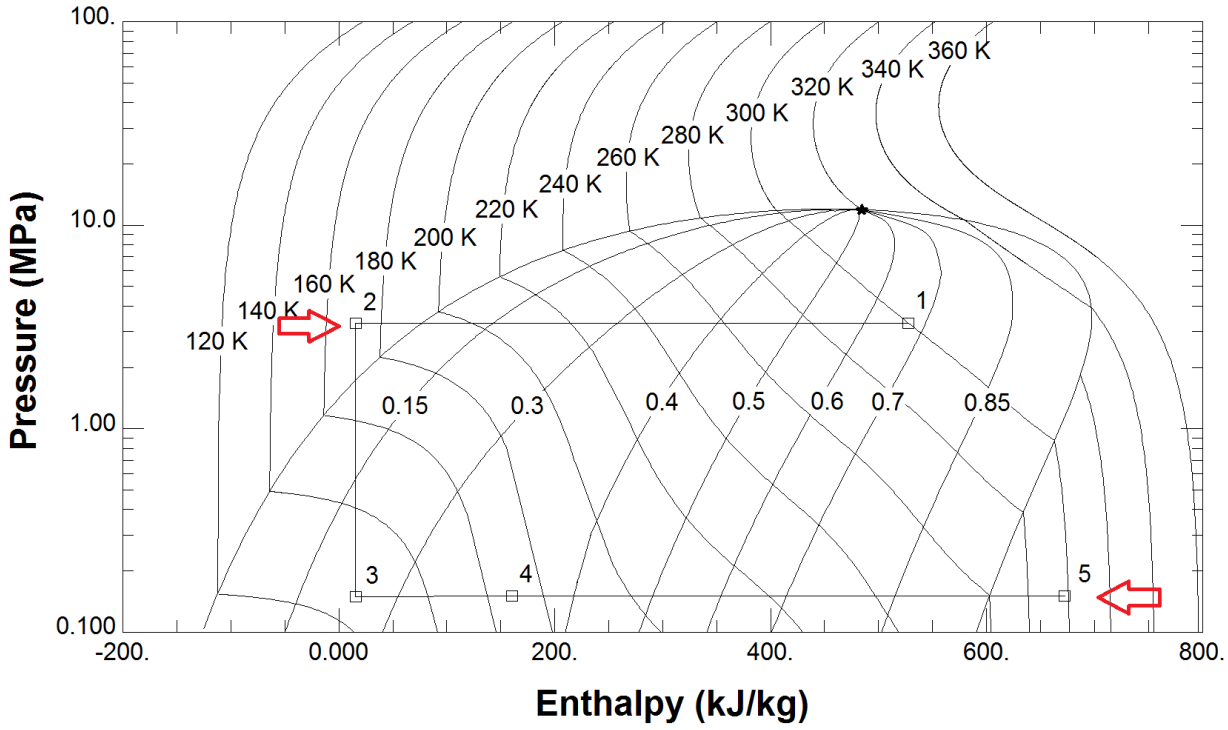


Figure 9 P-h Diagram showing the qualities that define the operation of the recuperator.

Figure 9 shows a basic JT cycle overlaid on a P-h diagram for a three component mixture of methane, propane, and pentane. The lines of constant quality make it clear that state 2 and 5 will define the limits of quality in the recuperator in most cases. In some extreme cases it is possible that state 1 could have a higher quality than state 5. Thus the quality is calculated at state 1 as well, and the higher quality between state 1 and state 5 is used. Therefore, instead of calculating the quality for each node in the recuperator, only three quality calculations are needed to define the range of qualities in the recuperator.

This model of the recuperator was used because of its simplicity and robustness when compared to the alternative which involves iteratively solving the heat transfer coefficients

and pressure drop in each sub-heat exchanger. Lookup tables can no longer be used if a pressure drop is included in a more complex model. Additionally, since this technique requires the iteration of both T_2 and the length of each sub-heat exchanger, it requires much more time to solve. Even with the additional accuracy of a pressure drop heat transfer model, it is almost impossible to match every parameter in the test to the real cycle. Effects such as different compressor parameters, selective condensation of components, improper JT valves, wrong precooling temperature, and many other parameters can affect the performance of the MR. As a result, the performance in the experimental tests will not perfectly match model anyway. The approach temperature difference method allows the model to quickly solve while providing a good approximation of the results.

This model is provided electronically, for information see item A and B in the appendix.

2.3 Interpolation Error

Since this is a numerical method, there is inherent error built into the simulation. One approximation is the number of entries in the interpolation tables that are used to calculate the MR properties in the recuperator. The other is the number of subsections in the recuperator. A parametric study was conducted that looked at the effect that each of these two parameters has on the cooling load of the cycle. The cooling load was calculated for three different mixture compositions. The parameters for the cycle are given in Table 2.

Table 2 Operating parameters for interpolation error calculations.

Parameter	Value
T_1	240 [K]
T_4	150 [K]
ΔT_{app}	0 [K]
$P_{suction}$	150 [kPa]

The mixture used was methane, ethane, and butane. For each run of the parametric study the cooling load was calculated for three mixtures. The mixtures used are given in Table 3.

Table 3 Mixture composition of 3 mixtures used in interpolation table testing

Mixture	Methane	Ethane	Butane
1	0.62	0.28	0.1
2	0.694	0.201	0.105
3	0.718	0.15	0.132

These mixtures correspond to the optimal mixtures that this model predicted using the methods described below. The test matrix is shown in Table 4.

Table 4 Test matrix for interpolation table error

		Number of Subsections				
		10	20	30	45	60
Number of Interpolation table entries	15	Test 1	Test 2	Test 3	Test 4	Test 5
	20	Test 6	Test 7	Test 8	Test 9	Test 10
	30	Test 11	Test 12	Test 13	Test 14	Test 15
	45	Test 16	Test 17	Test 18	Test 19	Test 20
	65	Test 21	Test 22	Test 23	Test 24	Test 25
	90	Test 26	Test 27	Test 28	Test 29	Test 30

It was assumed that the run with 60 subsections and 90 entries in the interpolation table represented that actual operation of the cycle. This is valid because when compared with test 25, fewer interpolation table entries, and test 29, fewer sub sections, the results were nearly identical. Figure 10 shows the maximum percent error of cooling load for the three mixtures at a given test parameters compared to test 30.

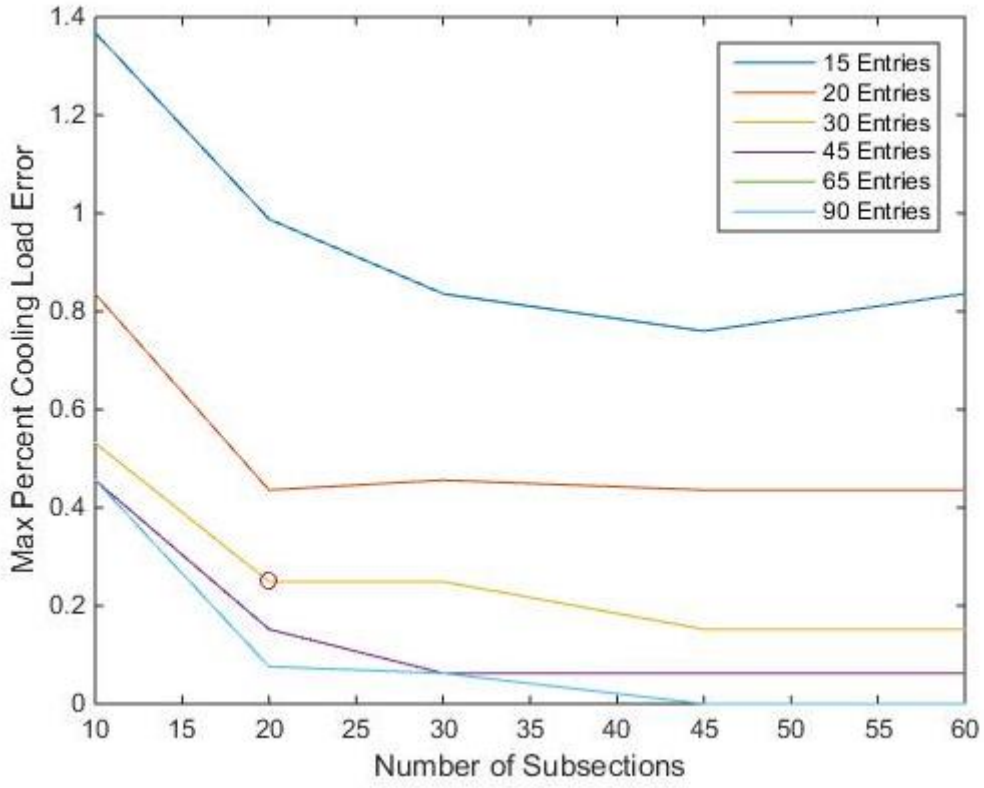


Figure 10 Cooling load error for parametric study of interpolation table entries and recuperator subsections.

It is clear from Figure 10 that the assumption that cooling load for 60 subsections and 90 table entries is very close to the actual value since the cooling does not change as the number of entries is reduced or if the number of subsections is decreased. The error in all cases is very low, and only exceeds one percent during test 1. It was decided that the model

would use 20 subsections in the recuperator and 30 entries in the lookup tables for the remainder of the calculations. These parameters provide a maximum cooling error of under 0.3 percent and keep the computation time for the model to a minimum. This point is indicated in Figure 10 by a circle placed on the graph.

3.0 Parametric Study

The model is used to conduct parametric studies to determine the effect that mixture selection has on a MRJT cycle. Mixtures are chosen to include components that have normal boiling points distributed between the high and low temperatures of the cycle. In general, to maintain two-phase conditions in the recuperator, at least one component with a boiling point below the load temperature, and one with a boiling point above the supply temperature are needed [15]. Since the constraint has been made that the mixed refrigerant in the recuperator must be in the two phase region it makes sense that components are needed with boiling points spanning the range of the recuperator. Otherwise with some given mixture components it would be impossible to uphold the constraint since they would always be subcooled or superheated depending on their normal boiling point. Table 5 shows the normal boiling points for the refrigerants used in the parametric studies.

Table 5 Normal boiling points of components used in hydrocarbon and synthetic mixtures. From calculated using REFPROP [11]

Hydrocarbons		Synthetics	
Refrigerant	Normal Boiling Point (K)	Refrigerant	Normal Boiling Point (K)
Methane	111.67	Argon	87.30
Ethane	184.57	R14	145.10
Propane	231.04	R218	236.36
Isobutane	261.40		
Pentane	309.21		

REFPROP has difficulty calculating the properties for some mixtures of synthetic refrigerants, so only mixtures that have been tested and found to work well were used. For this reason, certain components had to be left out (such as Nitrogen). However, REFPROP is able to accurately calculate the properties of hydrocarbon mixtures, and as a result were used as the basis for most of the parametric studies.

The parameter space is the composition of the MR and discharge pressure of the compressor. The composition of the MR is defined by the mole fractions of each component. The mole fractions of all components must always sum to one. This means the degrees of freedom for the mixture is one minus the number of components. For example, a three component mixture is defined by the mole fractions of two of the components. The mole fraction components of the parameter space are defined using a program written Matlab [16] that is listed in Appendix items E, F, and G. For a three component mixture the parameter space is shown in Figure 11.

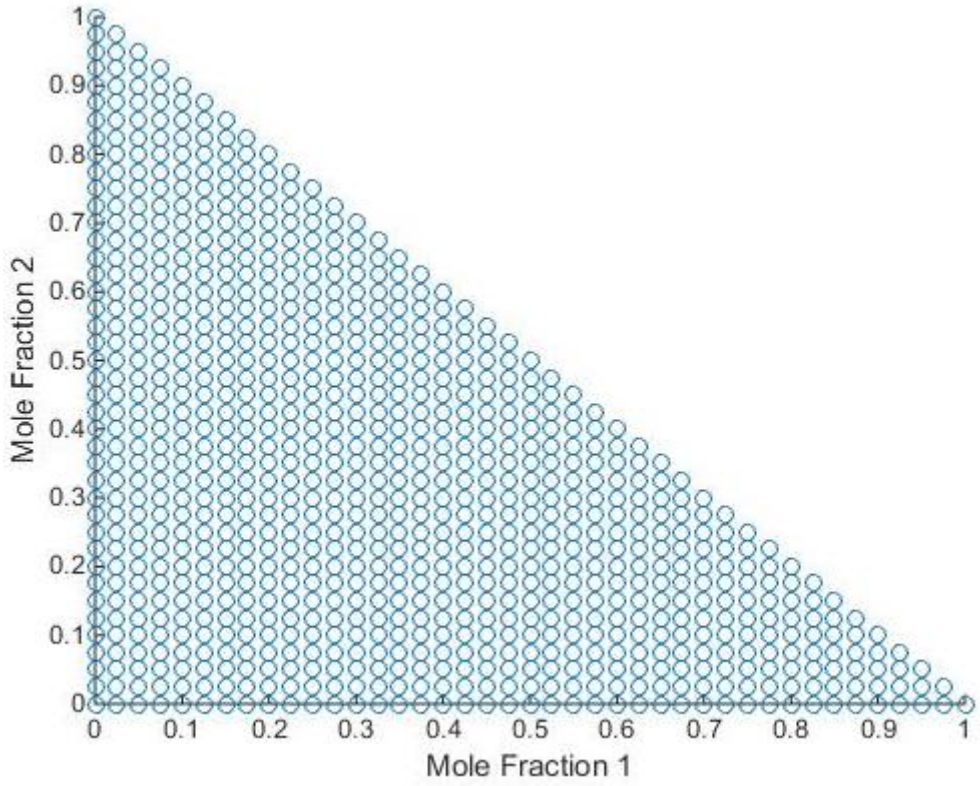


Figure 11 Mole fraction parameters space for 3 component mixtures.

Equal divisions in both the mole fraction 1 and mole fraction 2 directions and chosen, and only the combinations that sum to less than or equal to one are kept. As more components are added, the mole fraction parameters space becomes more complicated. With four components, it becomes a triangular pyramid and with five components it becomes a hyper triangular pyramid. Additionally the number of runs increases dramatically when adding more components. In order to keep the computational time reasonable the mole fraction divisions were made larger as the number of components increased.

The suction and discharge pressure need to be supplied to the model in order to calculate the cooling load. A constant suction pressure was used in these parametric studies

of 150 kPa. This value was chosen because it was representative of the value seen in the experimental test facility. The discharge pressure is part of the parameter space and added an additional degree of freedom to the problem. It was varied between 1200 kPa and 3700 kPa for most tests. It was observed that for a given MR the cycle would exhibits a maximum normalized cooling load when the discharge pressure was somewhere between these two limits. The discharge pressure was varied in increments of 100 kPa, for each discharge pressure all the mixture compositions in the parameter space need to be run. This ends up increasing the number of runs drastically but the result is not only an optimal mixture but also an optimal discharge pressure. The optimal discharge pressure is important because it defines the size of JT valve needed in the device.

3.1 Plotting

The parametric study data was logged for each mixture and used to create maps of the cycle performance. The quality of the high pressure and low pressure exits of the recuperator were used as the independent variables and plotted on the x and y axes respectively. In the parameter space a scatter plot of all the runs are plotted with their color corresponding to the thermodynamic cooling load. Using this method of plotting shows what the effects of constraining the quality to be in the enhanced heat transfer region are on the thermodynamic cooling load of the cycle. The data points for a representative study are shown in Figure 12 with the green points having a higher thermodynamic cooling load.

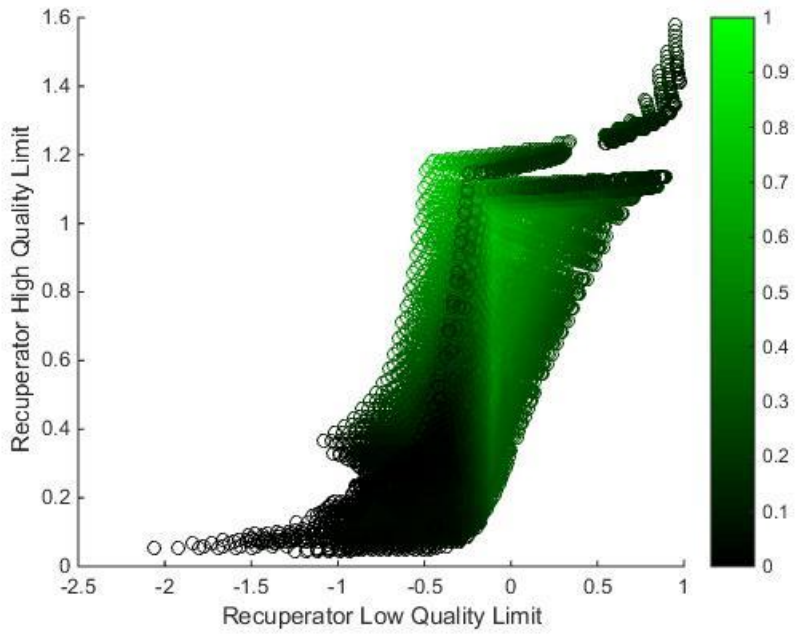


Figure 12 Scatter plot of a representative parametric study with color of points corresponding to cooling load of points

For each point in Figure 12 the cooling load is normalized according to Eqn (3.1).

$$\dot{Q}_{load,norm} = \frac{\dot{Q}_{load}}{\dot{V}_{disp}} \tag{3.1}$$

That is, the cooling load ($\dot{Q}_{load}$) is divided by the displacement rate of the compressor ($\dot{V}_{disp}$). The normalized cooling load ($\dot{Q}_{load,norm}$) can be used to compare the performance of the MR regardless of the size of the compressor used. Since it has the units of kW/(m^3/s) it can be used to estimate the cooling load for a cycle operating under the same conditions but with a different volumetric flow rate. To make the comparison of the different MR compositions easier, the cooling load is further normalized by the maximum cooling load that the

components of the MR can provide at the specified operating conditions. Equation (3.2) shows the formula used.

$$\dot{Q}_{load,norm,frac} = \frac{\dot{Q}_{load,vol}}{MAX(\dot{Q}_{load,vol})} \tag{3.2}$$

Figure 12 shows that the results of the parametric study do not result in a smooth surface, but rather a point cloud of data. However, the only points that are of interest are those that lie on the top surface of the cloud. These are the points that represent the maximum possible cooling load for a given constraint on quality. In a first attempt to plot the upper surface of the point cloud, Matlab's built in curve fit program was used to plot contours of constant normalized cooling load [16]. The results are seen in Figure 13.

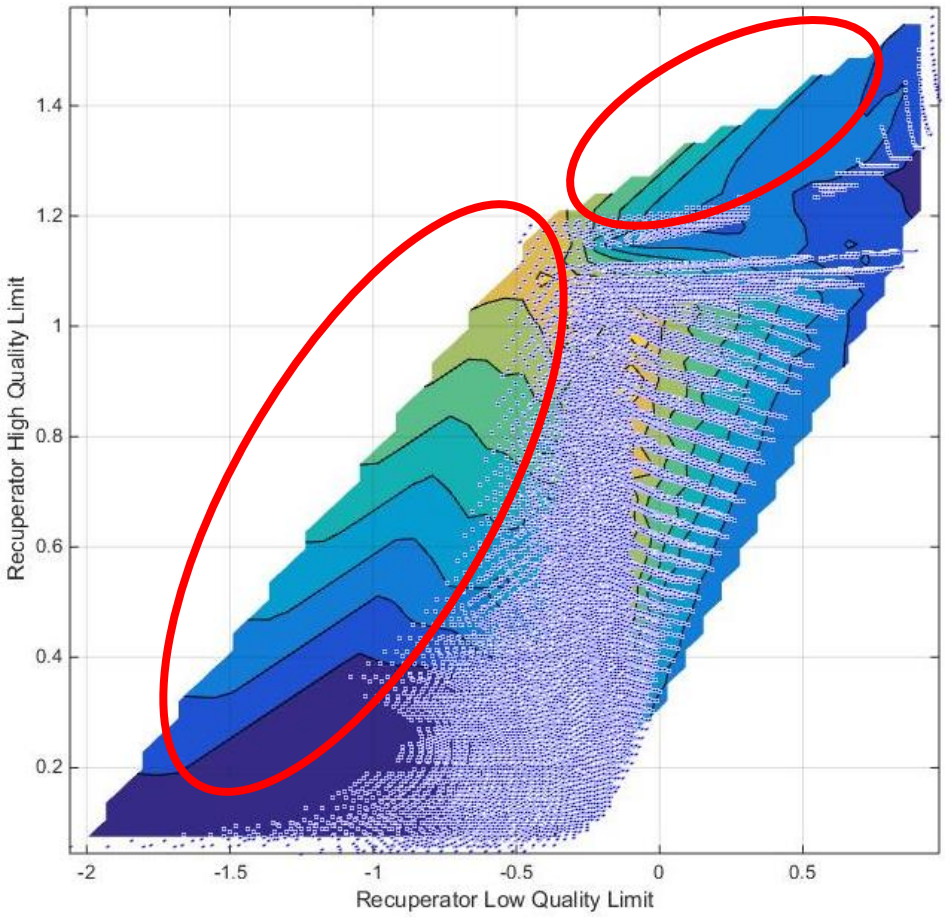


Figure 13 Cooling load Vs quality at the exits of the recuperator plotted using Matlab's curve fit tool. With red ovals indicating areas of error in plot.

Matlab does a good job at creating contours and fitting them to the general shape of the data, but it represents the average value of the cooling load at a given constraint instead of the maximum value. Figure 13 shows that half the points are above the surface, and half below. Additionally, Matlab does not handle concave regions of the plot well. Figure 13 shows this for the same representative parametric study, the red ovals show areas where the plot indicates it is possible to operate, however no data points appear there. What is needed instead is a way to define the outline of a concave shape. Matlab's curve fit tool cannot do this, so instead, alpha shapes are used. The Alpha Shapes function in Matlab takes an arrays

of points in either 2 or 3 dimensions and connects them to create shapes using Delaunay triangulation [16]. Using Alpha Shapes, a shape can be created that fully closes all the points without falsely representing the data.

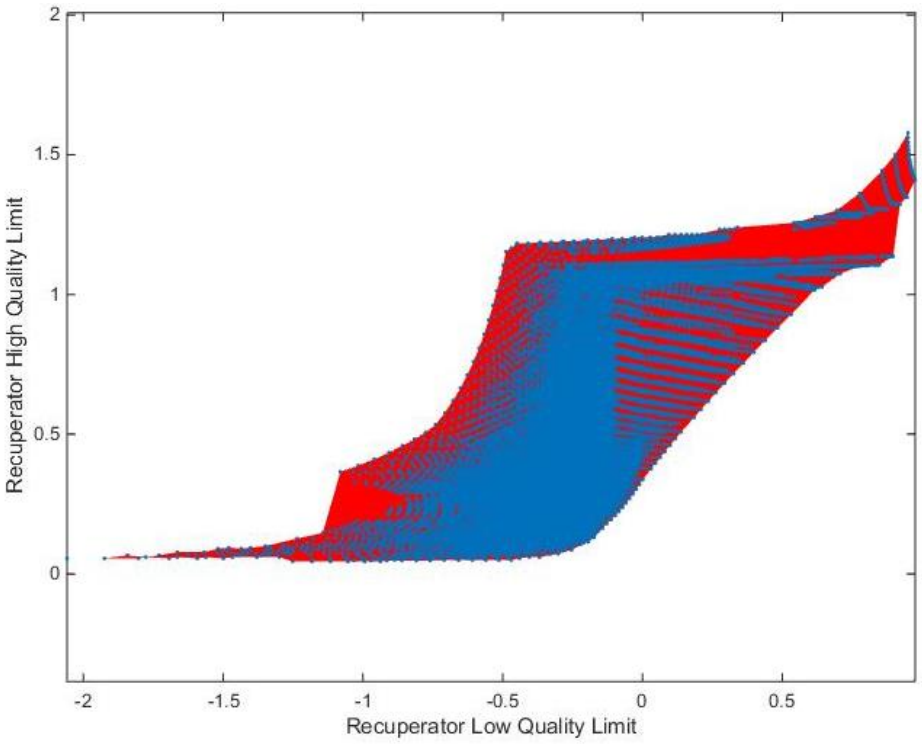


Figure 14 Alpha shape drawn to enclose all data points in representative parametric study.

Figure 14 shows the alpha shape drawn enclosing all the points of the parametric study with all data point overlaid. All of the points fit within the shape, and there is no issue with the area extending outside of the points. However, simply having the outline is not enough. Contours need to be drawn that indicate the maximum cooling at any point on the graph. One way to do this is to divide all the points into groups based on the cooling load they provide and create a shape that encloses all those points. Then any point within the shape would have at least as much cooling load as the cooling load constraint. When this is done for multiple

groups, contours are created, and the more groups used, the higher the accuracy. For this representative parametric study, the points were divided into five groups. The first group is all points with a positive cooling load as shown in Figure 14. The second is all points with a cooling load greater than 20 percent of the maximum normalized cooling load as shown in Figure 15.

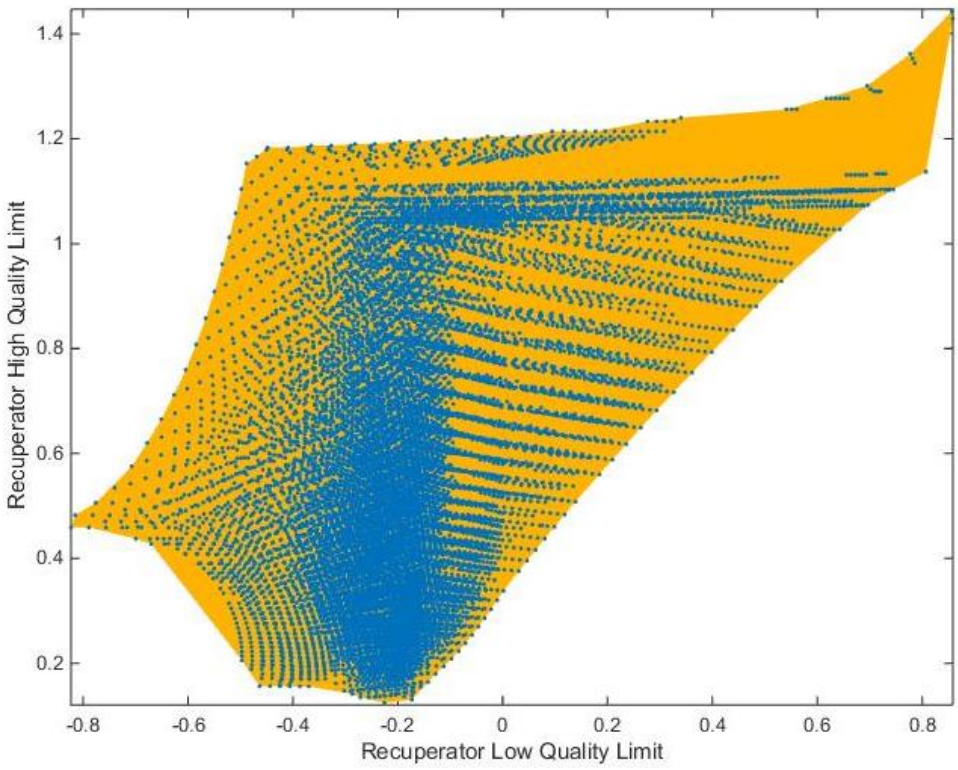
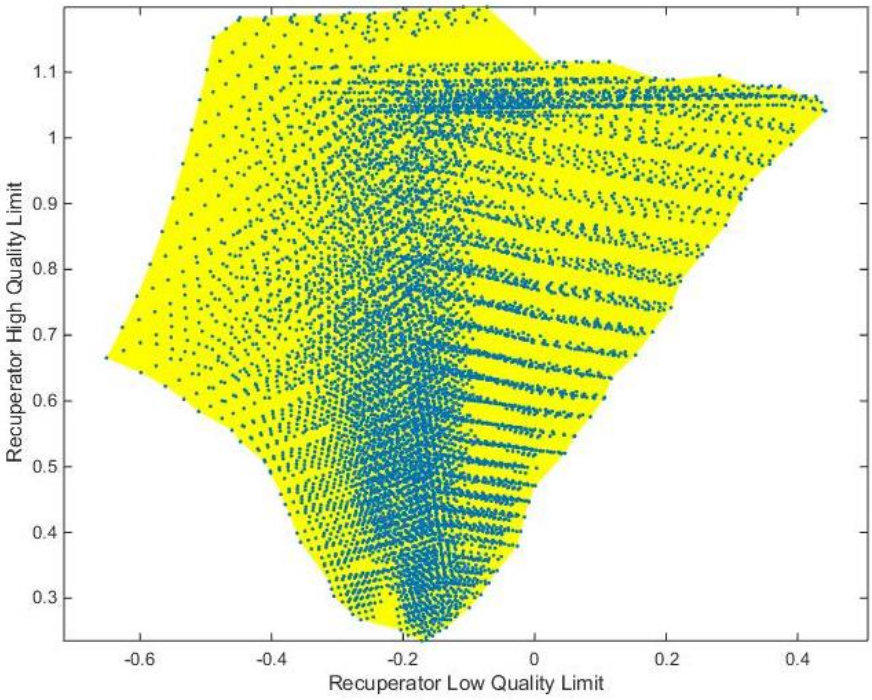
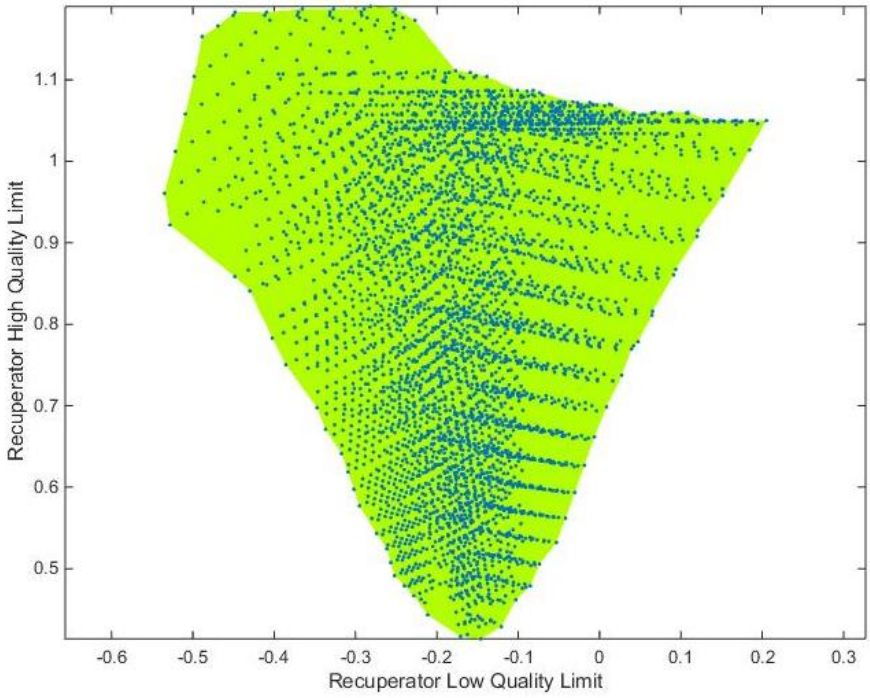


Figure 15 Points with at least 20 percent of the normalized cooling load of the best case

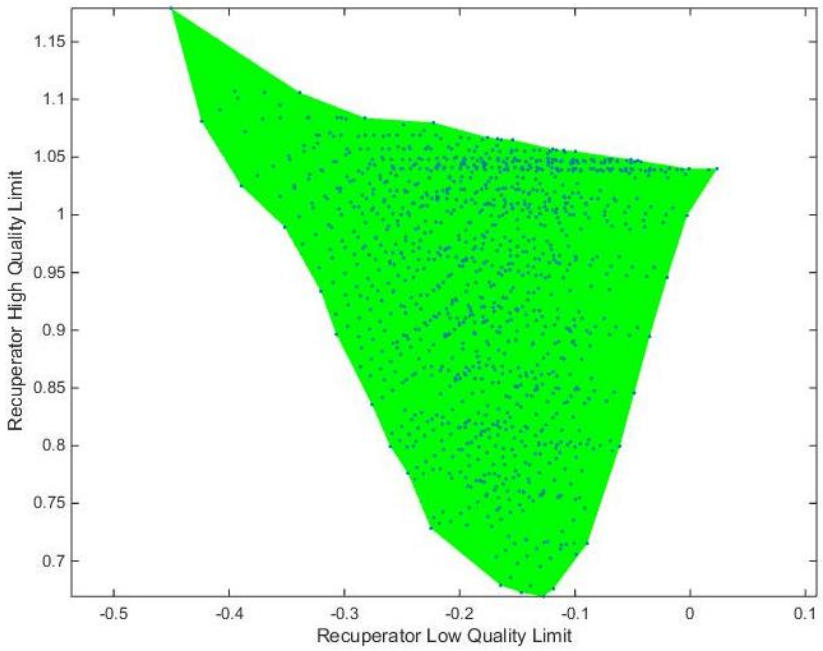
For any point inside the curve shown in Figure 15 there is a mixture composition that will result in a cooling load that is at least 20 percent of the maximum cooling load. More shapes are needed to more accurately define the contours. The same technique is applied for 40 percent, 60 percent, and 80 percent of the maximum cooling load, and the results are shown in Figure 16.


Recuperator High Quality Limit
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
-0.6
-0.4
-0.2
0
0.2
0.4
Recuperator Low Quality Limit

(a)


Recuperator High Quality Limit
1.1
1
0.9
0.8
0.7
0.6
0.5
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
Recuperator Low Quality Limit

(b)


Recuperator High Quality Limit
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
Recuperator Low Quality Limit

(c)

Figure 16 Alpha shapes created for 40 percent (a), 60 percent (b), and 80 percent (c) of the maximum cooling load

By making the constraint on cooling load higher, the area of the shapes gets smaller. By themselves these are shapes are not very useful, they need to be combined on one plot in order to make a contour plot. The result is shown in Figure 17.

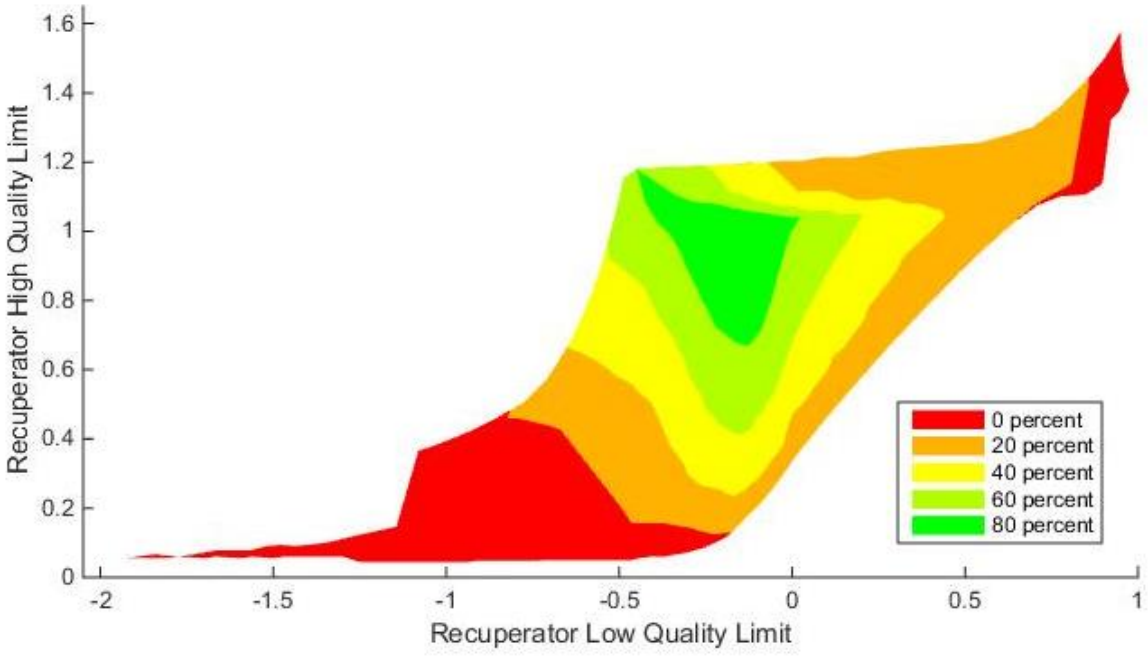


Figure 17 Contour plot created with alpha shape for representative parametric study

Figure 17 shows the result when the alpha shapes for the different cooling load constraints are overlaid on each other. Figure 17 does not represent quality constraints that are impossible to operate at, in contrast to Figure 13 where areas indicated with the red oval were impossible to operate at. Additionally, because the contours are obtained by taking a horizontal slice of the point cloud, the contour plot represents the upper surface of the data. This means that the maximum cooling loads for a given quality constraint are represented by the plot.

The contours made using this method are dependent on the parameters used to create the alpha shape. In the extreme case, it can result in plots that are no better than those initially created using the curve fit function. The alpha shapes use a variable called the alpha radius to control the concavity of the shapes. A large alpha radius will result in areas included that are not actually in the domain as seen in Figure 18.

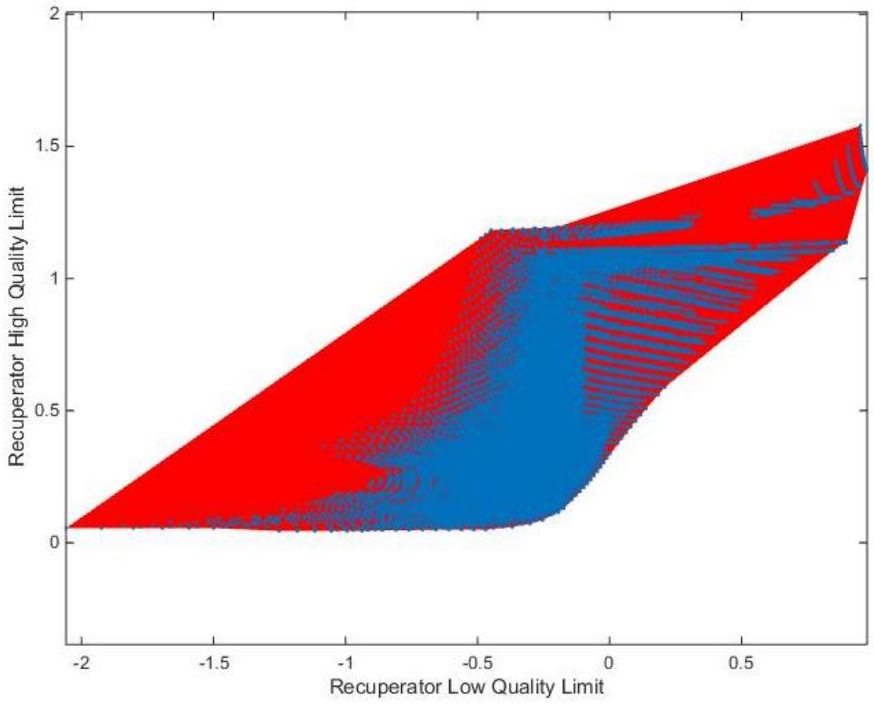


Figure 18 Alpha shape created from representative parametric study with an alpha radius much greater than the critical alpha radius.

A smaller alpha radius is needed for the alpha shape to fit the data better, however, if the alpha radius is reduced too much, the shape can exhibit too much concavity as seen in Figure 19.

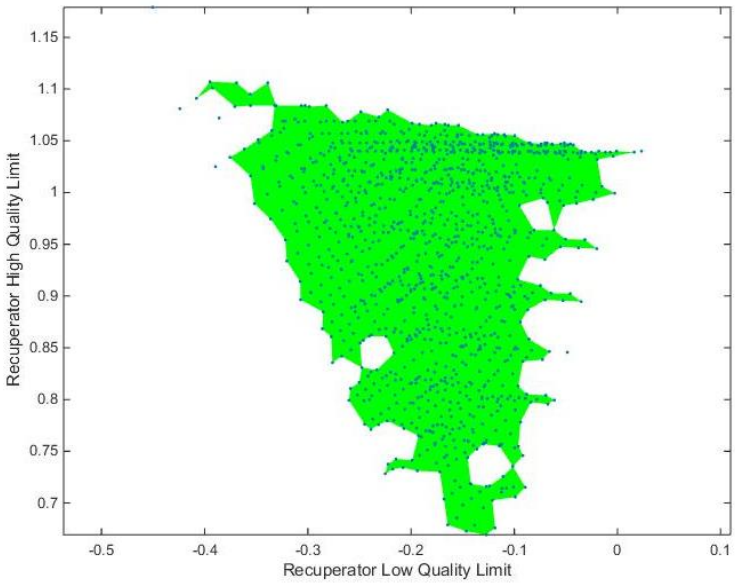


Figure 19 Alpha shape created from representative parametric study with an alpha radius below the critical alpha radius. All point with cooling load above 60 percent of the maximum.

For every alpha shape there is a value called the critical alpha radius. This the smallest value of alpha that encloses every point; using an alpha radius smaller than the critical alpha radius results in the excessive concavity shown in Figure 19. This is the absolute smallest value that will work for the alpha radius for graphing. However this value often causes too much concavity, so it is best to increase the alpha radius by a factor of about five when plotting the shapes to ensure that the data are accurately represented. For every parametric study graphed, the alpha radius was adjusted by a factor of five to best represent the data.

The Matlab code for this plotting technique is given in Appendix H.

3.2 Interpolating Data Points

Since the goal of the parametric study is to study at the interaction of cooling load and quality, the only real points of interest on the contour plot are those that lie on the upper

surface of the point cloud. As a result, it is important to identify the mixture/discharge pressure that provides the maximum cooling load for a given quality constraint which lies on the top surface, and not a mixture/discharge pressure that operates at the same quality constraint but with a lower cooling load. There needs to be a way to determine what the best mixture is when the quality in the recuperator is constrained. A method of determining which points lie of the top surface then interpolating between those points was used to determine the optimal mixture for a given quality constraint. The first step uses alpha shapes much like the plotting method, however in this case they were used in three dimensions to create a 3D surface of all the data points. Figure 20 shows the alpha shape created using this method.

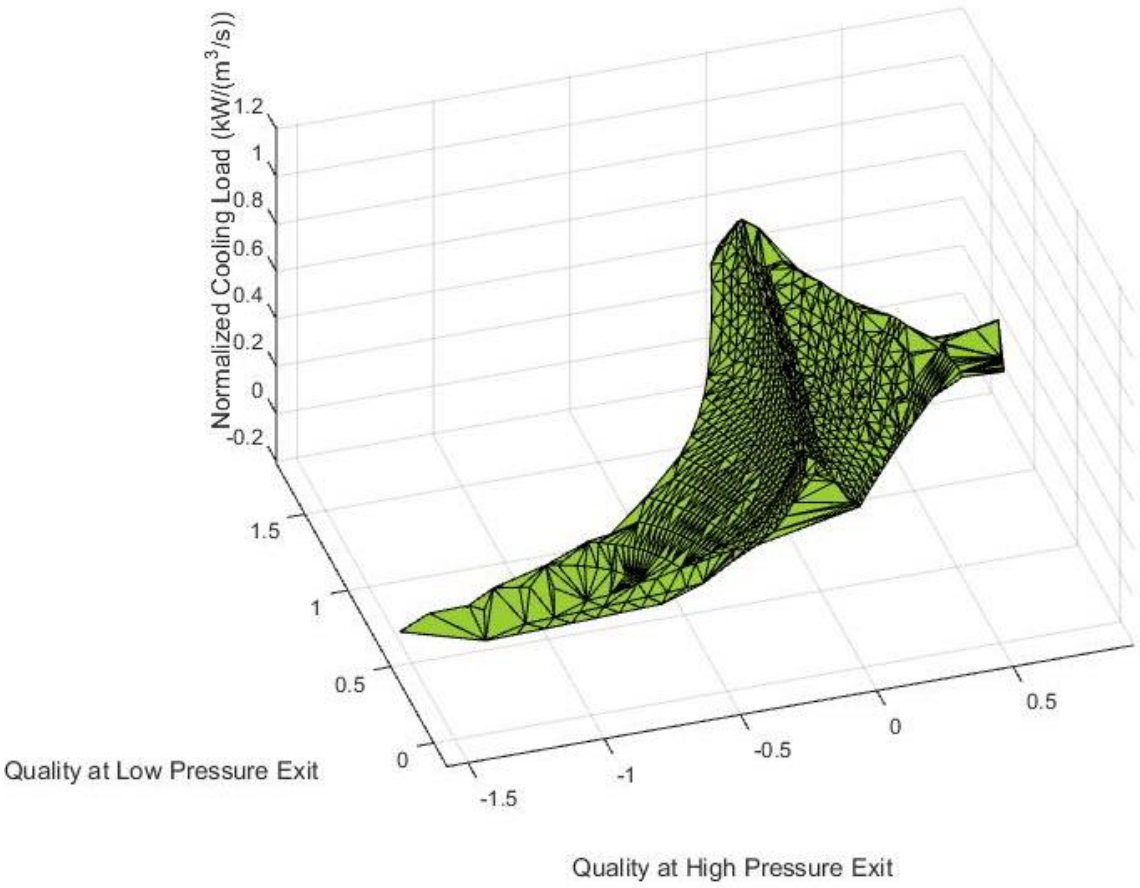


Figure 20 Alpha Shape surface for point cloud of cooling loads

The alpha shape divides the shape into points on the surface, and those beneath the surface. For this analysis the only points that matter are those that lie in the two phase region, or where the quality is between zero and one. Points outside this region do not experience the same enhanced heat transfer. The points of interest can be seen in Figure 21.

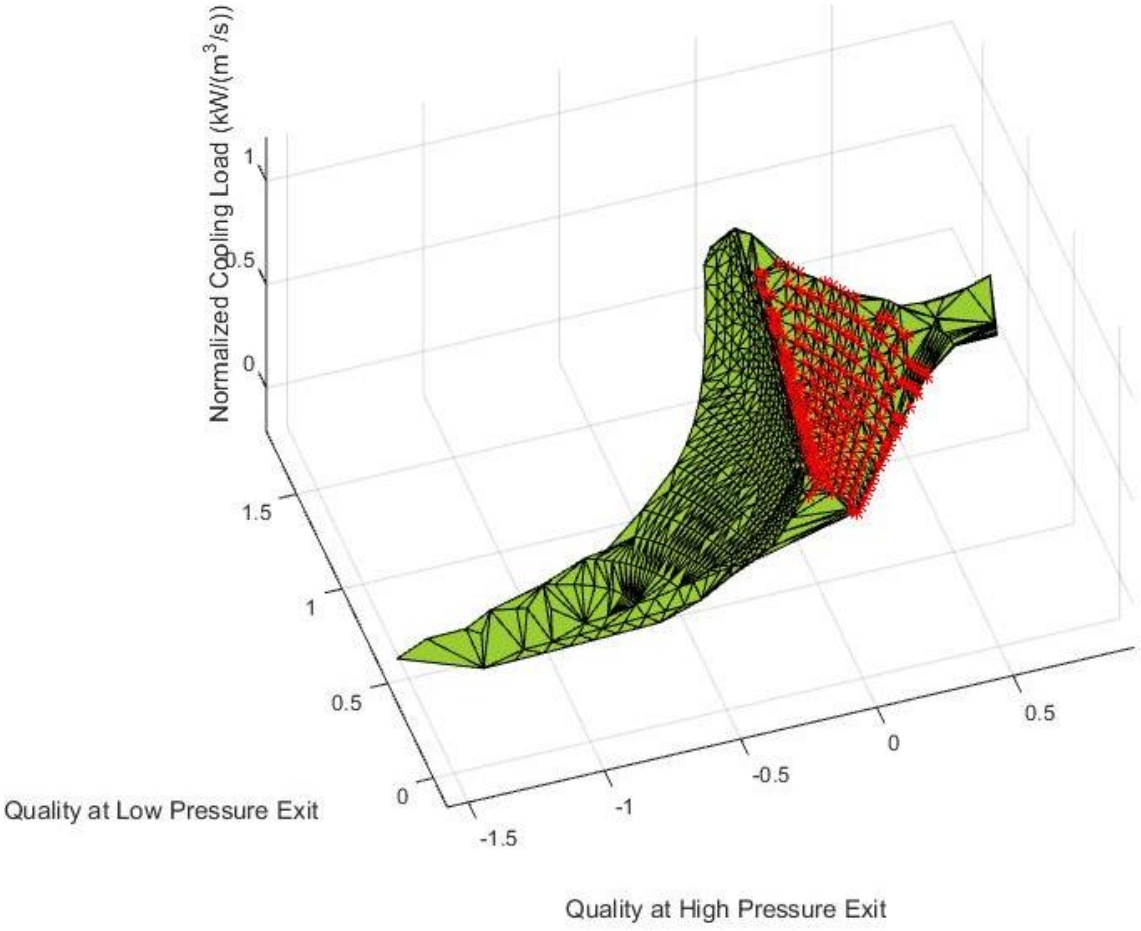


Figure 21 Alpha shape of surface with interpolation points identified in red

Much like the concavity in the alpha shapes when drawing shapes for plotting, the alpha radius has an impact on the surface created by the alpha shape. The surface represented by the red points in Figure 21 is slightly concave, meaning that a lower value of alpha radius will enclose more points while a larger value will enclose fewer as seen in Figure 22.

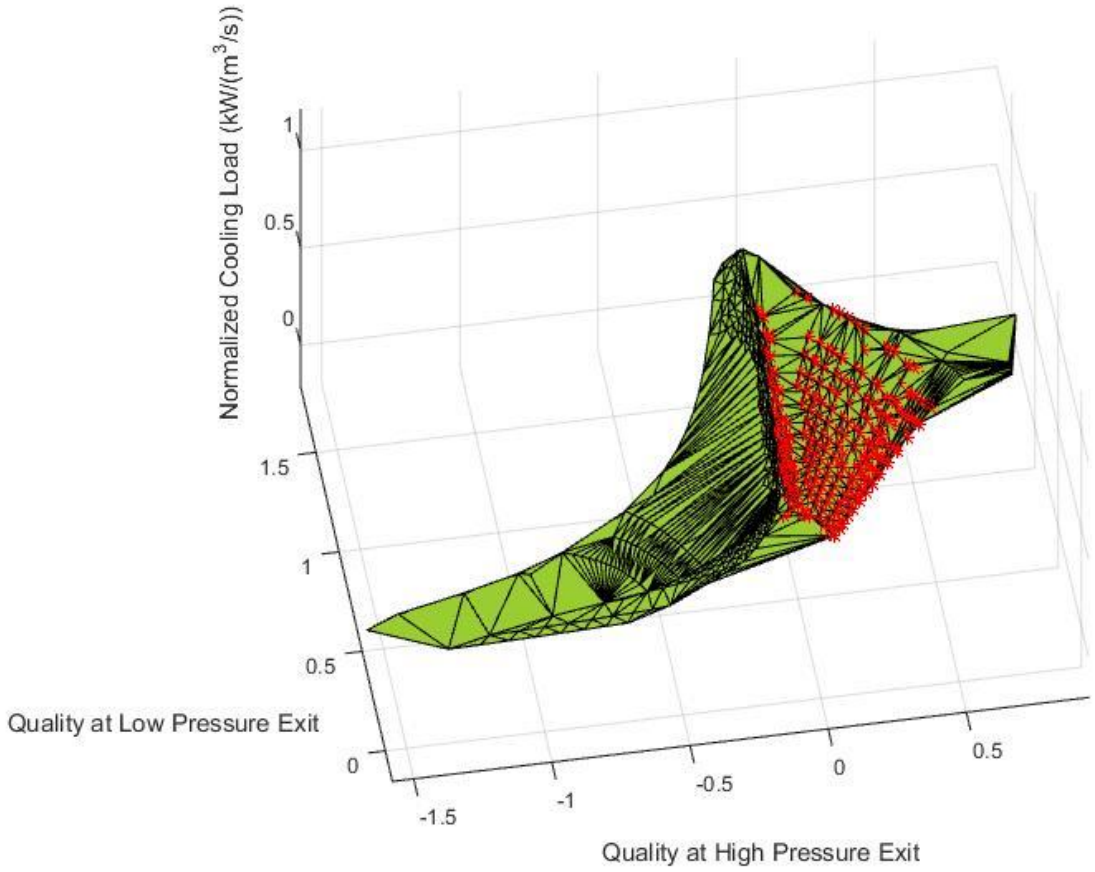


Figure 22 Alpha shape surface with points of interest in red. Higher alpha value.

The alpha radius was chosen so that all the points chosen are on the top surface of the shape. It was found that if the alpha radius was large enough, the bottom side of the point cloud would not contribute any points to the shape. Through experimentation, this alpha radius was determined to be 0.6 for most cases. Once the alpha radius is set, the points on the surface are found and can be used for interpolation.

Using the points on the top surface, it is a possible to interpolate the cooling load, mixture composition, and discharge pressure. Matlab's built in Grid Data function is used interpolate the values of interest for given values of quality. This function takes in the data

points on the top surface found using alpha shapes and a location and returns the interpolated result. To capture the curvature of the data in the region of interest, the v4 interpolation method was used. This method is based on work done by [17] and uses a biharmonic spline interpolation. This method is the same as the method ship builders used to draw complex lines. A flexible ruler or spline is placed on the paper and weights attached at certain points, by adjusting the weights, the spline can be fitted to the points. The same concept is used in the Matlab function, which can calculate the results for interpolated values in two dimensions. This method captures the curvature of the surface to accurately determine the cooling load, mixture composition, and discharge pressure.

The Matlab code for interpolating the results can be found in Appendix I.

3.3 Error Analysis

Since this parametric study is trying to create a surface, it is important to determine the minimum number of runs necessary to ensure accurate the representation of the surface. The two things that can affect the accuracy of the parametric study are the divisions between each mole fraction and the division between each discharge pressure. A parametric study was conducted to determine what divisions of mole fraction and discharge pressure are required for an accurate result. The division for the mole fractions tested were 0.05, 0.025, 0.02, and 0.015. The divisions for the discharge pressures tested were 300 kPa, 200 kPa, 150 kPa, 100 kPa, and 50 kPa. The cycle modeled was a room temperature (i.e., no precooling) cycle operating with a mixture of methane, propane, and pentane. The cooling load is interpolated at three points of interest, when quality is constrained within 0 and 100 percent, 10 and 90 percent, and 15 and 85 percent, using the procedure outlined above. The maximum percent

error in cooling load relative to the 50 kPa, 0.015 mole fraction division case is shown in Figure 23.

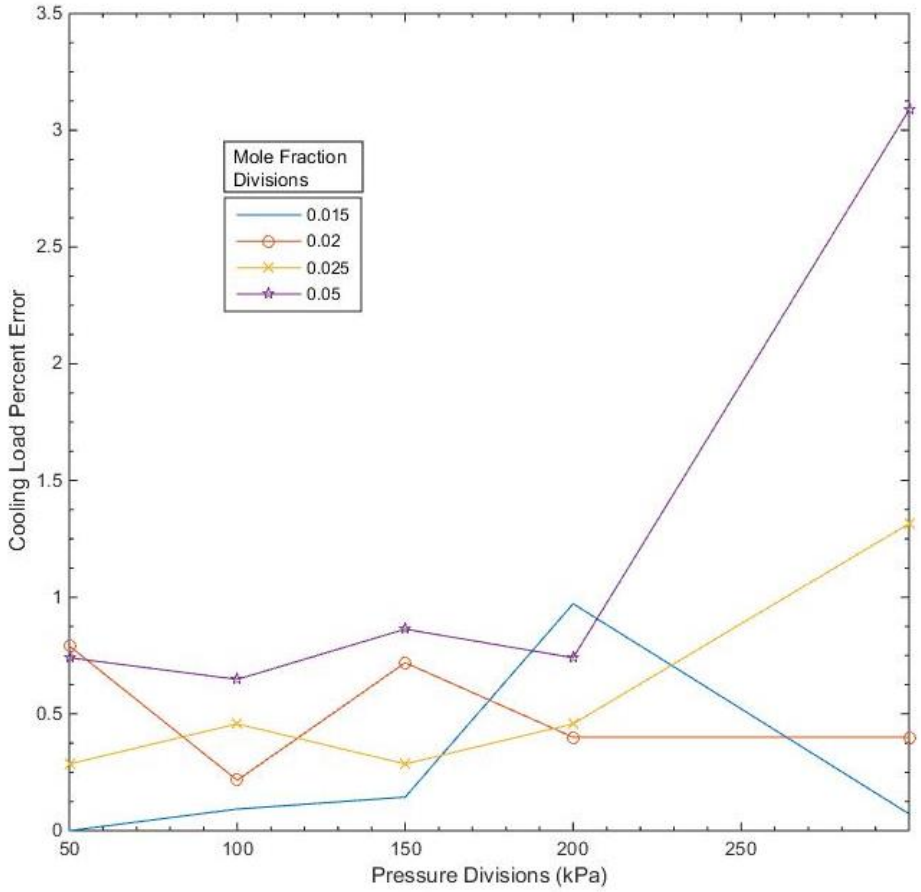


Figure 23 Percent Error for cooling load vs the pressure division and mole fraction divisions

Since this is a numerical method, the results do not exhibit a uniform response to increasing the number of points, however the general trend is that as the divisions get smaller the accuracy increases. For most of the cases the error in the interpolated cooling load is below one percent, which was deemed to be sufficiently accurate.

The interpolation of the mole fractions and the discharge pressure can also be affected by the discharge pressure and mole fraction divisions so maximum error for mole fraction

one, mole fraction two, and the discharge pressure are plotted on the same axes as Figure 23, using the same cycle.

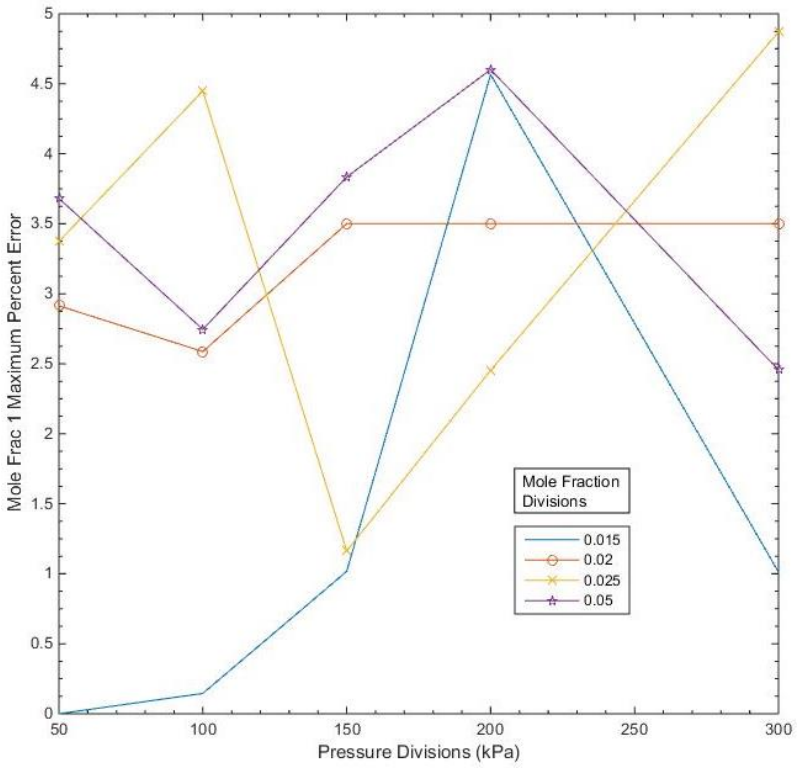


Figure 24 Maximum percent error in mole fraction one vs pressure divisions and mole fraction divisions

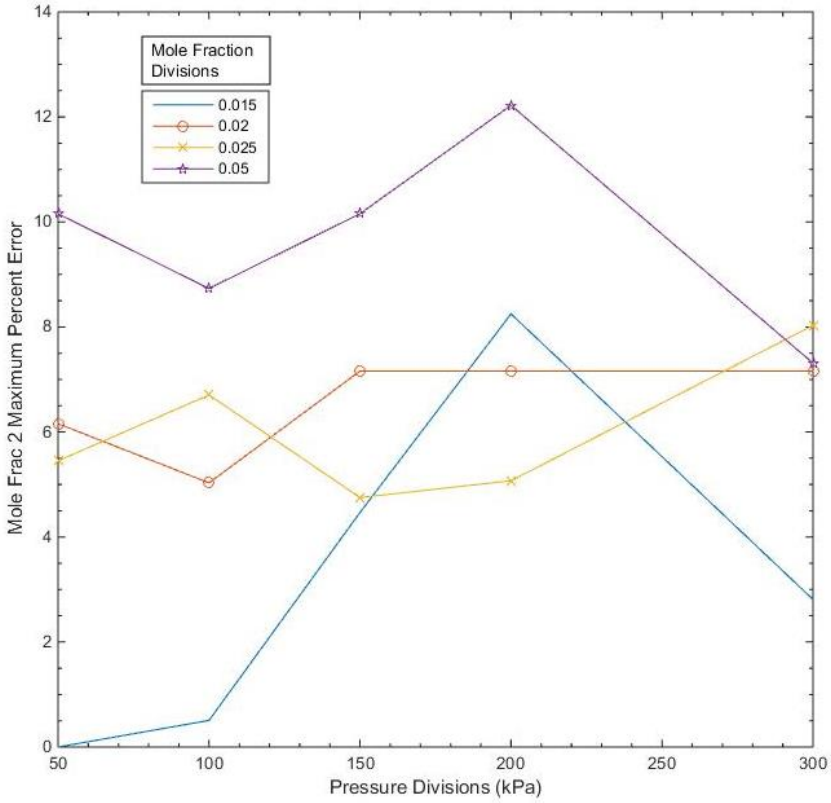


Figure 25 Maximum percent error in mole fraction two vs pressure divisions and mole fraction divisions

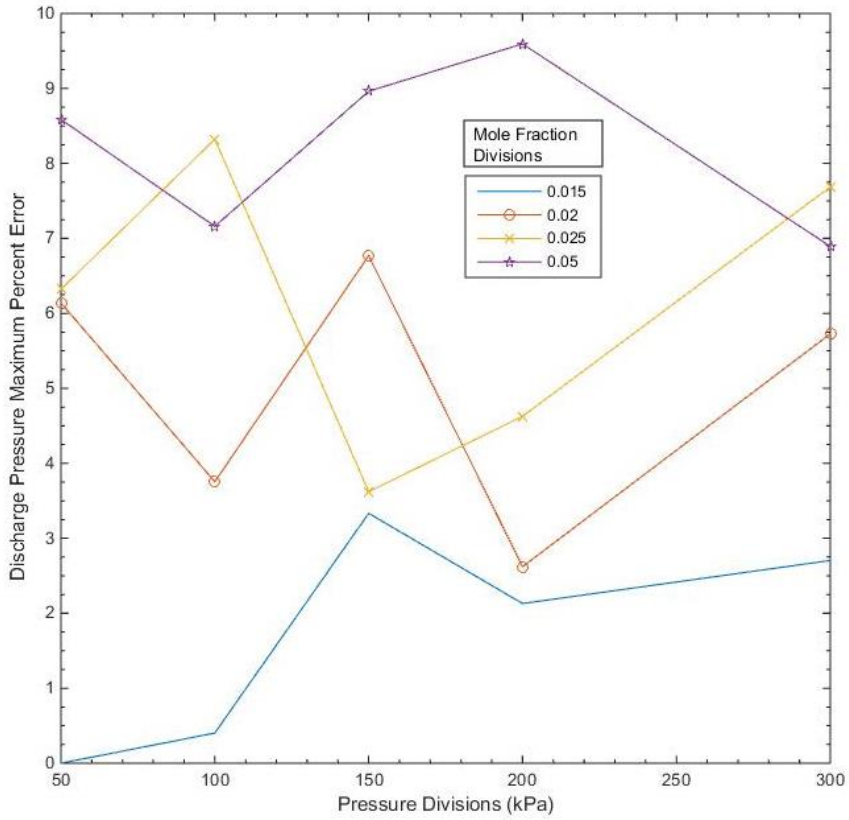


Figure 26 Maximum percent error in discharge pressure vs pressure divisions and mole fraction divisions

It is clear that from Figure 24, Figure 25, and Figure 26 that there is much more error in the composition and discharge pressure interpolation than in the cooling load interpolation. To minimize this error while also having a model that can be run in a reasonable amount of time, a mole fraction division of 0.02 and a pressure division of 100 kPa were used. These choices result in a maximum error of under three percent in mole fraction one, under six percent for mole fraction two, and under four percent for the discharge pressure. Since there was little to be gained by decreasing the mole fraction and pressure divisions, this was determined to be sufficiently accurate. Additionally, it is practically difficult to control the mixture composition and discharge pressure to within better than about five percent.

4.0 Model Predictions

To show the usefulness of this plotting technique, a sample mixture of methane, ethane, and isobutane was modeled. Hydrocarbon mixtures were chosen as the basis for the parametric study because the accuracy at which REFPROP can determine mixture properties is better for hydrocarbon mixtures than it is for synthetic mixtures. The parametric studies were conducted using the input parameters described in Table 6.

Table 6 Operating parameters for precooled MRJT cycle.

Parameter	Value
T_1	240 [K]
T_4	150 [K]
ΔT_{app}	0 [K]
$P_{suction}$	150 [kPa]

A precooled cycle was chosen due to the increased cooling load of a precooled cycle over a room temperature cycle. Also, a precooled cycle prevents the need for high boiling components to keep the mixture two phase in the recuperator, which makes experimental verification of the mixtures easier. High boiling components have a low saturation pressure at room temperature, and if the saturation pressure is below the required pressure for the mixture, the compressors need to be turned on and the mixture fed into the suction side of the compressor to get the proper filling pressure. This makes filling difficult as it is almost impossible to tell how much of the refrigerant has been added when feeding into a running cycle. Lower boiling point refrigerants can be used with a precooled cycle, and have a high

saturation pressure meaning the mixtures can be created more precisely using the partial pressure of each component without the need to run the compressors.

As mentioned above, the model does not explicitly model the precooling cycle, but rather assumes that the precooling cycle is sufficiently sized to reduce the temperature at state one to the specified supply temperature. The approach temperature difference for these trials have been set to 0 Kelvin which corresponds to an infinitely large heat exchanger. This means that the cooling loads calculated are the thermodynamic optimal cooling loads, i.e. the best possible cooling load that a given mixture can achieve at the given operating parameters. A mixture operating outside of the enhanced heat transfer region will operate with a large approach temperature difference and the performance will be greatly reduced from the thermodynamic cooling load predicted by the model. This means that the heat transfer coefficients have a large effect on the actual cooling load the mixture can provide. The plotting and interpolation method described above is used to create the plot in Figure 27.

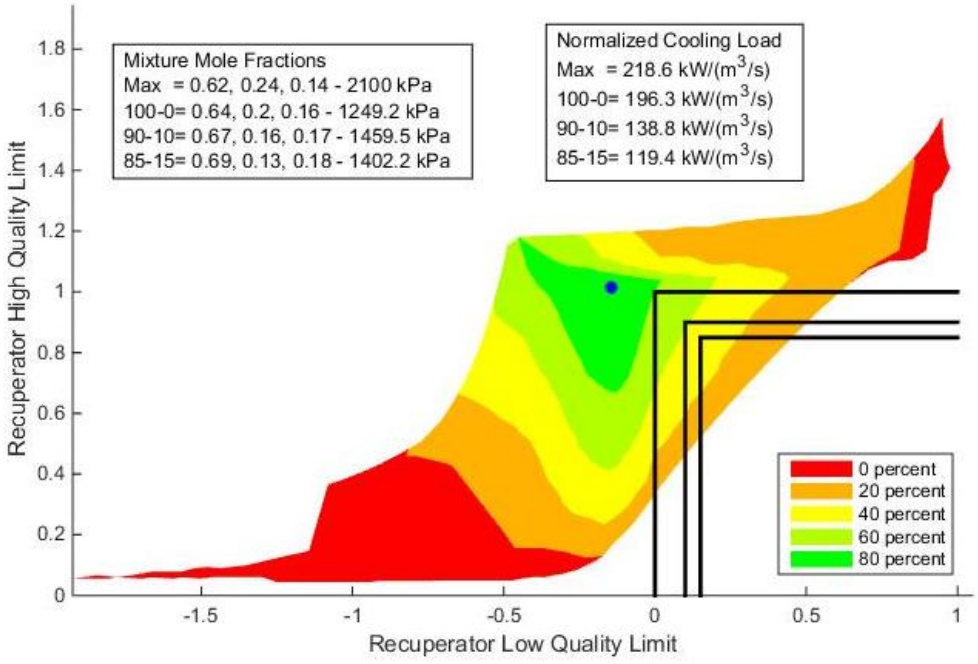


Figure 27 Cycle map of precooled cycle operating with parameters listed in Table 6. Mixture is methane, ethane, and isobutane.

The maximum thermodynamic cooling load is illustrated with a blue dot on the plot. It occurs when the mixture exits slightly sub-cooled at state 2, and slightly super-heated at state 5. However this is outside of the constraint that was set earlier that the quality in the recuperator must be between 15 percent and 85 percent in order to remain in the enhanced heat transfer region. This constraint is represented on the graph by the black lines in the lower right hand corner. Additional lines are added for the MR constrained to be within 10 and 90 percent quality, and between 0 and 100 percent quality. The cooling load at each of these points is interpolated from the data using the method described above, and listed in a box in the upper right corner of the plot. The mixture and discharge pressure that correspond to that cooling power is also interpolated from the data and listed in the box in the upper left corner of the plot.

The point with the highest thermodynamic cooling load in the 15-85 percent quality constrained region results in a cycle that operates with approximately half of the thermodynamically possible cooling load as the maximum point. If this constraint is relaxed to keep the MR between 10 and 90 percent quality then the thermodynamic cooling load is increased to about 60 percent of the maximum. Likewise, constraining the quality to only be between 0 and 100 percent results in a cooling load of 80 percent of the maximum. Figure 28 was created to visually show this behavior.

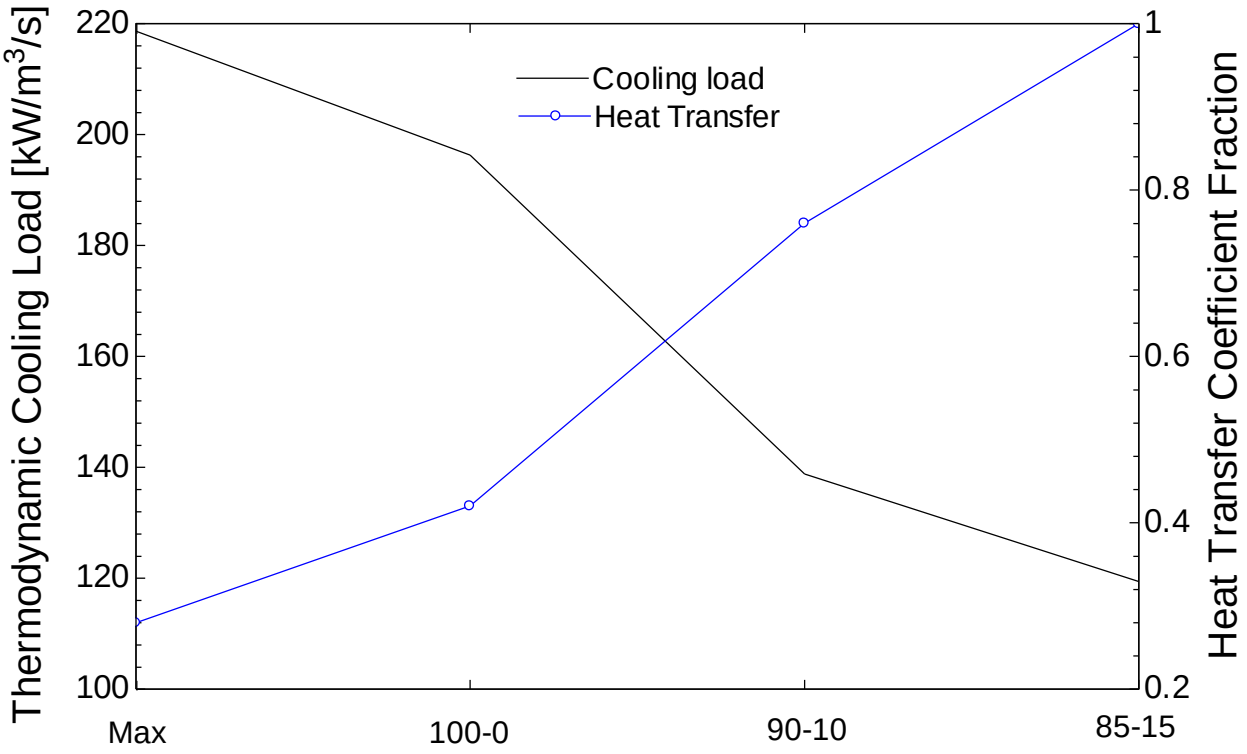


Figure 28 Effect of quality constraint on thermodynamic cooling load and heat transfer coefficient from [1].

Figure 28 shows the decrease in thermodynamic cooling load as a result of constraining the MR to be further in the vapor dome. The thermodynamic cooling load plotted is the maximum possible cooling load for a given quality constraint. The right axis

shows the heat transfer coefficient with respect to the quality constraint. Since the thermodynamic cooling load and heat transfer coefficient have opposite effects on the actual cooling load, i.e. increasing the quality constraint results in lower thermodynamic cooling load but high heat transfer coefficients, there must be a quality constraint that provides the maximum possible cooling load. Without a model that considers heat transfer and pressure drop it is impossible to find this exact optimal mixture. However, this simplified model can be used to determine a range of mixtures and discharge pressures that should operate well in the actual cycle. These mixtures must lie somewhere along the design line, which is a line that identifies the best mixtures for the range of quality constraints.

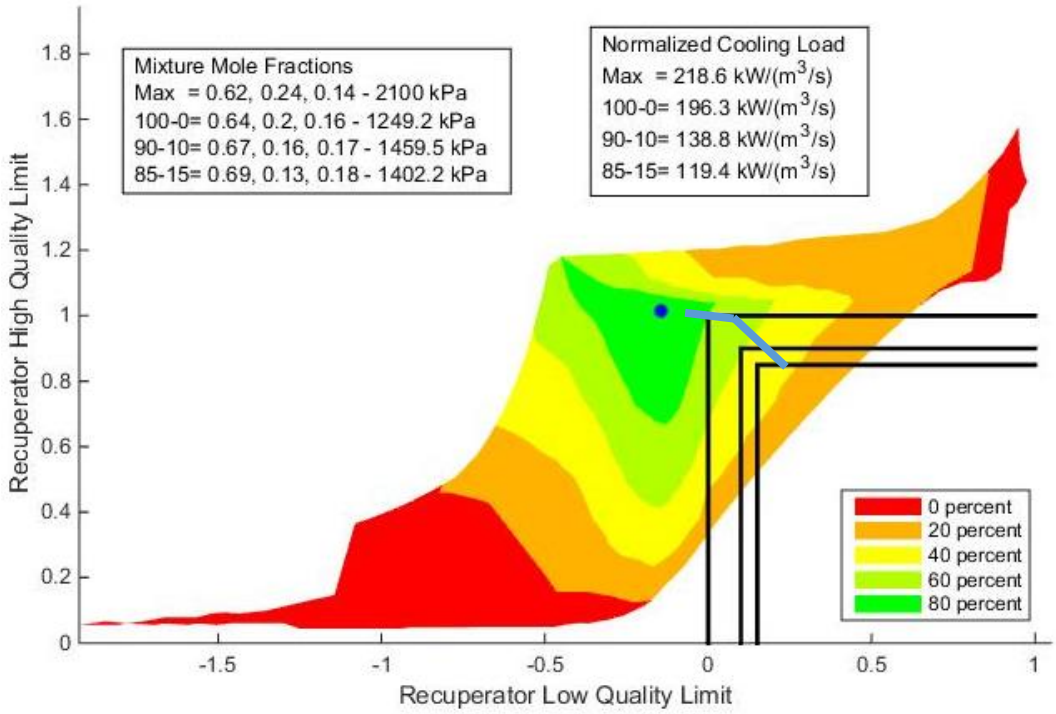


Figure 29 Cycle map with design line of precooled cycle operating with parameters listed in Table 6. Mixture is methane, ethane, and isobutane.

Figure 29 shows the same cycle map as above with the addition of the design line. It is along this line the best actual mixture must lie. The design line consists of the points that have the highest thermodynamic cooling load for a given constraint on quality. While it is impossible to perfectly match all the parameters from the cycle to the map, the points along this line should provide a good starting point for experimental testing.

This shape allows an educated guess at what the best MR will be, however, it is unknown if the surface created by this plotting technique is similar for all MRJT cycles. Therefore, to verify that this design method can be used for any MRJT cycle, additional parametric studies were conducted with different operating conditions.

4.1 Number of Components

The first test conducted was looking at the number of components used. The same conditions as the previous parametric study were used, as listed in Table 6. The mixture components chosen are listed in Table 7.

Table 7 Components of different hydrocarbon mixtures for parameters set according to Table 6

Number of Components		
3	4	5
Methane	Methane	Methane
Propane	Ethane	Ethane
Isobutane	Propane	Propane
	Isobutane	Isobutane
		Pentane

Choosing the additional components of the mixture is important to making sure the mixture can meet the constraint of being two phase in the recuperator. The fourth component is propane which has a normal boiling point between methane and isobutane. Since there are

not many hydrocarbon refrigerants with a normal boing point between the temperature span, pentane is added as the fifth component. This keeps the span of normal boiling points close to the operating temperatures in the recuperator to ensure two phase flow.

Adding an additional component also adds an additional degree of freedom to the parametric study. Thus the mole fraction divisions for the four and five component mixtures were reduced in order to decrease the number of runs needed. Even with this reduction, the amount of time required for the four and five component mixtures is significantly more than for the three component mixture. The mole fraction and pressure divisions used are listed in Table 8.

Table 8 Mole Fraction divisions, pressure divisions, and number of runs required for four and five component mixtures

	Number of Components		
	3	4	5
Mole Fraction Divisions	0.02	0.04	0.06
Pressure Divisions [kPa]	100	100	100
Number of Runs	31824	70200	116280

Since the 4 component mixture also includes all the mixtures that the three component mixture did, the results from the three component mixture study were included in the four component graph. Likewise, the results from the four component mixture study were included in the five component mixture graph. The result for the four component mixture is shown in Figure 30, while the results for the five component mixtures are shown in Figure 31.

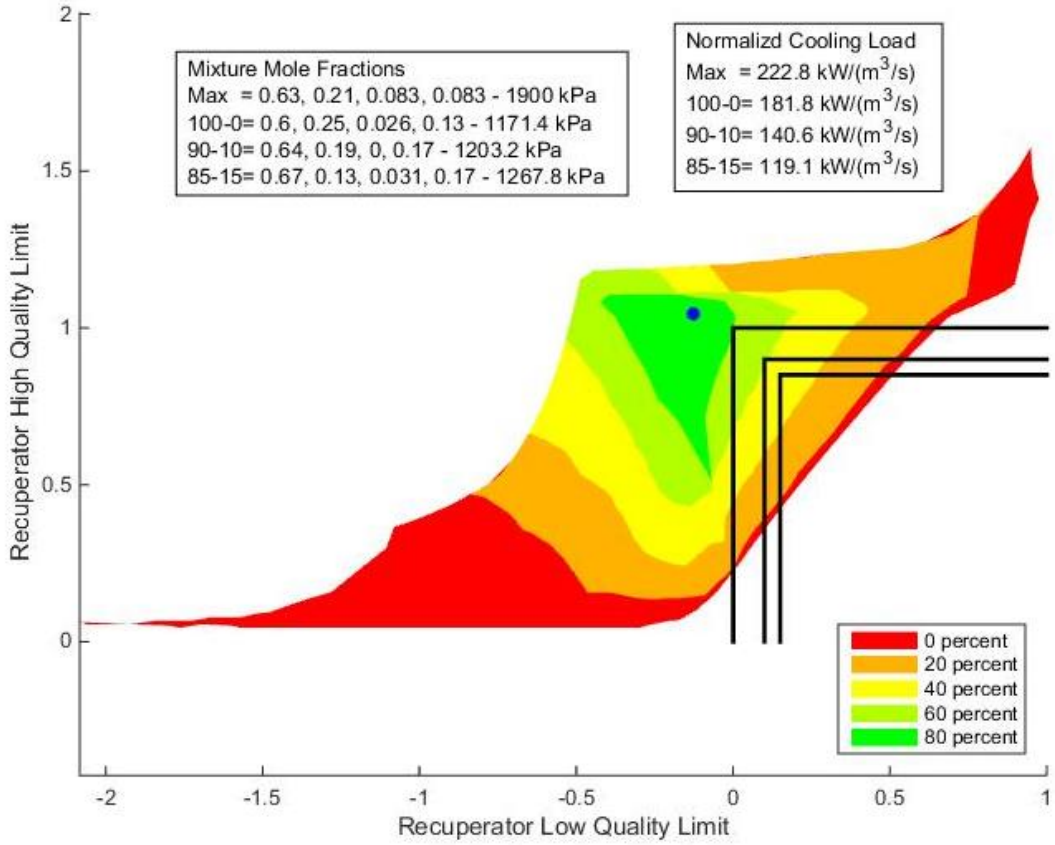


Figure 30 Cycle map of precooled cycle operating with parameters listed in Table 6. Mixture is methane, ethane, propane, and isobutane.

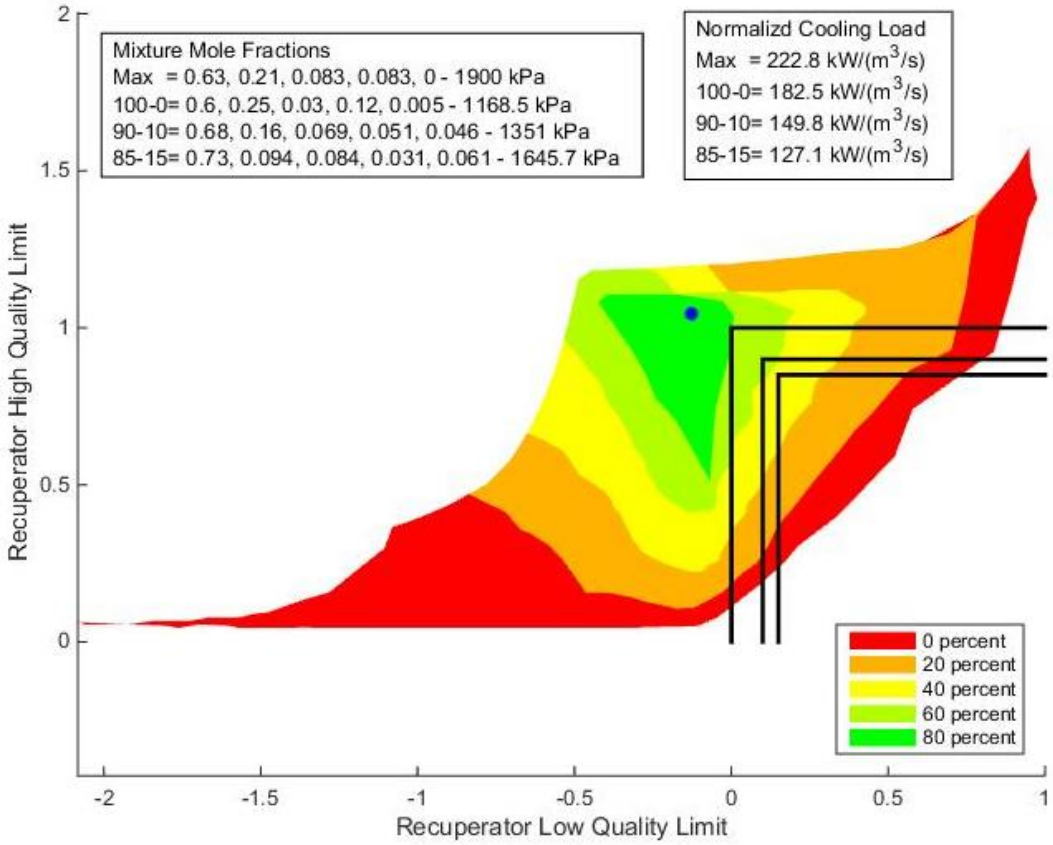


Figure 31 Cycle map of precooled cycle operating with parameters listed in Table 6. Mixture is methane, ethane, propane, isobutane, and pentane.

It is clear that the same general shape is retained when adding more components. Adding more components also expands the map further into the vapor dome, allowing the cycle to operate with a higher quality constraint. However, by examining the thermodynamic cooling load at the constraint points, there is little increase in performance when more components are added. A graph showing the maximum cooling load at each of the constrained points vs the number of components illustrates this conclusion.

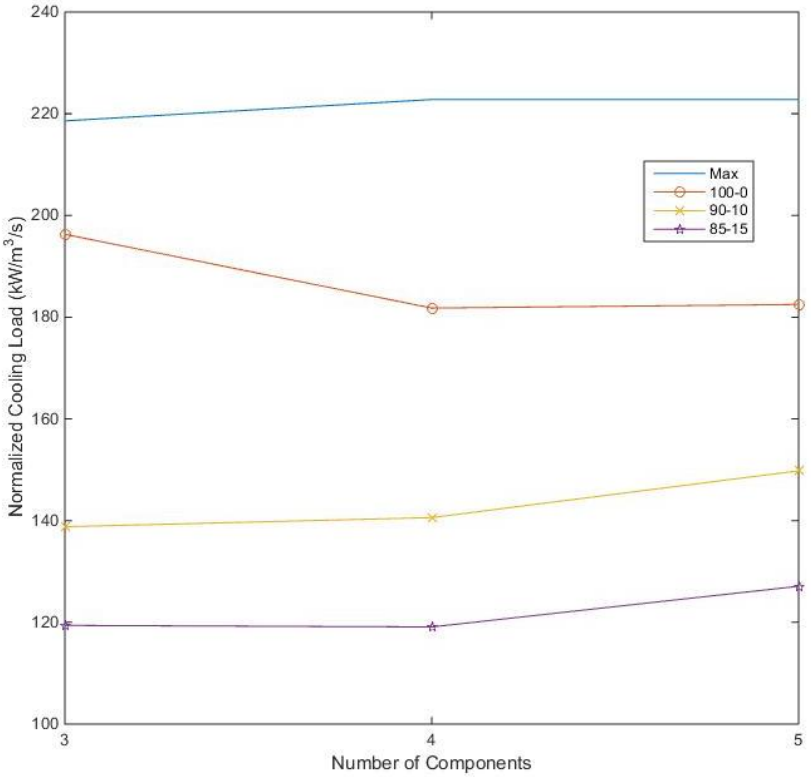


Figure 32 Maximum constrained cooling load Vs number of components for precooled cycle

Figure 32 clearly shows that increasing to four and five components does very little for the thermodynamic maximum cooling load of the cycle, with the thermodynamic maximum cooling load being the same for both the four and five component mixture, and only slightly increased over the three component case. The thermodynamic cooling load actually decreases when the mixture is constrained to be within 0 and 100 percent quality. This behavior is due to the interpolation technique, since the plot is concave down at that point, adding more points will cause the accuracy to increase slightly and results in a slightly lower thermodynamic cooling load. However, there is an increase in the thermodynamic

cooling load from four components to five components of 6.5 percent when constrained to be within 10 and 90 percent and 6.7 percent when constrained between 15 and 85 percent.

For this specific case, it appears that increasing the number of components to four or five is not necessary, as the gain in cooling load is small. Additionally, as more components are added, it becomes more difficult to match the mixture the cycle is operating with to the one specified by the model, because of selective absorption into the compressor oil, selective condensation of components at low temperature, and the difficulty of properly mixing a four or five component mixture.

However, in other cases four or five components may be required. This model can be used to determine how many component are required to provide a high cooling load. Multiple parametric studies can be done looking at the effects of adding components and using different combinations of components. If a four or five component mixture does not provide a dramatic increase in cooling load over three components then a three component mixture is sufficient. Additionally, if adding a certain component mixture dramatically increases the cooling load then an additional parametric study can be run to determine if that additional component can replace one of the components already in the mixture. The best option would be to replace the component with either the lowest mole fraction in the constrained cases, or the component that has a normal boiling point nearest the component to be added. To illustrate this method a mixture of methane, ethane, and propane was used to conduct a parametric study with the same parameters listed in Table 6. Figure 33 illustrates the cycle map that is created.

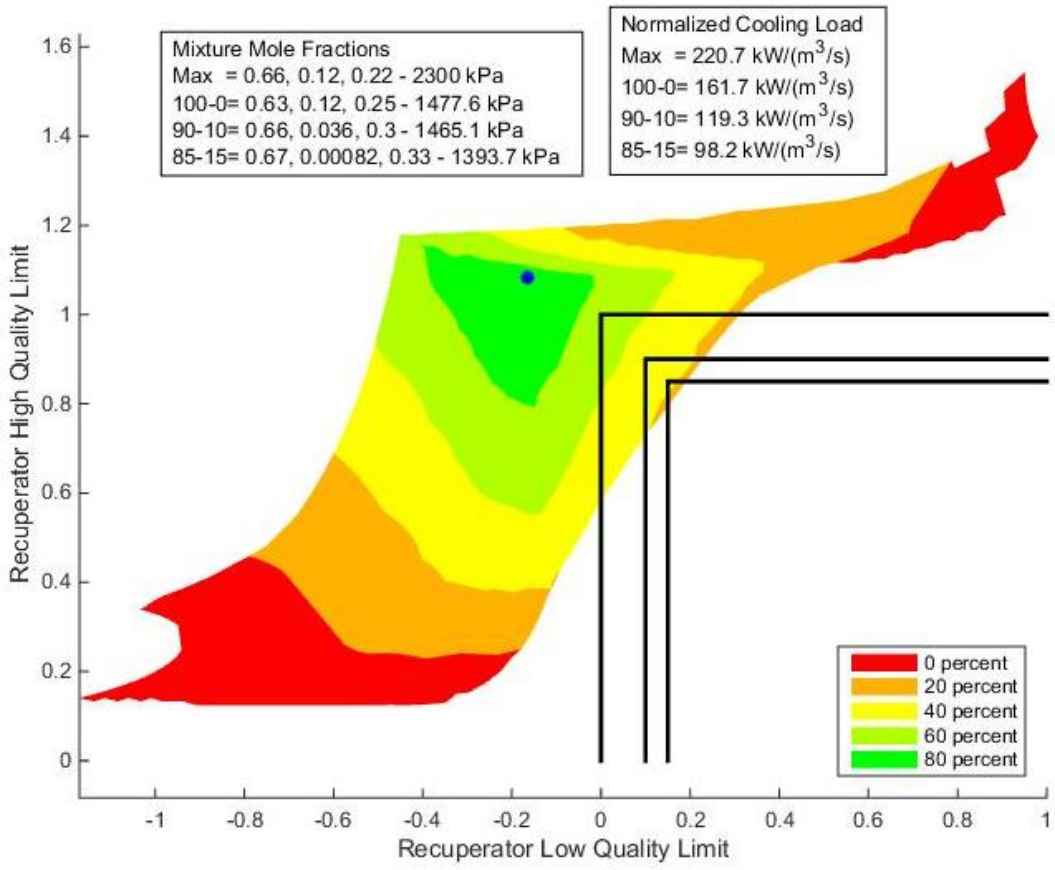


Figure 33 Cycle map of precooled cycle operating with parameters listed in Table 6. Mixture is methane, ethane, and propane.

Figure 33 shows that while the maximum cooling load is approximately the same as Figure 27 where the mixture was methane, ethane, and isobutane, there is a dramatic decrease when the mixture is constrained to be within the vapor dome. The next step in optimizing the mixture would be to run a model with four components. In this case isobutane is added, and the results can be seen in Figure 30. Here a large increase in cooling load in the vapor dome is realized by adding isobutane. Since the cycle map improved with the addition of isobutane, a test needs to be done to determine if the propane can be replaced with isobutane since it has the normal boiling point nearest isobutane. The result of this study is the initial result shown

in Figure 27, which show that replacing propane with isobutane results in better performance without the need for 4 components. This method can quickly determine not only the proper components to have in the mixture but also how many components are required for a given cycle. As a result, time can be saved during experimental testing knowing the minimum possible number of components are being used which reduces the degrees of freedom in the optimization process.

4.2 Suction Pressure

Another important parameter that can change for a MRJT cycle is the suction pressure. The suction pressure of a MRJT corresponds to both the filling pressure and orifice size used. A higher filling pressure will result in a higher suction pressure, while a larger orifice will also result in a high suction pressure and more mass flow. The suction pressure of 150 kPa was chosen as it is close to the value that the experimental test facility was operated at. The suction pressure was reduced to 100 kPa as shown in Table 9 to prove the plotting technique can be applied regardless of suction pressure.

Table 9 Operating parameters for precooled MRJT cycle with a lower suction pressure.

Parameter	Value
T_1	240 [K]
T_4	150 [K]
ΔT_{app}	0 [K]
$P_{suction}$	100 [kPa]

To save computation time, and since it has already been proved that mixtures with more components behave the same, a three component mixture was used. Since the temperature span is again 240 K to 150 K, the same mixture of methane, ethane, and isobutane was used.

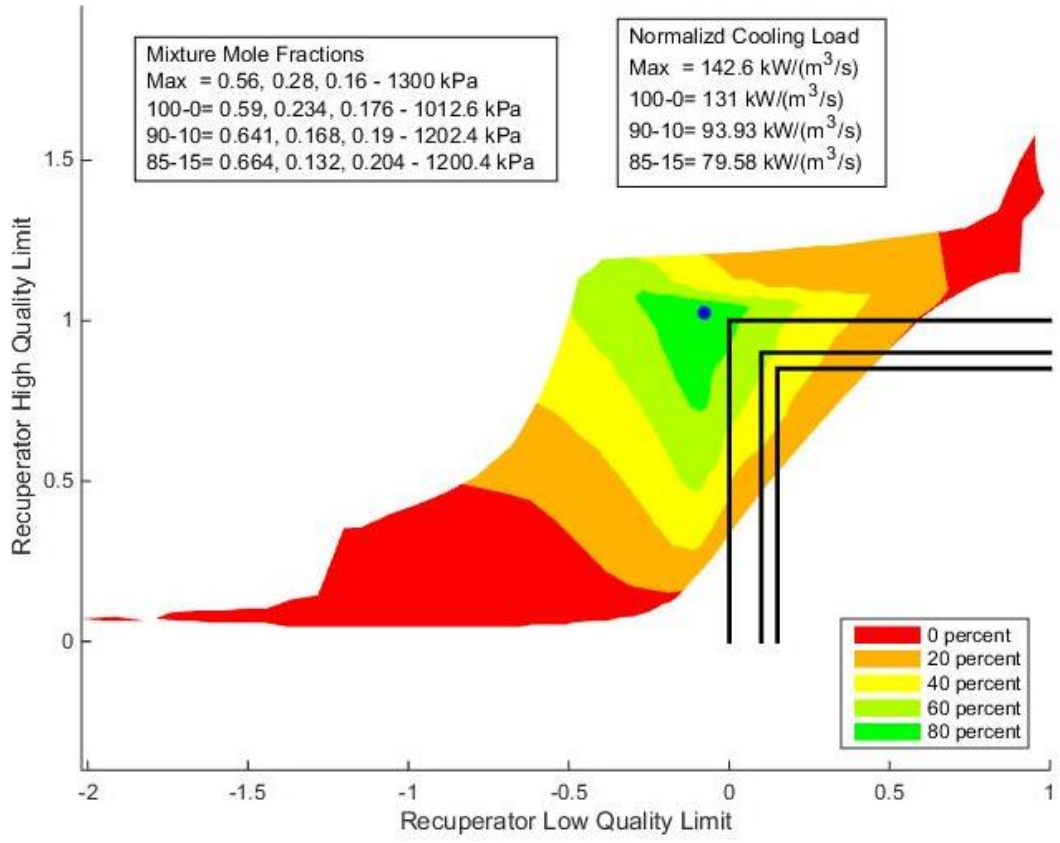


Figure 34 Cycle map of precooled cycle operating with low suction pressure, parameters listed in Table 9. Mixture is methane, ethane, and isobutane.

It is clear from Figure 34 that the same general shape is applicable to a cycle operating at a lower suction pressure. The most apparent difference between the low and high suction pressure results are the cooling load at each of the constrained points. In this case, the reduction of suction pressure results in an approximately 33 percent reduction in cooling load

across all of the constrained points. It should also be noted that the optimal discharge pressure is also drastically reduced. However, the pressure ratio is approximately the same in both cases. This means that the absolute pressure drop has the largest effect on cooling load, but the maximum cooling load occurs when the compressor operates with a pressure ratio around 12.

Because the absolute pressure difference is driving the cooling load of the cycle, and the cooling load is maximum when the pressure ratio is near 12, it is inferred that the cycle should be operated at the highest suction pressure possible, as this will result in a large absolute pressure difference and thus a high cooling load. But, there are limits to the pressures a reciprocating compressor can tolerate. The compressor used in the experimental test facility can only tolerate a maximum discharge pressure of 25 bar, or about 2500 kPa [18]. So increasing the suction pressure much above 150 kPa will result in an optimal pressure of greater than 2500 kPa and thus be unattainable by the compressor. So the suction pressure should in general be chosen to result in an optimal discharge pressure as close to the maximum discharge pressure as possible when the compressor has a pressure ratio of 12. This optimal value of 12 is dependent on the parameters used to model the compressor, and would change is the clearance volume fraction (C) was changed. This analysis should be run using each specific compressor parameters to determine which pressure ratio to aim for.

4.3 Operating Temperatures

A precooled cycle is a good way to achieve high cooling load at low temperature, but the precooling cycle doubles the complexity of the system. A JT cycle can also be operated as a single stage system. This greatly increases the simplicity of the cycle at the cost of

reducing cooling load at low temperatures. In order to prove that this model can also optimize a room temperature cycle, a cycle operating with the parameters in Table 10 was modeled.

Table 10 Operating parameters for room temperature MRJT cycle.

Parameter	Value
T_1	300 [K]
T_4	170 [K]
ΔT_{app}	0 [K]
$P_{suction}$	150 [kPa]

Since the operating range of temperatures in the recuperator is different, the mixture components also need to be changed to match the temperature range span. For this temperature range, the components used were methane, propane, and pentane. The propane and pentane have a higher normal boiling point than the ethane and isobutane so the mixture can remain two phase in the recuperator. The result of the parametric study are plotted using the same method, and the results are shown in Figure 35.

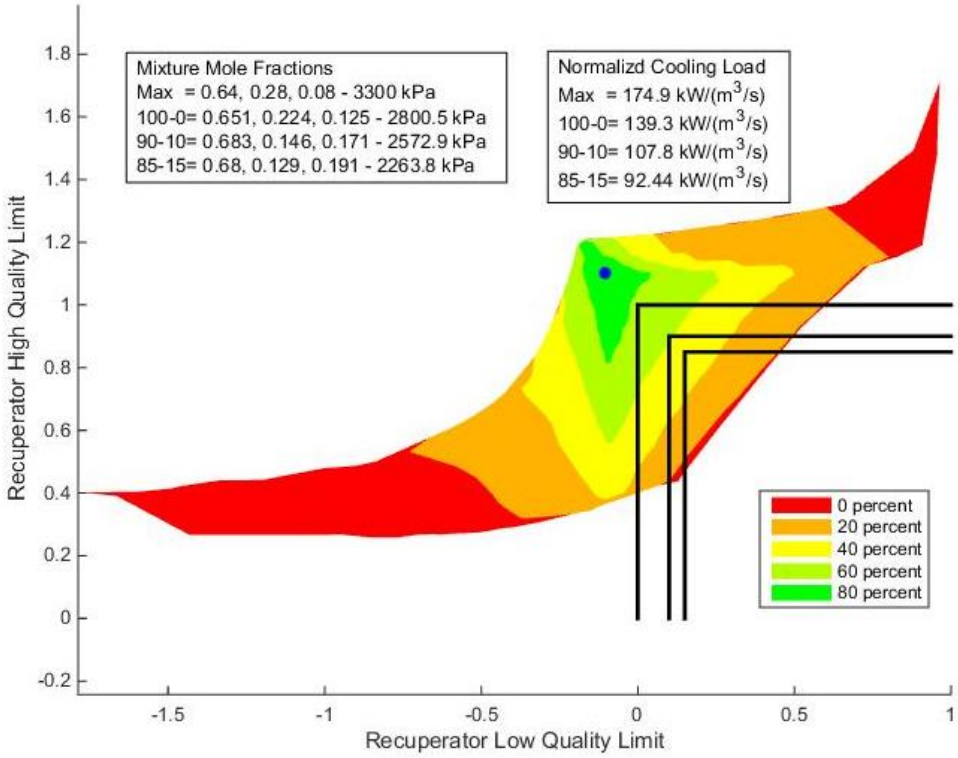


Figure 35 Cycle map of room temperature cycle operating with parameters listed in Table 10. Mixture is methane, propane, and pentane.

In this case, the map looks very similar to the precooled map. The obvious difference is the maximum cooling load is slightly more super-heated than the precooled cycle. Additionally, there is a dramatic increase in the optimal discharge pressure over the precooled cycle. For some of the constrained points, the pressure increases over the compressor maximum discharge pressure of 2500 kPa [18]. Reducing the pressure of the cycle to fall within in the operating range of the compressor would result in a decrease in the cooling load of the cycle. All the points that are attainable in a real world cycle are plotted in Figure 36.

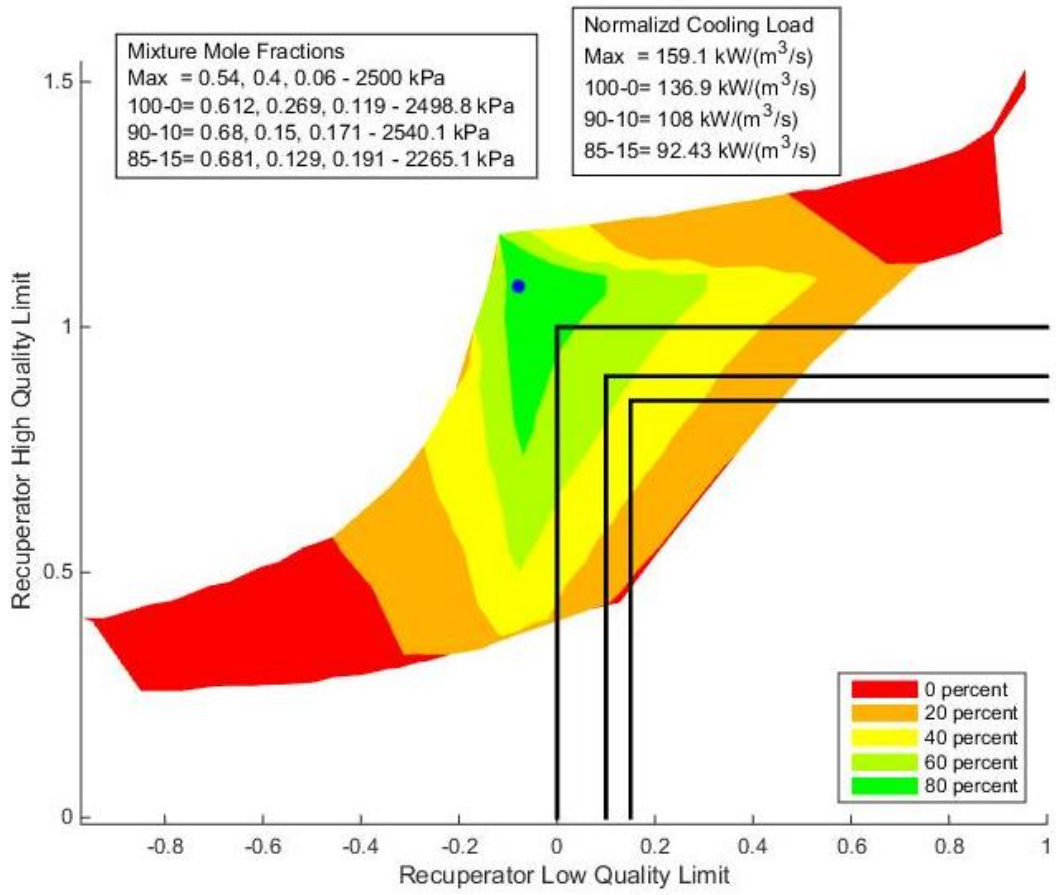


Figure 36 Cycle map of room temperature cycle with all data points under 2500 kPa discharge pressure, operating with parameters listed in Table 10. Mixture is methane, propane, and pentane.

Figure 36 shows that the same general shape is attainable, however, in this case the cooling load is greatly reduced. This shows the advantage to operating the cycle using a precooler, as lower temperatures and higher cooling loads are possible. Additionally since the same reduction in thermodynamic cooling load is seen with the constraint on quality, the same design line method can be used on room temperature cycles as well.

4.4 Synthetic Refrigerants

While there are some disadvantages to synthetic refrigerants in terms of ODP and GWP some also have the advantage of being chemically inert and non-flammable. In terms of safety this can be a significant advantage because regulators have not approved many hydrocarbons for use as a refrigerant [19]. The disadvantage to using synthetic refrigerants is REFPROP has more difficulty calculating the properties for synthetic refrigerants especially near the vapor dome. Because of this issue, only one parametric study with synthetic refrigerants was conducted. The mixture used was argon, R14, and R218. Argon has a normal boiling point of 87.3 K. R14's chemical formula is CF_4, and it has a normal boiling point of 145.1 K [11]. R218's chemical formula is C_3F_8, and has a normal boiling point of 236.36 K [11]. Mixtures of these components have been tested in REFPROP to ensure their properties can be accurately and reliably calculated for the required temperature span. Since the normal boiling point for all these components are low, a precooled cycle is modeled. The cycle operating parameters are shown in Table 11.

Table 11 Operating parameters for precooled MRJT cycle with synthetic refrigerants.

Parameter	Value
T_1	250 [K]
T_4	150 [K]
ΔT_{app}	0 [K]
$P_{suction}$	150 [kPa]

The parametric study is run and the results are plotted using the same plotting technique as above as seen in Figure 37.

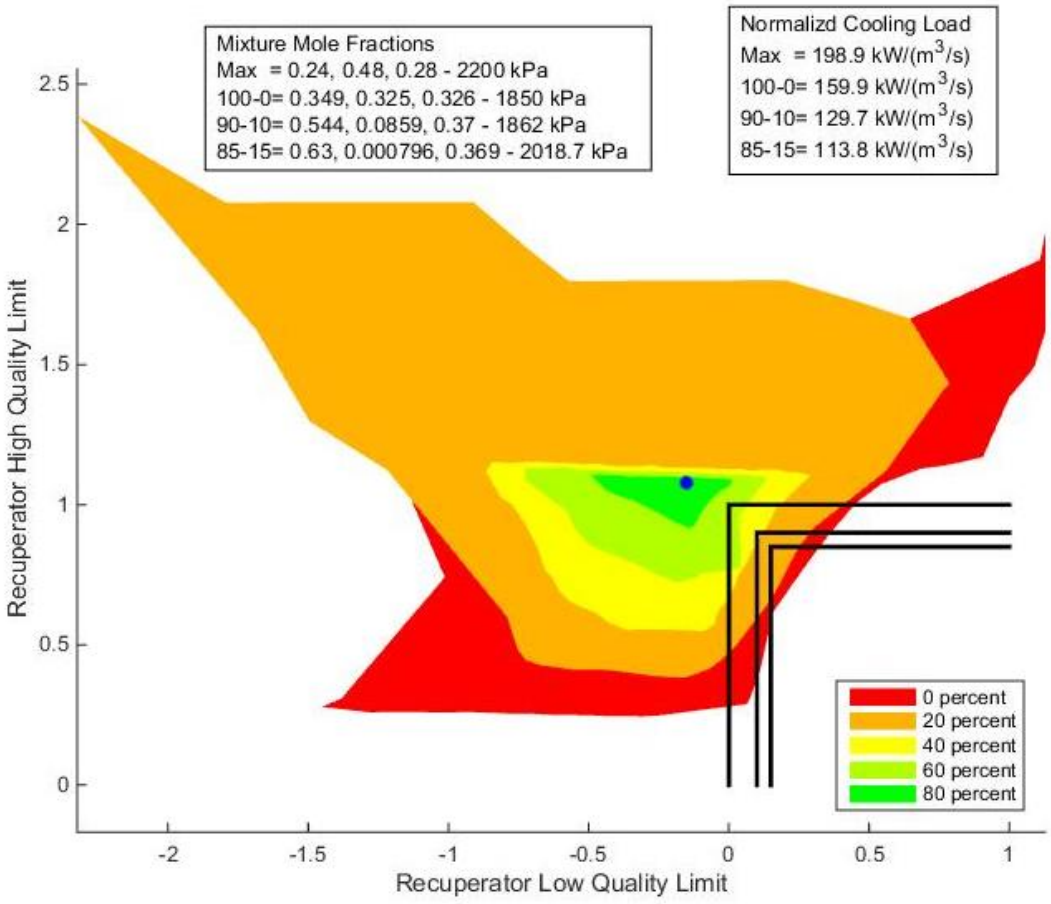


Figure 37 Cycle map of precooled cycle operating with parameters listed in Table 11. Mixture is argon, R14, R218.

Again, the same general shape is observed where constraining the quality results in a decreased thermodynamic cooling load. However in this case there is a large tail extending to cases where the quality span in very large. This is likely due to the vapor dome being much smaller for this mixture, meaning it is possible to transfer very little heat while causing a large change in quality.

This mixture operates with a very similar temperature span to the precooled hydrocarbon cycle shown in Figure 27, and even though the component normal boiling points are not matched closely to the operating temperature of the cycle, the cycle still has a high normalized cooling load. One of the reasons for this is the increased molar mass of R14 and R218. Which effectively increase the mass flow rate in the cycle and thus the cooling load. Figure 37 illustrates the same tradeoff between thermodynamic cooling load and heat transfer as the precooled hydrocarbon cycle meaning this method can be used to optimize synthetic refrigerants as well.

4.5 Non-zero Approach Temperature Difference

All the cycle presented previously have used an approach temperature difference of zero Kelvin, meaning that the recuperator is perfectly efficient. Because of this the cycles only show what would happen if the recuperator was sufficiently sized to allow the approach temperature difference to reach zero. However if a smaller recuperator is used, then the approach temperature difference would be significantly larger than zero which could affect the optimal mixture. To ensure that the same design method can be used if the recuperator is small a parametric study with a non-zero approach temperature difference was conducted. In this case an approach temperature difference of 5 Kelvin was arbitrarily chosen. The same mixture of methane, ethane, and isobutene was used with parameters listed in Table 12.

Table 12 Operating parameters for precooled MRJT cycle with non-zero approach temperature difference.

Parameter	Value
T_1	240 [K]
T_4	150 [K]
ΔT_{app}	5 [K]
$P_{suction}$	150 [kPa]

The cycle was plotted using the same method as above.

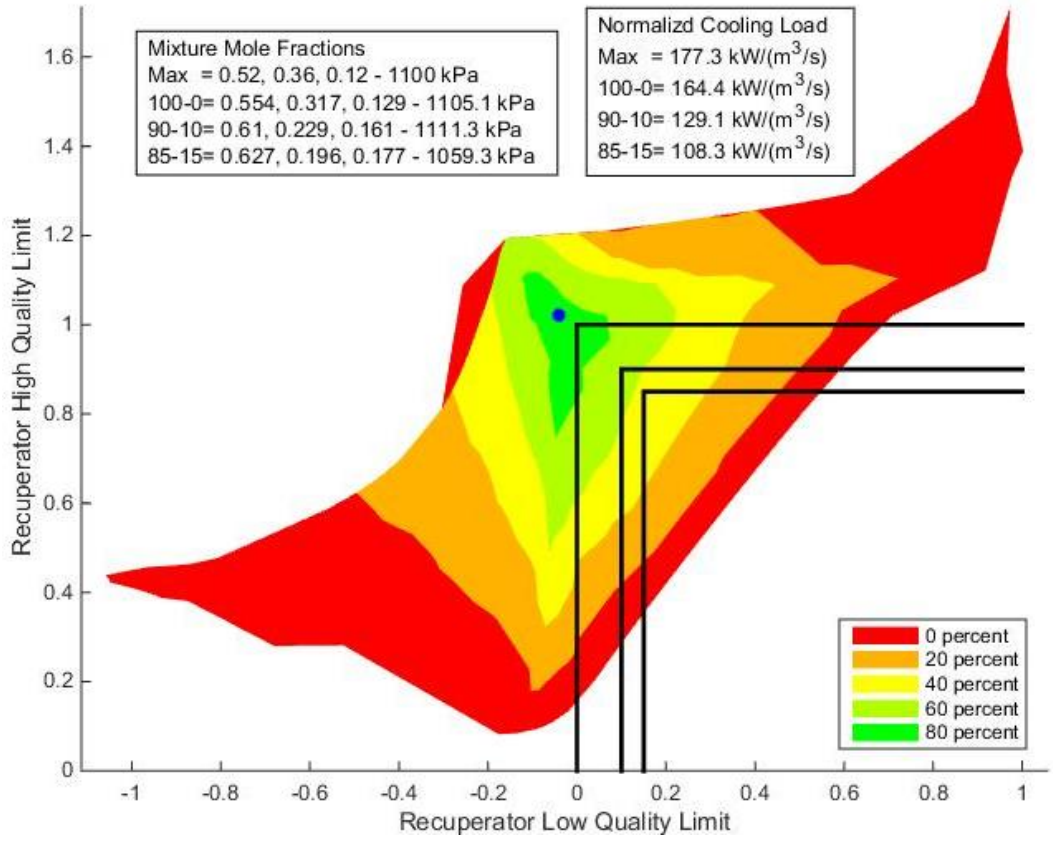


Figure 38 Cycle map of precooled cycle operating with 5 K approach temperature difference, parameters listed in Table 12. Mixture is methane, ethane, and isobutane.

Figure 38 shows the map created from a cycle with a non-zero approach temperature difference. It is clear the same general shape is retained, meaning that the method of using the design line to choose an optimal mixture is still valid. The first main distinction between the 5 K approach temperature difference case and the 0 K approach temperature difference is the maximum thermodynamic cooling load possible. For the 5 K case it is reduced about 17 percent from the 0 K case. This shows the importance of keeping the approach temperature difference in the recuperator as small as possible by keeping the MR in the enhanced heat transfer region.

The second main difference is the cycle discharge pressure of the optimal cases. For the 0 K case the optimal discharge pressure for the quality constrained cases is around 1400 kPa, while for the 5 K case this drops to about 1100 kPa. This is an advantage as it allows a lower discharge pressure to run and allows the compressor to operate within the safe operating pressure range. Along with the discharge pressure, the optimal mixture compositions also change when the approach temperature difference is increased to 5 K. The amount of methane is decreased by three percent while ethane is increased by approximately the same amount. This small change means that the cycle will probably behave close to the predictions given in Figure 27 even if the cycle is operating with a non-zero approach temperature difference. An important conclusion from this study is that it is not necessary to know the exact approach temperature difference, and that small changes in the approach temperature difference do not correspond to large changes in the optimal mixture composition. The 0 K approach temperature difference parametric study should be able to

determine an optimal mixture regardless of the actual approach temperature difference in the cycle.

4.6 COP

The COP is another important metric when looking at designing a system for cooling in certain instances where efficiency is more important than cooling load. The plotting method used above can be used to determine the mixture that will provide the maximum COP as well. The same plotting and interpolation method is used, however instead of normalized cooling load on the z axis, the COP of the cycle is used. To demonstrate this effect the same parametric study was used with the parameters listed in Table 6. The results of the plotting are shown in Figure 39.

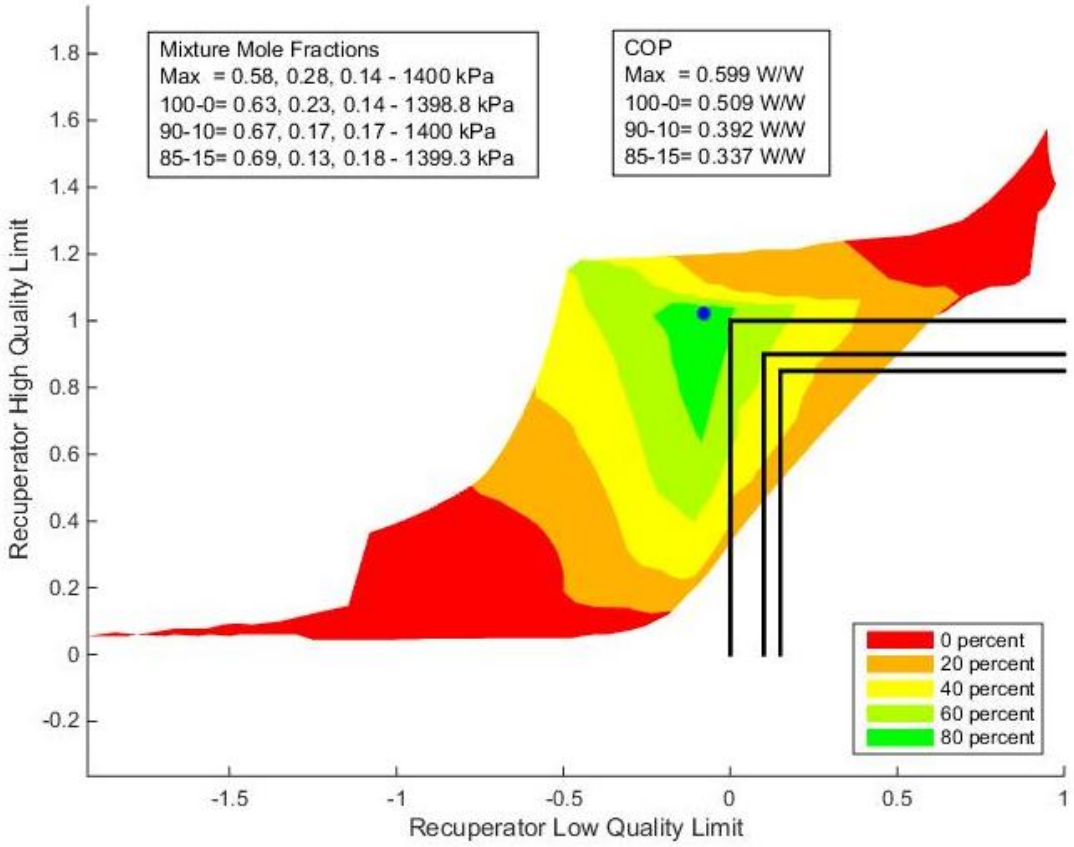


Figure 39 COP cycle map of precooled cycle operating with parameters listed in Table 6. Mixture is methane, ethane, and isobutane.

It is clear from the results that the COP is closely tied to the maximum cooling load as evident by the optimal mixtures for the quality constrained cases. These are almost identical mixtures and pressures to the case where the normalized cooling load is plotted. This means that the compressor is consuming approximately the same amount of power while the real change is in the amount of cooling available when operating at 1400 kPa. The optimal discharge pressure for the thermodynamic maximum point changes drastically from 2100 kPa when optimizing for normalized cooling load, to 1400 kPa when optimizing for COP. As the pressure is increased above 1400 kPa for this mixture the gains from having a higher pressure ratio are surpassed by the greater power needed for the compressor. This

shows that to operate at an optimal COP the cycle should be operated at a discharge pressure of around 1400 kPa while keeping the mixtures approximately the same.

4.7 Pressure Drop in Recuperator

One simplification of the model that can have a drastic impact on the cycle is the pressure drop in the recuperator. Experimental test results have shown that in some operating conditions the cycle can operate with almost all of the pressure drop of the cycle in the high pressure side of the precooler and recuperator. In order to see if this would affect the optimal mixture, the model above was modified to include a pressure drop. To make the model run quickly, a pressure drop fraction is set by the user which is the fraction of total pressure that is lost in the recuperator. This pressure drop is distributed evenly throughout all of the subheat exchangers. This is another approximation as more of the pressure drop would occur in the hot side of the recuperator because of the higher velocity. The pressure drop in each section was initially explicitly calculated but it proved to be too computationally expensive to use in a parametric study. Since the pressure at each node is no longer constant, a new way to look up temperature from enthalpy and pressure is needed. Instead of modifying the lookup tables to lookup temperature as a function of both enthalpy and pressure, Eqn (4.1) was used to calculate the temperature drop at constant enthalpy for a given change in pressure.

$$T = T(h, P_{high}) + \frac{dT}{dP}\Delta P \tag{4.1}$$

By keeping the enthalpy distribution the same, the energy balance at each node still applies, but now the temperature at each node is now degraded according to the degree of JT cooling.

It was quickly observed that $\frac{dT}{dP}$ is not constant throughout the range of pressures the cycle is spanning, as shown in Figure 40.

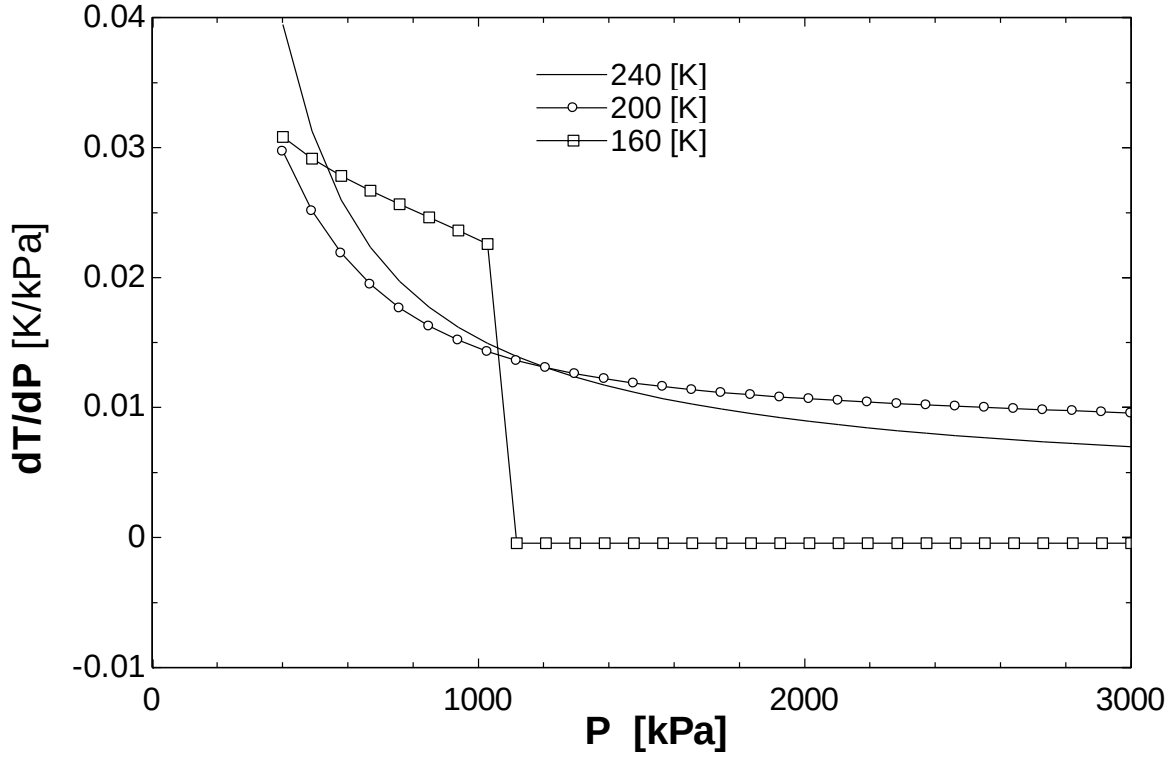


Figure 40 Joule Thompson Coefficient for mixture of methane, ethane, and isobutane.

At low pressure the JT effect is much more profound then at higher pressures. So, if $\frac{dT}{dP}$ were only calculated at the high pressure or low pressure of the cycle there would be significant error. The other interesting effect occurs when the mixture exits the vapor dome. Figure 40 shows the 160 K isotherm which starts in the vapor dome where the JT effect is large, however when the mixture gets to about 1000 kPa it exits the vapor dome and the JT effects falls drastically. Because of this behavior, it was assumed that if the mixture is outside of the vapor dome that the JT effect is negligible. In order to accurately predict the

temperature drop that will occur due to the JT effect, a curve fit is used to define $\frac{dT}{dP}$ as a function of pressure. A power fit is used of the form $\frac{dT}{dP} = a_1 P^{a_2}$ where a_1 and a_2 are the unknown parameters. To assess the accuracy of this method a plot was created that compares the temperature calculated from REFPROP at a given enthalpy and pressure to the temperature calculated using the integration of a curve fit and a linear fit.

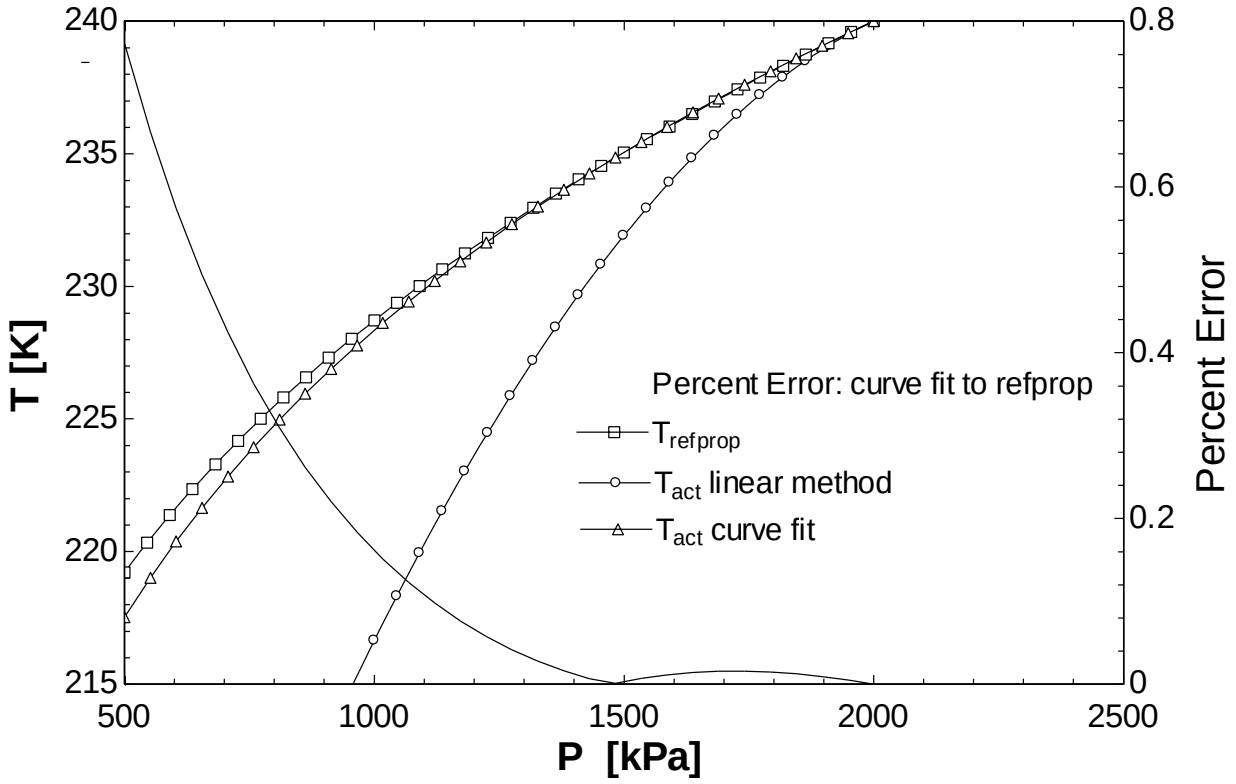


Figure 41 Temperature as function of pressure using REFPROP, linear fitting, and curve fitting

Figure 41 shows that the linear fit method diverges from the actual temperature calculated from REFPROP. However, the curve fit temperature matches very closely to the temperature provided by REFPROP throughout the range of pressures. The percent error

between the REFPROP results and curve fit results are plotted on the right axis, and it shows that the percent error is under one percent for the region shown, which represents the pressures that the cycle will experience. The curve fit was made with only three data points, so to have higher confidence in the results in the model, nine points were used to fit the curve for $\frac{dT}{dP}$. Since $\frac{dT}{dP}$ is no longer constant a new equation is needed to calculate the final temperature. This is shown in Eqn (4.2).

$$T = T(h, P_{high}) + \int_{P_{high}}^{P_i} \frac{dT}{dP} dP \tag{4.2}$$

The power function for $\frac{dT}{dP}$ was easily be integrated, and the limits substituted into Eqn (4.2).

$$T = T(h, P_{high}) + a_1 \frac{P_i^{(a_2+1)} - P_{high}^{(a_2+1)}}{(a_2+1)} \tag{4.3}$$

Eqn (4.3) is inserted into the model to replace the calculation for temperature at each state point on the high pressure side of the recuperator. The pressure drop is only modeled on the high pressure side of the recuperator because in the experimental test facility the low pressure side the mass flux is much lower. Here the MR is passing over the finned tube while on the high pressure side it is passing through it which causes a much larger pressure drop. This effect was confirmed in measurements from the experimental test facility.

The pressure drop is also modeled in the precooler since the mixture is again traveling through a finned tube. The pressure drop is especially high in the precooler because the vapor

is higher in quality and thus has a higher velocity which results in higher pressure drops. While the pressure drop in the precooler does not affect the recuperator, it does affect the mass flow rate through the compressor, as well as the supply pressure to the cold head. As an estimate, a pressure drop of 30 percent of the discharge pressure is assumed to occur within the precooler and is considered the model.

The modified model can be found in Appendix C and D.

Using this new method, the same precooled cycle operating with parameters listed in Table 13 and a mixture of methane, ethane, and isobutane was modeled.

Table 13 Operating parameters for precooled MRJT cycle with pressure drop.

Parameter	Value
T_1	240 [K]
T_4	150 [K]
ΔT_{app}	0 [K]
$P_{suction}$	300 [kPa]

The suction pressure was increased to more closely align with past experimental results where high pressure drop in the recuperator results in a high suction pressure [2]. Only results that would operate with a discharge pressure under 2500 kPa, the operating limit of the compressor are plotted. The results are shown in Figure 42.

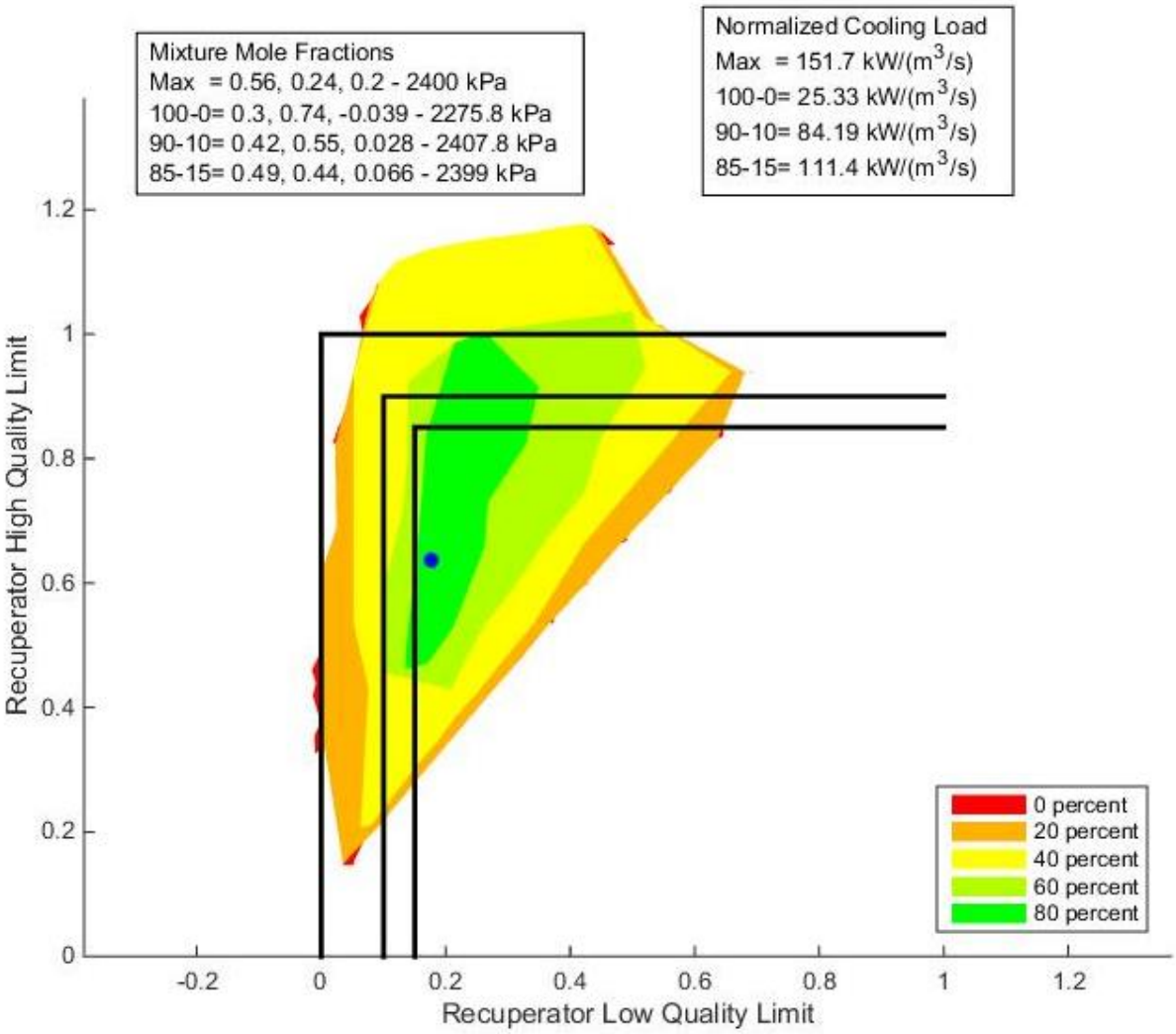


Figure 42 Cycle map of precooled cycle modeled with pressure drop in high pressure stream of recuperator, operating with parameters listed in Table 6. Mixture is methane, ethane, and isobutane.

The surprising result from this study is that the model predicts only a small decrease in thermodynamic cooling load from the idealized case shown in Figure 28. This is surprising since the maximum possible cooling load is the isothermal enthalpy difference from high pressure to low pressure at the load temperature. So a small pressure difference should mean a small enthalpy difference. It was quickly realized that even with the more accurate method

of calculating the temperature at each node the model still exhibited a significant amount of error. To illustrate this a P-h diagram of the maximum case is shown in Figure 43.

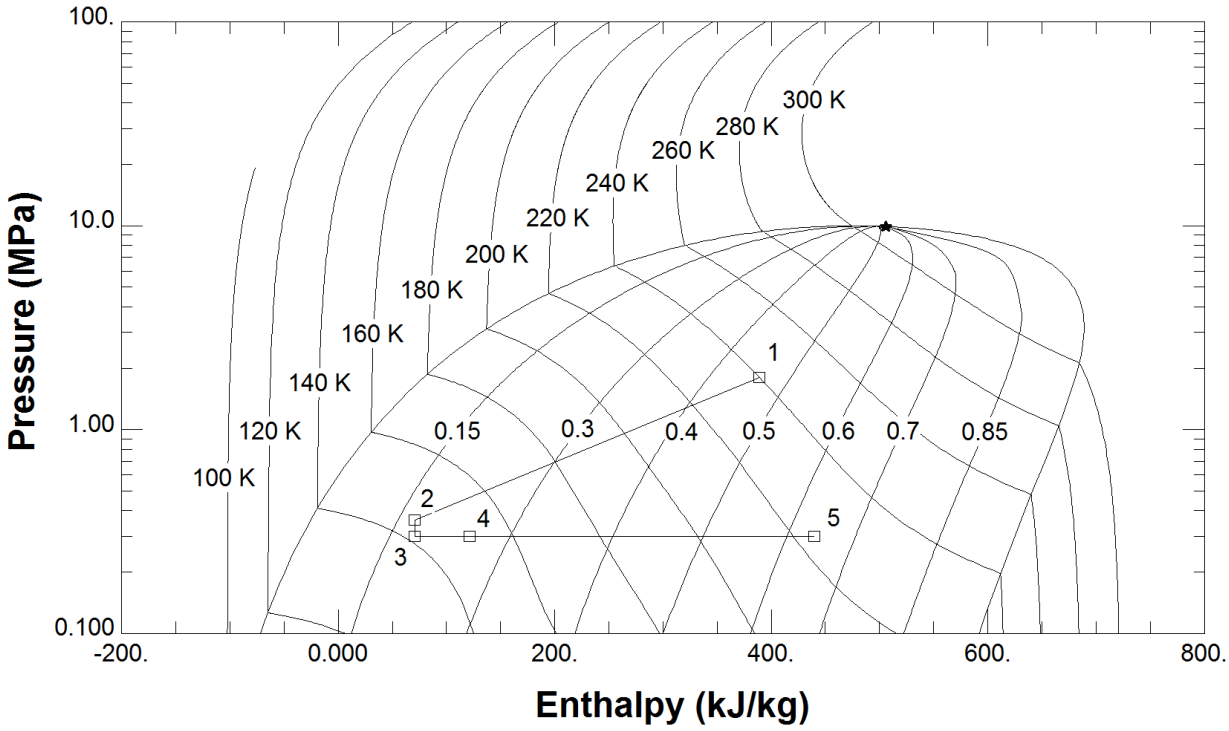


Figure 43 P-h diagram of precooled cycle with distributed pressure drop

Figure 43 shows the MR with the maximum cooling load from Figure 42 on a P-h diagram. It is clear that the cycle does not operate correctly since T_2 is lower than T_4. This would mean that the recuperator is transferring heat from low temperature to high temperature, which is physically impossible. However, this does not mean the map is completely useless. The isothermal enthalpy difference from high pressure to low pressure is the largest in the area where the optimal cycle is operating. It is in this area where the slope of an isotherm is the smallest meaning that there will be the greatest enthalpy drop for a given pressure drop. This

means that even though the cooling load is not accurately predicted the mixture identified should operate with a high cooling load.

Since the maximum thermodynamic cooling load mixture is entirely within the enhanced heat transfer region, it is the only mixture of interest. When compared to the cycle modeled with no pressure drop, the main difference is the decrease in methane, and increase in ethane. This is a result of the cycle trying to remain in the optimal region, and represents a significant difference in composition. In order to optimize for a cycle with significant pressure drop in the recuperator, the model should be run considering the pressure drop in the recuperator. However, having a large pressure drop in the recuperator greatly reduces the cooling power of the cycle. A cycle operating optimally will have little pressure drop in the recuperator and the pressure drop model will not be needed. If there is little pressure drop in the recuperator the idealized model should be used to determine the optimal mixture.

5.0 Conclusion

A MRJT cycle will only operate properly when the MR is chosen to properly match the parameters of the cycle. With the optimal mixture, it is possible to operate a MRJT cycle at a much lower pressure, and with a much higher cooling load than with a single component refrigerant. However, if the wrong MR is chosen the cycle performance will suffer, and in some cases not operate at all. This thesis describes a methodology to choose an optimal mixture based on the thermodynamic cooling load and enhanced heat transfer region concurrently.

The MRJT system is split into a compressor and cold head model. The compressor model accepts the inputs of suction pressure, discharge pressure, and mixture composition, and returns the mass flow rate and compressor work. The cold head model accepts inputs of supply temperature, load temperature, approach temperature difference, suction pressure, discharge pressure, and the mixture composition to determine the cooling load of the cycle, as well as the quality in the recuperator. By changing the supply temperature the model has the ability to model both a precooled and room temperature MRJT cycle. Additionally, by adjusting how the temperature at each point is calculated in the recuperator, the model can also be used to optimize a mixture while considering the pressure drop in the recuperator.

The model is used to conduct a parametric study that looks at the effect of varying the composition of the mixture and the discharge pressure. Using all the data points from the study, a contour plot was created that shows the cooling load as a function of the quality limits in the recuperator. The cycle 'map' clearly shows what the effect of constraining the quality is on the thermodynamic cooling load of the cycle. A design line was identified that along which the optimal mixture must lie. The composition and discharge pressure are given at several points along the design line. It was proposed that experimentally testing the mixtures along this line would result in a good starting point for further experimental testing.

The optimization method was testing with several variations to the cycle, including changing the operating suction pressure, the cycle operating temperature, and the approach temperature difference in the recuperator. Additionally testing with mixtures of synthetic refrigerants was also conducted. In each case it was determined that the cycle map followed the same general shape, meaning this optimization technique is valid for all MRJT cycles.

The model can also be used to determine the optimal number of mixture components, and which refrigerants should be in that mixture. This is done by running multiple parametric studies and comparing the results of mixtures with different components and number of components. This makes experimental testing since it is much easier to control the composition of a mixture with fewer components.

Instead of optimizing only the maximum possible cooling load for the cycle, the model was also used to see the effect of optimizing the MR to have the cycle operate with the maximum possible COP. It was found that the cycle map for COP looked very similar to the cycle map for the optimal cooling load. However it proved that the model and plotting technique could be used to determine an optimal mixture for different objective statements. For example the cooling load divided by the conductance of the recuperator could be used as the objective statement and the model could be used to find the optimal mixture in that case.

Finally, in order to replicate a real world cycle a parametric study was conducted assuming a uniform pressure drop in the high pressure side of the recuperator. The same method of sub-heat exchanger discretization was used, however the temperature at each node on the high pressure side of the recuperator is reduced according to the level of JT cooling. This model introduced significantly more error into the calculations, and the cooling load was not accurately calculated. However, it was shown that despite the accuracy issues the optimal mixture specified still operated in the region where the cooling load would be highest, so it could still be used to determine the optimal mixture for cases with high pressure drop. An ideal cycle will operate with little pressure drop in the recuperator so for most cases it is best to use the idealized cycle model to determine the optimal mixture.

The combination of the model and plotting method make this a powerful tool for quickly determining a good starting mixture for experimental testing. It recognizes the difficulties of matching the experimental parameters to the model and strives to make a good guess at the optimal mixture with the understanding that experimental testing is the only way to truly ensure the optimal mixture operation.

6.0 Future Work

In order to prove that this method does provide an accurate method for the initial selection of optimal MRs, experimental testing is needed. At the University of Wisconsin Madison, the same test facility that was used in the past for optimizing mixtures is available for use. This test facility was created by Skye, and also used by Passow and Rule in their testing. This cycle is based on a cryosurgical probe that has been modified to provide high accuracy measurements of temperature and pressure at specified state points in the cycle. The cooling load and temperature can also be highly controlled. The cycle consists of two stages, allowing for precooled cycles to be tested. More information on the test facility is available in their theses [2] [3] [5].

The model will first be used to determine a range of optimal mixtures based on previous cycle operating parameters. The mixture that has been modeled to provide the best cooling load when constrained to be within 15 and 85 percent quality will be tested. The actual operating parameters will be compared to those used to model the cycle. If there is a drastic difference, the model will be ran again with the actual operating parameters. The new mixture can then be experimentally tested again, and the parameters compared to the model

results. If they still differ drastically, this process can be repeated until the agreement is sufficient. Once the model and cycle parameters match, the load temperature and cooling load can be compared to the cooling load of previous optimization attempts. If the cycle is operating with a cooling load near the past optimized results, within 30 percent, than the model has provided a good starting point for optimizing the mixture. If the cycle does not perform as expected, other mixtures along the design line can be run. One of the mixtures along this line should provide an optimal cooling load and the experimental testing should show this. Once it is determined which constraint on quality provides the best cooling load, that point can be used as the starting point for testing on all other cycles as well.

Initial testing has been done using the model of a precooled mixture of methane, ethane, and isobutane. The parameters from past experimental testing were used as the starting point for the inputs to the model. The plot used to predict the optimal mixture is shown in Figure 27. First, the optimal mixture when quality was constrained to be between 15 and 85 percent was charged into the system. Matching the composition from the model to the actual mixture proved very difficult since isobutane absorbed strongly into the compressor oil. The mixture could only be created to be within about 10 percent of the desired composition. It was also observed that the mixture composition changed greatly when operating at steady state when compared to the mixture charged into the system, further adding to the error of the cycle.

When the mixture was actually run, it appeared that the increased heat capacity of the high pressure stream caused the precooling temperature to rise dramatically, about 10 K, above the modeled input parameter. The greatly changes the operation of the cycle as it now

has to drop an additional 10 K in the recuperator in order to reach the desired load temperature.

Finally, it was observed that almost no pressure drop occurred over the expansion valve. This is likely because of the way the optimized mixture is designed to operate. State 2 is the lowest quality in the system, meaning it has the lowest specific volume and thus the lowest velocity. This means that the pressure drop will be greatly reduced through the expansion valve. In order to match the idealized cycle the expansion orifice needs to be reduced drastically to increase the temperature drop and cooling load.

These challenges make it difficult to experimentally verify the results of the model. However, by making the changes required to the cycle, and matching the parameters from the actual experimental test facility to the model, the experimental results should closely match the predicted results.

Once the experimental test facility is operating correctly it can be used to determine which mixtures along the design line operate the best. The four mixtures identified on the cycle map should be tested in the experimental facility. However, it is likely that the actual mixture in the facility will not exactly match the MR defined by the model. The mixture should be corrected until all the mixtures along the design line have been tested. The discharge pressure should also be adjusted using the bypass valve or by changing the orifice to closely match the discharge pressure specified from the model. If possible all the operating conditions should remain the same for all tests, for example the filling pressure, the suction pressure, and the precooling temperature should all be the same regardless of the mixture being tested. Since the test facility can measure the temperature and pressure at state 2 and 5

the quality limits can be calculated using REFPROP and the measured mixture composition. Additionally the cooling load at the specified load temperature can also be measured.

From this data a surface plot can be created with the limits of quality on the x and y axes and measured cooling load on the z axis. This surface plot can be overlaid on the cycle map predicted by the model. The experimental plot should show a peak in cooling load much closer to the vapor dome than the thermodynamic cooling load predicted by the model. If this is the case it means that by operating in the enhanced heat transfer region the conductance of the recuperator has increased resulting in an increased effectiveness and cooling load.

This method can be tested for the three component mixture of methane, ethane, and propane as well. The surface plot that is created in that case can be compared to the mixture or methane, ethane, and isobutane. The model shows that there should be an increase in performance for the isobutane case, which should also be realized in the experimental tests. If the experimental tests proved this to be the case then it would be confirmed that the model can be used to choose which components will work well in the cycle.

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8.0 Appendix

Index of Programs available at http://sel.me.wisc.edu/

	Program Name
A	Model of Compressor and Cold Head W/Parametric Study Code
B	Macro for EES Code
C	Model of Compressor and Cold Head W/Parametric Study Code and Pressure Drop
D	Macro for EES Code with Pressure Drop
E	Mixture Mole Fraction Selection for 3 components
F	Mixture Mole Fraction Selection for 4 components
G	Mixture Mole Fraction Selection for 5 components
H	Matlab Alpha Shapes Plotting
I	Matlab Alpha Shape Interpolation
J	3 Component Sort Program
K	Program Guide