

INVESTIGATION OF VARIOUS THERMAL CAPACITANCE MODELS

by

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ABSTRACT

The effective thermal capacitance of structures is investigated in this paper. The effective thermal capacitance of a structure is the value of thermal capacitance which permits the accurate modelling of the distributed thermal mass of the structure as a single lumped capacitance. Two methods are used to determine effective thermal capacitance. The effective thermal capacitance of a homogeneous wall is determined by comparing the results of finite difference simulations to the analytic solution of the single capacitance model. The effective thermal capacitance of a structure is determined by comparing the results of transfer function model and modified degree-day model simulations.

It is shown that the effective thermal capacitance of a homogeneous wall with one sinusoidal ambient temperature boundary and one constant ambient temperature boundary is defined by two parameters: the average Biot modulus and the periodic Fourier modulus.

In Chapter Three it is shown that no satisfactory relationship exists between actual and effective thermal capacitance when actual weather data is used to represent ambient conditions. The modified degree-day model is compared to the transfer function model for fixed values of lumped capacitance. The lumped model can be fairly accurate in estimating total heating season loads when no solar gains are considered. A comparison of the constant room temperature and

floating room temperature transfer function models showed that the constant room temperature model over-predicts cooling and heating loads when the room temperature varies.

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Nomenclature

Symbols used in this thesis which do not appear below are defined locally in the text.

A_a	ambient temperature sine wave amplitude
A_c	collector area
A_i	interior temperature sine wave amplitude
a_i	general transfer function coefficient
B, C, D, E	Coefficients of general analytic solution of lumped model
Bi	Biot modulus
\overline{Bi}	average Biot modulus
b_n	ambient temperature transfer function equation coefficient
C_{DD}	modified degree-day model thermal capacitance
C_e	total effective thermal capacitance of a structure
C_{ew}	effective thermal capacitance of structure walls
C_i	thermal capacitance of node i
C_k	thermal capacitance of node k
c_n	interior temperature transfer function equation coefficient
c_p	material specific heat
C_r	lumped room thermal capacitance
C_s	actual thermal capacitance of structure walls
C_w	effective thermal capacitance of finite-difference wall
d_n	previous heat flow transfer function equation coefficient

T_i	average interior temperature
T_j, T_k, T_l	nodal temperatures
T_r	room temperatures
\overline{T}_r	average room temperature
T_{max}	maximum thermostat set temperature
$T_{max, eff}$	effective maximum thermostat set temperature
T_{min}	minimum thermostat set temperature
T'_{min}	night setback minimum thermostat set temperature
T_w	lumped model wall temperature
T_{wl}	ambient-side wall surface temperature
T_{wm}	capacitance-weighted mean wall temperature
UA	structure heat transfer coefficient
U_w	wall heat transfer coefficient
Δx_i	finite difference model nodal spacing

Greek Symbols

α	material thermal diffusivity
β	interior temperature phase lag
γ	dimensionless capacitance ratio
π	Archimedes number
ρ	material density
σ	standard deviation of monthly-average ambient temperature
τ	time

P	period of sine wave
\dot{Q}_{EX}	net rate of house energy gain
q''_{τ}	transfer function equation heat flow at time τ
q_a	rate of energy flow from ambient to structure walls
q_{in}	rate of energy flow into a node
\dot{q}_k	direct energy flow into node k
q_{out}	rate of energy flow out of a node
q_r	rate of energy flow from structure walls to interior space
q_{sol}	rate of direct solar energy gain by structure interior
R_a	lumped model ambient-side thermal resistance
$R_{c,i}$	transfer function equation coefficient relative importance
R_i	lumped model interior-side thermal resistance
R_{jk}, R_{kl}	finite-difference inter-nodal thermal resistances
r_a	simplified lumped model analytic solution response amplitude
S.E.	standard error
\bar{T}	time-average temperature in general analytic solution of lumped model
T_a	ambient temperature
\bar{T}_a	average ambient temperature
T_e	sol-air temperature
T_i	interior temperature
T_i	temperature of node i (Eqs. 2.3.17 and 2.3.18)

\dot{E}_{store}	rate of change of energy stored in wall
F_{CAP}	fractional thermal capacitance
F_L	relative standard error
F_o	Fourier modulus
F_o, ω	periodic Fourier modulus
GEN	rate of energy generation in room
\bar{H}_T	monthly-average daily radiation incident on a tilted surface
h_a	ambient convection coefficient
h_i	interior convection coefficient
k	material thermal conductivity
L	total heating load (Chapter 1)
L	wall thickness (Chapter 2)
L_{CRT}	constant room temperature transfer function model load
L_{DD}	modified degree-day model load
L_{FRT}	floating room temperature transfer function model load
L_{TF}	transfer function model load
M	number of previous heat flow transfer function equation coefficients
(mc_p)	material thermal capacitance
N	number of days in a month (Eq. 1.4.2 & 1.4.3)
N	total number of nodes for finite-difference model (Chapter 2)
N	number of ambient or interior temperature transfer function equation coefficients (Chapter 3)
N_{obs}	number of observations for calculation of standard error

ϕ_i	phase lag in simplified analytic solution of lumped model
ϕ_a	phase lag in simplified analytic solution of lumped model
ω_a	ambient temperature circular frequency
ω_i	interior temperature circular frequency
$\overline{(\tau\alpha)}$	monthly-average transmittance-absorptance product of the collector
$\Delta\tau$	time interval

1. INTRODUCTION

1.1 Introduction

In the past building design considerations depended largely on aesthetics and cost of construction. Little attention was paid to the energy requirements of the structure. Consideration was given mainly to aesthetically pleasing placement of windows, doors and building orientation at minimum construction costs. The effect of these parameters on the energy requirements of the building were virtually ignored. Energy for heating and cooling was inexpensive and plentiful, so building energy use was not a major concern in the design process.

Constantly rising energy costs, however, have forced people to more closely examine building energy requirements and to seek alternate methods to supply energy for heating and cooling. Proper use of insulation to reduce energy costs, placement of windows in order to use energy from the sun, the use of multi-stage thermostats to reduce energy consumption, and many other factors are now carefully considered in the design process.

Building energy use concerns and the need to evaluate the effect of design changes and energy conserving measures on heating and cooling loads necessitated the development of techniques to accurately model building energy consumption. Many methods have been developed to estimate building energy use. The complexity and amount of computation required to use these methods varies quite widely. The simple degree-day method requires only a minimum number of algebraic

calculations. Other methods, such as the finite difference method, are very complex and require access to a large computer to use the methods efficiently.

In the modelling of buildings, the thermal mass of the structure has a large effect on heating and cooling loads. The mass elements of the building can be used to store energy from the sun in conventional buildings as well as passive solar applications. Energy collected during the daylight hours can be stored in the building elements and used at a later time to reduce the energy required from conventional sources such as gas or oil. In cooling applications, the thermal mass of the building can absorb heat during the day which would otherwise become part of the cooling load. The energy required to operate an air conditioner is therefore reduced.

All of the elements of a structure; the walls, floors, ceilings, furniture, appliances, etc., contribute to the thermal mass. The primary mechanism by which thermal mass stores and releases energy is through temperature variation. The mass heats up when exposed to higher temperatures or solar radiation, thus storing energy. When the thermal mass is warmer than the surroundings, stored energy is released back to them.

The elements which contribute to the thermal mass do not all have equal storage capacity and are not all exposed to the same amount of solar radiation or ambient temperature variation and therefore do not store equal amounts of energy. For this reason, a rigorous building model considers the thermal response of each of the elements. These

elements contribute to the thermal mass individually as a part of a larger interconnected thermal system. This is a distributed thermal mass model. Models do exist which consider building elements in this manner [1, 2, 19, 20] and generally estimate building heating and cooling loads quite accurately. However, these models can be quite cumbersome to use, and usually require access to a large scale computing system.

A much simpler way to represent building thermal mass is to consider all mass as a single lumped thermal capacitance. This model is computationally much simpler, and generally easier to use than the more complex methods.

Modelling a distributed thermal capacitance system as a single thermal mass requires an accurate estimation of the lumped thermal capacitance in order to predict precise heating and cooling loads. If the capacitance is not correctly estimated, the model can generate results which are inaccurate and of little value. Estimation of this capacitance, the effective thermal capacitance, and the general accuracy of the lumped analysis method are the topics of this thesis.

1.2 Definition of Effective Thermal Capacitance

The effective thermal capacitance is defined as the capacitance which adequately represents the thermal response of the mass of a distributed capacitance system. It is a single value which represents the thermal mass of the entire structure for building modelling

purposes.

To quantify the energy stored in any thermal mass, a representative temperature or temperature swing must be associated with the capacitance. In a distributed capacitance system, each thermal mass is represented by its own temperature variation which is not necessarily the same as any other thermal mass in the system. The single capacitance of the lumped model is characterized by one temperature variation. Effective thermal capacitance must account for this difference in temperature variations. The physical and thermal properties of each of thermal masses in the distributed model can also differ, thus giving each mass its own thermal capacitance (mc_p). Proper estimation of effective thermal capacitance should also consider this.

Thus it can be stated that effective thermal capacitance is the single lumped thermal mass characterized by one temperature variation which accurately represents the combined thermal behavior of the individual thermal masses in the distributed capacitance system. The correct effective thermal capacitance used in the lumped model for heating and cooling load analysis will generate the same loads as a more rigorous model or as determined from actual data.

1.3 Effective Thermal Capacitance of Walls

Lumped capacitance modelling is not limited to buildings. Individual elements of a structure, such as a wall, are distributed capacitance systems and can be modelled as lumped capacitance sys-

tems. A homogeneous wall may not have a uniform temperature variation throughout. The different components of a composite (multi-layer) wall can have different thermal capacitances in addition to a non-uniform temperature variation.

Like the effective thermal capacitance of a building, the effective thermal capacitance of a wall accounts for non-uniform temperature variation and different thermal capacitances, and is represented by a single temperature variation. Similar to the structure effective thermal capacitance, the effective thermal capacitance of a wall is the single lumped thermal mass characterized by one temperature variation which accurately represents the distributed thermal mass of the wall.

1.4 Current Methods That Use Effective Thermal Capacitance

The physical meaning of effective thermal capacitance can be better understood by considering how the parameter is used in current simulation and design methods. Two cases in which the effective thermal capacitance is used are present here. One case is from the transient simulation program TRNSYS [2], and the other is a design method for direct gain passive solar energy systems.

The TRNSYS Type 12 component is an energy/degree-hour space heating model. When used in mode four, the component models a single thermal capacitance building. The component uses the lumped capacitance in the calculation of successive values of room temperature, shown as equation 1.4.1.

$$T_r^{T+\Delta\tau} = T_r^T + \int_{\Delta\tau} (\dot{Q}_{ex}/CAP) d\tau \quad 1.4.1$$

The parameter CAP which appears in the denominator of the integrand in the equation is called the load capacitance. TRNSYS documentation states that CAP is the sum of the thermal capacitances of the house structure and furnishings. For accurate load calculation, the value should be the effective thermal capacitance of the building.

In Monsen's direct gain unutilizability design method [3], the fraction of the monthly heating load which can be met by solar energy for a finite thermal capacitance structure is determined by three dimensionless parameters: $\bar{\phi}$, X, and Y. $\bar{\phi}$ is the monthly-average daily utilizability; X is the solar load ratio, equation 1.4.2; and Y is the storage-dump ratio, defined by equation 1.4.3.

$$X = \frac{\bar{H}_T(\tau\alpha)A_c N}{L} \quad 1.4.2$$

$$Y = \frac{C_h(\Delta T_r)N}{\bar{\phi} \bar{H}_T(\tau\alpha)A_c N} \quad 1.4.3$$

The storage-dump ratio is the ratio of the maximum solar energy stored in the house during a month to the amount of solar energy dumped from the house per month if the house had no thermal capacity. The parameter C_h , contained in the storage-dump ratio, is the effective thermal capacitance of the building.

For given values of $\bar{\phi}$ and X, the fraction of the monthly load provided by solar can be a strong function of C_h . Two plots of the

solar fraction as a function of X and Y are shown in Figure 1-1. For specific values of $\bar{\phi}$, X , and T_r (the allowable room temperature swing), the monthly solar fraction, F , is a function of C_h only. Considering a case where X is equal to 1.0 and $\bar{\phi}$ equals 0.3, the monthly solar fraction ranges from 0.6 to 1.0. For $\bar{\phi}$ equal to 0.7 the solar fraction ranges from 25 percent of the load to the entire load. The monthly solar fraction is a strong function of C_h and accurate estimation of the monthly solar fraction is closely related to accurate estimation of the effective thermal capacitance.

1.5 Existing Methods to Determine Lumped Capacitance

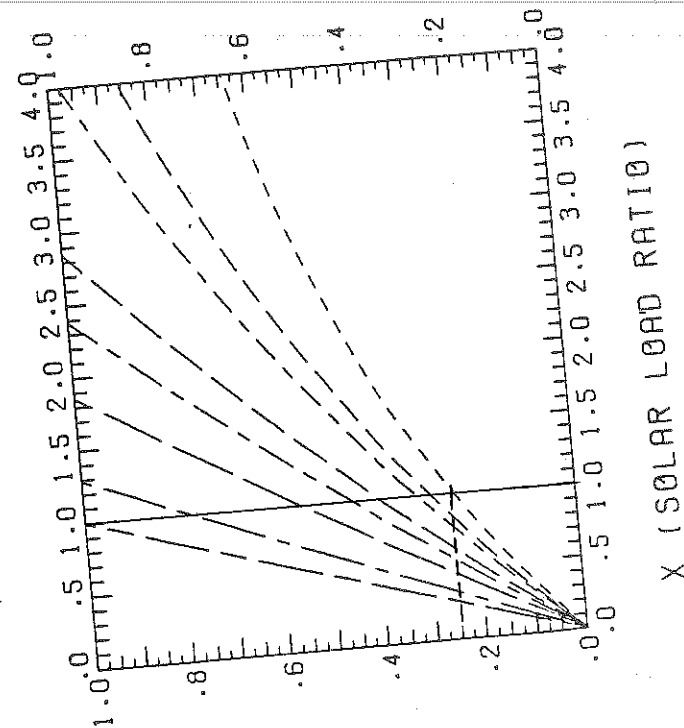
The most well known method to determine lumped capacitance is the diurnal heat capacity (dhc) method presented by Balcomb [4]. Diurnal heat capacity is essentially the same as the thermal admittance discussed by some authors [5, 6], and is defined as the amount of heat per unit surface area that is stored and released per unit of temperature swing.

Balcomb's dhc and Davies' [6] thermal admittance* are derived assuming that the variation in ambient temperature is sinusoidal, and that material properties are constant. With these assumptions analytic expressions for dhc are developed. The effective wall capacitance presented in Chapter Two of this paper is the same as dhc.

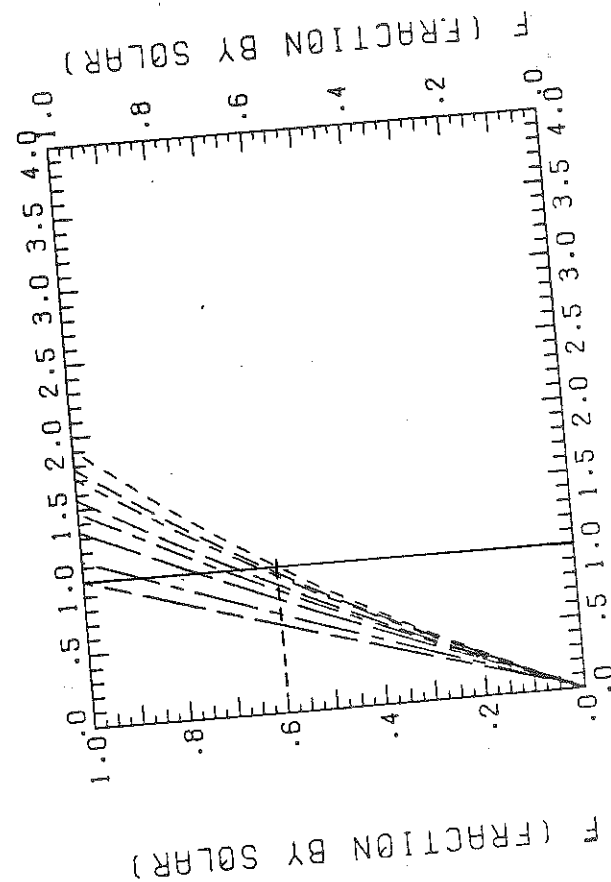
*dhc is equal to $\frac{P}{2\pi}$ times the thermal admittance where P is the period of oscillation of the heat flux.

--- Y=INF.
 --- Y=3.00
 --- Y=1.00
 --- Y=.50
 --- Y=.30
 --- Y=.10
 --- Y=.05
 --- Y=.00

PHIBAR = .700



PHIBAR = .300



X (SOLAR LOAD RATIO)

Figure 1-1 Examples of Mosen's Utilizability Design Method
(adapted from Reference 3)

Balcomb's method consists of determining the dhc of each of the elements of the structure. The dhc of the structure is the sum of the elemental values. The limitations of the method are that the dhc can only be calculated for homogeneous walls, ceilings, and floors, and the elements must either be insulated on one side, or have identical boundary conditions on both sides for the dhc method to be applicable.

The largest difference between dhc as evaluated by Balcomb and the effective thermal capacitance presented in Chapter Three is that actual weather data was used in this analysis. Non-homogeneous construction was also considered. Davies' thermal admittance method may be used for non-homogeneous constructions, but the method also assumes sinusoidal boundary conditions. Actual weather is not strictly sinusoidal, and does affect derived values of the effective thermal capacitance.

1.6 Objectives

The objective of this study is to find a method or means of estimating effective thermal capacitance. The method of Chapter Two shows that the effective thermal capacitance of homogeneous walls with simple boundary conditions can be determined. In Chapter Three it is shown that the effective thermal capacitance of a structure is not a constant value, or simply related to physical or thermal material properties when actual weather data is used to represent

ambient conditions. In many cases, the lumped capacitance model does not predict accurate loads. Thus the utility of lumped capacitance modelling, as it currently exists, may be limited.

2. THE USE OF FINITE DIFFERENCE MODELLING IN THE DERIVATION OF EFFECTIVE THERMAL CAPACITANCE

2.1 Introduction

In this chapter the method of finite-differences is used to model walls for the derivation of effective thermal capacitance. Two wall models which represent the thermal mass of the wall in different ways are presented. One model, the finite-difference model, represents the thermal mass of the wall as a series of smaller interconnected thermal masses. The second model, termed the lumped model, treats the thermal mass as a single lump. For a specified set of boundary conditions, the finite-difference model thermal response is generated by computer simulation. The thermal response of the lumped model is derived analytically for the same set of boundary conditions.

For periodic boundary conditions, the periodic steady-state responses of the two models are similar. The lumped model wall temperature response equation can be curvefitted to the capacitance-weighted mean wall temperature response of the finite-difference model. The effective thermal capacitance of the wall is derived from comparison of the curvefitted temperature response amplitude to the response amplitude derived in the analytic solution of the lumped model.

The method is used to generate the effective thermal capacitance of concrete walls of different thicknesses subjected to identical boundary conditions. Convection boundaries are considered. The

ambient temperature on one side of the wall is sinusoidal. The ambient temperature is constant on the other side of the wall.

Results are presented as a function of two parameters: the average Biot modulus and a periodic Fourier modulus. Presentation of the results in non-dimensional form is accomplished by defining the fractional thermal capacitance of the wall as the ratio of the effective thermal capacitance to the actual material capacitance.

The method is also used to derive the effective thermal capacitance of concrete walls when the amplitude or mean of the sinusoidal ambient temperature is different than in the initial derivation. The effective thermal capacitance of homogeneous walls constructed of materials other than concrete are also derived. The results generated in these two cases are compared to the results of the initial derivation. The fractional thermal capacitance is completely defined by characteristic average Biot and periodic Fourier moduli when one ambient-temperature is sinusoidal and the other is constant.

The difficulties perceived in the extension of the method to non-homogeneous construction and to other ambient and interior boundary conditions are the reasons this method was eventually abandoned.

2.2 Models Used in the Finite-Difference Method

Two wall models are used in this derivation. One model is a one-dimensional finite-difference representation of the wall, while the other model, termed the lumped model, represents the wall as a one-node thermal mass with two thermal resistances to the boundary

conditions.

2.2.1 Finite-Difference Wall Model

The finite-difference wall model represents the wall as a series of interconnected thermal masses or nodes approximated as a series of coupled first-order differential equations. The representation of the wall for simulation purposes is essentially that presented by Myers [7].

Equally spaced nodes are used in this model, with nodes at each surface if the wall is of homogeneous construction. In the simulation of multi-layer walls, the nodes are evenly-spaced within each layer, with a node at each surface and each material interface. The number of internal nodes, n , in each material of thickness δ , are selected so that the value ξ , defined by equation 2.2.1, is about the same for all materials in the wall.*

$$\xi = \frac{k(n+1)^2}{\rho c_p \delta^2} \quad 2.2.1$$

The model is a one-dimensional representation of the wall; the finite dimension is in the direction of energy flow. End effects are neglected and all nodal parameters are based on unit depth and

*Ceylan [8] recommended this criterion for selecting the number of nodes to use in his program RC1 which generates transfer function coefficients (see Chapter Three) from finite difference wall representations. No other criterion was found regarding nodal spacing in composite walls so equation 2.2.1 is recommended.

height. Graphic representations of the wall are shown in Figures 2-1 and 2-2.

The solution of this model for an N node representation requires the solution of N simultaneous ordinary first-order differential equations derived from nodal energy balances. All nodal energy balances are of the form of Equation 2.2.2, shown graphically in Figure 2-3a.

$$q_{in} - q_{out} + \dot{q} = \dot{E}_{store} \quad 2.2.2$$

The nodal differential equation is derived from the substitution of the appropriate finite-difference rate equations into Equation 2.2.2. For node k, bounded by nodes j and l (see Figure 2-3b), the nodal differential equation is Equation 2.2.3. R_{jk} and R_{kl} are the internodal resistances, \dot{q} is any direct energy flow into node k, and C_k is the thermal capacitance ($m c_p$) of the node.

$$\frac{T_j - T_k}{R_{jk}} - \frac{T_k - T_l}{R_{kl}} + \dot{q}_k = C_k \frac{dT_k}{dt} \quad 2.2.3$$

If nodes j, k and l are interior nodes, the resistances, R_{jk} and R_{kl} are conduction resistances. If either node j or k is a node representing the ambient or interior boundary condition, the respective resistance will be a boundary resistance. For example, if node j represents a convective boundary condition, R_{jk} will be a convection resistance.

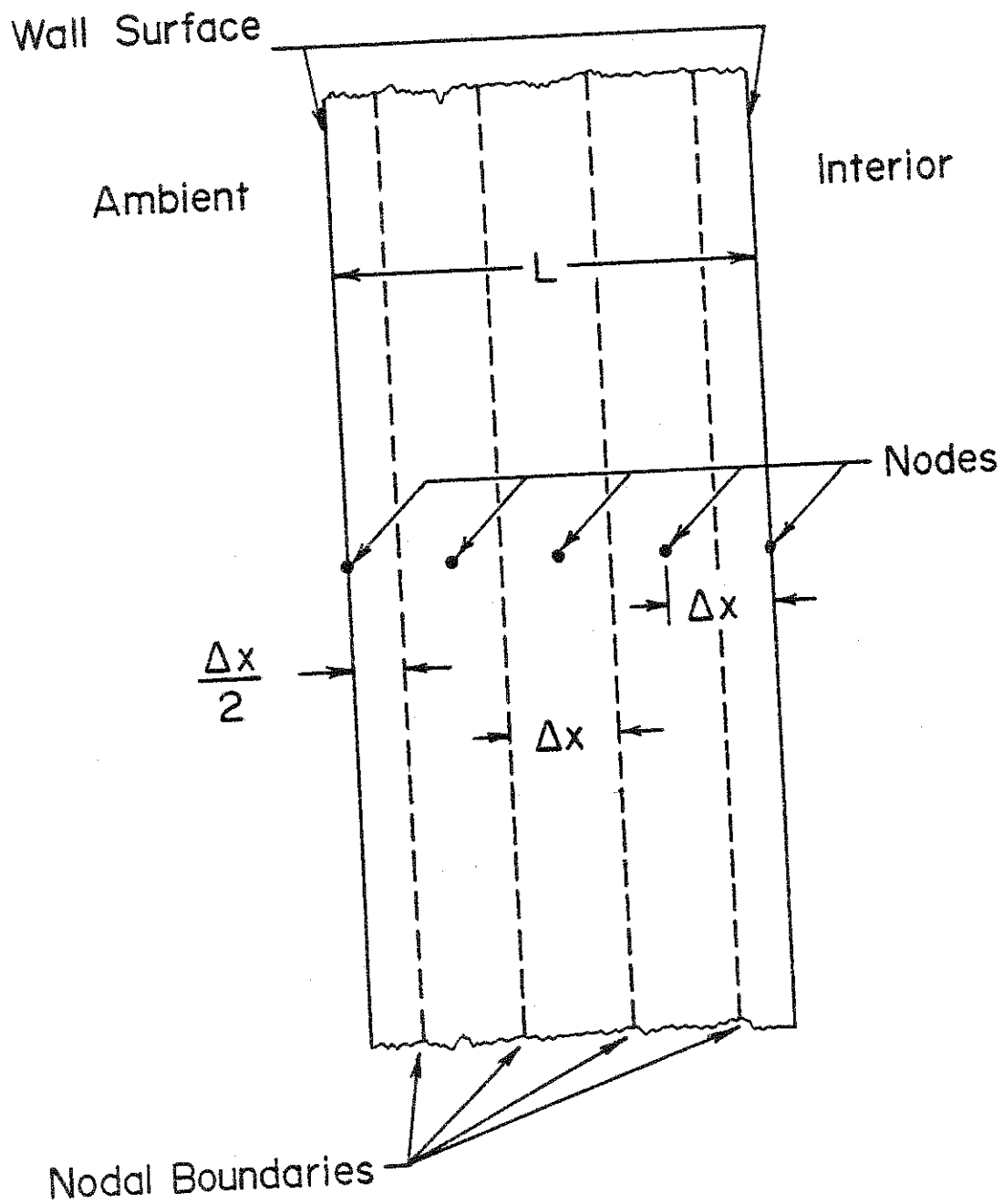


Figure 2-1 Finite-Difference Wall Model Representation

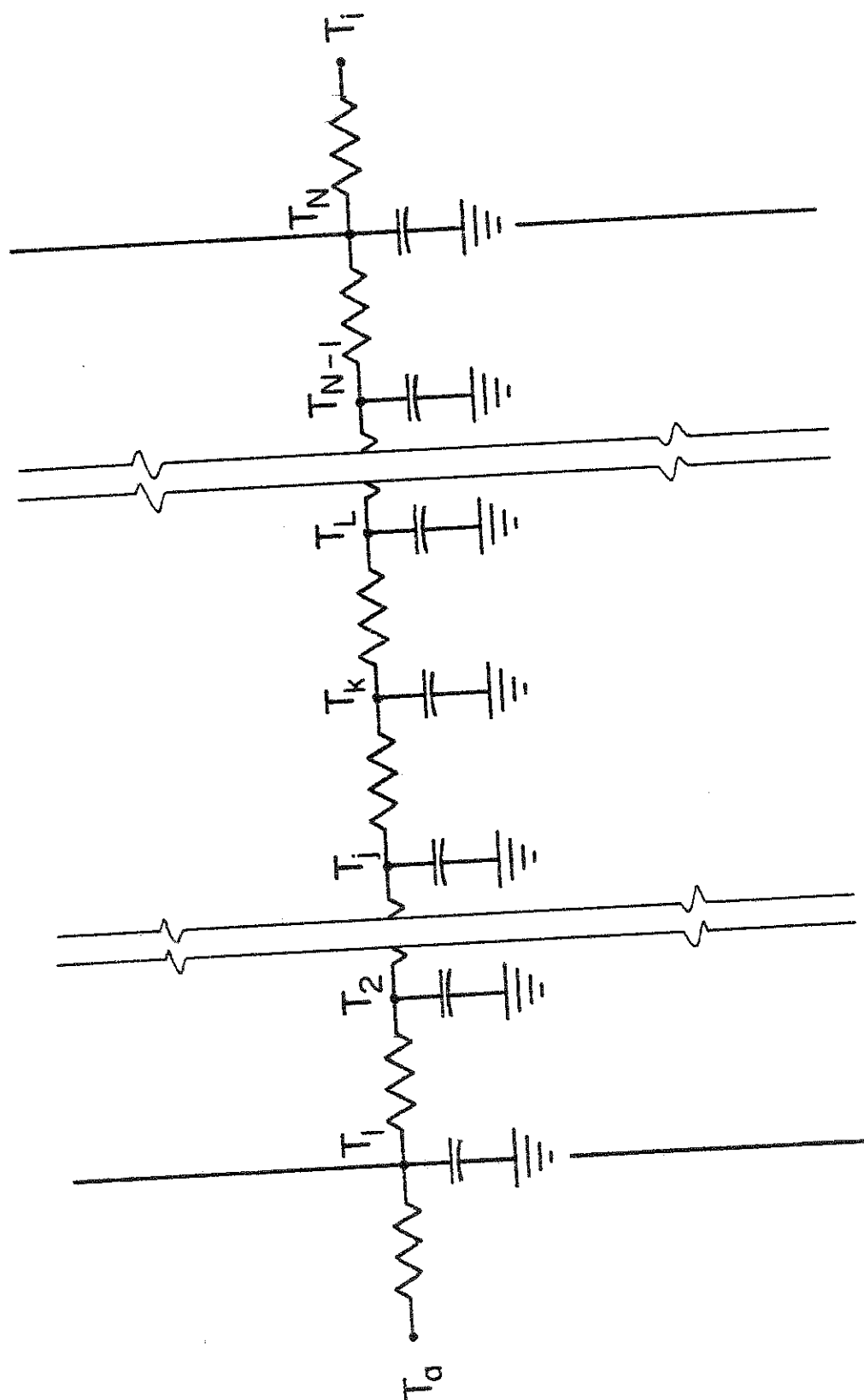
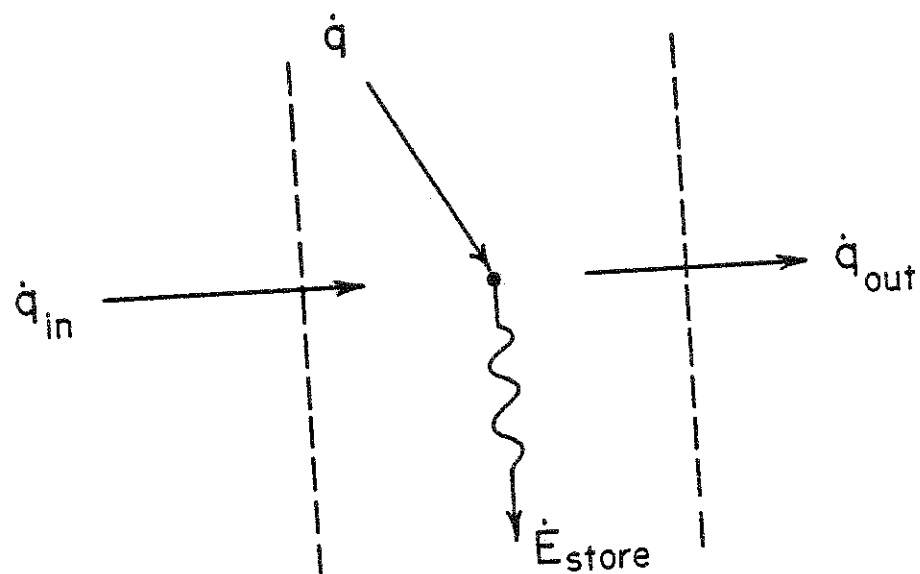
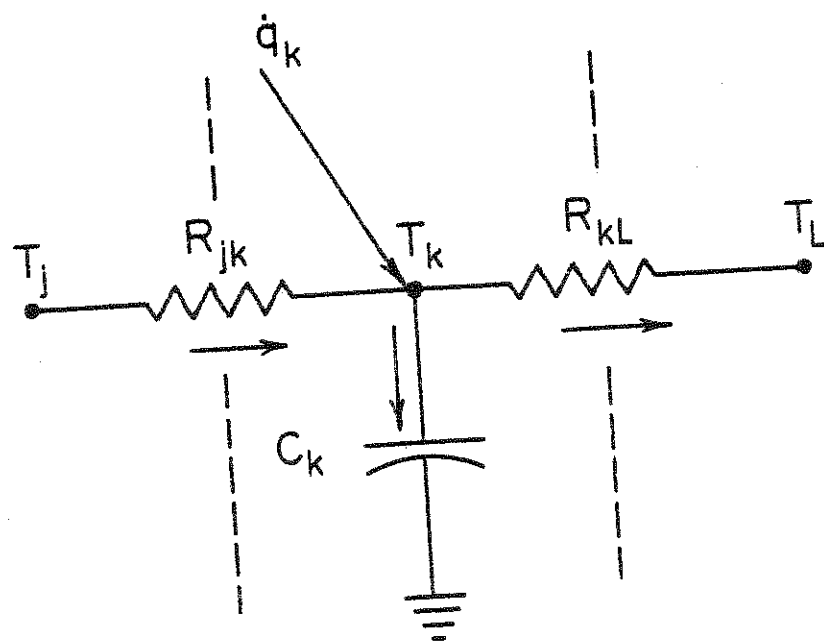


Figure 2-2 Finite-Difference Model Thermal Network



(a) General Nodal Energy Balance



(b) Nodal Thermal Network

Figure 2-3 Nodal Thermal Network and General Energy Balance

2.2.2 Lumped Model

The second wall model used in this derivation, the lumped model, represents the wall as a one-node thermal mass connected to the two boundaries by two resistances, as shown in Figure 2-4. The wall is characterized by a single temperature, as opposed to a set of N temperatures for an N node finite-difference model.

The lumped model is assumed to have the same physical dimensions as the finite difference model—finite only in the direction of energy flow. The thermal conductivity and density are also the same. The thermal capacitance in this model is different than the thermal capacitance of the finite-difference model. The thermal capacitance of the lumped model is the effective thermal capacitance, C_w , defined in Section 1.3.

In contrast to the finite-difference model which requires the solution of a set of N simultaneous ordinary differential equations, this model requires the solution of only one differential equation. The characteristic differential equation for this model is obtained by the application of the nodal energy balance, Equation 2.2.2, to the lumped system. The result is Equation 2.2.4.

$$\frac{T_a - T_w}{R_a} - \frac{T_w - T_i}{R_i} + \dot{q} = C_w \frac{dT_w}{d\tau} \quad 2.2.4$$

The thermal resistances of the finite-difference model are either internal conduction resistances or boundary resistances. This model consists of only a single node, so the lumped thermal resis-

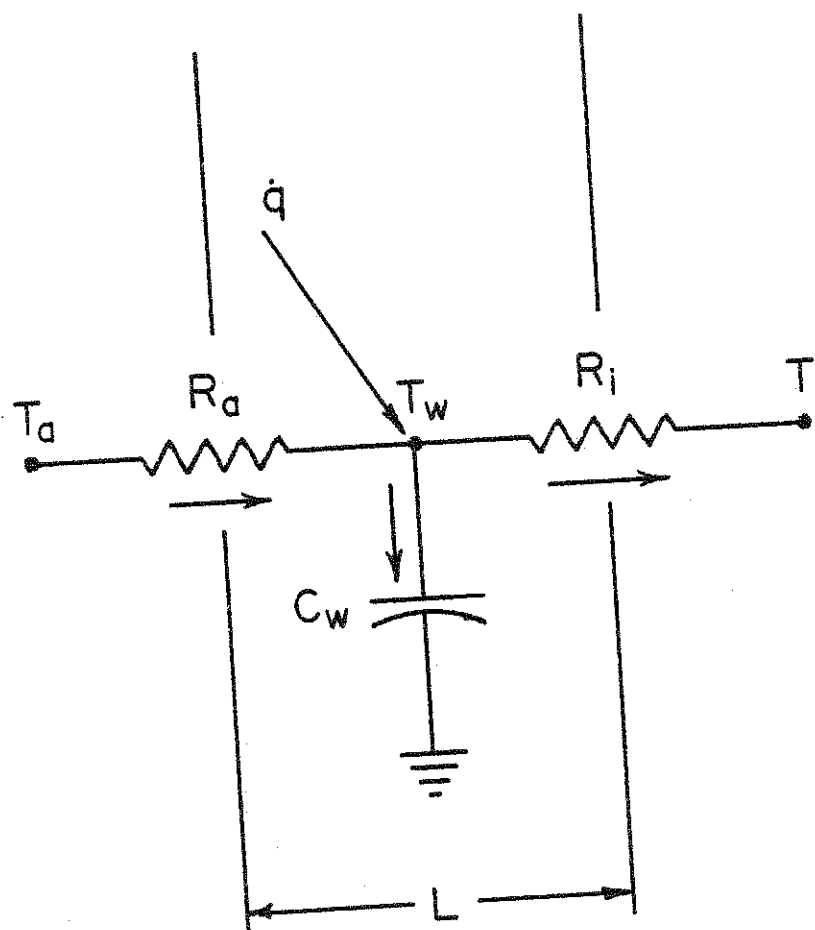


Figure 2-4 Lumped Model Representation

tances, R_i and R_a , are a combination of material (conduction) and boundary resistances. The two thermal resistances in series have the same overall resistance as that used in the finite difference model.

The lumped model is an approximation of the "real," distributed capacitance, finite-difference wall, as outlined in Section 1.3. Therefore, unlike the finite difference model where the sum of the nodal thermal capacitances is equal to the actual material mass times specific heat product, the thermal capacitance of this model is the effective thermal capacitance. The effective thermal capacitance is the capacitance which represents the distributed capacitance of the real system in a way such that the two models generate the same response for identical boundary conditions. The derivation of this value, C_w , is discussed in Section 2.3.

2.3 Derivation of Effective Thermal Capacitance From Finite-Difference--Lumped Model Comparisons

2.3.1 Assumptions

The assumptions made in this analysis are predominantly related to the nature of heat transfer through the wall. Conduction in the direction of the finite dimension is the only energy transfer considered within the wall. The assumption is made that any initial transient response of the wall can be ignored. The temperatures and energy flows of interest are those produced after the wall has reached a periodic steady-state response. Periodic steady-state is an approximation of diurnal temperature variation. All thermal and

physical properties of the wall are assumed to be constant, as well as any boundary convection coefficients. Finally, the assumption is made that the ambient boundary condition can be modelled as a sinusoidal input with a period of one day as reported in references 5 and 6.

2.3.2 Analytic Solution of the Lumped Model

The analytic solution of the lumped model consists of solving Equation 2.2.4 for known ambient and interior boundary conditions, thermal resistances R_i and R_a and known heat flow input \dot{q} . Equation 2.2.4 is solved for convective boundaries assuming sinusoidal ambient and interior temperature profiles. A special case of this solution, when only the ambient temperature profile is time-dependent is also presented.

The ambient temperature profile is defined by Equation 2.3.1 and the interior temperature profile by Equation 2.3.2. No heat flux input is considered.

$$T_a = \bar{T}_d + A_a \sin(\omega_a \tau) \quad 2.3.1$$

$$T_i = \bar{T}_i + A_i \sin(\omega_i \tau + \beta) \quad 2.3.2$$

The thermal resistance R_i and R_a are constant and are defined in Section 2.3.4.

Substituting the ambient and interior temperature conditions into Equation 2.2.4 and rearranging terms, Equation 2.3.3, the

lumped model characteristic differential equation is derived.

$$\begin{aligned} \frac{dT_w}{d\tau} + T_w \left(\frac{R_i + R_a}{C_w R_i R_a} \right) &= \frac{R_a \bar{T}_a + R_i \bar{T}_i}{C_w R_i R_a} + \frac{A \sin(\omega_a \tau)}{R_a C_w} \\ &+ \frac{A_i \sin(\omega_i \tau + \beta)}{R_i C_w} \end{aligned} \quad 2.3.3$$

The full solution of this equation consists of two parts: a transient solution, and a periodic steady-state solution. Since the transient response is ignored in this derivation, the corresponding portion of the solution is not presented here.

The general periodic steady-state solution of a differential equation of the form of Equation 2.3.3 is given in Equation 2.3.4.

$$\begin{aligned} T_w &= \bar{T} + B \sin(\omega_a \tau) + C \cos(\omega_a \tau) + D \sin(\omega_i \tau) \\ &+ E \cos(\omega_i \tau) \end{aligned} \quad 2.3.4$$

Substitution of Equation 2.3.4 and its first derivative into Equation 2.3.3, defines the parameters of Equation 2.3.4. Equations 2.3.5 through 2.3.10 give the results of this operation.

$$\bar{T} = \frac{R_i \bar{T}_a + R_a \bar{T}_i}{R_i + R_a} \quad 2.3.5$$

$$B = \frac{A_a m}{R_a C_w (m^2 + \omega_a^2)} \quad 2.3.6$$

$$C = \frac{-A_a \omega_a}{R_i C_w (m^2 + \omega_a^2)} \quad 2.3.7$$

$$D = \frac{A_i (\omega_i \sin\beta - m \cos\beta)}{R_i C_w (m^2 + \omega_i^2)} \quad 2.3.8$$

$$E = \frac{A_i (m \sin\beta - \omega_i \cos\beta)}{R_i C_w (m^2 + \omega_i^2)} \quad 2.3.9$$

where:

$$m = \frac{R_i + R_a}{R_i R_a C_w} \quad 2.3.10$$

Equation 2.3.4 can be presented in a simpler form with the application of trigonometry. The simplified form is presented in Equation 2.3.11.

$$T_w = \bar{T} + r_a \sin(\omega_a \tau + \phi_a) + r_i \sin(\omega_i \tau + \phi_i) \quad 2.3.11$$

where

$$\bar{T} = \text{see Equation 2.3.5}$$

$$r_a = \frac{A_a}{R_a C_w \sqrt{m^2 + \omega_a^2}} \quad 2.3.12$$

$$\phi_a = \tan^{-1} \left(\frac{-\omega_a}{m} \right) \quad 2.3.13$$

$$r_i = \frac{A_i}{R_i C_w \sqrt{m^2 + \omega_i^2}} \quad 2.3.14$$

$$\phi_i = \tan^{-1} \left(\frac{m \sin\beta - \omega_i \cos\beta}{\omega_i \sin\beta + m \cos\beta} \right) \quad 2.3.15$$

A special case of Equation 2.3.11 exists when the interior temperature, T_i , is constant at \bar{T}_i . The interior input amplitude A_i is equal to zero and Equation 2.3.11 reduces to:

$$T_w = \bar{T} + r_a \sin(\omega_a \tau + \phi_a) \quad 2.3.16$$

Equations 2.3.11 and 2.3.16 are the solutions of the lumped model for two different convective boundary conditions. The effective thermal capacitance of the wall is derived from the curve fit of the appropriate equation to the capacitance weighted mean wall temperature response of the finite-difference model. The capacitance weighted mean wall temperature must first be defined.

2.3.3 Capacitance-Weighted Mean Wall Temperature

The temperature response of the lumped model is characterized by a single temperature, T_w . The corresponding response of the finite-difference model is represented by a series of temperatures distributed throughout the wall. In order to compare the responses of the two models, a single temperature characteristic of the response of the finite-difference model must be defined. This temperature should be representative of the thermal storage capacity of the wall and the different storage capacities of the separate layers in a multi-layer wall.

The capacitance-weighted mean wall temperature, T_{wm} , is designated as the representative wall temperature of the finite-difference model. T_{wm} is the capacitance-weighted average nodal

temperature and is defined by Equation 2.3.17.

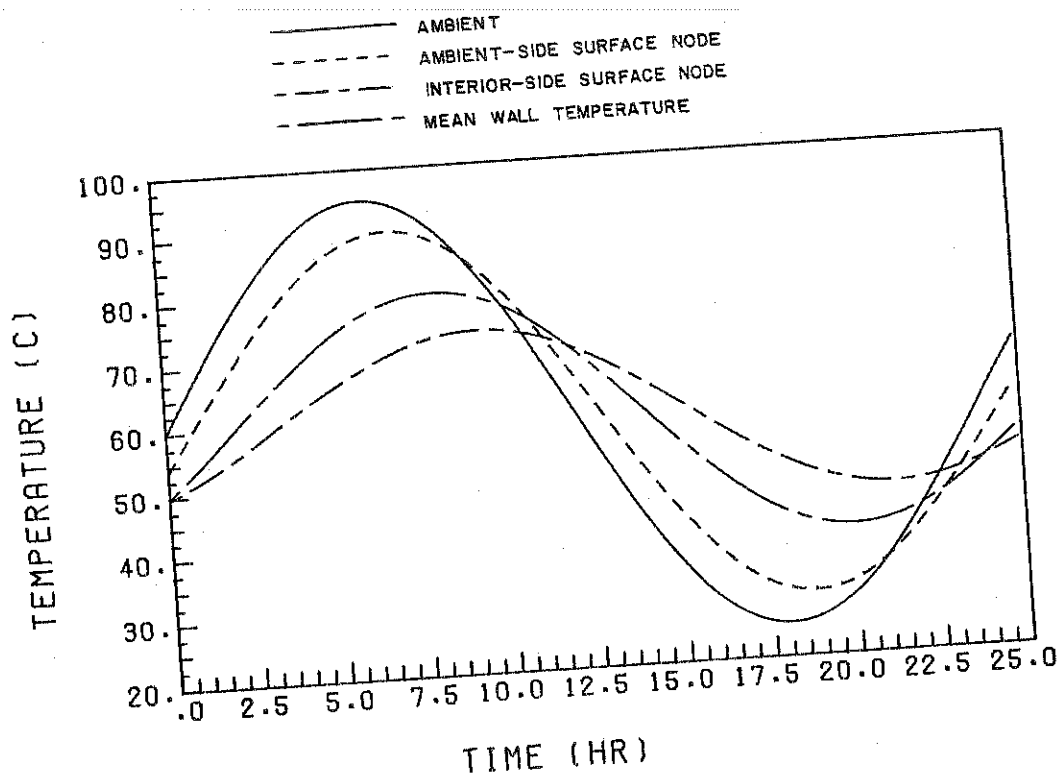
$$T_{wm} = \frac{\sum_{i=1}^N (C_i T_i)}{\sum_{i=1}^N C_i} \quad 2.3.17$$

If the wall consists of a single homogeneous slab, T_{wm} is given by Equation 2.3.18, a special case of Equation 2.3.17.

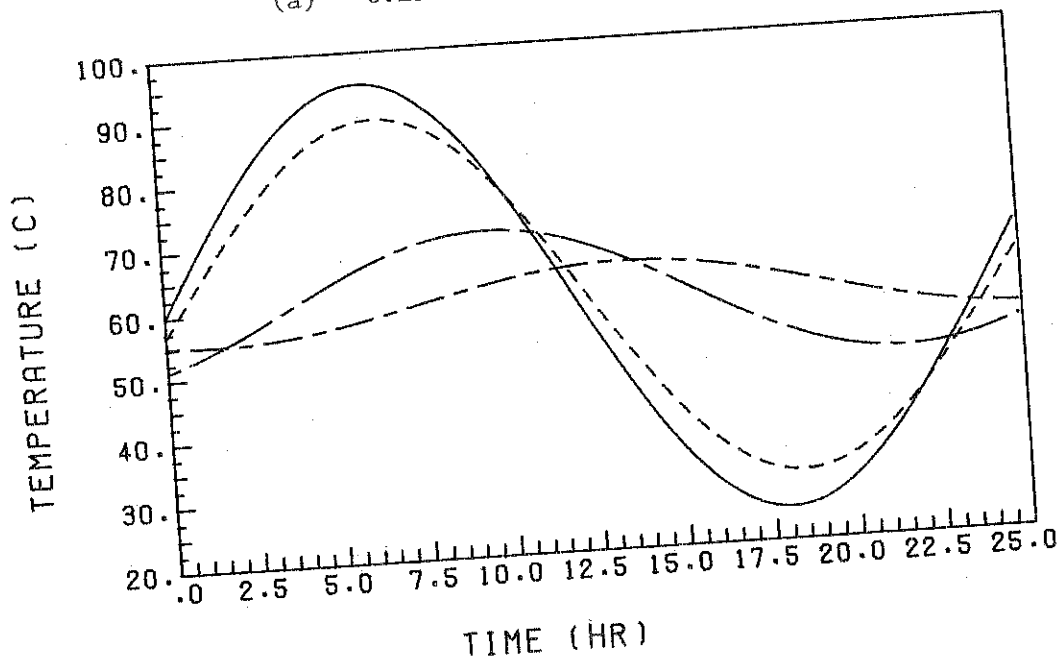
$$T_{wm} = \frac{1}{L} \sum_{i=1}^N (\Delta x_i T_i) \quad 2.3.18$$

2.3.4 Effective Thermal Capacitance of a Homogeneous Wall With Sinusoidal Ambient Temperature Boundary and Constant Interior Temperature Boundary

The underlying assumption in this method is that the periodic steady-state response of a real wall, as simulated by the finite-difference model, is of the same mathematical form as the response of the lumped model. Shown in Figures 2-5a, b, and c are the ambient, exterior surface node, interior surface node, and capacitance-weighted mean wall temperature responses of three homogeneous walls after periodic steady-state conditions have been reached. These results are from the finite-difference model solution. The wall material, thickness and boundary conditions are shown in Table 2-1. The three temperature responses plotted in each of the figures are sinusoidal, with the same characteristics as the ambient temperature

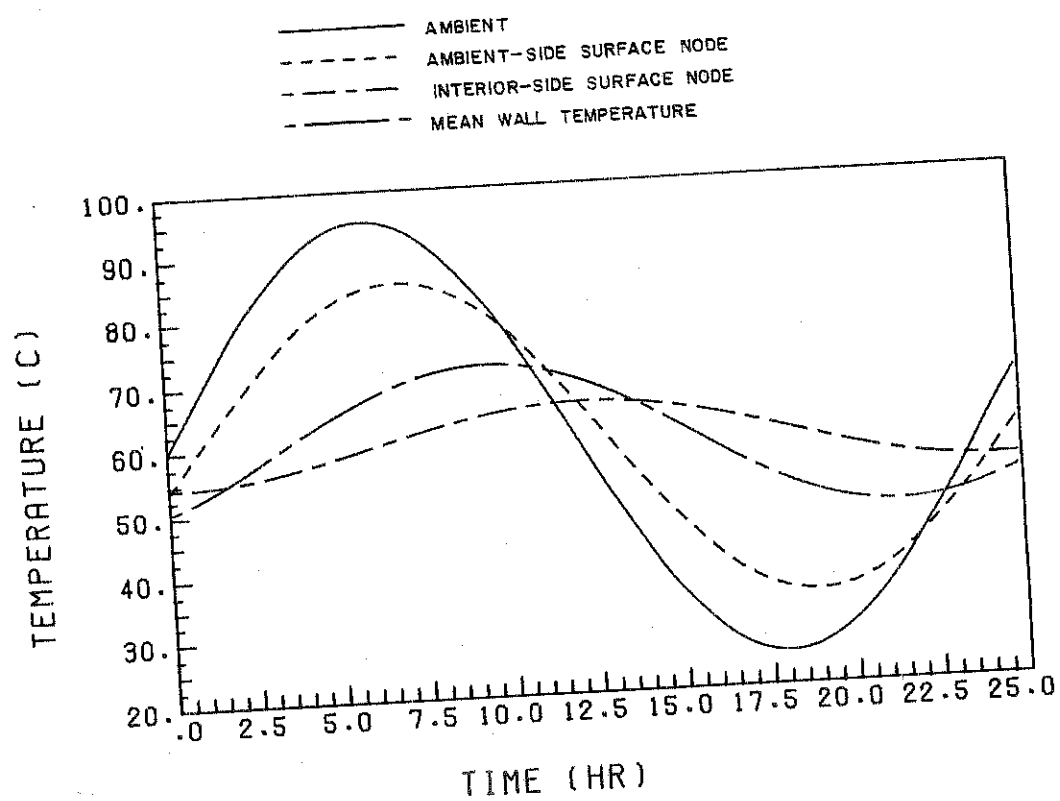


(a) 0.15 m Thick Concrete



(b) 0.30 m Thick Concrete

Figure 2-5 Examples of Periodic Steady State Temperature Response of Homogeneous Walls



(c) 0.42 m Thick Maple

Figure 2-5 (continued)

TABLE 2-1

Boundary Conditions and Composition of Finite-Difference Walls of
Figure 2-5

	2-5a	2-5b	2-5c
Figure			
Material	Concrete	Concrete	Maple
Thickness (m)	.153	.305	.419
T_a (C)	15.6	15.6	15.6
A_a (C)	19.4	19.4	19.4
ω_a (hr ⁻¹)	.262	.262	.262
$h_a \frac{W}{m^2 C}$	56.8	56.8	28.4
T_i (C)	15.6	15.6	15.6
A_i (C)	0	0	0
ω_i (hr ⁻¹)	Not applicable		
$h_i \frac{W}{m^2 C}$	8.52	8.52	8.52

forcing function. This is the same form as the corresponding lumped model response, Equation 2.3.16.

The similarity between the finite-difference and lumped model responses suggests that the effective thermal capacitance of the wall can be derived from a comparison of the two responses. The effective thermal capacitance of a wall was defined in Chapter 1 as the capacitance which makes the lumped system accurately represent the finite difference or real system.

Since the analytical form of the lumped model response is known, the lumped response equation can be curvefit to the finite-difference response. The effective thermal capacitance can be calculated from the application of the appropriate lumped model response parameter definition (Equations 2.3.5, 2.3.10, 2.3.12-2.3.15) to the value of the parameter found in the curvefit.

In this derivation, the effective thermal capacitance, C_w , can be obtained from two parameters of the lumped response, Equation 2.3.11: the amplitude, r_a , and the phase lag, ϕ_a . The two parameters do not yield the same value of C_w . Since the energy storage in a wall is related to the temperature swing of the wall, r_a was selected as the parameter from which C_w was to be determined.

Rearrangement of the definition of r_a , Equation 2.3.12, shows how C_w can be obtained if the other values in the definition are known. The result of this operation is presented in Equation 2.3.19.

$$C_w = \sqrt{\frac{\left(\frac{A_a}{R_a r_a}\right)^2 - \left(\frac{R_i + R_a}{R_i R_a}\right)^2}{\omega_a^2}} \quad 2.3.19$$

Before this equation can be applied, the thermal resistances, R_i and R_a must be defined. It was stated in Section 2.2.2 that R_i and R_a are both a combination of boundary and material resistances. The resistance due to the boundary is easy to define. It is simply the inverse of the respective convection coefficient. The definition of the material resistance is somewhat less clear. For lack of a better convention, one-half of the material resistance was assigned to each R_i and R_a , thus defining the two thermal resistances, as shown in Equations 2.3.20 and 2.3.21.

$$R_a = \frac{1}{h_a} + \frac{L}{2k} \quad 2.3.20$$

$$R_i = \frac{1}{h_i} + \frac{L}{2k} \quad 2.3.21$$

The response amplitude, r_a , (as well as \bar{T} and ϕ_a), is determined from a curvefit, as stated above. The period of the response is known. \bar{T} is found by averaging the response over a period. Secondly ϕ_a is determined. Finally r_a is obtained by applying the partially fitted equation at any known data point. All parameters of Equation 2.3.19 are now known and C_w may be determined. The results of this method are presented in Section 2.4.

2.4 Results of The Finite-Difference Method

The results of the derivation of the effective thermal capacitance of a homogeneous wall exposed to a sinusoidal ambient temperature boundary on one side, and to a constant temperature on the other are presented in this section. The fractional thermal capacitance, F_{CAP} , i.e., the ratio of the derived effective thermal capacitance to the actual capacitance (mc_p) of the wall, is presented graphically in Section 2.4.1 as a function of two dimensionless parameters, an average Biot modulus and a periodic Fourier modulus. The graph was generated from the analysis of concrete walls of various thicknesses exposed the same ambient temperature boundary. The fractional thermal capacitance of concrete walls exposed to other ambient temperature means and amplitudes and F_{CAP} for other homogeneous materials are presented in Section 2.4.2.

The computer program used to obtain the response of the finite-difference walls is listed in Appendix A. The routine used to curve-fit the lumped model response to the capacitance weighted mean wall temperature response is presented as subroutine VALUES in the Appendix.

2.4.1 Fractional Thermal Capacitance of A Homogeneous Concrete Wall

The results in this section were obtained from finite-difference simulations of concrete walls of various thicknesses exposed to a simulated diurnal temperature variation on the ambient side and to a constant temperature on the interior side. The ambient and in-

terior convection coefficients, h_a and h_i , were also varied in this analysis, with the condition that h_a was greater than or equal to h_i . The properties of the wall, the boundary temperature conditions and the convection coefficients of this analysis are given in Table 2-2.

It was found that the fractional thermal capacitance of a homogeneous concrete wall could be represented as a function of two dimensionless quantities: the average Biot number, defined by Equation 2.4.1, and a periodic Fourier modulus, defined by Equation 2.4.2.

$$\overline{Bi} = \frac{(h_a + h_i)L}{2k} \quad 2.4.1$$

$$Fo, \omega = \frac{\alpha}{\omega_a L^2} = \frac{\alpha P}{2\pi L^2} \quad 2.4.2$$

The interpretation of \overline{Bi} is analogous to the interpretation of the conventional Biot modulus defined in heat transfer analysis [10]. The value represents a comparison of the relative magnitudes of the conductive and convective resistances of the system. A very low value of \overline{Bi} indicates that internal conduction resistance is negligible in comparison to the surface convective resistance. This implies that the material is nearly at a uniform temperature throughout.

Normally, the Fourier modulus, $Fo, \left(\frac{\alpha T}{L^2}\right)$ is viewed as a comparison between a characteristic body dimension and an approximate temperature wave penetration at a specific time during the transient

TABLE 2-2
 Material Properties [9], Boundary Conditions and Parameter Ranges
 Used For Derivation of Figure 2-6

Material	Concrete
ρ	2080 kg/m ³
k	1.38 W/m-C
c_p	0.838 kJ/kg-C
α	7.91×10^{-7} m ² /s
L	.076-.914 m
\bar{T}_a	15.6 C
A_a	19.4 C
P	24 hr
\bar{T}_i	15.4 C (constant)
h_a	8.5-113.5 W/m ² -C
h_i	8.5-45.4 W/m ² -C

response of a system. This analysis is a periodic steady state analysis, and the Fourier modulus, Fo, ω , is both defined and interpreted somewhat differently. Fo, ω is a comparison between a characteristic body dimension, the slab thickness, and an approximate temperature wave penetration during one ambient temperature cycle.

Since the fractional thermal capacitance is a function of \overline{Bi} and Fo, ω limiting cases of F_{CAP} as a function of these two parameters can be determined. It is commonly assumed in heat transfer analysis that any system with a characteristic Biot modulus smaller a certain value is at a uniform temperature. This implies that the system is lumped. In this case the effective thermal capacitance is equal to the actual value.

A system is normally assumed to be lumped if Equation 2.4.3 is true [10]. Although an average Biot modulus is used to characterize this system, it is assumed that the same criterion holds. Therefore, when Equation 2.4.3 is true, F_{CAP} is given by Equation 2.4.4.

If

$$\overline{Bi} \leq 0.1 \quad 2.4.3$$

Then

$$F_{CAP} = 1.0 \quad 2.4.4$$

A second limiting case is given by Equations 2.4.5 and 2.4.6.

For

$$Fo, \omega = \infty \quad 2.4.5$$

2.4.6

$$F_{CAP} = 1.0$$

Equation 2.4.5 is true when either the thermal diffusivity or the ambient temperature period is infinite, or as the material thickness approaches zero. Physically these three situations have little meaning, but from a theoretical viewpoint, a limit is provided. If the thermal diffusivity is infinite, it is implied that all of the slab responds instantaneously. Similarly if the ambient temperature period is large enough, the entire slab will respond within that period. Finally, zero thickness indicates that the wall has no thermal capacitance. If the wall has no capacitance, the effective thermal capacitance must also be zero. The two capacitances are equal so F_{CAP} is equal to 1.0.

These two limits provide the basis for one point and one line in Figure 2-6, which presents F_{CAP} as a function of \overline{Bi} and Fo, ω . Equations 2.4.3 and 2.4.4 imply that the point 0.1, 1.0 is common for all values of Fo, ω . The plotted data indicate this is true. The line defined by Equations 2.4.5 and 2.4.6 appears to be an asymptotic limit.

2.4.2 Extension of Concrete Wall Results to Other Ambient Temperature Mean and Amplitudes and to Homogeneous Walls of Other Materials

The representation of the fractional thermal capacitance of a homogeneous slab presented in the last section was derived with

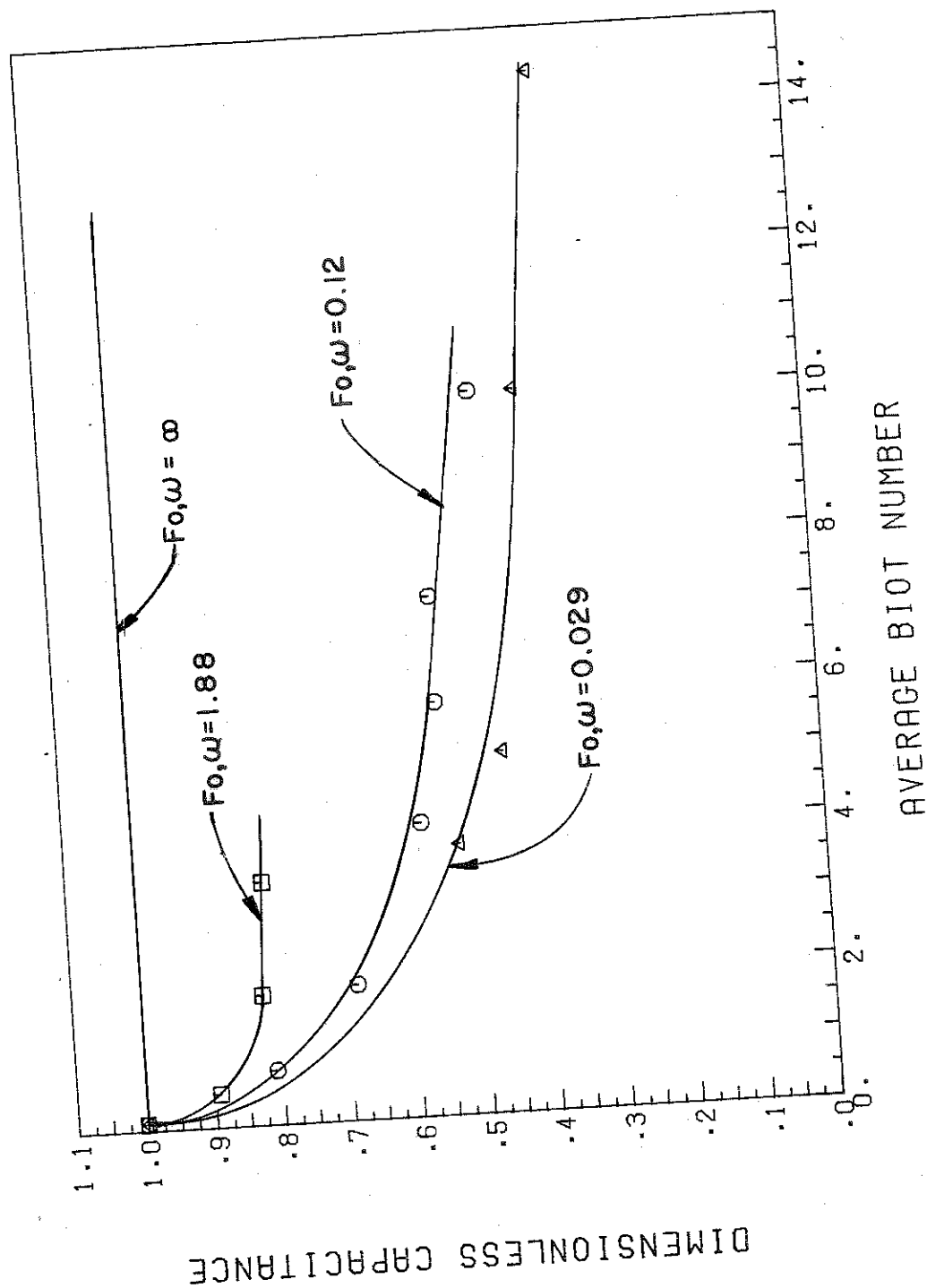


Figure 2-6 Fractional Thermal Capacitance of a Homogeneous Slab

minimal parameter variation. Only concrete walls and only one temperature boundary were considered in the derivation. One way to test the applicability of a representation like Figure 2-6 to other boundary conditions and to walls of other materials, is to compare the fractional thermal capacitance predicted from the figure or derived for the conditions presented in Table 2-2, to the fractional thermal capacitance derived for other materials or boundary conditions.

Investigation of the effect of the ambient temperature amplitude and mean on fractional thermal capacitance was conducted on the homogeneous concrete walls characterized by \overline{Bi} and Fo, ω presented in Table 2-3. Finite-difference simulations of the walls were conducted for the ambient temperature conditions given in Table 2-4, and the effective thermal capacitance was determined.

Table 2-4 also presents the derived and predicted values of the fractional thermal capacitance of these walls. The predicted values were read from Figure 2-6 for wall 1, and derived under the same conditions as those used in the derivation of the Figure 2-6 for wall 2 and wall 3. For the cases studied, variations in ambient temperature amplitude and mean have no effect of the fractional thermal capacitance of a concrete wall.

Homogeneous walls consisting of maple, granulated cork, and asbestos cement board were simulated to see if a representation such as Figure 2-6 is applicable to other homogeneous wall constructions. The properties of these three materials are presented in Table

TABLE 2-3

Biot and Fourier Moduli of Concrete Walls Simulated For Study of Ambient Temperature Variation Effects

	\overline{Bi}	Fo, ω
Wall 1	10.	.12
Wall 2	1.0	.18
Wall 3	10.0	.18

TABLE 2-4

Derived and Predicted Values of Fractional Thermal Capacitance of Concrete Walls for Various Ambient Temperature Amplitudes and Means

	Ambient Temperature		Fractional Thermal Capacitance	
	T_a (C)	A_a (C)	Derived	Predicted
Wall 1	21.1	19.6	.48	.48
Wall 1	26.7	13.9	.48	.48
Wall 1	- 1.1	19.6	.48	.48
Wall 2	10.0	16.7	.78	.78
Wall 3	10.0	16.7	.48	.48

$T_i = \overline{T}_i = 15.4^\circ \text{C}$ for all cases.

2-5. The ambient and interior temperatures in this analysis are given in Table 2-2.

The predicted and derived fractional thermal capacitances of the walls are presented in Table 2-6. Similar to the result obtained above, for the cases studied, a representation such as Figure 2-6, derived for one material, is applicable to other homogeneous constructions. Any differences between the predicted and derived values are probably due to numerical error generated by the finite difference approximation.

Comparison of C_w to Balcomb's diurnal heat capacity (dhc) [4] and to Davies' thermal admittance [6] shows that C_w is the same as dhc and $P/2\pi$ times the thermal admittance. In Balcomb's dhc method, the diurnal heat capacity of walls with convection boundaries can only be determined if one side of the wall is insulated or if the two convection boundaries are identical. Figure 2-6 is valid when the two convection boundaries are not identical. The values of C_w obtained from the figure may be used with Balcomb's design method. The figure should only be used for one-periodic--one-constant temperature boundaries cases, since the two periodic temperature boundary condition case has not been analyzed.

2.5 Discussion and Problems of The Finite-Difference--Lumped Solution Curvefit Method

In this chapter, the method of derivation of the effective ther-

TABLE 2-5

Properties of Maple, Cork, and Asbestos Cement Board [9]

Material	Density $\left(\frac{\text{kg}}{\text{m}^3}\right)$	Thermal Conductivity $\left(\frac{\text{W}}{\text{m}\cdot\text{C}}\right)$	Specific Heat $\left(\frac{\text{kJ}}{\text{kg}\cdot\text{C}}\right)$	Thermal Diffusivity $\left(\frac{\text{m}^2}{\text{s}} \times 10^7\right)$
Maple	721.	1.90	1.26	20.9
Cork	86.5	0.048	2.03	2.7
Asbestos Cement Board	1520.	5.48	1.00	28.5

TABLE 2-6

Predicted and Derived Fractional Thermal Capacitance of Maple,
Cork and Asbestos Cement Board Walls

Material	Bi	Fo, ω	Fractional Thermal Capacitance	
			Derived	Predicted
Maple	1	0.311	0.78	0.80
Maple	10	0.311	0.48	0.50
Cork	10	0.51	0.48	0.48
Asbestos Cement Board	0.76	0.12	0.81	0.81

mal capacitance of a homogeneous wall from the comparison of two temperature responses was presented for a specific set of boundary conditions. The results presented showed that the fractional thermal capacitance, the ratio of the effective to actual value, is defined by two parameters, the average Biot modulus and a periodic Fourier modulus, for a homogeneous concrete wall. Additional results showed that this same representation applies to other homogeneous wall constructions, and to other values of ambient temperature amplitudes and means.

One basic underlying premise in this method became the major factor which led to the abandonment of this method. The effective thermal capacitance derived in this chapter is based on system temperature response. The effective thermal capacitance is the value which produces the same lumped model response as a designated temperature response of the finite difference model. Wall temperature response does reflect the energy stored in a wall. A more direct way to consider energy stored in the wall is to model energy flows rather than temperatures.

In building energy analysis, it is not the temperature response of the building elements that is not generally of interest, but rather the energy contribution of the elements to the heating and cooling requirements. To the designer the temperature response of added thermal mass is not desired; it is desired to estimate the effect of this extra thermal mass on the heating and cooling requirements of the building. For example, Monsen's Direct-Gain Unutiliza-

bility Design Method [3] requires the estimation of the room thermal capacitance in order to estimate the amount of solar energy which can be used to reduce a conventional heating load.

Therefore, the effective thermal capacitance should be based on energy quantities opposed to temperature response. The effective thermal capacitance should be the value which results in identical energy requirements for both the lumped and "real" systems.

Like the methods of Balcomb [4] and Davies [6], the ambient temperature in this method was modelled as a diurnal sinusoid. Although daily temperature variation is basically sinusoidal, it is not a perfect sinusoid. The effect of actual ambient temperature variation on effective thermal capacitance should be investigated. Discrete temperature data is compatible with the finite difference model, but a simple (if any) analytic solution of the lumped model can not be derived for discrete data. The lumped model would have to be simulated by a computer with only an estimate of the effective thermal capacitance and then compared to the finite-difference solution. This would be an iterative process requiring multiple simulations of the lumped model. The finite-difference and lumped models could be adapted to this situation, but existing computer programs, such as TRNSYS [1], are less costly to use.

Even if further investigation proved that a simple representation such as Figure 2-6 was accurate enough to estimate effective thermal capacitance for "real" conditions for homogeneous walls, it would be difficult to estimate the effective thermal capacitance of a

nonhomogeneous wall. Composite walls are not characterized by single values of thermal conductivity, density and specific heat. Thus it would be difficult to estimate \overline{Bi} and Fo, ω since they require single values of thermal and physical parameters. A simple representation such as Figure 2-6 may not be found.

These problems: The effective thermal capacitance defined by temperature response, opposed to energy response, periodic steady-state assumptions, and difficulties anticipated in the estimation of the Biot and Fourier moduli for composite walls, were the reasons this method was abandoned in favor of the transfer function method of Chapter 3. The results produced here are meaningful but the method of Chapter Three better suits the goals of this study.

3. EFFECTIVE THERMAL CAPACITANCE OF A ONE-ZONE STRUCTURE

3.1 Introduction

In this chapter, two models are developed and used to derive the effective thermal capacitance of a simple one-zone structure that consists of only four walls. One model consists of transfer function wall representations coupled to a room model. Transfer functions model energy flow through a wall as an algebraic function of the boundary conditions and previous heat flows. The second model is a modified degree-day load model connected to a room.

Two transfer function equations exist which can be used to model conduction through walls. The current TRNSYS [1] transfer function wall model uses the equation presented in the ASHRAE Handbook of Fundamentals [9]. The equation was derived assuming a constant interior temperature boundary condition. However, the equation is applied in the TRNSYS program for situations in which the room temperature varies between specified limits. The application of the ASHRAE equation is not strictly correct in these situations.

A second transfer function equation exists for a floating interior temperature boundary condition [8, 11], and is used in this derivation. The drawback of this equation is that the transfer function coefficients must be determined by the execution of computer programs [8, 12]. The coefficients for the constant room temperature

equation are readily available in published form for many walls [9].

Computer simulations are conducted to compare the heating and cooling loads predicted by the two transfer function wall model equations. The errors generated through the use of the constant room temperature equation in floating room temperature situations are assessed.

The effective thermal capacitance of a simple one-zone structure is determined through comparison of transfer function model and modified degree-day model results. For given boundary conditions, weather data, wall construction, and room capacitance, a distinct set of heating and cooling requirements are generated by simulation using the transfer function model. The modified degree-day model loads are a function of the thermal capacitance used in the model for the same conditions. The effective thermal capacitance of the structure is the value, when used in the modified degree-day model, that causes the degree-day model loads to agree with the transfer function model loads.

The effective thermal capacitance is a function of the construction material, boundary conditions, and type of load. The effective thermal capacitance of a structure derived for one set of simulation conditions may not be the same as the value derived for a different set. The values derived from heating and cooling load comparisons may also differ. The value may even change from month to month. The utility of the effective thermal capacitance concept is diminished if no one single value or fraction of the actual capacitance

can be used for a wide range of conditions. The errors generated by the use of a specified fraction of the real capacitance as the effective capacitance are assessed to determine the feasibility of the concept.

3.2 One-Dimensional Transfer Function Wall Models

The transfer function approach to modelling the thermal response of walls is a method which enables the replacement of the time consuming and expensive numerical solution of the governing differential equations. The heat flow at the surface of a wall is expressed algebraically as a function of previous heat flows and temperatures in the transfer function approach. The method is well documented [11, 12, 13, 14, 15, 16] and information about the method not contained here can be obtained from these sources.

The transfer function representation of a plane wall with two time dependent boundary conditions, s_l and s_r , is given by Equation 3.2.1.*

$$u_\tau = \sum_{n=0}^N b_n s_{l,\tau-n\Delta} - \sum_{n=1}^M d_n u_{\tau-n\Delta} - \sum_{n=0}^N c_n s_{r,\tau-n\Delta} \quad 3.2.1$$

*A word should be said about the sign convention of this equation and the other transfer function equations in this Chapter. Equation 3.2.1 uses the sign convention of Mitalas, Stephenson and Arsenault (MSA). The equivalent equation contained in the works of Ceylan uses a plus (+) sign instead of a minus sign in front of the s_r and u summations. As a consequence, the c_n and d_n coefficients derived from Ceylan's program have the opposite sign of those generated by the MSA program. In this Chapter, unless noted otherwise, the sign convention of MSA is followed.

If only one time dependent boundary condition exists and the other is constant, the transfer function representation of the wall is a special case of Equation 3.2.1 given by Equation 3.2.2.

$$u_{\tau} = \sum_{n=0}^N b_n s_{l,\tau-n\Delta} - \sum_{n=1}^M d_n u_{\tau-n\Delta} - s_r \sum_{n=0}^N c_n \quad 3.2.2$$

The transfer function coefficients (b_n , c_n , d_n) depend on the particular wall construction, the output desired (u), and the boundary conditions (s_l , s_r). Mitalas and Arsenault [12] have developed a calculation procedure to determine the coefficients of a wall using Z-transfer functions. Ceylan [8] has developed an alternate method to calculate the coefficients. In Ceylan's method, the coefficients are determined from the analytic solution of the first-order differential equations generated from the discretization of the wall by finite-differences. Computer programs are available for both the Z-transfer function method [12], and Ceylan's method. Pawelski [17] developed a third method to find the coefficients. In his method the coefficients are determined from a linear least squares regression technique applied to a transient response generated by an alternate method, such as finite-differences or experimental data.

The transfer function coefficients required for the calculation of heat flow through a wall as a function of time dependent ambient temperature and constant interior temperature have been published for many exterior wall and flat roof types in the ASHRAE Handbook of Fundamentals, and thus are readily available. These coefficients were derived from the Z-transfer function method. The TRNSYS Type 17 wall

and Type 18 roof models are compatible with these coefficients.*

The Types 17 and 18 models are often coupled to a type 19 room model to simulate a one-zone structure. The interior temperature in the room model is allowed to float between user specified limits. The use of the ASHRAE coefficients is not strictly correct in this case. Coefficients derived for a time-dependent interior boundary condition should be used.

However, computer programs must be used to obtain the floating room temperature (FRT) coefficients. Therefore, during this study, the differences in the loads calculated by the two models were assessed. Presented in the following two subsections are the applicable transfer function equations for the two different interior temperature conditions. General comments about the differences expected between the two models are also presented.

3.2.1 Constant Interior Temperature Heat Flow Equation

Equation 3.2.2 is used to calculate the heat flow through a wall with a constant interior temperature. The ambient conditions are characterized by the sol-air temperature, T_e , and the room temperature is constant at T_r . Inserting these conditions into Equation 3.2.2, the constant room temperature transfer function heat flow equation, Equation 3.2.3, is defined.

*The components were written using the sign convention of Mitalas, Stephenson, and Arsenault. Coefficients generated by Ceylan's program may be used with these components if the negative of the c_n and d_n are input to the components.

$$q''_{\tau} = \sum_{n=0}^N b_n T_{e,\tau-n\Delta} - \sum_{n=1}^M d_n q''_{\tau-n\Delta} - T_r \sum_{n=0}^N c_n \quad 3.2.3$$

There is a constraint between the coefficients of this equation and between the coefficients of the equation presented in the next section which must be satisfied. If the sol-air temperature is constant with time, for every value of n

$$T_{e,\tau-n\Delta} - T_r = T_e - T_r \quad 3.2.4$$

Equation 3.2.3 simplifies to

$$q''_{\tau} (1.0 + \sum_{n=1}^M d_n) = T_e \sum_{n=0}^N b_n - T_r \sum_{n=0}^N c_n \quad 3.2.5$$

This is steady-state condition. Under these conditions, the heat flow can also be represented by the overall wall conductance (U-value) as

$$q''_{\tau} = U_w (T_e - T_r) \quad 3.2.6$$

Comparing Equations 3.2.5 and 3.2.6 establishes that

$$U_w = \frac{\sum_{n=0}^N b_n}{1 + \sum_{n=1}^M d_n} = \frac{\sum_{n=0}^N c_n}{1 + \sum_{n=1}^M d_n} \quad 3.2.7^*$$

and

$$\sum_{n=0}^N b_n = \sum_{n=0}^N c_n \quad 3.2.8^*$$

These two constraints may be useful to check the accuracy of any coefficients derived from computer simulation.

3.2.2 Floating Interior Temperature Transfer Function Heat Flow

Equation

The heat flow through a wall to a room with floating interior temperature is calculated from the application of Equation 3.2.1. Like the CRT model, the ambient conditions are represented by the sol-air temperature T_e , and the room temperature is T_r . Inserting

*The alternate sign convention of Ceylan dictates that

$$U_w = \frac{\sum_{n=0}^N b_n}{1 - \sum_{n=1}^M d_n} = \frac{-\sum_{n=0}^N c_n}{1 - \sum_{n=1}^M d_n}$$

and

$$\sum_{n=0}^N b_n = - \sum_{n=0}^N c_n$$

these conditions into Equation 3.2.1 gives the FRT transfer function heat flow equation, Equation 3.2.9.

$$q''_{\tau} = \sum_{n=0}^N b_n T_{e,\tau-n\Delta} - \sum_{n=1}^M d_n q''_{\tau-n\Delta} - \sum_{n=0}^N c_n T_{r,\tau-n\Delta} \quad 3.2.9$$

The same constraints (Equations 3.2.7 and 3.2.8) apply to this equation as Equation 3.2.3.

Equation 3.2.9 can also be applied if the room temperature, T_r , is constant. Equation 3.2.3 is a special case of the more general FRT equation, Equation 3.2.9. The coefficients in the two equations are exactly the same.

Since the CRT heat flow equation is a special case of the more general FRT equation, the energy flows predicted by the two models will be equal when the room temperature is constant. However, when the room temperature is changing with respect to time, the previous room temperatures become important. The CRT equation does not account for the interior-temperature history. The current room temperature is the only important value in the CRT model. Therefore the model over-emphasizes the importance of the current room temperature.

The different manner in which the room temperature history is treated in the two models will cause the heat flows calculated by the two equations to be different when the room temperature is not constant. The amount of disagreement between the two models will be a function of the allowable limits on the room temperature. As the allowable temperature range of the room decreases, the heat

flows predicted by the two models should approach the same value. In the limit of no room temperature variation, the two models predict the same results. The disagreement between the two models will also be a function of the wall construction since the transfer function coefficients depend on the physical properties of wall.

In the calculation of monthly loads, the differences between the results of the two models will be related to the amount of variation of the room temperature during the month. If conditions are such that the room temperature shows little variation during the month, the loads predicted by the two models should be relatively close. During a month when the room temperature variation is quite large, greater differences are expected between the models. A similar relationship should be true for seasonal or annual loads.

3.3 Simulation Models of A Simple One-Zone Structure

This section describes the simulation models used to compare the CRT and FRT transfer function heat flow equations. A description of the modified degree-day model used to determine the effective thermal capacitance of a simple structure is also presented.

The structure considered consists only of four walls, all of the same construction. A window is included in the south wall of the structure when solar gains are considered. The roof and floor of the structure are assumed to be adiabatic. The structure is assumed to be tightly constructed and no energy gains or losses occur due to air infiltration.

The models used in this derivation are "constructed" using existing components of TRNSYS. However, some changes have been made to the components. The modifications, and qualitative component descriptions are presented below.

The models presented in this section are described by component only, so a brief overview of the system models is presented. Three system models are employed in this analysis: the two transfer function models, and the modified degree-day model. The two transfer function models are identical, except one model uses Equation 3.2.3 to model heat flow through the wall (CRT model) and the other uses Equation 3.2.9 (FRT model). The transfer function models consist of the transfer function wall models connected to a floating temperature room model. The modified degree-day model uses degree-hour calculations to model conduction through the walls. Similar to the transfer function models, the walls are connected to a floating temperature room.

3.3.1 Transfer Function Wall Models

The transfer function wall models use the transfer function method described in Section 3.2 to calculate the heat flow through the walls. The TRNSYS transfer function wall model permits modeling a single wall, or four walls, if all walls are of the same construction. Since this study involved modelling a structure constructed of four identical walls, the four wall option was selected. It is possible to specify windows in any of the walls.

When solar gains were considered, a window was specified in the south wall of the structure.

The present TRNSYS Type 17 transfer function wall model uses Equation 3.2.3 to calculate the heat flow through the walls. A second transfer function wall model was written which uses Equation 3.2.9. Ceylan's program, RC1 [8] was used to generate the coefficients for the FRT model. All other aspects of the model are the same as the current model. The FRT model Fortran program listing is presented in Appendix B.

3.3.2 Room Model

The room model used in this study is the TRNSYS Type 19, originally developed by Pawelski [17], with some modifications. Two different types of control strategies may be used with this model, namely, energy-rate control, and temperature-level control. Energy-rate control is used in this study.

Energy-rate control is a strategy which calculates the load independent of auxiliary energy sources. In this control strategy it is assumed that the furnace, air conditioner and ventilation are of sufficient capacity and are controlled in such a manner that when heating or cooling is required the rate of energy addition or removal meets the load exactly. In addition, the room temperature is fixed at either the upper or the lower set point. The heating and cooling loads generated by this model with energy-rate control are the nominal heating and cooling requirements of the structure.

Temperature-level control is a strategy which takes into account the performance characteristics and transient behavior of the devices (furnace, air conditioner, etc.) which add or remove energy from the room, and the interaction between these devices and the load. With this control strategy, the rate of energy addition to or subtraction from the room is device dependent, related to the capacity of the device. The rate of energy addition or removal does not exactly meet the load in most cases. Excess energy may therefore be added or removed from the room. The energy requirements of the room are device-dependent when temperature-level control is used.

As written, the minimum and maximum allowable room temperatures used with energy-rate control were input to the model as fixed parameters. To consider the effects of night set back, the model was modified to allow the minimum and maximum temperatures to vary by designating these two values as inputs. The values may then be controlled by an external source such as a forcing function.

A second modification made to the room model involved the time distribution of the room energy gains. In the model, the loads due to solar heat gain, conduction, equipment, people, and lights, were time-distributed by a transfer function which calculated the current load from each source as a function of the current gain and previous loads and gains. The transfer function coefficients used to distribute the gains are a function of the type of gain and construction weight, as outlined in the ASHRAE Handbook of Fundamentals [9]. The values of the transfer function coefficients have been

changed as subsequent volumes were published. The model as written used coefficients from the 1972 Handbook. The values in the 1977 Handbook were different. Although some heat gains do not immediately become part of the cooling or heating load, the uncertainty of the magnitude of the transfer function coefficients led to the decision to disable the time distribution feature of the model.

The solution of the current heat flow and temperature when the Type 17--Type 19 wall--room combination is used is an iterative process. In order to calculate the current conduction heat flow through the walls, the current room temperature is required (see Equations 3.2.3 and 3.2.9). Yet to calculate the current room temperature, the current heat flow through the walls is required. As written, the type 19 room model allowed iteration, but the time distribution of gains, which is a function of the current gain, was only calculated at the beginning of the time-step.

The time distributed gains are used to calculate the room temperature. This new value of room temperature was input to the wall model and a new heat flow was calculated. When this new heat flow was input back to the room model, a new time distributed gain was not calculated, and the same room temperature was predicted.

In essence, a "forced convergence" was present. Elimination of the time distribution of the gains alleviated this problem.

The modified room component is used in the transfer function structure models and in the modified degree-day model. Computationally, the component serves the same purpose in both structure repre-

sentation. However, the definition of the thermal mass of the room, C_r , differs in the two structure models. In the transfer function model C_r represents the thermal capacitance of the interior space only; i.e., the sum of the thermal capacitances of the interior partitions, furnishings, appliances, etc. The thermal capacitance of the exterior walls are accounted for in the transfer function representation. The modified degree-day wall representation does not account for walls capacitance. In this model, the thermal capacitance is a single lumped or effective thermal capacitance characterized by a single temperature, the room temperature, C_r , when the Type 19 room is used in the modified degree-day model is the sum of the interior capacitance and the effective thermal capacitance of the walls. C_r in the modified degree-day model is designated as C_{DD} .

3.3.3 Modified Degree-Day Loads

The modified degree-day load model used in this study is a quasi-steady-state model. The instantaneous conduction heat transfer load is calculated as the product of the structure thermal conductance (UA value) and a temperature difference. The structure UA is assumed to be constant.

The ambient conditions in the transfer function wall model are represented by the sol-air temperature. The same representation is used in the model. The instantaneous conduction heat transfer load is given by Equation 3.3.1.

$$q_{\tau} = UA (T_{e,\tau} - T_{r,\tau}) \quad 3.3.1$$

A TRNSYS component has been written to calculate the sol-air temperature and the instantaneous conduction heat transfer load for the structure modelled in this analysis. The component Fortran listing is presented in Appendix B.

3.4 Determination of Effective Thermal Capacitance From the Transfer Function Method

The effective thermal capacitance, C_e , is defined as the value of room capacitance which causes the modified degree-day model to predict the same loads as the transfer function model. C_e consists of two parts; the effective thermal capacitance of the room, i.e., the furnishings, appliances, etc., and the effective thermal capacitance of the walls. In this analysis it is assumed that the effective thermal capacitance of the room is known. It is the value of room capacitance, C_r , used in the transfer function model room.

The effective thermal capacitance of the walls is the value which is derived here. The effective thermal capacitance of the walls* is the effective thermal capacitance, C_e , minus the room thermal capacitance of the transfer function model.

*"The effective thermal capacitance of the walls" will be referred to as "effective thermal capacitance" in the remainder of this Chapter.

$$\begin{aligned}
 C_{ew} &= C_e - C_r \\
 &= C_{DD} - C_r
 \end{aligned}
 \tag{3.4.1}$$

when $L_{DD} = L_{TF}$

The determination of C_{ew} is a trial and error process. Modified degree-day model simulations are conducted for various values of C_{DD} and the loads are determined. The transfer function model is also used to determine the loads for the same conditions. The effective thermal capacitance of the structure is determined through comparison of the resultant loads of the two models.

The simplest way to estimate C_{ew} is to plot the ratio of the transfer function and modified degree day model loads as a function of dimensionless capacitance. The load ratio is the modified degree-day model load divided by the transfer function load. The dimensionless capacitance is defined by Equation 3.4.2.

$$\gamma = \frac{C_{DD} - C_r}{C_s}
 \tag{3.4.2}$$

The value of γ when the load ratio is equal to one defines C_{ew} . Theoretically, if C_{ew} is a single value or fixed fraction of the actual structure capacitance C_s , C_{ew} could be determined from monthly, seasonal, or annual loads. In this study, annual loads were used.

3.5 Comparison of Floating and Constant Room Temperature Transfer Function Models.

Simulation results of the CRT and FRT transfer function models described in Sections 3.2 and 3.3 are compared in this section. Two types of wall construction were considered. One construction was a frame wall (ASHRAE exterior wall #26) and the other wall was a 0.3 m thick concrete wall (ASHRAE exterior wall #11). The construction of the two walls and the physical properties of the wall elements are shown in Table 3-1. The transfer function coefficients are shown in Table 3-2.

The two transfer function models described in Section 3.3 were simulated for five different simulation conditions and basic simulation parameters listed in Table 3-3. The five conditions are as follows. Conduction heat transfer only was considered in the first situation. Thus T_e in Equations 3.2.3 and 3.2.9 is replaced by T_a , the ambient temperature. The second case consisted of imposing a 4 C setback on the minimum room temperature from 10 P.M. to 6 A.M. (solar time) daily. Constant internal generation where GEN/UA was equal to 5 C was considered in the third case. Night setback was not considered in this case.

The fourth and fifth cases consisted of replacing one-half the south-facing wall (3.7 m^2) of the structure with a double-glazed window to consider solar gains. In addition, T_e was used in Equations 3.2.3 and 3.2.9. The allowable room temperature limits in the fourth case were as presented in Table 3.3. The 4 C night setback

TABLE 3-1

ASHRAE Exterior Walls 11 and 26 Construction

Wall 26 (Frame Wall)					
Description	Thickness (m)	Thermal conduc- tivity $\left(\frac{W}{m-C}\right)$	Density $\left(\frac{kg}{m^3}\right)$	Specific heat $\left(\frac{kJ}{kg-C}\right)$	Thermal resistance $\left(\frac{m^2-C}{W}\right)$
Outside Surface Resistance	---	---	---	---	0.059
Finish	0.013	0.42	1250	1.09	0.031
Insulation	0.051	0.043	91.3	0.847	1.18
Finish	0.013	0.42	1250	1.09	0.031
Inside Surface Resistance	---	---	---	---	0.121
Wall 11 (.3m heavy weight concrete)					
Description	Thickness (m)	Thermal conduc- tivity $\left(\frac{W}{m-C}\right)$	Density $\left(\frac{kg}{m^3}\right)$	Specific heat $\left(\frac{kJ}{kg-C}\right)$	Thermal resistance $\left(\frac{m^2-C}{W}\right)$
Outside Surface Resistance	---	---	---	---	0.059
Stucco	0.025	0.69	1860	0.84	0.037
HW Concrete	0.30	1.73	2240	0.84	0.18
Plaster or gypsum	0.049	0.73	1600	0.84	0.026
Inside Surface Resistance	---	---	---	---	0.121

TABLE 3-2
ASHRAE Exterior Wall 11 and Exterior Wall 26 Transfer Function
Coefficients

$$\Delta\tau = 1.0 \text{ hr}$$

<u>Wall 26</u>			
n	$\left(\frac{b_n}{W}\right)$ $\frac{1}{m^2-C}$	d_n	$\left(\frac{c_n}{W}\right)$ $\frac{1}{m^2-C}$
0	0.11161	--	4.09833
1	0.35908	-0.25682	-3.76556
2	0.06097	0.01052	0.19996
3	0.00033	-0.000002	-0.00030
$\Sigma c_n = 0.53243$			

<u>Wall 11</u>			
n	$\left(\frac{b_n}{W}\right)$ $\frac{1}{m^2-C}$	d_n	$\left(\frac{c_n}{W}\right)$ $\frac{1}{m^2-C}$
0	0.000004	--	6.26317
1	0.00189	-1.87840	-12.67611
2	0.01748	1.10990	8.22389
3	0.02252	-0.22354	-1.90178
4	0.00572	0.01217	0.13523
5	0.00028	-0.00012	-0.00257
6	0.000002	0.0000002	0.00001
$\Sigma c_n = 0.04784$			

TABLE 3-3

Basic Simulation Parameters for Transfer Function Model Comparison

Floor Area	9.3 m ²	
Wall Area	29.7 m ²	
Minimum Room Temp.	18 C	
Maximum Room Temp.	25 C	
Structure, UA	(no solar)	(solar)
Wall 11	71.1 W/C	74.8 W/C
Wall 26	21.0 W/C	31.0 W/C
Room Capacitance		
Wall 11	600-1500 kJ/C	
Wall 26	800-1000 kJ/C	

was imposed on the room temperature in the fifth case. SOLMET TMY data [18] for Madison, WI, Columbia, MO, and Albuquerque, NM were used to represent ambient conditions for all cases.

Shown in Figures 3-1 and 3-2 are the monthly loads calculated for the two wall constructions, three locations and five simulation conditions. The FRT model loads are plotted as a function of the CRT model loads. The heating season is defined as October through April and the cooling season as May through September.

The general trend shown in the figures is that the CRT model tends to predict larger monthly loads than the FRT model. If the two models predicted the same loads, all points in the figures would lie on a straight line denoted by the solid line in each figure.

The difference in the dependency of the conduction heat transfer on the room temperature causes the disagreement between the two models. The CRT model is dependent only on the current value of the room temperature, while the dependency of the FRT model is "time smoothed" by the previous temperatures required in the model. The greater dependence of the CRT model on the current room temperature causes the iterative solution of conduction heat flow and room temperature to generate larger temperature swings and conduction heat flows in the model.

The effect of the greater dependency of the CRT model of the room temperature is shown graphically in Figures 3-3 and 3-4 for the two types of wall construction. The figures show the room temperature response and conduction heat flow in response to the same periodic

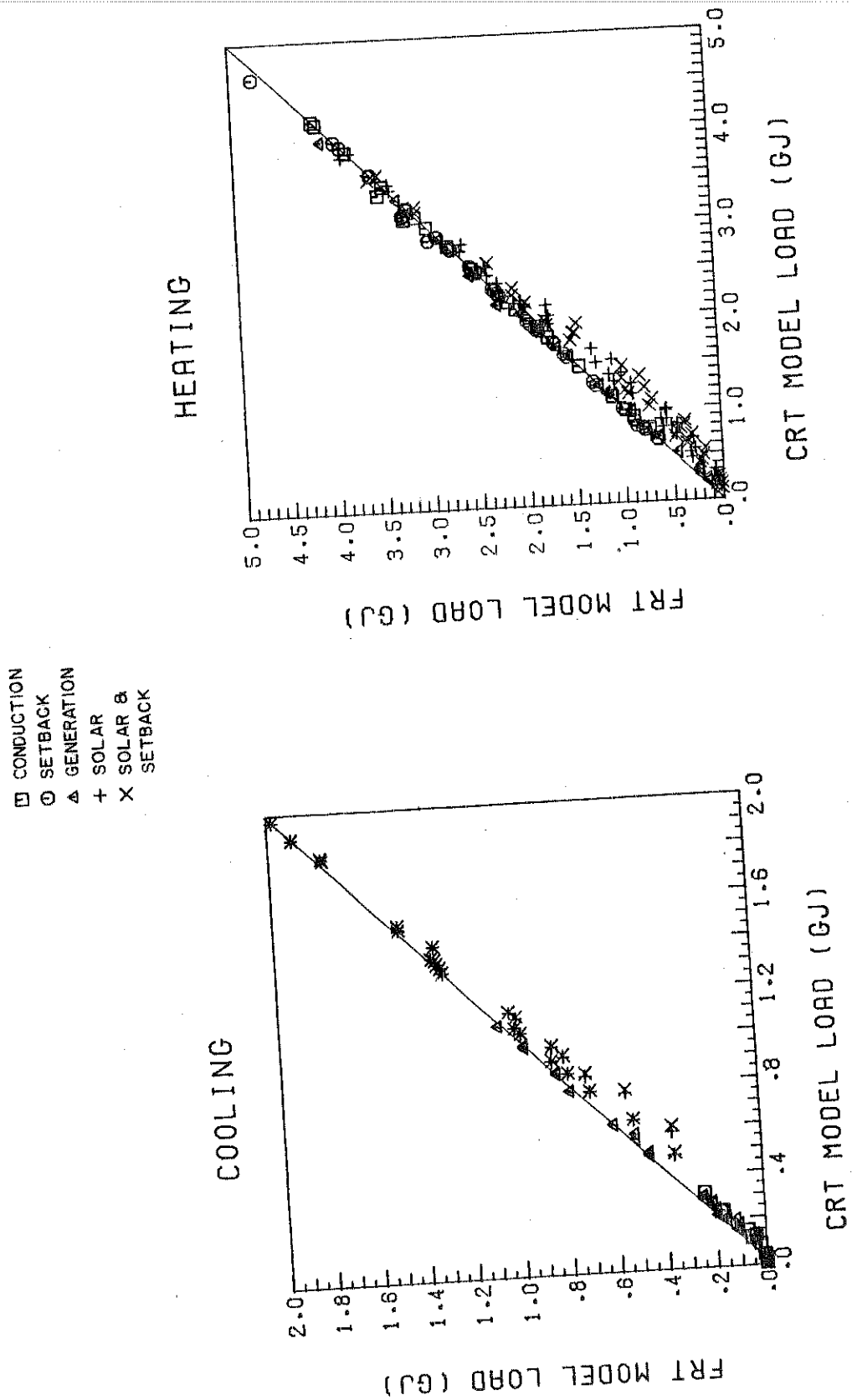


Figure 3-1 Comparison of Constant Room Temperature and Floating Room Temperature Transfer Function Model Monthly Loads--ASHRAE Wall 11

□ CONDUCTION
 ○ SETBACK
 ▲ GENERATION
 + SOLAR
 × SOLAR &
 SETBACK

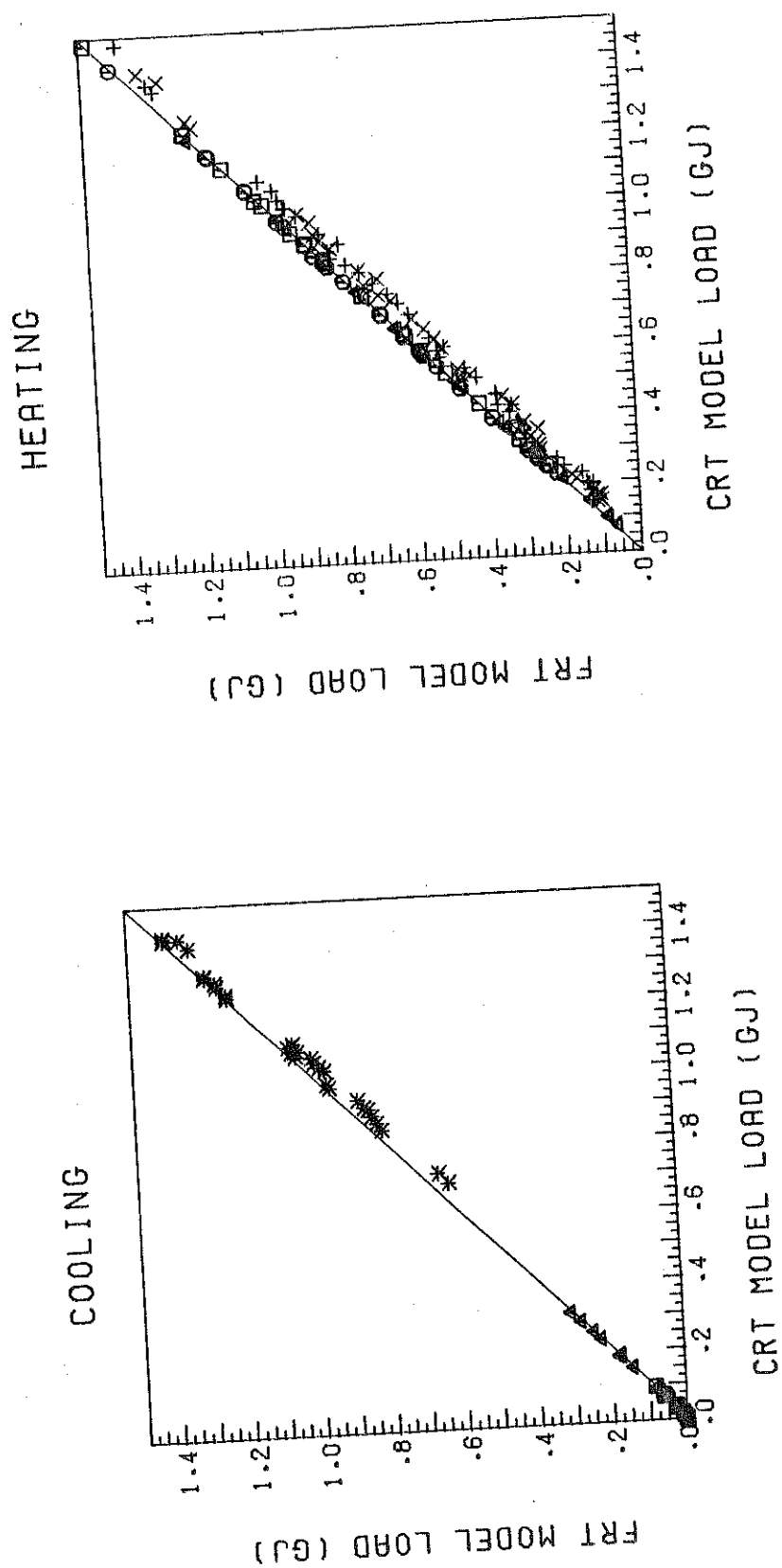


Figure 3-2 Comparison of Constant Room Temperature and
 Floating Room Temperature Transfer Function Model
 Monthly Loads---ASHRAE Wall 26

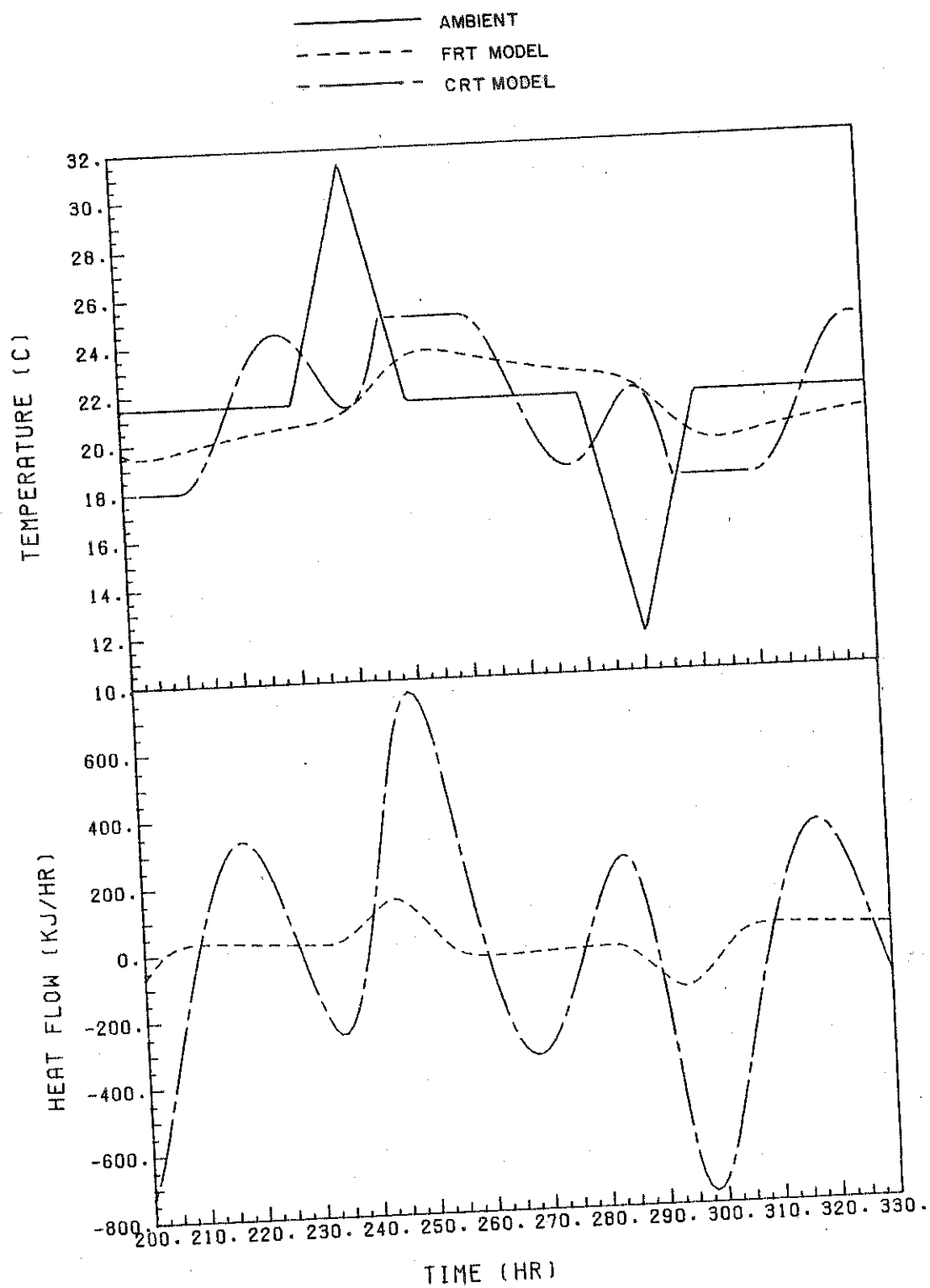


Figure 3-3 ASHRAE Wall 11 Temperature and Heat Flow Response to a Periodic Ambient Temperature Forcing Function

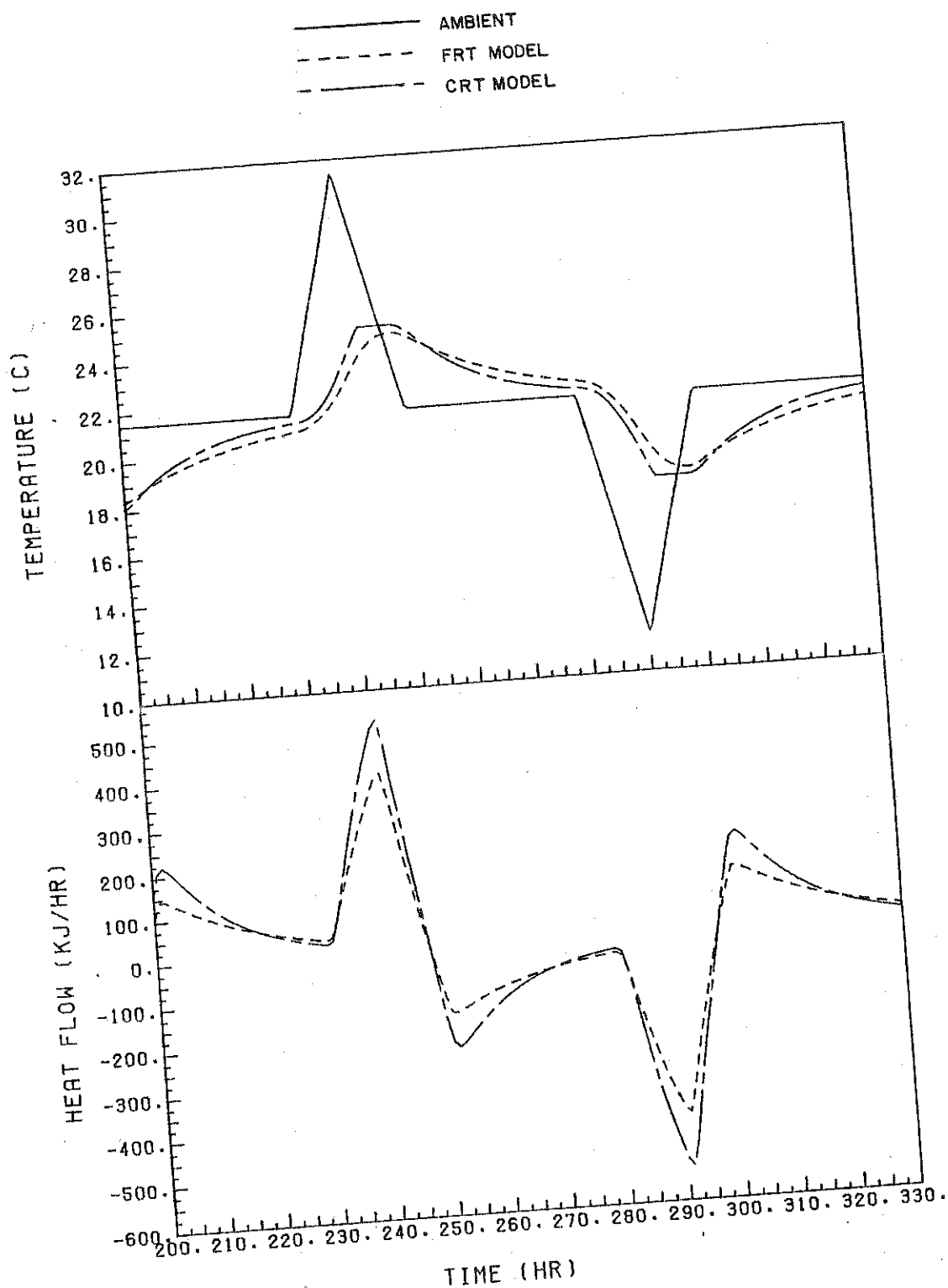


Figure 3-4 ASHRAE Wall 26 Temperature and Heat Flow Response to a Periodic Ambient Temperature Forcing Function

ambient temperature input. The simulation parameters are presented in Table 3-3.

The room temperature response of the CRT model is faster than the FRT model for both types of wall construction. Hence, the CRT model reaches the room temperature minimum or maximum (in this case the FRT model room temperature does not reach either limit) before the FRT model room temperature does. Consequently, cooling and heating are required for longer periods of time in the CRT model and larger loads are predicted.

The oscillation of the CRT model response of Wall 11 in Figure 3-3 is caused by the use of the model designed for constant room temperature in a floating room temperature situation. The instability is caused by the incorrect use of the model, not by any simulation errors.

Examples of the differences between the two models are shown in Tables 3-4 and 3-5, which compare the heating and cooling season loads predicted by the two models. The average load shown in the tables is the mean of the loads for the three locations and two room capacitances simulated with the FRT model. The standard error is defined by Equation 3.5.1 and F_L is the ratio of the standard error to the average load.

$$S.E. = \sqrt{\frac{\sum (L_{CRT} - L_{FRT})^2}{N_{obs}}} \quad 3.5.1$$

The differences in the average heating season loads for the two models are minimal for the conduction, setback, and generation

TABLE 3-4

Seasonal Average Loads and Standard Errors for Wall 11

	Heating			Cooling		
	Average Load (GJ)	Standard Error (GJ)	F_L	Average Load (GJ)	Standard Error (GJ)	F_L
Conduction	18.11	0.12	0.007	0.26	0.18	0.69
Setback	16.27	0.20	0.012	0.26	0.18	0.69
Generation	12.09	0.07	0.006	2.12	0.19	0.09
Solar	9.25	1.48	0.16	6.14	0.33	0.05
Solar and Setback	7.86	1.73	0.22	6.14	0.36	0.05

TABLE 3-5

Seasonal Average Loads and Standard Errors for Wall 26

	Heating			Cooling		
	Average Load (GJ)	Standard Error (GJ)	F_L	Average Load (GJ)	Standard Error (GJ)	F_L
Conduction	5.36	0.003	0.001	0.11	0.03	0.27
Setback	5.03	0.03	0.006	0.11	0.03	0.27
Generation	3.63	0.02	0.004	0.67	0.03	0.04
Solar	4.30	0.41	0.09	5.25	0.18	0.03
Solar and Setback	3.79	0.37	0.10	5.25	0.18	0.03

scenarios. In these cases, the largest portion of the seasonal load occurs at times when the room temperature is essentially constant, and the two models predict the same loads when the room temperature does not vary. The error for the setback case is slightly higher than the conduction and generation cases because the thermostat setback allows the room temperature to decay to the night-time set point. A larger difference is not noticed because the room temperature decay is rapid in response to the low ambient temperatures.

The addition of solar gains causes the differences between the models to increase during the heating season. Direct solar gain by the room causes the room temperature to rise during the daytime hours. At night the room temperature decreases as the energy absorbed during the day is released to offset conduction losses. Hence, a floating room temperature condition exists and the differences between the models are larger.

In contrast to the heating season, the cooling season loads showed the best agreement when solar gains were considered. For these cases, the relatively high ambient temperatures and the solar gains caused the room temperature to stay near the maximum setpoint for a major portion of the cooling season. The nearly constant room temperature caused the two models to generate essentially identical loads.

The agreement for the generation case is relatively good during the cooling season. The standard error was 9 percent of the average load for wall 11, and only 4 percent for wall 26. The constant rate

of generation effectively reduces the ambient temperature above which cooling is required. Without generation and solar gains, cooling is not required until the ambient rises beyond the maximum set point. Internal generation effectively reduces the maximum set point to $T_{\max, \text{eff}}$ defined by Equation 3.5.2. Now the ambient temperature needs to only rise past $T_{\max, \text{eff}}$ before the room temperature is pinned at the maximum value and a constant room temperature condition exists.

$$T_{\max, \text{eff}} = T_{\max} - \frac{\text{GEN}}{UA} \quad 3.5.2$$

Generally, the agreement between the two models is better for wall 26, the frame wall, than wall 11, the concrete wall. The better agreement occurs because the room temperature history is not as important for wall 26 as wall 11.

The difference in the importance of the room temperature history can be shown in two ways. First, the number of previous room temperatures required to calculate the heat flow through the walls is less for wall 26. Only three previous room temperatures are required for wall 26. Six are required for wall 11. The lesser number of temperatures required indicates the temperature history is not as important. The second reason the room temperature history of wall 26 is not as important as wall 11 is related to the relative importance of the room temperature transfer function coefficients (c_n).

The importance of a coefficient is defined as the ratio of the magnitude of a coefficient to the sum of the absolute values of all coefficients of a particular summation as defined in Equation 3.5.3.

$$R_{c,i} = \frac{|a_i|}{\sum_{i=0}^N |a_i|} \quad 3.5.3$$

This relative importance, $R_{c,i}$, is a fractional measure of the contribution of a particular term in a summation.

The relative importance distribution of the room temperature transfer function coefficients for walls 11 and 26 is shown in Figure 3-5. The most important room temperature for wall 26 is the current value, the previous temperatures contribute less than the current value. In contrast, the most important room temperature for wall 11, the concrete wall, is that of the previous time step. The room temperature two time decrements is even more important than the present value. The greater importance of the two previous room temperatures is a significant cause of the greater disagreement between the CRT and FRT models for the concrete wall.

Similar to the seasonal loads, the monthly loads showed the largest disagreement when the room temperature was not constant. The largest errors, as a fraction of the monthly average load, occurred during the months of March, April, and October for heating, and during May and September for cooling.

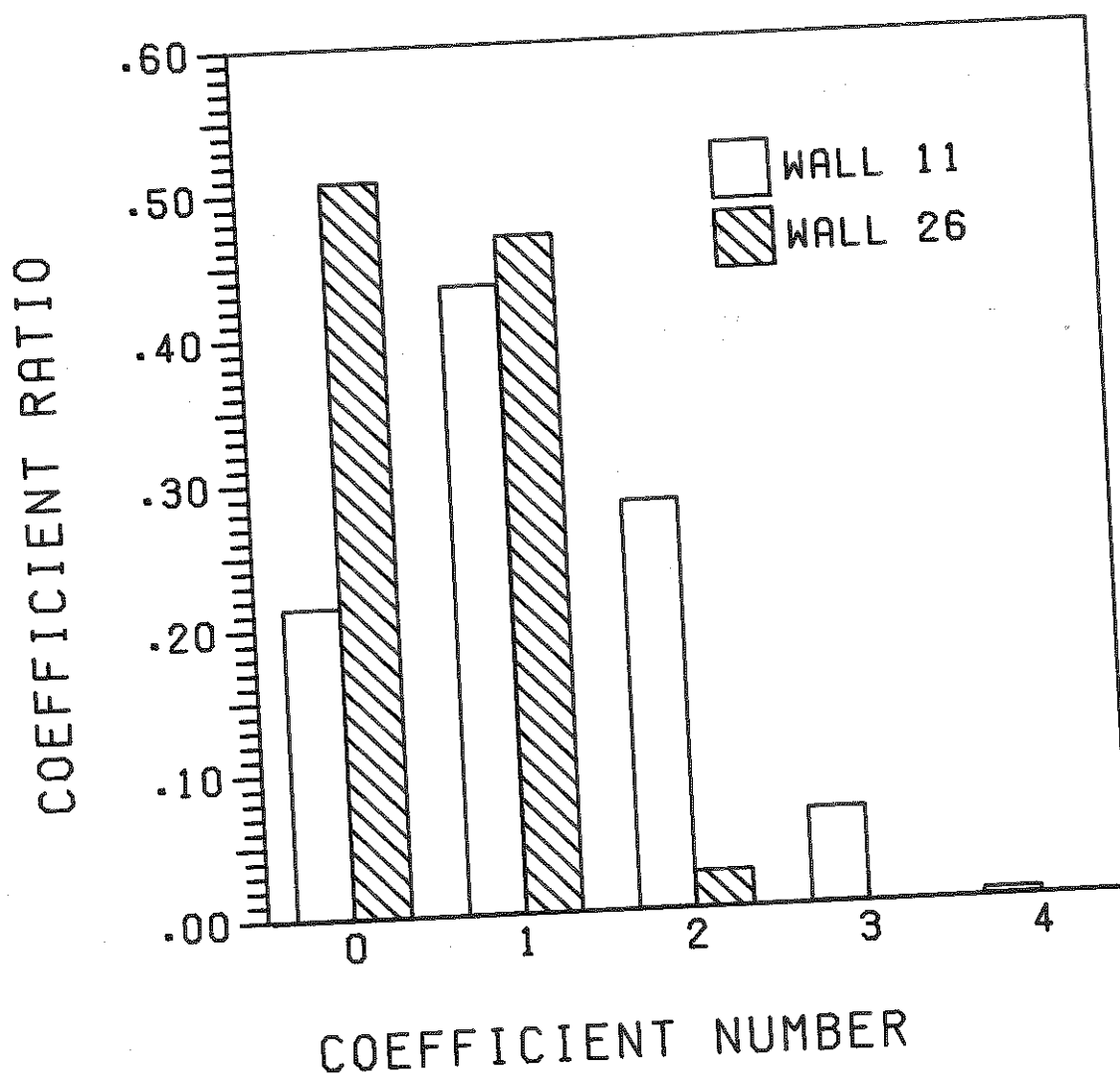


Figure 3-5 Comparison of ASHRAE Wall 11 and ASHRAE Wall 26 Room Temperature Transfer Function Coefficients

The loads during these months are relatively small compared to the seasonal total. Therefore, the errors in these months do not contribute significantly to the overall seasonal error. However, if precise values of the monthly loads are desired, the use of the CRT model to predict them can produce relatively large errors. During the heating season the errors were less than 3 percent for the non-solar cases. When solar gains were considered, the CRT model predicted twice the load of the FRT model in some months. The range during the cooling season was less than 7 percent for the solar cases to situations where the CRT model predicted loads and the FRT model did not.

In conclusion, the comparison of the CRT and FRT transfer function models show that the two models agree quite well if the room temperature does not vary. Significant differences can occur when the room temperature is not constant. The overall significance of the errors depends on the desired results. Although monthly errors may be relatively large, the seasonal totals may not be affected. If monthly loads are desired, the differences between the two models can be significant.

3.6 Transfer Function Method Effective Thermal Capacitance Derivation Results

The method used to derive the effective thermal capacitance of a simple one-zone structure was presented in Section 3.4. The results of the derivation are presented here. The effective thermal

capacitance is not a specific fraction of the capacitance of the elements of the structure. It is a function of the interior and ambient conditions, and the structure material, and cannot be easily derived. An assessment of the errors generated by the use of a specific fraction of the actual capacitance in the modified degree-day model is made.

In this section, like Section 3.5, structures constructed of either ASHRAE exterior wall 11 or 26 were considered. The FRT transfer function model was used. The five simulation conditions: conduction, setback, generation, solar and solar with setback were the same as Section 3.5. Ambient conditions for Madison, WI, Columbia, MO, and Albuquerque, NM were used in this study. The basic simulation parameters are presented in Table 3-3.

3.6.1 Discussion of Effective Thermal Capacitance

The effective thermal capacitance of a structure is defined mathematically by Equation 3.4.1. Effective thermal capacitance is meaningful however only when the thermal mass of the structure is active. The thermal mass is active when the modified degree-day model loads are a function of the value of C_{DD} used in the model. If the loads predicted by the degree-day model are the same for any value of C_{DD} , the transfer function and modified degree-day models predict identical loads and effective thermal capacitance is meaningless. In this situation the room temperature does not vary in either model.

Presented in Table 3-6 are the transfer function and modified degree-day model heating loads of the structure in this study for $C_r = 600 \text{ kJ/C}$ and three values of C_{DD} . The loads are for the month of March in Madison, WI. The structure consists of four ASHRAE exterior wall 11. Conduction heat transfer is the only energy gain or loss considered. Neglecting the small (less than one percent) numerical error, the four loads are identical. The modified degree-day model loads are not a function of C_{DD} and the effective thermal capacitance is meaningless.

A second example is shown in Table 3-7. In this case, the same structure was simulated with ambient weather conditions for Albuquerque, NM in July. Conduction and solar gains were considered in this situation. The loads predicted by the modified degree-day model are not a function of C_{DD} , and neglecting numerical error (three percent), equal to the transfer function model load. The low values of T_{\min} and T_{\max} were chosen only to show that the relationship between the modified degree-day model loads and C_{DD} is true for cooling as well as heating.

Examination of Tables 3-6 and 3-7 shows that in addition to the equivalence of the modified degree-day and transfer function model loads, the average monthly room temperatures are constant at the appropriate room temperature set point in all case models and values of C_{DD} . Whenever the effective thermal capacitance is meaningless, the transfer function and modified degree-day model average room temperatures will be equal and constant at either the minimum

TABLE 3-6

March Heating Loads and Monthly Average Room Temperature---
Conduction Only

Location: Madison, WI

Construction: ASHRAE Exterior Wall 11

UA = 71.05 W/C

$T_{\min} = 18 \text{ C}$

$T_{\max} = 25 \text{ C}$

$\bar{T}_a = -1.9 \text{ C}$

Model	C_r (kJ/C)	C_{DD} (kJ/C)	LOAD (GJ)	\bar{T}_r (C)
Transfer Function	600.	-	3.81	18.0
Degree-Day	-	600.	3.78	18.0
Degree-Day	-	8966.	3.78	18.0
Degree-Day	-	19550.	3.78	18.0

TABLE 3-7

July Cooling Loads and Monthly Average Room Temperature--
Conduction and Solar Gains

Location: Albuquerque, NM

Construction: ASHRAE Exterior Wall 11

UA = 74.89 W/C

$T_{\min} = 3.0 \text{ C}$

$T_{\max} = 10.0 \text{ C}$

$\bar{T}_a = 28.01 \text{ C}$

Model	C_r (kJ/C)	C_{DD} (kJ/C)	LOAD (GJ)	\bar{T}_r (C)
Transfer Function	600.	-	5.54	10.0
Degree-Day	-	600.	5.71	10.0
Degree-Day	-	8061.	5.71	10.0
Degree-Day	-	17180.	5.71	10.0

set point (heating) or the maximum set point (cooling). The room temperature is constant because the direction of energy flow to or from the room is essentially constant over the time period.

The loads presented in Tables 3-8 are for the same ambient conditions and wall construction as Table 3-6. Solar gains are now considered. The room temperature varies in this situation, and the modified degree-day model loads are a function of C_{DD} . The thermal mass of the structure is active, and effective thermal capacitance is meaningful.

The transfer function model room temperature does not necessarily have to vary for effective thermal capacitance to be meaningful. Shown in Table 3-9 are the transfer function and modified degree-day model cooling loads for the same conditions as Table 3-7, except the minimum room temperature set point is 18C and the maximum set point is 25C. The transfer function model room temperature is constant at the maximum set point. The modified degree-day model room temperature does vary in this case however, and the loads are a function of C_{DD} . Effective thermal capacitance is meaningful. This condition only occurs when the room temperature is at the maximum set point.

It may not be obvious how the thermal mass of the structure could be active when the room temperature is constant in the transfer function model. Reference to Figure 3-6, the thermal circuit representation of the transfer function model, will help explain this apparent paradox.

TABLE 3-8

March Heating Loads and Monthly Average Room Temperature--
Conduction and Solar Gains

Location: Madison, WI

Construction: ASHRAE Exterior Wall 11

UA = 74.89 W/C

$T_{\min} = 18.0 \text{ C}$

$T_{\max} = 25.0 \text{ C}$

$\bar{T}_a = 1.9 \text{ C}$

Model	C_r (kJ/C)	C_{DD} (kJ/C)	LOAD (GJ)	\bar{T}_r (C)
Transfer Function	600.	-	1.77	20.7
Degree-Day	-	600.	2.53	19.8
Degree-Day	-	8061.	1.67	19.7
Degree-Day	-	17181.	1.38	19.1

TABLE 3-9

Example of Meaningful Effective Thermal Capacitance for Constant
Transfer Function Model Room Temperature

Location: Albuquerque, NM

Construction: ASHRAE Exterior Wall 11

Type of Load: Conduction and Solar Gains; Cooling

Month: July

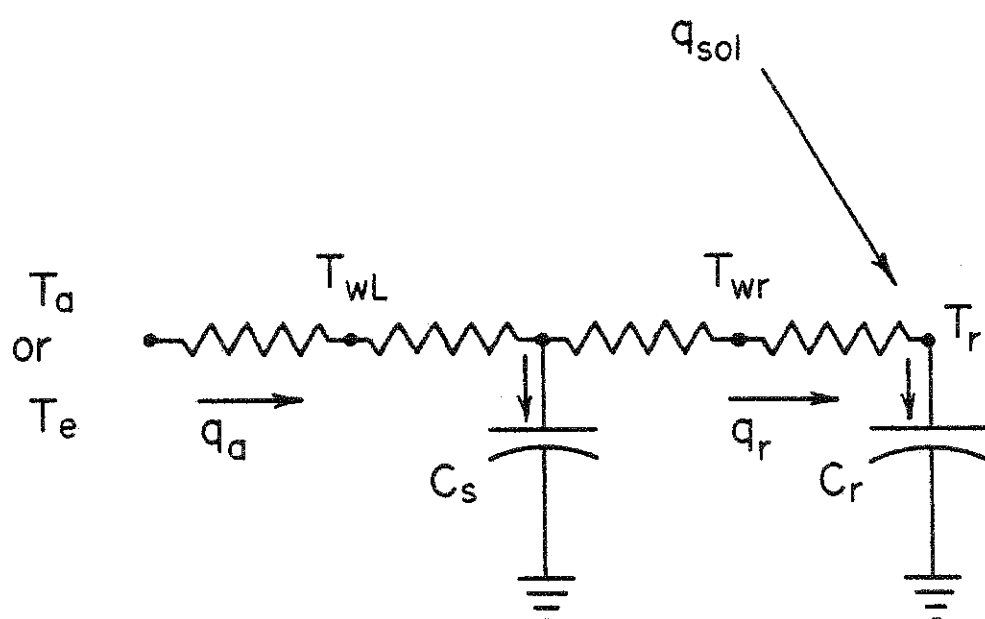
UA = 74.89 W/C

$T_{\min} = 18.0 \text{ C}$

$T_{\max} = 25.0 \text{ C}$

$\bar{T}_a = 28.01 \text{ C}$

Model	C_r (kJ/C)	C_{DD} (kJ/C)	LOAD (GJ)	\bar{T}_r (C)
Transfer Function	600.	-	2.07	25.0
Degree-Day	-	600.	2.50	23.83
Degree-Day	-	8060.	2.32	24.73
Degree-Day	-	17180.	2.29	24.96



Note: C_s is actually a distributed thermal capacitance

Figure 3-6 Transfer Function Model Thermal Circuit Representation

A constant transfer function model room temperature at the maximum set point implies that the direction of the net energy flow into the room, $q_r + q_{sol}$, is constant. However, the direction of the energy flow to the wall from the ambient, q_a , is not necessarily constant. This energy flow causes the thermal mass of the structure, C_s , to be active.

When the ambient temperature, T_a or T_e , is greater than the wall surface temperature, T_{wl} , the direction of q_a is as indicated in the figure. Energy flows into the wall from the ambient, but due to the finite thermal diffusivity $\left(\frac{k}{\rho c_p}\right)$ of the wall, this energy is not "seen" by the room until a later time. If the ambient temperature falls below T_{wl} , some of the energy which went into the wall before will flow back to the ambient, and never be "seen" by the room. The thermal mass of the wall is actively discharging energy. This reduces the cooling load of the room.

3.6.2 Results

The initial attempt to derive the effective thermal capacitance consisted of using the graphical estimation method of Section 3.4 for annual loads. Generally, the graphical technique generated curves similar to curve A of Figure 3.7. This curve shape indicates that the modified degree-day model loads decreased as the thermal mass increased.

Thermal mass reduces auxiliary energy requirements by storing

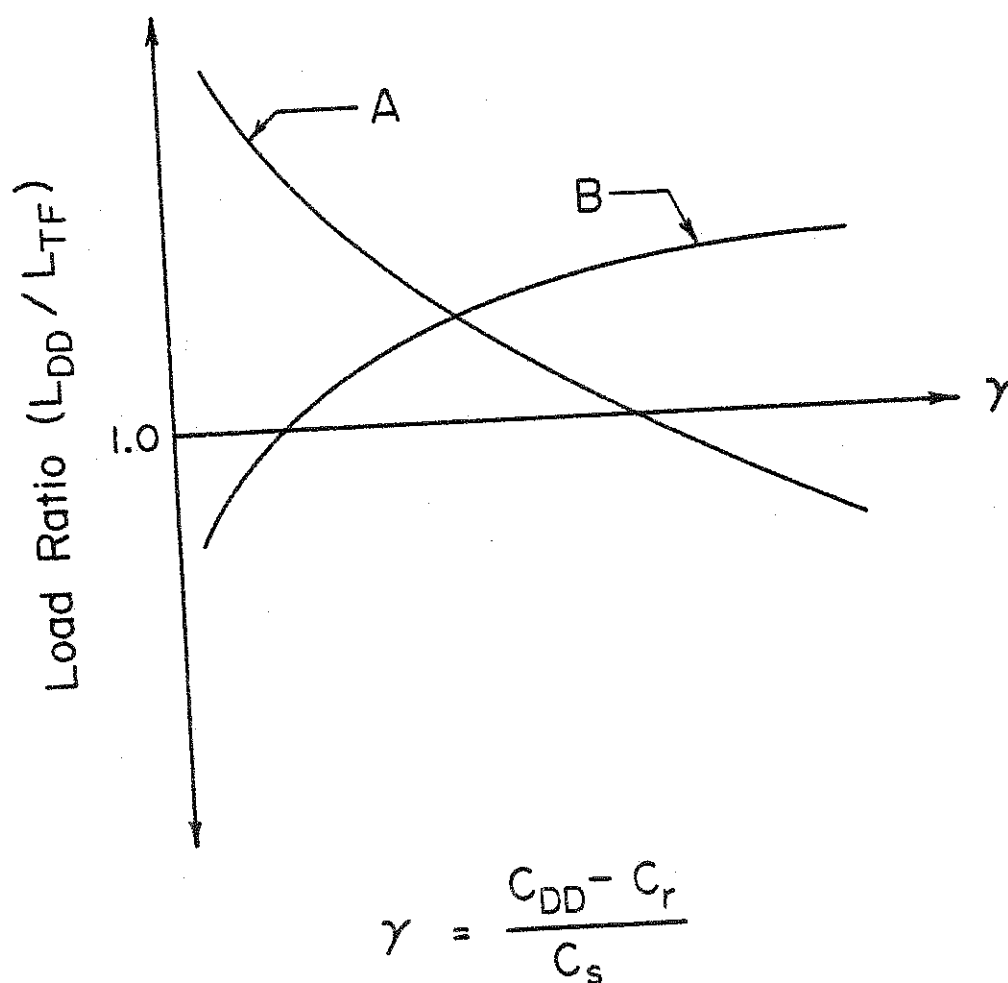


Figure 3-7 General Form of Graphs of Load Ratio Versus Dimensionless Capacitance

energy at times when excess energy is available, and releasing the energy at a later time. As thermal mass increases, more energy can be stored. Consequently, the auxiliary energy requirements decrease as thermal mass increases.

On an annual basis, the modified degree-day model loads decreased as the thermal mass increased for all ambient and interior conditions except for heating loads when night setback was considered. When night setback was considered, the ratio of the annual heating loads as a function of dimensionless capacitance was characterized by curves similar to curve B of Figure 3-7. The shape of curve B indicates that the modified degree-day loads increased as the thermal mass of the model increased.

With night setback, the room temperature decays to the night thermostat set point when the setpoint is reduced. The rate of room temperature decay decreases as the thermal capacitance increases. Thus a model with a large thermal mass has higher average night-time room temperature than a model with less thermal mass. The higher night-time room temperature results in increased conduction losses and larger loads.

It was found that the effective thermal capacitance of a structure varied with the interior conditions, ambient conditions, wall construction, and type of load, and could not be simply related to the actual structure capacitance. Figures 3-8, 9 and 10 graphically illustrate the variability of the effective thermal capacitance.

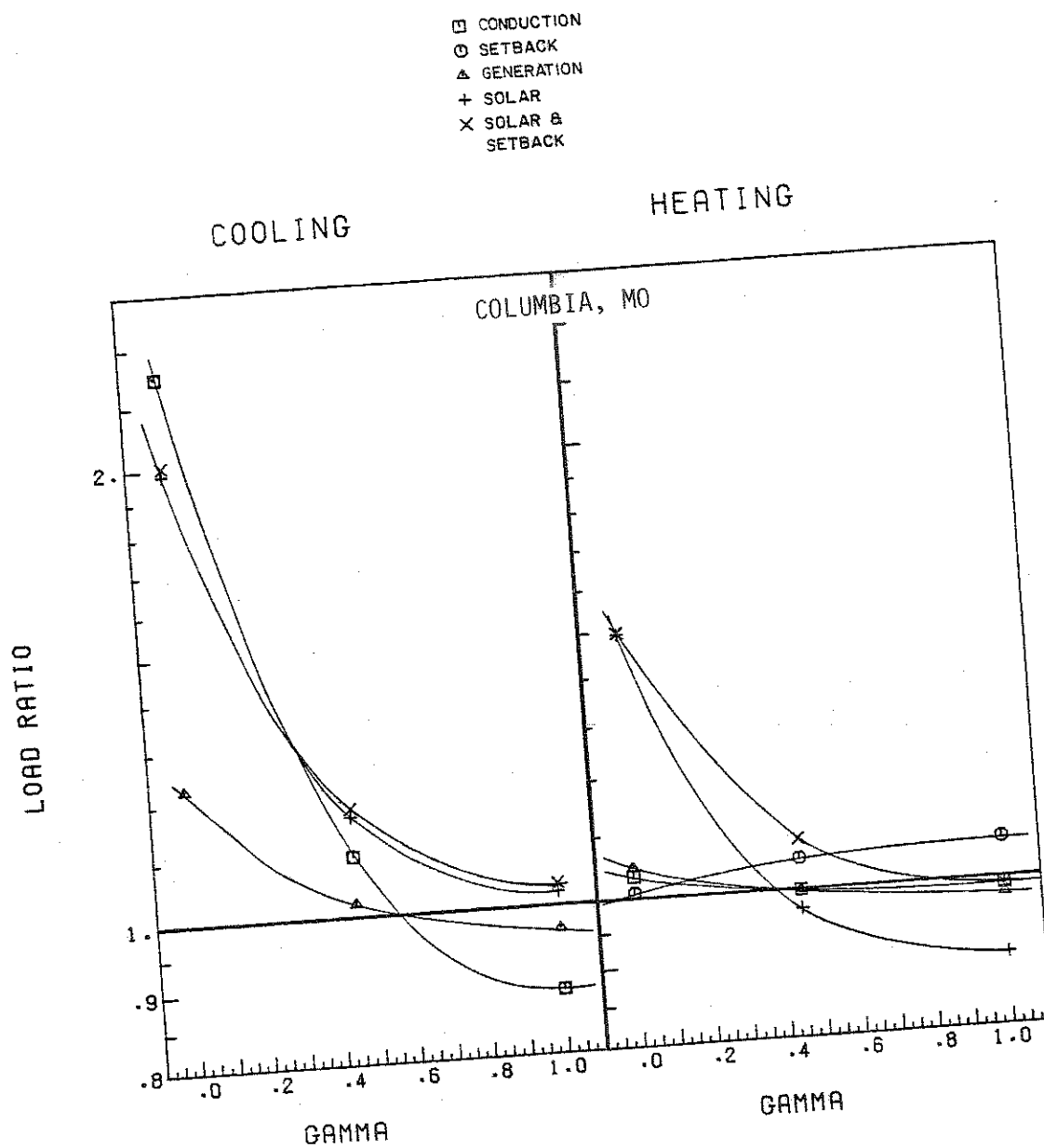


Figure 3-8 Example of Annual Load Ratio Versus Dimensionless Capacitance for ASHRAE Wall 11

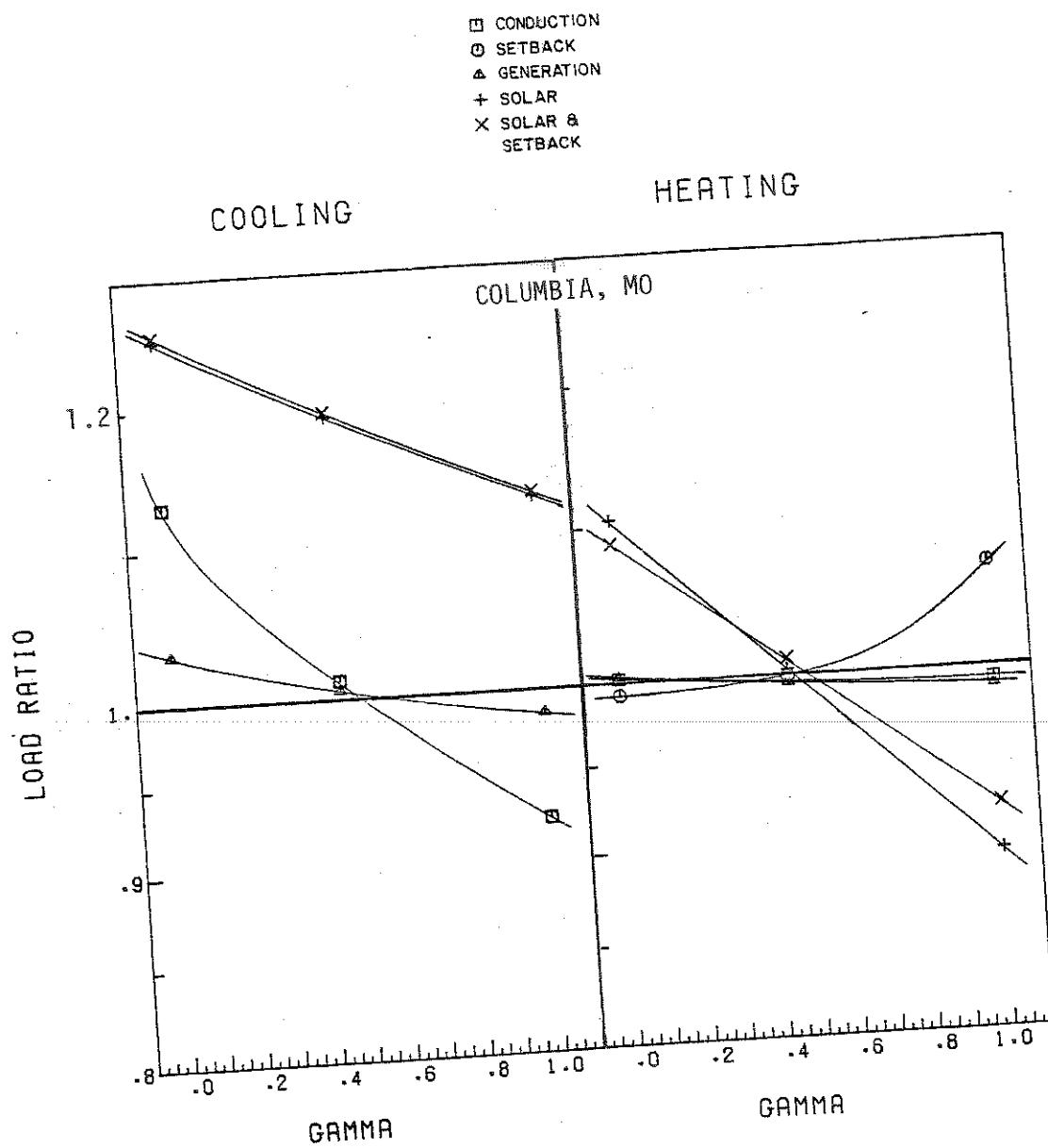


Figure 3-9 Example of Annual Load Ratio Versus Dimensionless Capacitance for ASHRAE Wall 26

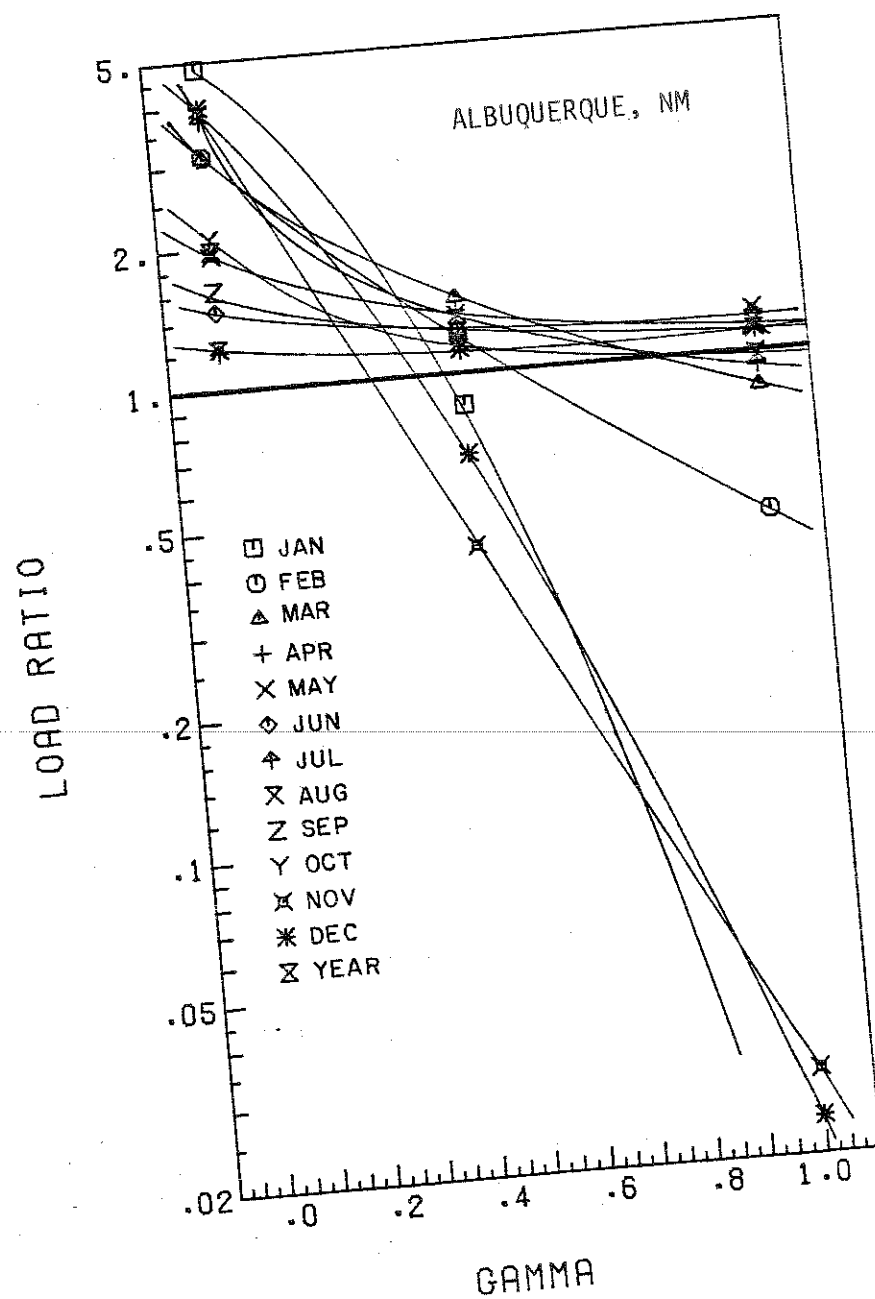


Figure 3-10 Example of Monthly Variance Effective Thermal Capacitance

Shown in Figures 3-8 and 3-9 are plots of the ratio of annual modified degree-day model loads to the annual transfer function model loads versus the dimensionless capacity γ . The ambient conditions were for Columbia, MO.

The figures show that even on an annual load basis, the effective thermal capacitance, which is defined by the value of γ when the load ratio equation to 1.0, can be different for cooling and for heating, and for different simulation conditions. It is even shown that in some cases, the effective thermal capacitance is greater than the actual capacitance of the structure.

The effective thermal capacitance can even vary from month to month for a particular type of simulation condition and type of load. Figure 3-10 shows the load ratio as a function of γ for solar cooling using Albuquerque, NM ambient weather data. The value of γ at which the load ratio is equal to one, defining C_{ew} , ranges from 0.3 to greater than 1.0 for the different months of the year. The effective thermal capacitance is not constant from month to month.

The original goal of this research was to find effective thermal capacitance. Tables 3-10 and 3-11 show the derived values of C_{ew} based on comparison of annual loads. The usefulness of these values is somewhat limited however, because of the variability of C_{ew} demonstrated above. The values in Tables 3-10 and 3-11 also exhibit this variability.

The effective thermal capacitance derived for cooling loads for the conduction and setback cases are identical because the trans-

TABLE 3-10

Wall 11 Effective Thermal Capacitance Based on Annual Loads

Wall Thermal Capacitance: 18950 kJ/C (non-solar)

16580 kJ/C (solar)

Room Thermal Capacitance: 600 kJ/C

	<u>Madison</u>		<u>Columbia</u>		<u>Albuquerque</u>	
	$\frac{C_{ew}}{C_s}$	C_{ew} (kJ/C)	$\frac{C_{ew}}{C_s}$	C_{ew} (kJ/C)	$\frac{C_{ew}}{C_s}$	C_{ew} (kJ/C)
Conduction Heating	0.50	9475	0.45	8528	0.45	8528
Conduction Cooling	0.50	9475	0.56	10610	0.62	11750
Setback Heating	Small	Small	Small	Small	Small	Small
Setback Cooling	0.50	9475	0.56	10610	0.62	11750
Generation Heating	0.48	9096	0.45	8528	0.45	8528
Generation Cooling	0.57	10800	0.53	10040	0.57	10800
Solar Heating	0.35	6632	0.40	7580	0.40	7580
Solar Cooling	>1.0	>16580	>1.0	>16580	0.90	14920
Solar & Setback Heating	1.0	16580	0.80	13260	0.55	9120
Solar & Setback Cooling	>1.0	>16580	>1.0	>16580	1.0	16580

TABLE 3-11

Wall 26 Effective Thermal Capacitance Based on Annual Loads

Wall Thermal Capacitance: 7142 kJ/C (non-solar)
999 kJ/C (solar)

Room Thermal Capacitance: 1000 kJ/C

	Madison		Columbia		Albuquerque	
	$\frac{C_{ew}}{C_s}$	C_{ew} (kJ/C)	$\frac{C_{ew}}{C_s}$	C_{ew} (kJ/C)	$\frac{C_{ew}}{C_s}$	C_{ew} (kJ/C)
Conduction	0.40	457	0.07	80	0.10	114
Conduction Cooling	0.46	525	0.50	571	0.50	571
Setback Heating	0.45	514	0.50	571	0.54	617
Setback Cooling	0.46	525	0.50	571	0.50	571
Generation Heating	0.40	457	0.07	80	0.17	194
Generation Cooling	0.48	548	0.57	651	0.57	651
Solar Heating	0.40	400	0.47	470	0.60	600
Solar Cooling	>1.0	>999	>1.0	>999	>1.0	>999
Solar & Setback Heating	0.42	420	0.50	500	0.70	700
Solar & Setback Cooling	>1.0	>999	>1.0	>999	>1.0	>999

fer function and modified degree-day model loads are identical for the two cases. Night thermostat setback has no effect on cooling loads, only heating loads. The small differences (the loads vary by less than one percent) between the solar cooling and solar with set back cooling are due to numerical error.

The effective thermal capacitance was found to be greater than the actual structure capacitance for the solar and solar with set-back cooling cases in several situations. Intuitively this does not seem correct; it is physically impossible for a greater amount of thermal mass than exists in the system to be active. The apparent anomaly is not caused by simulation or derivation error, but by the way the structure capacitance is represented in the modified degree-day model.

The thermal circuit representation of the transfer function model is shown in Figure 3-6. A similar representation of the modified degree-day model is shown in Figure 3-11. The solar gains, q_{sol} , are the same for both models. The difference between the models is that the modified degree-day model contains only one thermal capacitance, while the transfer function model contains two. The structure (C_s) and room (C_r) thermal capacitances are separate in the transfer function model, and combined, or lumped (C_{DD}) in the modified degree-day model.

C_r in the transfer function model, and C_{DD} in the modified degree-day model are characterized by the room temperature, T_r . The temperature extremes of the thermal mass are the room temperature minimum and maximum set points. C_s , however, does not have con-

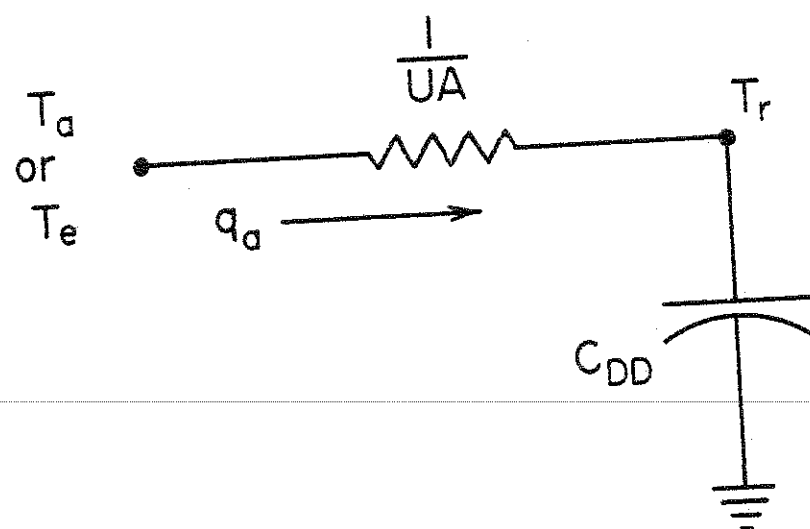


Figure 3-11 Modified Degree-Day Model Thermal Circuit Representation

stant temperature limits associated with it. The temperature limits of this thermal mass are controlled by the combination of the ambient temperature extremes and the room temperature thermostat set points.

In the modified degree-day model, any ambient temperature fluctuation immediately generates energy flow into or out of the room. A time lag exists between an ambient temperature disturbance and any corresponding energy flow to or from the room in the transfer function model. In a cooling situation, it is possible that some of the energy which flowed into the wall in the transfer function model does not reach the room. If the ambient temperature drops to a value lower than the ambient side wall surface temperature, energy flows from the wall back to the ambient and never is seen by the room.

The structure is acting as thermal storage in a way that the modified degree-day model can not characterize. During periods of cooling the room temperature is at the maximum set point so the modified degree-day model thermal capacitance can store no energy. The transfer function structure capacitance C_s can store energy, however, because the upper temperature limit of this thermal mass is not exclusively controlled by the maximum set point.

Shown in Table 3-12 are the net conduction cooling loads of the structure for the transfer function and modified degree-day models. The loads are for June, July, and August for an ASHRAE exterior wall structure located in Albuquerque, NM. In all cases the modified degree-day model over-predicts the net conduction load, yet the room

TABLE 3-12

Transfer Function and Degree Day Model Solar Cooling Conduction
Loads

Location: Ambuquerque

Construction: ASHRAE Exterior Wall 11

$$C_r = 600 \text{ kJ/C}$$

$$C_{DD} = 17180 \text{ kJ/C}$$

$$C_s = 16580 \text{ kJ/C}$$

$$T_{\min} = 18.0 \text{ C}$$

$$T_{\max} = 25.0 \text{ C}$$

Month	\bar{T}_a	Transfer function model		Degree-Day Model	
		Net Conduction (GJ)	\bar{T}_n (C)	Net Conduction (GJ)	\bar{T}_R (C)
June	27.57	1.57	25.00	1.80	24.94
July	28.01	1.64	25.00	1.82	24.96
August	26.90	1.39	25.00	1.61	24.95

temperature in the two models is essentially constant. The thermal mass of the wall in the transfer function model is storing energy during the day by heating up beyond the maximum room temperature set point, and releasing the energy to the ambient at night resulting in a lower conduction cooling load for this model. The modified degree-day model cannot account for this thermal activity and therefore over-predicts the load even for this case where $C_{DD} = C_s + C_r$.

Analysis of monthly simulation results indicated that C_{ew} may be related to the ambient temperature variation. An attempt was made to correlate C_{ew}/C_s to the dimensionless temperature variation, t^* , given by Equation 3.6.2.

$$t^* = \frac{T_{SET} - \bar{T}_a}{\sigma} \quad 3.6.2$$

where

$$\begin{aligned} T_{SET} &= T_{min} \text{ for heating} \\ &= T_{max} \text{ for cooling.} \end{aligned}$$

No consistent relationship was found.

3.6.3 Comparison of Transfer Function Modified Degree-Day Model Loads at Fixed Values of Degree-Day Model Thermal Capacitance

The results contained in Section 3.6.2 showed that the effective thermal capacitance is not a constant value or fraction of the actual capacitance, even for a single wall construction in a single location. The value of C_{ew} can be different for heating and cool-

ing loads, and the value for annual loads is not necessarily the correct effective thermal capacitance of the individual monthly loads.

Since the effective thermal capacitance can not be easily defined, an alternative method to assess the utility of lumped thermal capacitance modelling is to estimate the errors in load estimation generated by the use of the modified degree-day model. The loads generated by the modified degree-day model for a specific value of the C_{DD} can be compared to the analogous transfer function model loads and an assessment of the differences between the two models can be made.

Conducting this comparison for a wide range of C_{DD} enables one to assess the sensitivity of the errors to the value of C_{DD} used in the modified degree-day model. In a case where little error is shown by the use of the modified degree-day model at any value of C_{DD} , lumped capacitance modelling is useful, and although, C_{ew} , may not be easily definable, any value of C_{DD} generates accurate results. Conversely, if C_{DD} has a large effect on the error in load prediction, lumped capacitance modelling may be of little use.

Simulations were conducted to assess the sensitivity of errors and the accuracy of lumped capacitance modelling. The transfer function and modified degree-day model loads were compared for three values of C_{DD} :

1. $C_{DD} = C_r$
2. $C_{DD} = C_r + 0.45 C_s$
3. $C_{DD} = C_r + C_s$

The first value represents the minimum capacitance of the room and structure. The third capacitance is the total actual thermal capacitance of the system. The second value was chosen based on the results presented in Tables 3-10 and 3-11. In these tables a large number of the cases show C_{ew} to be in the range of 40 to 55 percent of C_s . A representative fraction of 45 percent was chosen.

ASHRAE exterior walls 11 and 26 were modelled. Weather data for Madison, WI, Albuquerque, NM, and Columbia, MO were used to represent ambient conditions. The basic simulation parameters are presented in Table 3-3. The simulation conditions are as in the last section: conduction, night setback, generation, solar, and solar with setback.

The two models were compared for both total season loads and monthly loads within the respective season. The heating season is defined as October through April; the cooling season, May through September. The statistic used to compare the two models is the standard error (S.E.) defined in Equation 3.5.1.

Tables 3-13 and 3-14 present the results of this analysis for the two walls for seasonal total loads. The standard errors are presented in normalized form as F_L the ratio of the standard error to the transfer function model average load. The transfer function average load is the mean load predicted by the transfer function model for the two room capacitances and three locations simulated for a particular simulation condition.

TABLE 3-13

Wall 11 Relative Standard Errors for Three Values of C_{DD} --
Seasonal Total Loads

Condition	Average Load (GJ)	Cooling Season		
		F_L (fraction of load)		
		$C_{DD} = C_r$	$C_{DD} = C_r + .45C_s$	$C_{DD} = C_r + C_s$
Conduction	0.26	(+) 2.17	(+) 0.14	(-) 0.14
Setback	0.26	(+) 2.17	(+) 0.14	(-) 0.14
Generation	2.12	(+) 0.29	(+) 0.03	(-) 0.04
Solar	6.14	(+) 0.55	(+) 0.21	(+) 0.15
Solar & Setback	6.14	(+) 0.55	(+) 0.21	(+) 0.15

Heating Season

Conduction	18.11	(+) 0.01	(-) 0.002	(-) 0.004
Setback	16.72	(-) 0.01	(+) 0.04	(+) 0.06
Generation	12.09	(+) 0.03	(0) 0.000	(-) 0.01
Solar	9.15	(+) 0.51	(-) 0.04	(-) 0.16
Solar & Setback	7.86	(+) 0.53	(+) 0.06	(-) 0.07

(+) indicates over prediction of loads by modified degree-day model.

(-) indicates under prediction of loads by modified degree-day model.

(0) indicates agreement of loads.

TABLE 3-14

Wall 26 Relative Standard Errors for Three Values of C_{DD} --
Seasonal Total Loads

Condition	Average Load (GJ)	Cooling Season F_L (fraction of load)		
		$C_{DD} = C_r$	$C_{DD} = C_r + .45C_s$	$C_{DD} = C_r + C_s$
Conduction	0.11	(+) 0.18	(-) 0.03	(-) 0.14
Setback	0.11	(+) 0.18	(-) 0.03	(-) 0.14
Generation	0.67	(+) 0.04	(+) 0.01	(-) 0.03
Solar	5.25	(+) 0.21	(+) 0.18	(+) 0.15
Solar & Setback	5.25	(+) 0.21	(+) 0.18	(+) 0.15

Heating Season

Conduction	5.36	(+) 0.001	(-) 0.003	(-) 0.004
Setback	5.03	(-) 0.008	(+) 0.003	(+) 0.01
Generation	3.63	(+) 0.002	(-) 0.005	(-) 0.01
Solar	4.30	(+) 0.09	(-) 0.03	(-) 0.12
Solar & Setback	3.79	(+) 0.09	(+) 0.03	(-) 0.10

(+) indicates over prediction of loads by modified degree-day model.

(-) indicates under prediction of loads by modified degree-day model.

The values of F_L can be used to assess the accuracy of the modified degree-day model, and to estimate how sensitive the accuracy of the model is to C_{DD} . A small value of F_L indicates that the degree-day model is relatively accurate in predicting loads. If F_L is about the same for any value of C_{DD} , the accuracy of the degree-day model is not affected by the value of C_{DD} . A large F_L indicates the degree-day model is not very accurate, and if F_L varies greatly for different values of C_{DD} , the accuracy of the model will be affected by the C_{DD} specified.

On a seasonal basis, the conduction, setback and generation heating cases showed the best agreement between the transfer function and modified degree-day models for both walls. The accuracy of the modified degree-day model was relatively insensitive to C_{DD} . Generally, the modified degree-day model was quite accurate for these three cases.

The result may be somewhat misleading however. For these three cases, the largest portion of the seasonal load occurs when the room temperature in both models is essentially constant at the minimum setpoint. It was shown above that the models will predict the same load in this situation, and effective capacitance is meaningless. Therefore, the accuracy shown here is biased by the months when the room temperature is constant.

For the solar and solar with setback heating season cases, the modified degree-day model was most accurate for the middle values of C_{DD} . Both large and small values of C_{DD} increased the error of the

modified degree-day model. Small values of C_{DD} caused the modified degree-day model to over-predict the season total load, and larger volumes caused the model to under-predict.

These results are not biased by any months with constant room temperature as found in the conduction, setback, and generation cases. The solar gains cause the room temperature to vary even during the coldest months. The modified degree-day model accuracy is sensitive to C_{DD} because the room temperature is varying.

The modified degree-day model accuracy exhibited a close dependency on the value of C_{DD} during the cooling season in all cases, indicating that the accuracy of the model is quite sensitive to the value of C_{DD} used. However, with the exception of the solar and solar with setback cases, the middle value of C_{DD} generated the least error. The large errors for the solar and solar with setback cases are present because of the incorrect modified degree-day model representation of the structure capacitance (C_s) discussed in Section 3.6.2.

These results suggest that, for most cases, a modified degree-day model capacitance of about one-half the structure capacitance plus the room capacitance will cause the modified degree-day model to generate relatively accurate seasonal total loads for any simulation condition except cooling with solar gains. This conclusion is valid for these limited cases.

It was shown in Section 3.6.2 that the effective thermal capacitance is not necessarily the same for monthly loads as annual loads.

For this reason, the middle values of C_{DD} may generate large errors in monthly loads even though season total loads are relatively accurate.

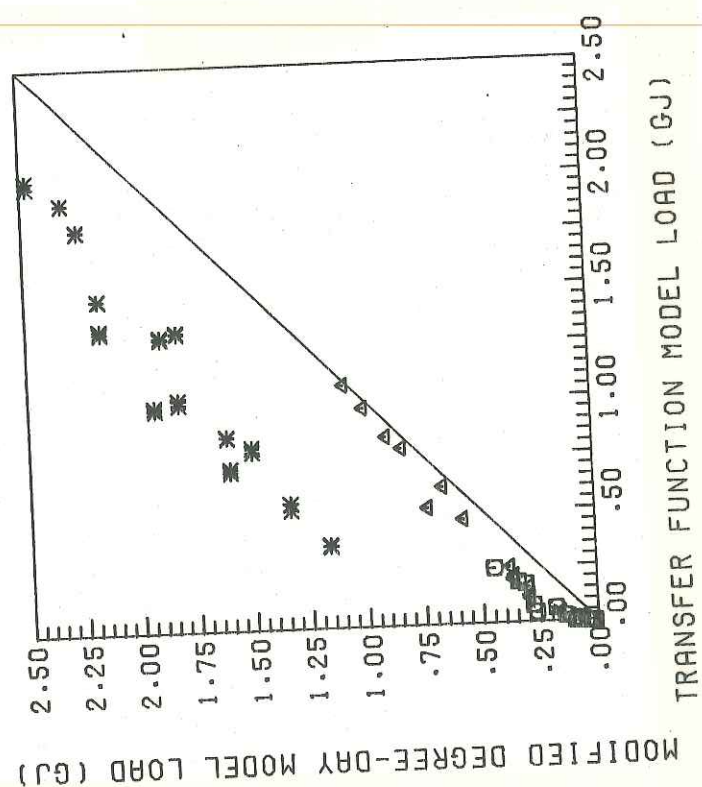
Shown in Figures 3-12 through 3-17 are the seasonal monthly loads generated by the two models for the three values of C_{DD} . The same general trends regarding the accuracy of the modified degree-day model are present for the monthly loads as the seasonal total loads. The accuracy of the modified degree-day model is more sensitive to C_{DD} on a monthly basis.

Generally, the modified degree-day model was most accurate when the middle value of C_{DD} was used. The monthly loads for this case are shown in Figures 3-14 and 3-15. The largest relative standard errors for this case are shown in Tables 3-15 and 3-16. The tables show that the largest errors generally occurred during the transition months: April, May, September, and October. The loads in these months are generally small compared to the total season load, so the errors generated do not greatly affect the accuracy of the modified degree-day model on a seasonal total basis.

If monthly loads instead of seasonal loads are of interest, the modified degree-day model errors can be significant, as shown in Tables 3-15 and 3-16. The accuracy of the modified degree-day model is also more sensitive to C_{DD} for monthly loads. Therefore, the utility of the modified degree-day model is limited if accurate monthly loads are required.

□ CONDUCTION
 ○ SETBACK
 ▲ GENERATION
 + SOLAR
 × SOLAR &
 SETBACK

COOLING



HEATING

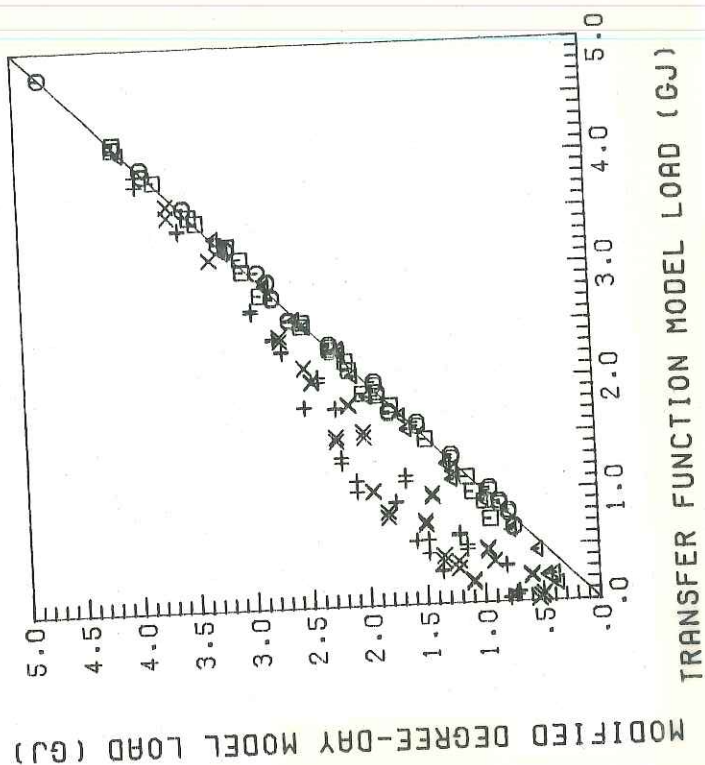
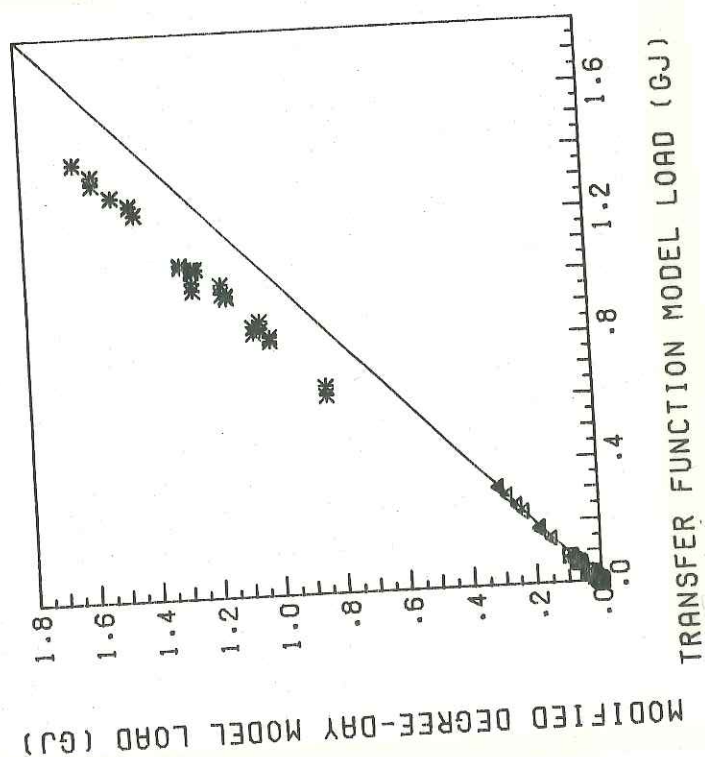


Figure 3-12 Comparison of Modified Degree-Day and Transfer Function Model Monthly Loads for ASHRAE Wall 11 when $C_{DD} = C_r$

□ CONDUCTION
 ○ SETBACK
 ▲ GENERATION
 + SOLAR
 × SOLAR &
 SETBACK

COOLING



HEATING

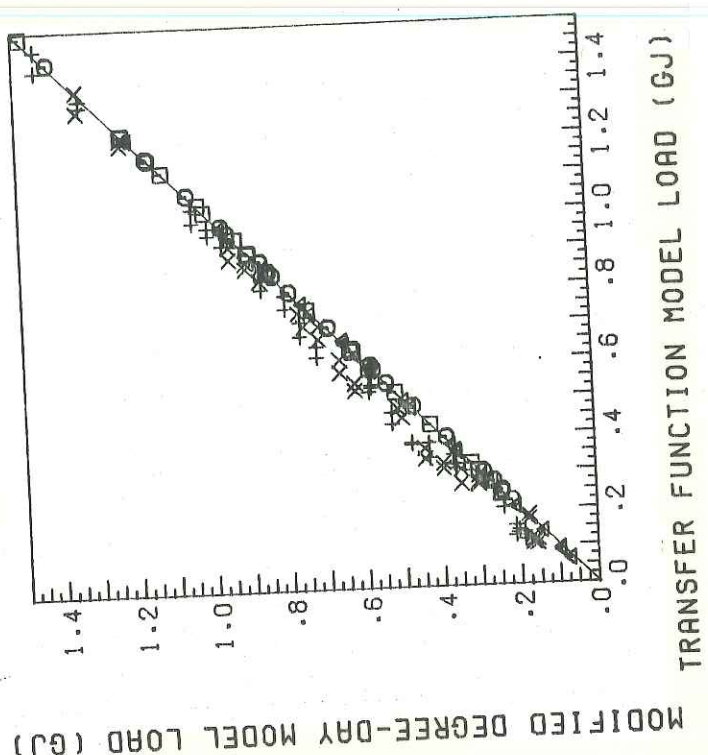
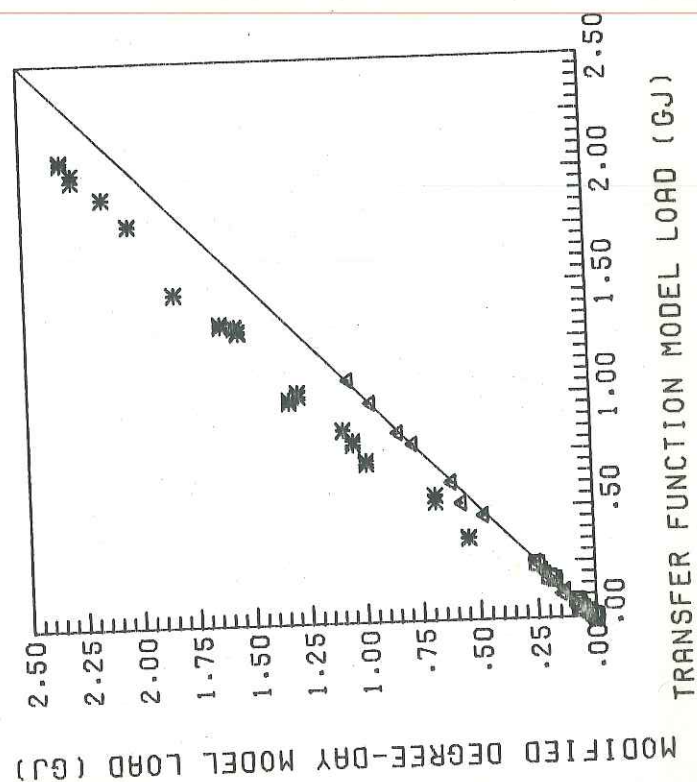


Figure 3-13 Comparison of Modified Degree-Day and Transfer
 Function Model Monthly Loads for ASHRAE Wall 26
 when $C_{DD} = C_r$

- CONDUCTION
- SETBACK
- △ GENERATION
- + SOLAR
- x SOLAR & SETBACK

COOLING



HEATING

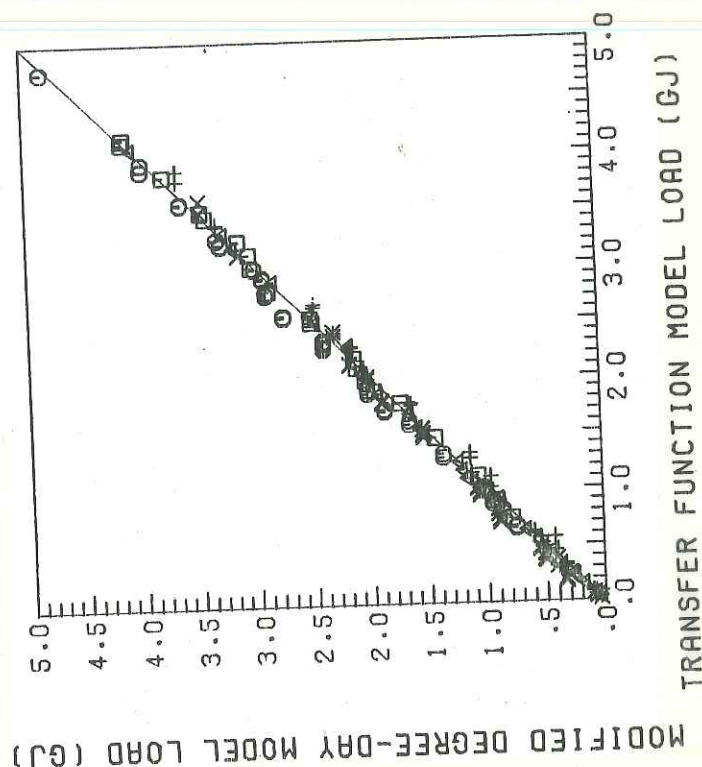
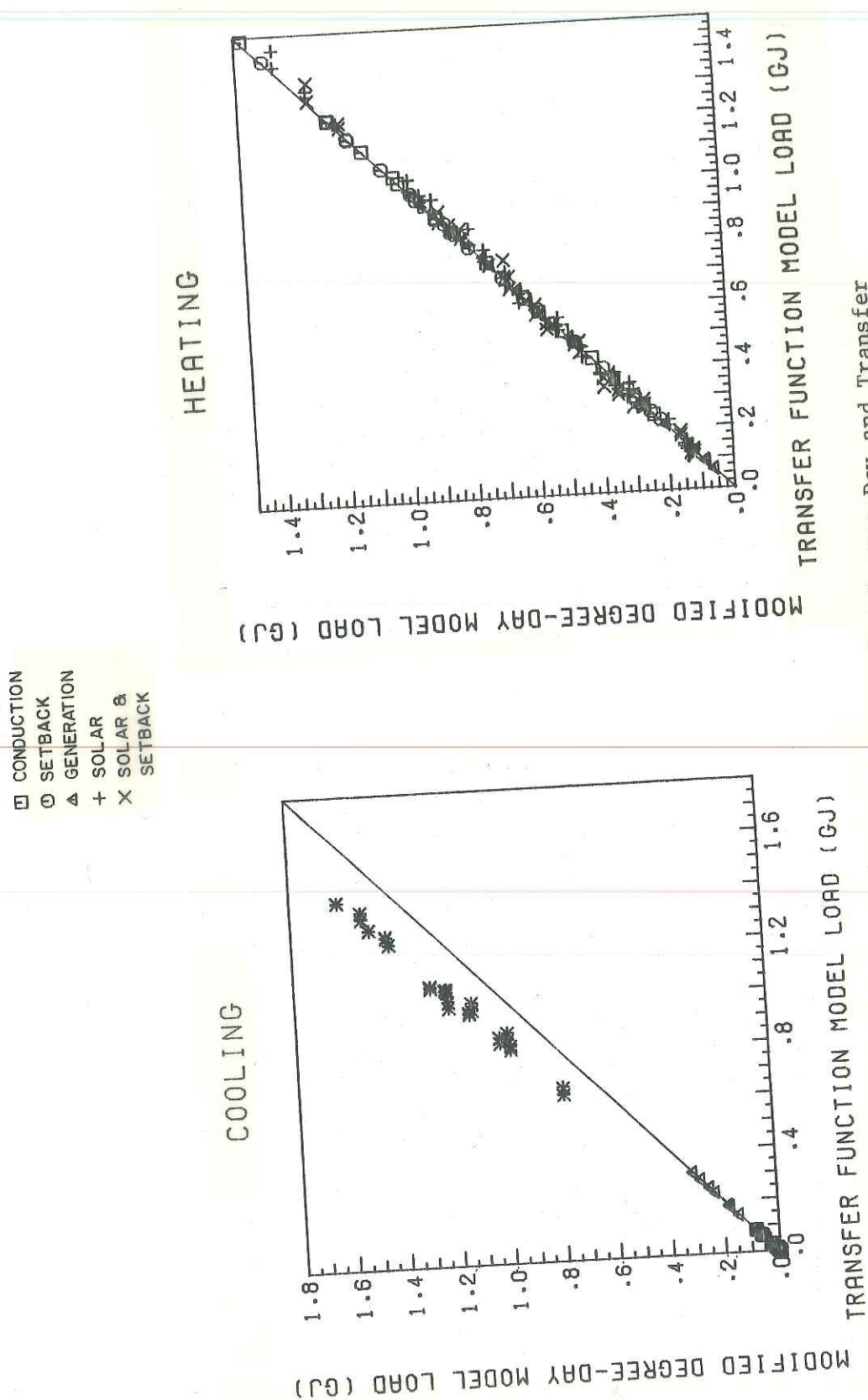
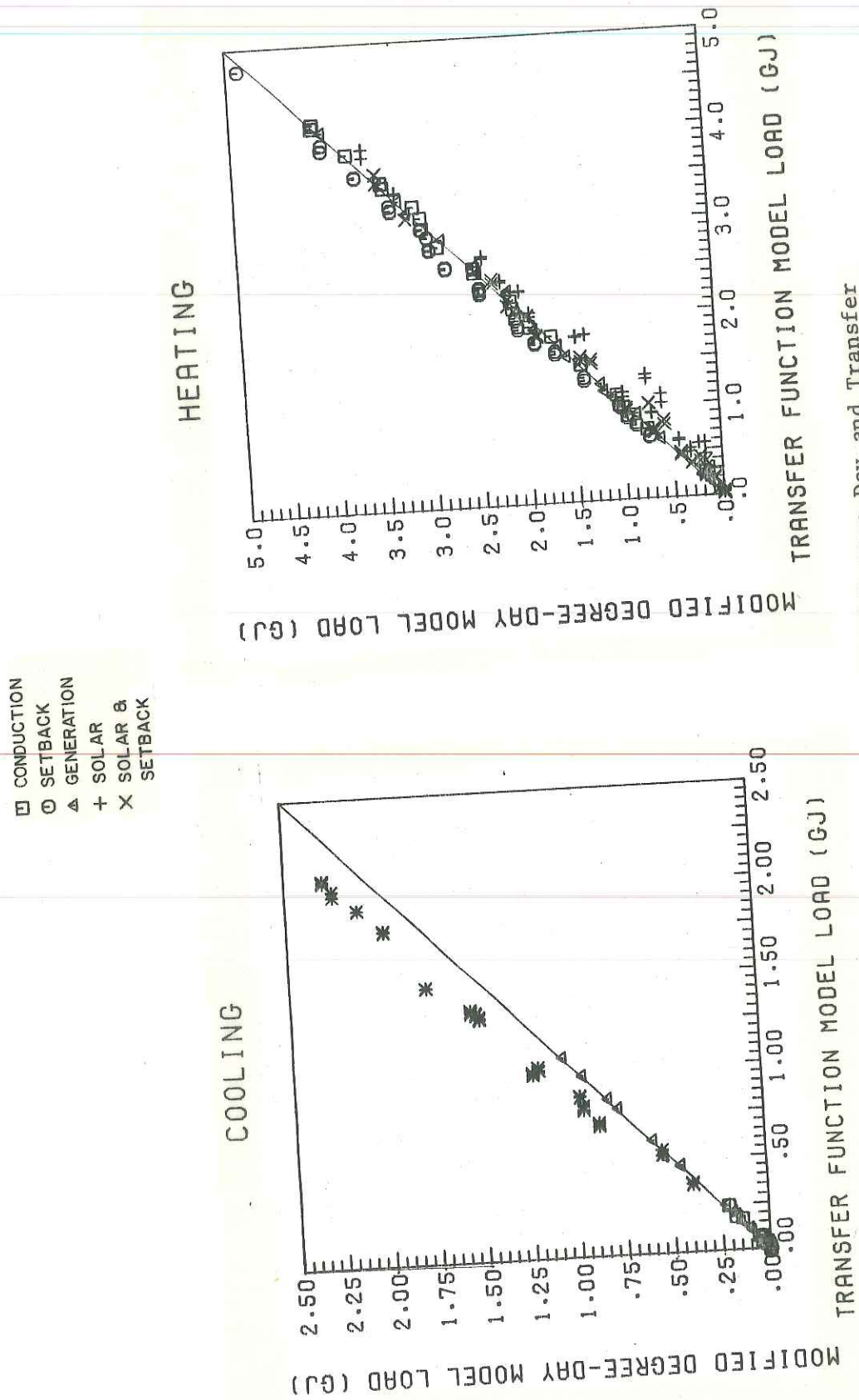


Figure 3-14 Comparison of Modified Degree-Day and Transfer Function Model Monthly Loads for ASHRAE Wall 11 when $C_{DD} = C_r + 0.45 C_s$





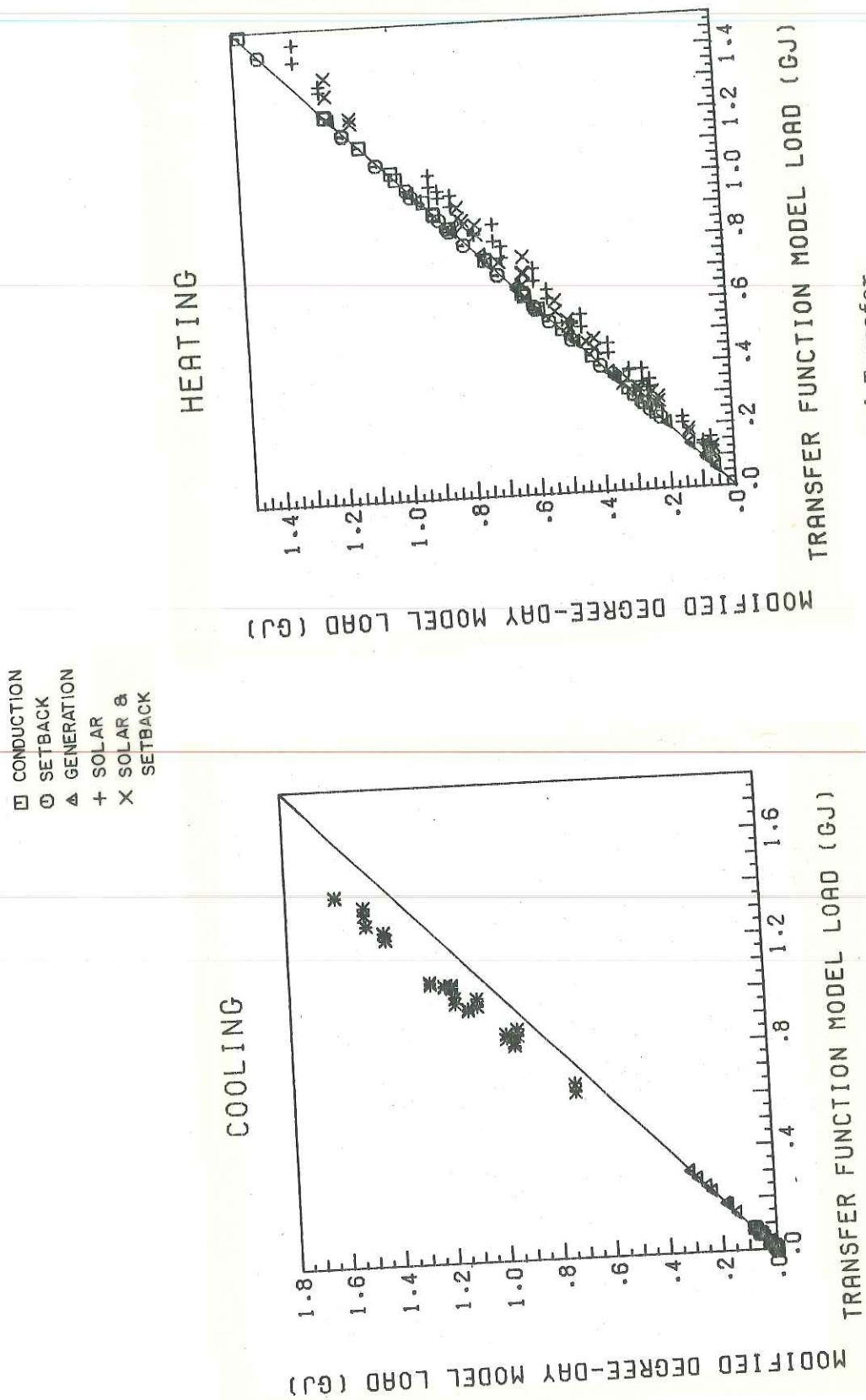


TABLE 3-15

Wall 11 Maximum Relative Standard Errors of Monthly Loads for

$$C_{DD} = C_r + 0.45 C_s$$

Cooling Season

Condition	Month	Average Load (GJ)	F_L (fraction)
Conduction	May	0.04	(+) 0.47
Setback	May	0.04	(+) 0.51
Generation	May	0.11	(+) 0.22
Solar	May	0.68	(+) 0.40
Solar & Setback	May	0.68	(+) 0.40

Heating Season

Conduction	November	2.25	(+) 0.03
Setback	October	0.92	(+) 0.09
Generation	November	0.34	(+) 0.08
Solar	October	0.22	(+) 0.14
Solar & Setback	October	0.15	(+) 0.54

(+) indicates over-prediction by modified degree-day model.

(-) indicates under-prediction by modified degree-day model.

TABLE 3-16
 Wall 26 Maximum Relative Standard Errors of Monthly Loads for
 $C_{DD} = C_r + 0.45 C_s$

Cooling Season			
Condition	Month	Average Load (GJ)	F_L (fraction)
Conduction	May	0.002	(-) 0.18
Setback	May	0.002	(+) 0.18
Generation	September	0.14	(+) 0.01
Solar	May	0.83	(+) 0.22
Solar & Setback	May	0.83	(+) 0.22
Heating Season			
Conduction	April	0.38	(-) 0.005
Setback	November	0.64	(+) 0.006
Generation	October	0.12	(+) 0.02
Solar	April	0.23	(-) 0.08
Solar & Setback	April	0.19	(-) 0.08

(+) indicates over-prediction by modified degree-day model.
 (-) indicates under-prediction by modified degree-day model.

3.7 Implications

The value of the lumped thermal capacitance used in the modified degree-day model was shown to affect building heating and cooling load prediction in Section 3.6. In this section the effect of lumped thermal capacitance on the prediction of building auxiliary energy requirements using direct-gain passive solar energy design methods is assessed.

Two one-zone passive structures were considered in this analysis. One building was constructed of ASHRAE exterior wall 11 and the other of ASHRAE exterior wall 26. Both buildings were characterized by the same physical dimensions and equal size direct-gain windows. Monthly average temperature and solar radiation data for Madison, WI, Columbia, MO, and Albuquerque, NM were used to represent ambient conditions. The parameters required for the design methods are shown in Table 3-17.

The annual auxiliary energy requirements and fraction of the load met by solar energy were predicted for each structure for three values of lumped thermal capacitance. One value of capacitance for each structure was derived using Davies' thermal admittance method [6] and appropriately scaled to represent the equivalent of Balcomb's diurnal heat capacity (dhc) [4]. The second value corresponded to 45 percent of the structure capacitance as found representative in Section 3.6. The third value was equal to the total building capacitance.

TABLE 3-17

Design Method Parameters Used for Assessment of Effect of Lumped
Thermal Capacitance on Auxiliary Energy Requirements

Window Area	9.3 m ²	
Number of Glazings	2	
($\tau\alpha$) (normal incidence)	0.75	
Window Conductance	4.17 W/m ² -C	
Thermostat Minimum Set Point	18.3 C	
Allowable Room Temperature Swing	5.6 C	
	Wall 11	Wall 26
Building UA	165.4 W/C	48.7 W/C

The annual auxiliary energy requirements and solar fractions predicted by the design method are shown in Table 3.18. The effect of thermal capacitance on auxiliary energy requirements was least when Madison, WI weather data was used. The range of capacitance studied represented a 3 percent variation in auxiliary energy requirements for wall 11 and a 6 percent variation for wall 26, but represented a 15 percent effect on the solar fraction. The greatest effect of the thermal capacitance was found in Albuquerque, NM. The variation in the auxiliary energy required was 21 percent for wall 11 and 16 percent for wall 26. The effect on the solar fraction was 20 percent for wall 26 and 24 percent for wall 11.

The auxiliary energy predicted by the Balcomb-Davies' method was always larger than the energy predicted by the 45 percent effective thermal capacitance estimate. The over-prediction ranged from 2 to 15 percent for wall 11, and from 2 to 9 percent for wall 26.

These results show that the value of thermal capacitance used in the design method can affect the predicted energy requirements and solar fraction of passive structures. This variation should be considered in light of the uncertainty surrounding the value of thermal capacitance to use in a design method.

TABLE 3-18

Annual Auxiliary Energy Requirements and Solar Fractions
 Predicted by Direct Gain Method

Wall 11						
Thermal Capac- itance (MJ/C)	Auxiliary Energy Required (GJ)			Solar Fraction		
	4.5	19.8	44.1	4.5	19.8	44.1
Location	(dhc)	(0.45C _s)	(C _s)	(dhc)	(0.45C _s)	(C _s)
Madison	59.6	58.0	57.7	0.21	0.23	0.24
Columbia	36.9	35.1	34.4	0.26	0.29	0.31
Albuquerque	25.5	22.1	20.8	0.39	0.48	0.51

Wall 26						
Thermal Capac- itance (MJ/C)	Auxiliary Energy Required (GJ)			Solar Fraction		
	0.7	1.2	2.7	0.7	1.2	2.7
Location	(dhc)	(0.45C _s)	(C _s)	(dhc)	(0.45C _s)	(C _s)
Madison	22.9	22.4	21.3	0.30	0.31	0.34
Columbia	14.6	14.1	13.2	0.32	0.34	0.38
Albuquerque	11.0	10.5	9.4	0.39	0.42	0.48

4. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

4.1 Summary and Conclusions

The goal of this research was to find the relationship between lumped, effective thermal capacitance and the actual value. This would increase the utility and accuracy of design and simulation methods which employ lumped capacitance modelling. This study showed, that for the situations and types of wall construction considered, no satisfactory relationship exists between actual and effective thermal capacitance for representative walls and actual weather data. The method appears to work for homogeneous walls with simple forcing functions.

In Chapter Two the effective thermal capacitance of a homogeneous wall subject to a mathematically describable forcing function was derived. The analytic solution of the lumped model was presented. The response of the same wall was generated by the method of finite differences. The effective thermal capacitance was derived from the comparison of the two responses.

The results of the finite difference methods show that for simple cases the effective thermal capacitance of a homogeneous wall could be determined. For a sinusoidal ambient temperature forcing function the effective thermal capacitance is defined by two parameters: the average Biot number and the periodic Fourier modulus.

The effective thermal capacitance of a simple structure was investigated by comparing transfer function and modified degree-day model simulations in Chapter Three. No simple relationship between

actual and effective thermal capacitance exists. The effective thermal capacitance was found to be different not only for different types of wall construction, but for heating and cooling, and for different interior and ambient boundary conditions. Effective thermal capacitance was found to vary even from month to month.

Analysis of results using the modified degree-day model show that errors in the load for the heating season for conduction, night setback, and generation cases were generally quite small for any value of lumped capacitance. Cooling loads and heating loads with solar gains are quite sensitive to the lumped capacitance used in the modified degree day model. For any case, monthly loads where the room temperature is not constant are quite sensitive to the lumped capacitance.

A lumped capacitance equal to the sum of about one-half the structure capacitance plus the lumped room capacitance produced the most accurate results for the modified degree-day model. Substantial error can still result when this value is used. Therefore caution should be exercised whenever lumped capacitance modelling is employed.

Two transfer function wall models were compared in Chapter 3. The use of transfer functions based on constant room temperature were found to over-predict loads and room temperature swings when used in situations in which the room temperature floated. The constant room temperature model is a special case of the more general floating room temperature transfer function model. The floating

room temperature model can be used for any room temperature condition. Use of the model for all cases will eliminate any over-prediction.

The effect of lumped thermal capacitance on building energy requirements predicted by passive solar energy design methods was shown in Section 3.7. The thermal capacitance was found to have an effect on both the auxiliary energy requirements and solar fraction for passive structures. The range in auxiliary energy requirements was from 2 percent for Madison, WI to over 20 percent for Albuquerque, NM. The range of thermal-capacitance studied caused the solar fraction to change by 15 to 24 percent.

It was also shown that the diurnal heat capacity--thermal admittance method tended to predict larger auxiliary energy requirements than the estimate of effective thermal capacitance which was found to generate the best results. The amount of over-prediction ranged from 2 percent for Madison weather data to 9-15 percent with Albuquerque weather data.

4.2 Recommendations

The following recommendations are made for future use of lumped capacitance modelling of structures and transfer function modelling of structures.

1. Lumped capacitance modelling of structures should be used with caution. The lumped thermal capacitance of a structure cannot be easily defined. Incorrect values of lumped

capacitance can cause large errors in load estimation.

2. If lumped capacitance modelling is used, the lumped capacitance should be taken as the sum of the room thermal capacitance plus about one-half the structure thermal capacitance. This value produced the best results for the most cases on a seasonal basis.
3. Lumped capacitance models should only be used to estimate seasonal or annual total loads. The monthly loads generally showed more error than the seasonal loads.
4. A floating room temperature transfer function model should be incorporated into TRNSYS. This is a more general case of the model in the current version. The floating room temperature model also works for constant room temperature conditions.
5. The coefficients for the floating room temperature transfer function equation should be published for standard wall construction as now exist in the ASHRAE Handbook of Fundamentals for the constant room temperature model. This would facilitate the use of the model and eliminate having to execute a computer program to determine the coefficients for many standard constructions.
6. Lumped capacitance modelling should be investigated further. Conceivably a two capacitance model could be developed where one capacitance represents the internal portion of the structure, and the other represents the walls.

APPENDIX A. Finite-Difference Wall Model Computer Program Listing

```

COMMON /R/ OMEGAL,OMEGAR,TBARR,AMPL,TBARR,AMPR,PHI
C *****
C
C THIS IS THE MAIN CALLING PROGRAM SET UP TO SIMULATE
C A ONE-DIMENSIONAL WALL VIA FINITE-DIFFERENCES WHEN
C TWO SINUSOIDAL BOUNDARY CONDITIONS EXIST.
C
C THE PROGRAM IS DESIGNED TO SIMULATE A WALL OF UP TO
C 10 DIFFERENT LAYERS. THE PROGRAM IS LIMITED TO 50
C TOTAL NODES.
C
C THE MAXIMUM NUMBER OF NODES THE USER MAY SPECIFY
C IS LIMITED TO 49 - THE NUMBER OF LAYERS IN THE
C WALL.
C
C THE PROGRAM MAY BE USED FOR EITHER TRANSIENT OR
C PERIODIC STEADY-STATE RESPONSE.
C
C THIS MAIN PROGRAM MAY ALSO BE USED FOR ONE SINUSOI-
C DAL AMBIENT BOUNDARY TEMPERATURE AND ONE CONSTANT
C AMBIENT BOUNDARY TEMPERATURE BY SPECIFYING THE
C APPROPRIATE AMPLITUDE (AMPL OR AMPR) AS 0.0 .
C
C TO USE THIS PROGRAM WITH OTHER TYPES OF BOUNDARY
C CONDITIONS, THIS MAIN PROGRAM MUST BE ALTERED TO
C SUIT THE USER DESIRED BOUNDARY CONDITIONS. SUB-
C ROUTINE RCALC MUST ALSO BE ALTERED TO SUIT THE
C DESIRED BOUNDARY CONDITONS
C
C IF IT IS DESIRED TO USE DISCRETE DATA FOR THE
C BOUNDARY CONDITIONS, SUBROUTINE RCALC SHOULD BE
C MODIFIED TO READ THE APPROPRIATE DATA FROM A FILE.
C
C THE SUBROUTINES USED BY THIS PROGRAM ARE:
C 1      SRAND      SOLVES A*X=B
C 2      DBAND      CHOLESKY DECOMPOSITON
C           OF A MATRIX
C 3      YAXPB      SOLVES Y=AX+R
C 4      HTRCES     CALCULATES MATRICES FOR
C           FINITE-DIFFERENCE SOLUTION
C 5      CRANK      CRANK-NICOLSON INTEGRATION
C 6      RCALC      CALCULATES R VECTOR
C 7      OTCALC     CALCULATES SIMULATION TIME
C           INCREMENT
C 8      VALUES    FITS ANALYTIC SOLUTION OF

```

```

C                                     LUMPED MODEL FOR SPECIFIC CASE
C
C   FOR THE SINUSOIDAL BOUNDARY CONDITIONS, THE
C   FOLLOWING INPUTS ARE REQUIRED:
C
C       TBARL      LEFT-HAND SIDE (LHS) AMBIENT
C                   TEMPERATURE MEAN
C       AMPL      LHS AMPLITUDE
C       PDL      PERIOD OF LHS SINE WAVE
C       TBARR      RIGHT-HAND SIDE (RHS)
C                   AMBIENT TEMPERATURE MEAN
C       AMPR      RHS AMPLITUDE
C       PDL      PERIOD OF RHS SINE WAVE
C       XLAG      RHS PHASE LAG
C       TIME      SIMULATION START TIME
C       TFINAL     SIMULATION STOP TIME
C
C   MATERIAL PHYSICAL AND THERMAL PROPERTIES AND
C   NODAL SPECIFICATIONS ARE INPUT IN SUBROUTINE MTRCES.
C
C   *****
C   COMMON /TIM/ IBW,TIME,DTIME,TFINAL,DTIME2
C   WRITE(*,1)
C
C   INPUTS FOR SINUSOIDAL BOUNDARY CONDITIONS
C
C   1 FORMAT(/,15X,'GIVE ME THE GOODS FOR AMB TEMPS.',/)
C   READ(*,*) TBARL,AMPL,PDL
C   READ(*,*) TBARR,AMPR,PDR,XLAG
C
C   START AND STOP TIME
C
C   READ(*,*) TIME,TFINAL
C
C   ECHO PRINT
C
C   WRITE(*,2)
C   2 FORMAT(/,15X,'LEFT-HAND SIDE AMB TEMP CONDS')
C   WRITE(*,3) TBARL,AMPL,PDL
C   3 FORMAT(/,5X,'MEAN TEMP ',F10.4,/,5X,'AMPLITUDE ',
C   $,F10.4,/,5X,'PERIOD ',F10.4,/)
C   WRITE(*,5)
C   5 FORMAT(/,15X,'RIGHT-HAND SIDE AMB TEMP CONDS')
C   WRITE(*,6) TBARR,AMPR,PDR,XLAG
C   6 FORMAT(/,5X,'MEAN TEMP ',F10.4,/,5X,'AMPLITUDE ',
C   $,F10.4,/,5X,'PERIOD ',F10.4,/,5X,'PHASE LAG ',F10.4,/)
C   PI=3.14159
C   OMEGAR=2.*PI/PDR
C   OMEGAL=2.*PI/PDL
C   PHI=XLAG/57.3
C   CALL MTRCES

```

```
CALL CRANK  
CALL VALUES  
STOP  
END
```

```

SUBROUTINE MTRCES
C *****
C
C THIS SUBROUTINE CALCULATES THE C AND S MATRICES
C FOR THE  $CTOT + ST = R$  REPRESENTATION OF THE
C FINITE-DIFFERENCE WALL. THE C AND S MATRICES AND
C THE INITIAL R VECTOR ARE PRINTED IN ECONOMY-
C BANDED FORM.
C THE INPUTS REQUIRED FOR THIS ROUTINE ARE:
C
C 1      TITLE      SIMULATION TITLE
C 2      UNITS      USER UNIT SYSTEM
C 3      NMAT        NUMBER OF LAYERS IN WALL
C 4      NBCL        LEFT-HAND SIDE BOUNDARY
C           CONDITION
C 5      NBCR        RIGHT-HAND SIDE BOUNDARY
C           CONDITION
C 6      IBW         BANDWIDTH OF MATRIX
C           (IBW=2 FOR 1-D WALL)
C 7      TI          INITIAL TEMPERATURE OF NODES
C           (NUMBER OF USER SPECIFIED NODES + NMAT
C           + 1 VALUES OF TI)
C
C           FOR EACH WALL LAYER:
C
C 1      XK          THERMAL CONDUCTIVITY
C 2      RHO         MATERIAL DENSITY
C 3      CP          SPECIFIC HEAT
C 4      THK         LAYER THICKNESS
C 5      NNODE       NUMBER OF INTERNAL NODES
C
C           FOR CONVECTION BOUNDARY CONDITIONS:
C
C 1      HLEFT       LHS CONVECTION COEFFICIENT
C
C           AND OR
C
C 2      HRIGHT      RHS CONVECTION COEFFICIENT
C
C THE BOUNDARY CONDITIONS (NBCL, NBCR) ARE DEFINED BY:
C
C 1      SPECIFIED HEAT FLUX BOUNDARY
C 2      SPECIFIED SURFACE TEMPERATURE BOUNDARY
C 3      SPECIFIED COVECTION BOUNDARY
C
C *****
C *****
COMMON /SIM/ C(50,10),S(50,10),R(50),T(50),Y(50),NBCL,NBCR,SAVE1,
$SAVE2,NODTOT
COMMON /R/ OMEGAL,OMEGAR,TBARR,ANPL,TBARR,AMPR,PHI
COMMON /TIM/ IBW,TIME,DTIME,TFINAL,DTIME2

```



```

COMMON /H/ HLEFT,HRIGHT
COMMON /Q/ XKMAT(50,10),GEN(50)
COMMON /PROP/ RHO(10),XK(10),CP(10),THK(10)
DIMENSION DELX(10),H(50),UNITS(5),TITLE(20),NNODE(10)

```

C

C READ AND ECHOPRINT INPUTS

C

```

      READ(*,1) TITLE
1  FORMAT(20A4)
      WRITE(*,2) TITLE
2  FORMAT(/,10X,20A4,/)
      READ(*,10) UNITS
10 FORMAT(5A5)
      WRITE(*,10) UNITS
      WRITE(*,3)
3  FORMAT(5X,'NMAT',2X,'NBCL',2X,'NBCR')
      READ(*,*) NMAT,NBCL,NBCR
      WRITE(*,*) NMAT,NBCL,NBCR
      TOTTHK=0.0
      DO 100 I=1,NMAT
      READ(*,*) XK(I),RHO(I),CP(I),THK(I),NNODE(I)
      WRITE(*,*) XK(I),RHO(I),CP(I),THK(I),NNODE(I)
      TOTTHK=TOTTHK+THK(I)
100 CONTINUE
      READ(*,*) IBW
      WRITE(*,101) IBW
101 FORMAT(/,10X,'THE BANDWIDTH IS',3X,13,/)
      HLEFT=0.0
      HRIGHT=0.0
      IF(ABS(NBCL),LE,2) GO TO 110
      READ(*,*) HLEFT
      WRITE(*,4) HLEFT
4  FORMAT(/,5X,'HLEFT=',F8.3,/)
110 IF(ABS(NBCR),LE,2) GO TO 120
      READ(*,*) HRIGHT
      WRITE(*,5) HRIGHT
5  FORMAT(/,5X,'HRIGHT=',F8.3,/)
      NODTOT=0.0
      DO 130 I=1,NMAT
      NODTOT=NODTOT+NNODE(I)
130 CONTINUE

```

C

C CHECK NUMBER OF NODES

C

```

      NODTOT=NODTOT+NMAT+1
      IF(NODTOT,LE,50) GO TO 140
      WRITE(*,6) NODTOT
6  FORMAT(/,5X,'ERROR ERROR, YOU SPECIFIED',I3,1X,'NODES,',
      * ' WHICH IS GREATER THAN THE MAXIMUM ALLOWED!!',/)
      IF(NODTOT,GT,50) STOP
140 WRITE(*,7) NODTOT

```

```

7 FORMAT(/,5X,I3,2X,'TOTAL NODES FOR SIMULATION',/)
C
C CLACULATE NODAL SPACING
C
  DO 150 J=1,NMAT
    DELX(J)=THK(J)/FLOAT(NNODE(J)+1)
150 CONTINUE
C
C INITIALIZE C AND S MATRICES
C
  DO 160 J=1,10
    DO 160 I=1,50
      C(I,J)=0.0
      S(I,J)=0.0
160 CONTINUE
C
C DETERMINE C MATRIX
C
  NLL=0
  NUL=1
  DO 200 J=1,NMAT
    NUL=NLL+NNODE(J)+1
    IF(J.EQ.1) NUL=NUL+1
    NLL=NLL+1
    DO 190 K=NLL,NUL
      C(K,1)=RHO(J)*CP(J)*DELX(J)
      IF(J.EQ.1.AND.K.EQ.1) C(K,1)=C(K,1)/2.0
      IF(J.EQ.NMAT.AND.K.EQ.NUL) C(K,1)=C(K,1)/2.0
      IF(K.EQ.NUL.AND.J.NE.NMAT) C(K,1)=C(K,1)/2.0+(RHO(J+1)*CP(J+1)*
        $DELX(J+1)/2.0)
190 CONTINUE
    NLL=NUL
200 CONTINUE
C
C PRINT C MATRIX
C
  WRITE(*,17)
17 FORMAT(/,15X,'THE CAPACITANCE MATRIX, ECONOMY-BANDED STORAGE
  $',/)
  DO 220 I=1,IBW
    WRITE(*,*) (C(J,I),J=1,NODTOT)
220 CONTINUE
C
C DETERMINE S MATRIX
C
  DO 240 I=1,50
    H(I)=0.0
240 CONTINUE
    NKLL=0.0
    NKUL=0.0
    DO 270 J=1,NMAT

```

```

NKUL=NKLL+NMODE(J)+1
IF(J.EQ.1) NKUL=NKUL+1
NKLL=NKLL+1
DO 250 K=NKLL,NKUL
S(K,1)=2.0*XK(J)/DELX(J)
IF(K.EQ.NKUL) GO TO 245
S(K,2)=-XK(J)/DELX(J)
245 IF(K.EQ.NKUL.AND.J.NE.NMAT) S(K,1)=(XK(J)/DELX(J))+
$(XK(J+1)/DELX(J+1))
IF(K.EQ.NKUL.AND.J.EQ.NMAT) S(K,1)=XK(J)/DELX(J)
IF(K.EQ.NKUL.AND.J.NE.NMAT) S(K,2)=-XK(J+1)/DELX(J+1)
IF(K.EQ.1.AND.J.EQ.1) S(K,1)=XK(J)/DELX(J)
250 CONTINUE
NKLL=NKUL
270 CONTINUE
DO 275 I=1,NODTOT
DO 275 J=1,IBW
XKMAT(I,J)=S(I,J)
275 CONTINUE
IF(ABS(NBCL).GE.3) H(1)=HLEFT
IF(ABS(NBCR).GE.3) H(NODTOT)=HRIGHT
DO 280 I=1,NODTOT
S(I,1)=S(I,1)+H(I)
280 CONTINUE
SAVE1=0.0
SAVE2=0.0
IF(ABS(NBCL).NE.2) GO TO 285
SAVE1=S(1,2)
S(1,2)=0.0
285 IF(ABS(NBCR).NE.2) GO TO 290
288 SAVE2=S(NODTOT-1,2)
S(NODTOT-1,2)=0.0

C
C PRINT S MATRIX
C
290 WRITE(*,8)
8 FORMAT(/,15X,'S MATRIX, ECONOMY-BANDED STORAGE',/)
DO 300 J=1,IBW
WRITE(*,*) (S(I,J),I=1,NODTOT)
300 CONTINUE

C
C ESTIMATE TIME STEP FOR INTEGRATION
C
CALL DTCALC

C
C DETERMINE INITIAL R VECTOR AND PRINT OUT
C
CALL RCALC
WRITE(*,9)
9 FORMAT(/,15X,'INITIAL R VECTOR',/)
WRITE(*,*) (R(I),I=1,NODTOT)

```

```
C
C READ INITIAL TEMPERATURE DISTRIBUTION
C AND PRINT OUT
C
  READ(*,*) (T(I),I=1,NDDTOT)
  WRITE(*,12)
12 FORMAT(/,15X,'THE INITIAL TEMPERATURE DISTRIBUTION',/)
  WRITE(*,*) (T(I),I=1,NDDTOT)

C
C PRINT TIME STEP
C
  WRITE(*,13) DTIME
13 FORMAT(/,5X,'THE ESTIMATED TIMESTEP IS',F8.4)
  WRITE(*,14) DTIME2
14 FORMAT(5X,'THE TIMESTEP USED IS ',F8.4,/)
  RETURN
  END
```

SUBROUTINE RCALC

```

C *****
C
C THIS ROUTINE IS A USER SPECIFIC SUBROUTINE WHICH
C CALCULATES THE R VECTOR FOR THE SOLUTION OF
C  $CTDOT + ST = R$ , AND SHOULD BE WRITTEN TO
C ACCOMMODATE USER SPECIFIED BOUNDARY CONDITIONS.
C
C ANY INTERNAL GENERATION OR HEAT FLUX INPUTS SHOULD
C BE INPUT IN THIS ROUTINE,
C
C AS PRESENTED HERE, THE ROUTINE CALCULATES R FOR
C EITHER TWO SINUSOIDAL AMBIENT TEMPERATURE BOUN-
C DARIES, OR FOR ONE SINUSOIDAL AND ONE CONSTANT
C AMBIENT TEMPERATURE BOUNDARIES WITHOUT INTERNAL
C GENERATION OR DIRECT HEAT FLUX INPUT,
C
C FOR ANY OTHER BOUNDARY CONDITION OR DISCRETE DATA
C INPUT, THIS ROUTINE AND THE MAIN CALLING PROGRAM
C BE MODIFIED.
C *****
C COMMON /SIN/ C(50,10),S(50,10),R(50),T(50),Y(50),NBCL,NBCR,SAVE1,
  $SAVE2,NODTOT
  COMMON /R/ OMEGAL,OMEGAR,TBARI,AMPL,TBARR,AMPR,PHI
  COMMON /FIN/ IBW,TIME,DTIME,TFINAL,DTIME2
  COMMON /H/ HLEFT,HRIGHT
  COMMON /Q/ XKHAT(50,10),GEN(50)
  DIMENSION QFLUX(50),HTINF(50)
C
C HEAT FLUX AND GENERATION INPUTS
C
  DO 10 J=1,NODTOT
    R(J)=0.0
    GEN(J)=0.0
    QFLUX(J)=0.0
    HTINF(J)=0.0
  10 CONTINUE
C
C CONVECTION INPUTS
C
  HTINF(1)=(TBARI+AMPL*SIN(TIME*OMEGAL))*HLEFT
  HTINF(NODTOT)=(TBARR+AMPR*SIN(TIME*OMEGAR+PHI))*HRIGHT
C
C DETERMINE R
C
  DO 20 J=1,NODTOT
    R(J)=GEN(J)+QFLUX(J)+HTINF(J)
  20 CONTINUE
  R(1)=R(1)-SAVE1

```

```
R(NDITOT)=R(NDITOT)-SAVE2  
RETURN  
END
```

```

      SUBROUTINE DTCALC
C *****
C
C   THIS ROUTINE ESTIMATES THE MINIMUM TIME STEP
C   REQUIRED FOR THE CRANK-NICOLSON INTEGRATION
C   SCHEME IN SUBROUTINE CRANK.
C
C   THE TIME STEP USED IN THE SIMULATION IS TAKEN
C   AS THE NEXT SMALLEST VALUE SUCH THAT PRINTING
C   MAY OCCUR AT AN INTEGER NUMBER TIME STEPS.
C
C   THE MINIMUM TIME STEP IS ESTIMATED FROM THE
C   NODAL VALUES OF C(I,1) AND S(I,J).
C *****
      DIMENSION THETA(50)
      COMMON /SIM/ C(50,10),S(50,10),R(50),T(50),Y(50),
      *NRCL,NBCR,SAVE1,SAVE2,NODTOT
      COMMON /R/ OMEGAL,OMEAGR,TBARL,AMPL,TBARR,AMPR,PHI
      COMMON /TIM/ IBW,TIME,DTIME,TFINAL,DTIME2
      COMMON /DT/ NOM
C
C   ESTIMATE TIME STEP
C
      DTIME=1.00+R5
      DO 10 I=1,NODTOT
      SUM=0.0
      DO 5 J=1,IBW
      SUM=SUM+ABS(S(I,J))
      5 CONTINUE
      THETA(I)=2.0*C(I,1)/SUM
      10 DTIME=AMIN1(DTIME,THETA(I))
C
C   PICK INTEGER FRACTION
C
      XIN=1.0/DTIME
      NOM=IFIX(XIN)
      DTIME2=1.0/FLOAT(NOM)
      RETURN
      END

```

SUBROUTINE CRANK

```

C *****
C
C THIS ROUTINE USES THE CRANK-NICOLSON INTEGRATION
C METHOD TO SOLVE  $CYDOT + ST = R$ 
C
C THE ROUTINE ALSO PRINTS:
C
C 1      TIME      SIMULATION TIME
C 2      QINL      HEAT FLUX AT LHS SURFACE
C              (POSITIVE INTO WALL)
C 3      QOUTR     HEAT FLUX AT RHS SURFACE
C              (POSITIVE OUT OF WALL)
C 4      TEMPL     LHS SURFACE TEMPERATURE
C 5      TEMPR     RHS SURFACE TEMPERATURE
C 6      TMEAN     CAPACITANCE-WEIGHTED MEAN
C              WALL TEMPERATURE
C
C AT 1/TIME STEP INTERVALS.
C
C THE TIME STEP IS CALCULATED IN SUBROUTINE DTCALC.
C
C *****
C COMMON /SIN/ C(50,10),S(50,10),R(50),T(50),Y(50),NBCL,NBCR,SAVE1,
  $SAVE2,NODTOT
  COMMON /R/ OMEGAL,OMEGAR,TBARR,AMPL,TBARR,AMPR,PHI
  COMMON /FIN/ IBW,TIME,DTIME,TFINAL,DTIME2
  COMMON /Q/ XKHAT(50,10),GEN(50)
  COMMON /DT/ NOM
  COMMON /VAL/ TMEAN(1000)
  DIMENSION AC(50,10),B(50,10),RNDW(50),XL(50),
  $QINL(1000),QOUTR(1000),T1(1000),TN(1000)
  DATA NR0W/50/,NCOL/10/
  NPRINT=0
  NHR=INT(TFINAL)
  NCOUNT=1
  QSUM1=0.0
  QSUMN=0.0
C
C CALCULATE INITIAL HEAT FLOWS AND TEMPS.
C
  DO 1 J=1,IBW
    QSUM1=QSUM1+(XKHAT(1,J)-S(1,J))*T(J)
    N1=NODTOT+1-J
    QSUMN=QSUMN+(XKHAT(N1,J)-S(N1,J))*T(N1)
  1 CONTINUE
  QINL(1)=QSUM1+R(1)-GEN(1)
  QOUTR(1)=- (QSUMN+R(NODTOT)-GEN(NODTOT))
  TSUM=0.0
  CSUM=0.0
  DO 2 I=1,NODTOT

```



```

      TSUM=TSUM+T(I)/C(I,1)
      CSUM=CSUM+C(I,1)
2    CONTINUE
      TMEAN(1)=TSUM/CSUM
C
C    PRINT INITIAL TEMPS. AND HEAT FLOWS
C
      WRITE(*,3) TIME
3    FORMAT('1',15X,'TIME = ',F10.4,/)
      WRITE(*,4)
4    FORMAT(8X,'QINL',8X,'QOUTR',8X,'TEMPL',8X,'TEMPR',
     $8X,'TMEAN')
      WRITE(*,5) QINL(1),QOUTR(1),T(1),T(NODTOT),TMEAN(1)
5    FORMAT(1X,1PE11.3,4(2X,1PE11.3),/)
      WRITE(13,13) TIME,QINL(1),QOUTR(1),T(1),T(NODTOT),TMEAN(1)
      T1(1)=T(1)
      TN(1)=T(NODTOT)
      DT2=DTIME/2.0
      TIME0=TIME
C
C    TRANSIENT SOLUTION
C
      DO 6 I=1,NODTOT
      DO 6 J=1,IBW
        AC(I,J)=C(I,J)+S(I,J)*DT2
        B(I,J)=C(I,J)-S(I,J)*DT2
6    CONTINUE
C
C    CHOLESKY DECOMPOSITION
C
      CALL DBAND(NROW,NCOL,NODTOT,IBW,AC,NBGO)
      IF(NBGO.EQ.1) GO TO 99
7    IF(NCOUNT-1.GE.NHR) GO TO 999
      NPRINT=NPRINT+1
      DO 8 I=1,NODTOT
6      RNOW(I)=R(I)
      TIME=TIME+DTIME2
      CALL RCALC
      DO 9 I=1,NODTOT
6      XL(I)=(RNOW(I)+R(I))*DT2
9    CONTINUE
C
C    SOLVE Y = BT + XL
C
      CALL YAXPB(NROW,NCOL,NODTOT,IBW,B,XL,T,Y)
C
C    SOLVE AC*Y = T
C
      CALL SBAND(NROW,NCOL,NODTOT,IBW,AC,Y,T)
      IF(NPRINT.NE.NOM) GO TO 7
      NPRINT=0

```

```

        NCOUNT=NCOUNT+1
        TSUM=0.0
        DO 10 I=1,NODTOT
            TSUM=TSUM+T(I)*C(I,1)
10    CONTINUE
C
C    CALCULATE MEAN WALL TEMPERATURE
C
        TMEAN(NCOUNT)=TSUM/CSUM
C
C    CALCULATE SURFACE HEAT FLOWS
C
        QSUM1=0.0
        QSUMN=0.0
        DO 11 J=1,IRW
            QSUM1=QSUM1+(XKMAT(1,J)-S(1,J))*T(J)
            N2=NODTOT-J+1
            QSUMN=QSUMN+(XKMAT(N2,J)-S(N2,J))*T(N2)
11    CONTINUE
        QINL(NCOUNT)=QSUM1+R(1)-GEN(1)
        QOUTR(NCOUNT)=- (QSUMN+R(NODTOT)-GEN(NODTOT))
        T1(NCOUNT)=T(1)
        TN(NCOUNT)=T(NODTOT)
C
C    OUTPUT
C
        WRITE(*,12) TIME
12    FORMAT(15X,'TIME = ',F10.4,/)
        WRITE(*,4)
        WRITE(*,5) QINL(NCOUNT),QOUTR(NCOUNT),T1(NCOUNT),TN(NCOUNT),
            $TMEAN(NCOUNT)
        WRITE(13,13) TIME,QINL(NCOUNT),QOUTR(NCOUNT),T1(NCOUNT),
            $TN(NCOUNT),TMEAN(NCOUNT)
13    FORMAT(1X,1PE11.3,5(2X,1PE11.3))
        GO TO 7
99    WRITE(*,100)
100   FORMAT('0',15X,'DBAND FAILED. LOOK FOR INPUT ERRORS. ')
        IF(NOGO.EQ.1) STOP
999   RETURN
        END

```

```

      SUBROUTINE YAXPB(NROW,NCOL,N,IBW,A,B,X,Y)
C *****
C THIS ROUTINE WAS WRITTEN BY PROFESSOR G. E. MYERS
C AND IS USED WITH HIS PERMISSION.
C *****
C *****
C * THIS SUBROUTINE COMPUTES  $Y = AX + B$  WHEN A IS A REAL, *
C * SYMMETRIC MATRIX *
C * INPUT: NROW = ROW DIMENSION, DEFINED IN CALLING PROGRAM *
C *          NCOL = COLUMN DIMENSION, DEFINED IN CALLING PROGRAM *
C *          N = NUMBER OF ROWS IN MATRIX *
C *          IBW = BANDWIDTH OF MATRIX (THIS IS ALSO THE NUMBER OF *
C *                COLUMNS NEEDED TO STORE DIAGONAL AND UPPER BAND *
C *                OF MATRIX IN AN N BY IBW RECTANGULAR ARRAY.) *
C *          A = REAL, SYMMETRIC MATRIX (DIAGONAL AND UPPER BAND *
C *                STORED AS AN N BY IBW ARRAY) *
C *          B = RIGHT-HAND-SIDE VECTOR TO BE ADDED TO  $AX$  *
C *          X = RIGHT-HAND-SIDE VECTOR TO BE MULTIPLIED BY A *
C * OUTPUT: A = SAME AS INPUT *
C *          B = SAME AS INPUT *
C *          X = SAME AS INPUT *
C *          Y = LEFT-HAND-SIDE VECTOR =  $AX + B$  *
C *****
      DIMENSION A(NROW,NCOL),B(NROW),X(NROW),Y(NROW)
      DO 3 I=1,N
        Y(I) = B(I)
        JMAX = N - I + 1
        IF (JMAX.GT.IBW) JMAX = IBW
        DO 1 J=1,JMAX
          II = I + J - 1
          Y(I) = Y(I) + A(I,J)*X(II)
1        IF (I.EQ.1) GO TO 3
          JMAX = IBW
          IF (I.LT.IBW) JMAX = I
          DO 2 J=2,JMAX
            II = I - J + 1
            Y(I) = Y(I) + A(II,J)*X(II)
2          CONTINUE
3        CONTINUE
      RETURN
      END

```

```

      SUBROUTINE SBAND (NROW,NCOL,N,IBW,A,B,X)
C *****
C THIS ROUTINE WAS WRITTEN BY PROFESSOR G. E. MYERS
C AND IS USED WITH HIS PERMISSION.
C *****
C *****
C * THIS SUBROUTINE SOLVES  $A \cdot X = B$  WHEN GIVEN THE CHOLESKY
C * DECOMPOSITION OF A
C * INPUT: NROW = ROW DIMENSION, DEFINED IN CALLING PROGRAM
C *          NCOL = COLUMN DIMENSION, DEFINED IN CALLING PROGRAM
C *          N = NUMBER OF ROWS IN MATRIX
C *          IBW = BANDWIDTH OF MATRIX (THIS IS ALSO THE NUMBER OF
C *                COLUMNS NEEDED TO STORE DIAGONAL AND UPPER BAND
C *                OF MATRIX IN AN N BY IBW RECTANGULAR ARRAY.)
C *          A = CHOLESKY DECOMPOSITION OF MATRIX A (DIAGONAL AND
C *                AND UPPER BAND STORED AS AN N BY IBW ARRAY)
C *          B = RIGHT-HAND-SIDE VECTOR
C * OUTPUT: A = SAME AS INPUT
C *          B = SAME AS INPUT
C *          X = SOLUTION VECTOR
C *****
      DIMENSION A(NROW,NCOL),B(NROW),X(NROW)
      DOUBLE PRECISION DSUM,D1,D2
      DO 2 I=1,N
        J = I - IBW + 1
        IF (I+1.LE.IBW) J = 1
        DSUM = B(I)
        K1 = I - 1
        IF (J.GT.K1) GO TO 2
        DO 1 K=J,K1
          IMKP1 = I - K + 1
          D1 = A(K,IMKP1)
          DSUM = DSUM - D1*X(K)
1      X(I) = DSUM/A(I,1)
2      DO 4 I1=1,N
        I = N - I1 + 1
        J = I + IBW - 1
        IF (J.GT.N) J = N
        DSUM = X(I)
        K2 = I + 1
        IF (K2.GT.J) GO TO 4
        DO 3 K=K2,J
          KMIP1 = K - I + 1
          D2 = A(I,KMIP1)
          DSUM = DSUM - D2*X(K)
3      X(I) = DSUM/A(I,1)
4      RETURN
      END

```

```

      SUBROUTINE DBAND (NROW,NCOL,N,IBW,A,NOGO)
C *****
C   THIS ROUTINE WAS WRITTEN BY PROFESSOR G. E. MYERS
C   AND IS USED WITH HIS PERMISSION.
C *****
C *****
C   * THIS SUBROUTINE COMPUTES THE CHOLESKY DECOMPOSITION OF A REAL, *
C   * SYMMETRIC, POSITIVE-DEFINITE MATRIX *
C   * INPUT:  NROW = ROW DIMENSION, DEFINED IN CALLING PROGRAM *
C   *          NCOL = COLUMN DIMENSION, DEFINED IN CALLING PROGRAM *
C   *          N = NUMBER OF ROWS IN MATRIX *
C   *          IBW = BANDWIDTH OF MATRIX (THIS IS ALSO THE NUMBER OF *
C   *                COLUMNS NEEDED TO STORE DIAGONAL AND UPPER BAND *
C   *                OF MATRIX IN AN N BY IBW RECTANGULAR ARRAY.) *
C   *          A = REAL, SYMMETRIC, POSITIVE-DEFINITE MATRIX (DIAGONAL *
C   *                AND UPPER BAND STORED AS AN N BY IBW ARRAY) *
C   *          NOGO = 0 IF DECOMPOSITION WORKS *
C   *                = 1 IF DECOMPOSITION FAILS *
C   * OUTPUT: A = DIAGONAL AND UPPER BAND OF THE DECOMPOSITION *
C *****
      DIMENSION A(NROW,NCOL)
      DOUBLE PRECISION DSUM,D1,D2,D3,DSQRT
      NOGO = 0
      DO 6 I=1,N
        IP = N - I + 1
        IF (IP.GT,IBW) IP = IBW
        DO 6 J=1,IP
          IQ = IBW - J
          IF (IQ.GT,I-1) IQ = I - 1
          DSUM = A(I,J)
          IF (IQ.LT,1) GO TO 2
          DO 1 K=1,IQ
            IMK = I - K
            KP1 = K + 1
            JPK = J + K
            D1 = A(IMK,KP1)
            D2 = A(IMK,JPK)
            DSUM = DSUM - D1*D2
          1
          2 IF (J.GT,1) GO TO 5
            IF (DSUM) 3,3,4
          3   NOGO = 1
            RETURN
          4   D3 = DSQRT(DSUM)
            A(I,J) = D3
            GO TO 6
          5   A(I,J) = DSUM/D3
          6   CONTINUE
      RETURN
      END

```

SUBROUTINE VALUES

```

C *****
C
C THE ANALYTIC SOLUTION OF THE LUMPED MODEL
C TEMPERATURE RESPONSE IS CURVEFITTED TO
C THE FINITE-DIFFERENCE GENERATED MEAN WALL
C TEMPERATURE RESPONSE IN THIS ROUTINE. THE
C ROUTINE IS FOR THE ONE SINUSOIDAL--ONE-
C CONSTANT AMBIENT TEMPERATURE SITUATION FOR
C PERIODIC STEADY-STATE RESPONSE.
C
C THE ROUTINE ALSO CALCULATES THE EFFECTIVE THERMAL
C CAPACITANCE OF THE WALL BASED ON THE AMPLITUDE
C OF THE CURVEFITTED RESPONSE FOR A HOMOGENEOUS
C WALL.
C
C *****
C COMMON /PROP/ RHO(10),XK(10),CP(10),THK(10)
C COMMON /R/ OMEGA,OMEGA*,TBARL,AMPL,TBARR,AMPR,PHI
C COMMON /VAL/ TMEAN(1000)
C COMMON /H/ HLEFT,HRIGHT
C DIMENSION TH(50),TPRD(50),TIME(50)
C
C STARTING AND ENDING TIME OF POINTS TO
C BE FIT
C
C DATA NSTART/457/,NSTOP/481/
C
C CALCULATE LUMPED RESISTANCES
C
C RL=1./HLEFT+THK(1)/(2.*XK(1))
C IF(HRIGHT.LE.0.0) GO TO 13
C RR=1./HRIGHT+THK(1)/(2.*XK(1))
C RRATIO=RL/RR
C
C FIND AVERAGE
C
C TBAR=(RR*TBARL+RL*TBARR)/(RL+RR)
C GO TO 15
13 RR=1.E+23
C TBAR=TBARL
C RRATIO=RL/RR
15 DO 1 J=NSTART,NSTOP
C I=J-NSTART+1
C TH(I)=TMEAN(J)
C TIME(I)=FLOAT(I-1)
1 CONTINUE
C N2=NSTOP-NSTART
C
C FIND PHASE LAG

```

```

C
  DO 2 J=1,N2
    N=J
    IF(TM(J).LT.TBAR.AND.TM(J+1).GT.TBAR) GO TO 3
  2 CONTINUE
  3 SLOPE=(TM(N+1)-TM(N))/(TIME(N+1)-TIME(N))
    XINT=TIME(N)-(TM(N)-TBAR)/SLOPE
    PHIO=-XINT*OMEGAL
    IF(XINT.LE.12.) NADD=6
    IF(XINT.GT.12.) NADD=-12
C
C  FIND AMPLITUDE
C
    BTIME=TIME(N+NADD)
    B=(TM(N+NADD)-TBAR)/SIN(OMEGAL*BTIME+PHIO)
    IF(B.LT.0.) B=ABS(B)
    N3=N2+1
    SUM=0.0
C
C  PREDICT TEMPERATURE RESPONSE
C
    DO 4 J=1,N3
      TPRD(J)=B*SIN(OMEGAL*TIME(J)+PHIO)+TBAR
      SUM=SUM+(TPRD(J)-TM(J))**2
    4 CONTINUE
C
C  CALCULATE STANDARD ERROR
C
    SIGMA=SQRT(SUM/FLOAT(N3))
C
C  AVERAGE BIOT NUMBER
C
    BIOT=(HLEFT+HRIGHT)*THK(1)/(2.0*XK(1))
    IF(HRIGHT.LE.0.0) GO TO 17
C
C  CALCULATE EFFECTIVE CAPACITANCE
C
    RHOC2=((AMPL/(B*RL))**2-((RL+RR)/(RL*RR))**2)/(OMEGAL**2)
    GO TO 18
  17 RHOC2=((AMPL/B)**2-1)/(OMEGAL*RL)**2)
  18 RHOCLE=SQRT(RHOC2)
    RHOCLEA=RHO(1)*CP(1)*THK(1)
    RHORAT=RHOCLE/RHOCLEA
C
C  INVERSE FOURIER MODULUS
C
    ALPHA=XK(1)/(RHO(1)*CP(1))
    FORIN=OMEGAL*(THK(1)**2)/ALPHA
C
C  PRINT IT OUT
C

```

```

WRITE(*,5)
5 FORMAT(/,15X,'HERE'S WHAT YOU'VE BEEN LOOKING FOR !!!',/)
WRITE(*,6) BIOT,RL,RR,RRATIO,FORIN
6 FORMAT(/,5X,'BIOT NUMBER = ',F8.4,/,5X,'LEFT-HAND SIDE RESISTANCE
$= ',F8.4,/,5X,'RIGHT-HAND RESISTANCE = ',F8.4,/,5X,'RESISTANCE ',
$'RATIO = ',F9.6,/,5X,'1/FOURIER = ',F9.6,/)
WRITE(*,7) RHOCLE,RHOCRA,RHORAT
7 FORMAT(/,5X,'EFFECTIVE RHOCLE = ',F8.4,/,5X,'ACTUAL RHOCLE = ',F8.4
$,/,5X,'EFFECTIVE/ACTUAL = ',F9.6,/)
WRITE(*,8)
8 FORMAT(/,5X,'CALCULATED TEMP',10X,'TIME',10X,'PREDICTED TEMP',/)
DO 10 J=1,N3
WRITE(*,9) TPRD(J),TIME(J),TM(J)
9 FORMAT(/,6X,F8.4,14X,F8.4,11X,F8.4)
10 CONTINUE
WRITE(*,11) SIGMA
11 FORMAT(/,5X,'SIGMA = ',1PE11.3)
WRITE(*,12) TBAR,PHIO,B
12 FORMAT(/,5X,'TBAR = ',F8.4,/,5X,'PHASE LAG = ',F8.4,/,5X,
$'AMPLITUDE = ',F8.4,/)
RETURN
END

```


APPENDIX B. Modified TRNSYS Component Computer Program Listings

```

SUBROUTINE TYPE12(TIME,XIN,DUT,T,DTDT,PAR,INFO)
C *****
C
C THIS SUBROUTINE MODELS HEAT TRANSFER THROUGH WALLS AS
C UA*(AMBIENT OR SOL-AIR TEMPERATURE - ROOM TEMPERATURE),
C AND CAN BE COUPLED TO A ROOM MODEL TO SIMULATE A ONE-ZONE
C STRUCTURE.
C
C THE PROGRAM CAN BE USED TO SIMULATE A SINGLE WALL OR FOUR
C WALLS IF ALL OF THE WALLS ARE OF THE SAME CONSTRUCTION.
C
C WINDOWS AND WINDOW SHADING ARE PERMITTED IN THE MODEL
C IF IT IS DESIRED TO CALCULATE DIRECT SOLAR GAIN.
C
C BELOW IS A LIST OF PARAMETERS, INPUTS, AND OUTPUTS.
C
C ***** PARAMETERS *****
C
C PARAMETER NUMBER      DEFINITION
C
C      1      BUILDING OR WALL UA (INCLUDING WINDOWS)
C      2      WALL SOLAR ABSORPTANCE
C      3      WALL INFRARED EMITTANCE
C      4      AREA OF SOUTH WALL
C      5      AREA OF EAST WALL
C      6      AREA OF NORTH WALL
C      7      AREA OF WEST WALL
C      8      FRACTION OF SOUTH WALL THAT IS WINDOW
C      9      FRACTION OF EAST WALL THAT IS WINDOW
C     10      FRACTION OF NORTH WALL THAT IS WINDOW
C     11      FRACTION OF WEST WALL THAT IS WINDOW
C     12      FRACTION OF SOUTH WINDOW THAT IS SHADED
C     13      FRACTION OF EAST WINDOW THAT IS SHADED
C     14      FRACTION OF NORTH WINDOW THAT IS SHADED
C     15      FRACTION OF WEST WINDOW THAT IS SHADED
C     16      WINDOW TRANSMITTANCE
C
C ***** INPUTS *****
C
C INPUT NUMBER      DEFINITION
C
C      1      AMBIENT TEMPERATURE

```

```

C      2      SOLAR RADIATION INCIDENT ON SOUTH WALL
C      3      SOLAR RADIATION INCIDENT ON EAST WALL
C      4      SOLAR RADIATION INCIDENT ON NORTH WALL
C      5      SOLAR RADIATION INCIDENT ON WEST WALL
C      6      WIND SPEED
C      7      ROOM TEMPERATURE
C
C
C ***** OUTPUTS *****
C
C      OUTPUT NUMBER      DEFINITION
C
C      1      TOTAL CONDUCTION AND SOLAR GAINS
C      2      TOTAL CONDUCTION ONLY
C      3      TOTAL DIRECT SOLAR GAINS
C
C *****
C      DIMENSION PAR(15),XIN(10),OUT(20),INFO(10)
C      INTEGER UA,ALPHA,EPS,AS,AE,AN,AW,FSW,FEW,FNW,FWW
C      ,FSS,FES,FNS,FWS,TAU2
C      DATA UA/1/,ALPHA/2/,EPS/3/,AS/4/,AE/5/,AN/6/,AW/7/,
C      ,FSW/8/,FEW/9/,FNW/10/,FWW/11/,
C      ,FSS/12/,FES/13/,FNS/14/,FWS/15/,TAU2/16/
C      IF(INFO(7).GE.0) GO TO 7
C      INFO(6)=3
C      INFO(9)=1
C      CALL TYPECK(1,INFO,7,16,0)
C 7 TA=XIN(1)
C   HTS = XIN(2)
C   HTE = XIN(3)
C   HTN = XIN(4)
C   HTW = XIN(5)
C   W = XIN(6)
C   TR = XIN(7)
C
C
C      CALCULATE TOTAL WALL AREA AND UNSHADED WINDOW AREAS
C
C
C      ATOTAL=PAR(AS)*(1.0 - PAR(FSW)) + PAR(AE)*(1.0 - PAR(FEW)) +
C      , PAR(AN)*(1.0 - PAR(FNW)) + PAR(AW)*(1.0 - PAR(FWW))
C      FS2 = PAR(AS)*(1.0 - PAR(FSW))/ATOTAL
C      FE2 = PAR(AE)*(1.0 - PAR(FEW))/ATOTAL
C      FN2 = PAR(AN)*(1.0 - PAR(FNW))/ATOTAL
C      FW2 = PAR(AW)*(1.0 - PAR(FWW))/ATOTAL
C
C
C      CALCULATE SOL-AIR TEMPERATURE
C

```

C

```

HO=53.15 + 13.72*W
TSAS = TA + PAR(ALPHA)*HTS/HO
TSAE = TA + PAR(ALPHA)*HTE/HO
TSAN = TA + PAR(ALPHA)*HTN/HO
TSAW = TA + PAR(ALPHA)*HTW/HO
TSA = FS2*TSAS + FE2*TSAE + FN2*TSAN + FW2*TSAW

```

C

C

C

C

C

```

CALCULATE HEAT TRANSFER TO ROOM AND SET OUTPUTS

```

```

QCOND=PAR(UA)*(TSA-TR)
QSOLS=HTS*PAR(TAU2)*PAR(AS)*PAR(FSM)*(1.0 - PAR(FSS))
QSOLE=HTE*PAR(TAU2)*PAR(AE)*PAR(FEW)*(1.0 - PAR(FES))
QSOLN=HTN*PAR(TAU2)*PAR(AN)*PAR(FNW)*(1.0 - PAR(FNS))
QSOLW=HTW*PAR(TAU2)*PAR(AW)*PAR(FWW)*(1.0 - PAR(FWS))
QSHG=QSOLS + QSOLE + QSOLN + QSOLW
OUT(1)=QCOND+QSHG
OUT(2)=QCOND
OUT(3)=QSHG
RETURN
END

```

```

      SUBROUTINE TYPE17(TIME,XIN,OUT,T,DTDT,PAR,INFO)
C *****
C
C THIS SUBROUTINE USES THE TRANSFER FUNCTION METHOD TO
C MODEL A SINGLE WALL, FOUR WALLS OF THE SAME CONSTRUCTION,
C OR A FLAT ROOF.
C
C THIS SUBROUTINE IS ESSENTIALLY THE SAME AS TRNSYS
C TYPE17 IN VERSION 11.1 OF THE PROGRAM, EXCEPT THIS
C SUBROUTINE USES THE TRANSFER FUNCTION EQUATION FOR A
C FLOATING ROOM TEMPERATURE.
C
C THE UNITS OF THE BN AND CN COEFFICIENTS INPUT TO THIS
C MODEL MUST BE IN UNITS OF KJ/(HR-M**2-C). THE OUTPUTS
C ARE IN UNITS OF KJ/HR.
C
C ***** NOTE !!!!! *****
C
C THE SIGN CONVENTION OF CEYLAN IS USED IN THIS
C SUBROUTINE. IF THE COEFFICIENTS OF MITALAS,
C STEPHENSON, AND ARSENAULT ARE USED WITH THE
C MODEL, THE NEGATIVE OF THE DN AND CN COEFFICIENTS
C SHOULD BE INPUT.
C
C *****
C THE INPUTS AND OUTPUTS OF THIS SUBROUTINE ARE
C THE SAME AS THE CURRENT (VERSION 11.1) TRNSYS
C TYPE17 AND ARE DEFINED IN THE TRNSYS USER'S
C MANUAL.
C
C PARAMETERS 1 THROUGH 5 ARE THE SAME IN BOTH
C MODELS. PARAMETER 6 IN THIS MODEL (NBR) IS THE
C NUMBER OF ROOM TEMPERATURE COEFFICIENTS (CN)
C REQUIRED.
C
C PARAMETERS 7 THROUGH (20 + NR + ND) ARE THE
C SAME AS PARAMETERS 6 THROUGH (19 + NR + ND)
C RESPECTIVELY IN THE CURRENT TRNSYS VERSION OF
C THE MODEL.
C
C THE CN COEFFICIENTS ARE INPUT AS PARAMETERS
C (21 + NR + ND) THROUGH (21 + NR + ND + NBR).
C
C THE PREVIOUS HEAT FLOWS AND SOL-AIR TEMPERATURES
C ARE STORED IN THE OUT ARRAY. THE PREVIOUS ROOM
C TEMPERATURES ARE STORED IN THE S ARRAY.
C

```

```

C *****
C   INTEGER B,D,TSA,Q,ALPHA,AREA,TAU1,TAU2,FW,FS,EPS,AS,AE,AN,AW,
C   , FSW,FEW,FNW,FWW,FSS,FES,FNS,FWS,TIS
C   DIMENSION XIN(10),OUT(20),PAR(45),INFO(10),UW(3)
C   COMMON /SIH/ TIME0,TFINAL,DELT
C   COMMON /STORE/ NSTORE,IAV,S(200)
C   DATA ALPHA/2/,EPS/3/,AREA/7/,TAU1/8/,FW/10/,FS/11/,AS/7/,
C   , AE/8/,AN/9/,AW/10/,TAU2/11/,FSW/13/,FEW/14/,FNW/15/,FWW/16/,
C   , FSS/17/,FES/18/,FNS/19/,FWS/20/,TSA/3/,Q/11/
C   DATA UW/21,7,12,3,8.2/,QT/0./,QW/0./,QSHG/0./,Q0/0./
C   DATA RDCONV/0.0174533/,SIGMA/2.041092E-07/

```

C
C
C

```

IF (INFO(7) .GE. 0) GO TO 30

```

C
C
C

```

FIRST CALL OF SIMULATION

```

```

INFO(6)=3
INFO(9) = 1
INFO(10)=11
DELT3 = DELT*.3
N=PAR(1)
IF (N .NE. 1 .AND. N .NE. 2) GO TO 400
IF (N .EQ. 2) GO TO 12

```

C
C
C

```

CHECK MODE 1 PARAMETERS

```

```

NPAR=11. + PAR(4) + PAR(5) + PAR(6)
CALL TYPECK(1,INFO,4,NPAR,0)
TIS=INFO(10)-1
NB = PAR(4)
ND = PAR(5)
NBR=PAR(6)
NG = PAR(9)
IF (NB.LT.1 .OR. NG.GT.3) GO TO 400
SLOPE = PAR(12)
IF (SLOPE.LT.-1.5 .OR. SLOPE .GT. 90.5) GO TO 400
B = 12
DO 5 I = 1,NB
5 OUT(TSA+I) = 20.0
DO 6 I = 1,ND
6 OUT(Q + I) = 0.0
NP = B+NB+NBR
DO 7 I=1,NBR
7 S(TIS+I)=20.0
GO TO 170

```

C
C
C

```

CHECK MODE 2 PARAMETERS

```

```

12 NPAR=20. + PAR(4) + PAR(5) + PAR(6)

```

```

      CALL TYPECK(1,INFO,7,NPAR,0)
      TIS=INFO(10)-1
      NB = PAR(4)
      ND = PAR(5)
      NBR=PAR(6)
      NG = PAR(12)
      IF (NG.LT.1 .OR. NG.GT.3) GO TO 400
      B = 20
      DO 13 I = 1,NB
13    OUT(TSA+I)=20.0
      DO 14 I = 1,ND
14    OUT(Q + I) = 0.0
      DO 15 I=1,NBR
15    S(TIS+I)=20.0
      GO TO 170
C
C DO CALCULATIONS ONCE PER HOUR
C
30  ELTIME = TIME - TIME0
      TIS=INFO(10)-1
      IF (ABS(ELTIME - AINT(ELTIME+DELT3)) .GT. DELT3) RETURN
      N=PAR(1)
      IF (N .EQ. 2) GO TO 100
C
C MODE 1 -- TRANSFER FUNCTION WALL OR FLAT ROOF
C
      TIS=INFO(10)-1
      TA = XIN(1)
      HT = XIN(2)
      W = XIN(3)
      TR = XIN(4)
      NB=PAR(4)
      ND=PAR(5)
      NBR=PAR(6)
C
C SHIFT PREVIOUS HEAT FLOWS AND TEMPERATURES ON FIRST
C CALL TO COMPONENT IN TIME STEP ONLY
C
      IF(INFO(7).GT.0) GO TO 35
      DO 311 II=2,NB
      I=NB+2-II
311  OUT(TSA + I) = OUT(TSA + I - 1)
      DO 321 II=2,ND
      I=ND+2-II
321  OUT(Q + I) = OUT(Q + I - 1)
      DO 331 II=2,NBR
      I=NBR+2-II
331  S(TIS+I)=S(TIS+I-1)
      OUT(Q + I) = S(TIS+II)
35  NG = PAR(9)
      SLOPE = PAR(12)

```

```

      R = 12
      D = B + NB
      NB2=D+ND
      HO=53.15 + 13.72*W
      OUT(TSA + 1) = TA + PAR(ALPHA)*HT/HO
      IF (SLOPE ,LT, 0.) GO TO 40
C
C   COMPUTE LOSS TO SKY AND GROUND FOR FLAT ROOF.
C
      TAMBK = TA + 273.16
      TSKY = 0.0552*(TAMBK**1.5)
      TSURR = TAMBK + 10.0
      FVSKY = (1. + COS(SLOPE*RDCONV))/2.0
      FVGND = 1. - FVSKY
      DRSKY = FVSKY*(TSKY**4 - TAMBK**4)
      DRGND = FVGND*(TSURR**4 - TAMBK**4)
      OUT(TSA+1) = OUT(TSA+1) + PAR(EPS)*SIGMA*(DRSKY+DRGND)/HO
      S(TIS+1)=XIN(4)
40    S1 = 0.0
      S2 = 0.0
      S3=0.0
      DO 45 I=1,NB
        S1 = S1 + PAR(B + I)*OUT(TSA + I)
45    CONTINUE
      DO 50 I=1,ND
50    S2 = S2 + PAR(D + I)*OUT(Q + I)
C
C   FLOATING ROOM TEMPERATURE CALCULATIONS
C
      DO 60 I=1,NBR
60    S3=S3+PAR(NB2+I)*S(TIS+I)
      QO=S1 + S2 + S3
      QWALL=QO*PAR(AREA)*(1.0 - PAR(FW))
      QSHG=HT*PAR(TAU1)*PAR(AREA)*PAR(FW)*(1.0 - PAR(FS))
      QWINDC=UW(NG)*PAR(AREA)*PAR(FW)*(TA - TR)
      QW=QWALL + QWINDC
      QT=QW + QSHG
      GO TO 170
C
C   MODE 2 -- TRANSFER FUNCTION HOUSE (4 WALLS)
C
100   TA = XIN(1)
      TIS=INFO(10)-1
      HTS = XIN(2)
      HTE = XIN(3)
      HTN = XIN(4)
      HTW = XIN(5)
      W = XIN(6)
      TR = XIN(7)
      NB=PAR(4)
      ND=PAR(5)

```

```

      NBR=PAR(6)
C
C  SHIFT PREVIOUS HEAT FLOWS AND TEMPERATURES ON FIRST
C  CALL TO COMPONENT IN TIME STEP ONLY
C
      IF(INFO(7).GT.0) GO TO 110
      DO 312 II=2,NB
      I=NR+2-II
312  OUT(TSA + I) = OUT(TSA + I - 1)
      DO 322 II=2,ND
      I=ND+2-II
322  OUT(Q + I) = OUT(Q + I - 1)
      OUT(Q + 1) = S(TIS+1)
      DO 332 II=2,NBR
      I=NBR+2-II
332  S(TIS+I)=S(TIS+I-1)
110  NG = PAR(12)
      B = 20
      D = B + NB
      NB2=D+ND
      ATOTAL=PAR(AS)*(1.0 - PAR(FSW)) + PAR(AE)*(1.0 - PAR(FEW)) +
      . PAR(AN)*(1.0 - PAR(FNW)) + PAR(AW)*(1.0 - PAR(FWW))
      FS2 = PAR(AS)*(1.0 - PAR(FSW))/ATOTAL
      FE2 = PAR(AE)*(1.0 - PAR(FEW))/ATOTAL
      FN2 = PAR(AN)*(1.0 - PAR(FNW))/ATOTAL
      FW2 = PAR(AW)*(1.0 - PAR(FWW))/ATOTAL
      HO=53.15 + 13.72*W
      TSAS = TA + PAR(ALPHA)*HTS/HO
      TSAE = TA + PAR(ALPHA)*HTE/HO
      TSAN = TA + PAR(ALPHA)*HTN/HO
      TSAW = TA + PAR(ALPHA)*HTW/HO
      OUT(TSA + 1) = FS2*TSAS + FE2*TSAE + FN2*TSAN + FW2*TSAW
      S(TIS+1)=XIN(7)
      S1 = 0.0
      S2 = 0.0
      S3 = 0.0
      DO 145 I=1,NB
      S1 = S1 + PAR(B + I)*OUT(TSA + I)
145  CONTINUE
      DO 150 I=1,ND
150  S2 = S2 + PAR(D + I)*OUT(Q + I)
C
C  FLOATING ROOM TEMPERATURE CALCULATIONS
C
      DO 160 I=1,NBR
160  S3=S3+PAR(NB2+I)*S(TIS+I)
      QO=S1 + S2 + S3
      QWALL=QO*ATOTAL
      QSOLS=HTS*PAR(TAU2)*PAR(AS)*PAR(FSW)*(1.0 - PAR(FSS))
      QSOLE=HTE*PAR(TAU2)*PAR(AE)*PAR(FEW)*(1.0 - PAR(FES))
      QSOLN=HTN*PAR(TAU2)*PAR(AN)*PAR(FNW)*(1.0 - PAR(FNS))

```



```

      QSOLW=HIW*PAR(TAU2)*PAR(AW)*PAR(FWW)*(1.0 - PAR(FWS))
      QSHG=QSOLS + QSOLE + QSOLN + QSOLW
      QWINDC = UW(NG)*(PAR(AS)*PAR(FSW) + PAR(AE)*PAR(FEW) + PAR(AN)*
      .   PAR(FNW) + PAR(AW)*PAR(FWW)) * (TA - TR)
      QW=QWALL + QWINDC
      QT=QW + QSHG
C
C  SET OUTPUTS
C
170   OUT(1) = QT
      OUT(2) = QW
      OUT(3) = QSHG
      S(TIS+11)=Q0
      RETURN
C
C  PARAMETER ERROR
C
400   CALL TYPECK(4,INFO,0,0,0)
      RETURN
      END

```

```

C
C
C
      SUBROUTINE TYPE19(TIME,XIN,OUT,T,DTDT,PAR,INFO)
C *****
C THIS ROUTINE MODELS THE INTERIOR OF A SPACE TO BE HEATED
C OR COOLED. MODE 1 IS COMPATIBLE WITH ENERGY RATE CONTROL
C AND MODE 2 IS COMPATIBLE WITH TEMPERATURE LEVEL CONTROL.
C
C THIS MODEL IS ESSENTIALLY THE SAME AS THE TRNSYS VERSION
C 11.1 TYPE19 ROOM MODEL, EXCEPT TIME DEPENDENT HEAT GAINS
C ARE NOT DISTRIBUTED IN THIS ROUTINE, AND THE MINIMUM AND
C MAXIMUM THERMOSTAT SET TEMPERATURES FOR MODE 1 ARE INPUTS
C INSTEAD OF PARAMETERS.
C
C PARAMETERS 14 AND 15 ARE DUMMY PARAMETERS IN MODE 1 AND
C ARE NOT USED IN THE ROUTINE FOR THIS MODE. THE TWO PARA-
C METERS ARE THE SAME AS SPECIFIED IN THE TRNSYS USERS
C MANUAL IN MODE 2.
C
C THE MINIMUM AND MAXIMUM THERMOSTAT SET TEMPERATURES
C ARE INPUTS 11 AND 12 IN MODE 1.
C
C ALL OTHER PARAMETERS, INPUTS, AND OUTPUTS ARE DEFINED
C IN THE TRNSYS USERS MANUAL (VERSION 11.1)
C *****
      REAL KT,KJPEPL
      DIMENSION XIN(15),OUT(20),T(2),DTDT(2),PAR(19),INFO(10)
      DIMENSION FC(4),DST(4)
      COMMON /SIM/ TIME0,TFINAL,DELT
      COMMON /STORE/ NSTORE,IAV,S(200)
      EQUIVALENCE (DUM1,CHIN),(DUM2,EFF),(WR,WRI)
      DATA KJPEPL/230./,RHOAIR/1.204/,CPAIR/1.012/,HTVAP/2468./
      DATA IUNIT/0/
C
C SET INTERNAL VARIABLES TO PARAMETER VALUES IF THE UNIT NUMBER
C HAS CHANGED. NOTE THAT THE LAST THREE OF FIVE NAMES FOR
C MODE TWO ARE EQUIVALENCED TO THOSE FOR MODE ONE.
C
      IF (INFO(1) .EQ. IUNIT) GO TO 200
      IUNIT = INFO(1)
      MODE = INT(PAR(1)+0.1)
      VOL   = PAR(2)
      RATEIF = PAR(3)
      AREA  = PAR(4)
      IC    = INT(PAR(5) + 0.1)
      CAPAC = PAR(6)
      UA    = PAR(7)

```

```

IBASE = INT(PAR(8)+SIGN(0.1,PAR(8)))
DEPTH = PAR(9)
PERIM = PAR(10)
TGRD = PAR(11)
QGEN = PAR(12)
PEPL = PAR(13)
DUM1 = PAR(14)
DUM2 = PAR(15)
TSTART = PAR(16)
IFC = PAR(17)
WDOT = PAR(18)
WR = PAR(19)
IF(MODE.EQ.2) CAPUM = PAR(20)
IF(MODE.EQ.2) CP = PAR(21)
NI = INFO(3)
IF (INFO(7) .NE. -1) GO TO 200
C
C FIRST CALL OF SIMULATION
C
IF (MODE .NE. 1 .AND. MODE .NE. 2) CALL TYPECK(4,INFO,0,0,0)
OUT(15) = TSTART
IF (MODE .EQ. 2) GO TO 20
C
C MODE 1
C
INFO(6) = 4
NI = MAX0(NI,12)
CALL TYPECK(1,INFO,NI,19,0)
GO TO 200
C
C MODE 2
C
20 INFO(6) = 7
OUT(17) = WRI
NI = MAX0(NI,14)
CALL TYPECK(1,INFO,NI,21,0)
C
C SET INPUT VARIABLES
C
200 IF (MODE .EQ. 2) GO TO 220
K = 0
THI = 0.
GO TO 230
220 K = 2
THI = XIN(1)
FLWH = XIN(2)
230 TVENT = XIN(K+1)
FLVENT = XIN(K+2)
WVENT = XIN(K+3)
TAMB = XIN(K+4)
WAMB = XIN(K+5)

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```

      WDOTI = XIN(K+6)
      QST(2) = XIN(K+7)
      QST(1) = XIN(K+8)
      QST(3) = XIN(K+9) + QGEN
      QST(4) = XIN(K+10) + PEPL*KJPEPL
      THIN = XIN(K+11)
      TMAX = XIN(K+12)
      QSTSUM = QST(1) + QST(2) + QST(3) + QST(4)
      QL = 0.
      FLINF = RATEIF*VOL*RHOAIR
      K = K+13
      IF (K .GT. NI) GO TO 300
      DO 240 I = K,NI
240    QL = QL + XIN(I)
      C
      C DO TRANSFER FUNCTION CALCULATIONS ONCE PER HOUR
      C
      300    ELTIME = TIME - TIME0
            IF (ABS(ELTIME-AINT(ELTIME+0.001)) .GT. 0.001)
               GO TO 400
            IF (QSTSUM .LE. 0. .OR. IFC .EQ. 0) GO TO 310
            KT = UA/PERIM*0.161
            FC(1)=1.0-0.019*KT
            FC(2)=1.0-0.016*KT
            FC(3)=1.0-0.022*KT
            FC(4)=1.0-0.025*KT
            GO TO 320
      310    DO 315 I = 1,4
      315    FC(I) = 1.0
            320    QLD = 0.
            DO 350 J = 1,4
            QLD = QLD + QST(J)
      350    CONTINUE
            OUT(10) = QLD
      C
      C ANALYTICAL SOLUTION TO 1-NODE ROOM DIFFERENTIAL EQUATION FOR
      C TEMPERATURE
      C
      400    UAB = 0.
            TSBAR = 0.
            IF(IBASE) 410,430,420
      410    UAB = 2.1633*PERIM
            TSBAR = TAMB
            GO TO 430
      420    UAB = 1.136*(PERIM*DEPTH+AREA)
            TSBAR = 1.136*(PERIM*DEPTH*(TGRD+TAMB)/2.+AREA*TGRD)/UAB
      430    ECHIN = 0.
            IF(MODE.EQ.2 .AND. FLWH.GT.0.0) ECHIN = EFF*CMIN
            QLD = OUT(10)
            AA = -(ECHIN+FLINF*CPAIR+FLVENT*CPAIR+UAB)/CAPAC
            BB = (ECHIN*THI + CPAIR*(FLINF*TAMB+FLVENT*TVENT)

```

```

      + UAB*TSBAR + QLD + QL)/CAPAC
      IF(INFO(7) .EQ. 0) OUT(15) = OUT(14)
      TRLAST = OUT(15)
      CALL DIFFEQ(TIME,AA,BB,TRLAST,TR,TRBAR)
      IF(MODE .EQ. 2) GO TO 450
C
C   MODE 1 - ROOM TEMPERATURE STAYS WITHIN COMFORT ZONE
C
      DT = DELT
      IF(TR.GT.TMIN .AND. TR.LT.TMAX) GO TO 450
C
C   FIX THE TEMPERATURE AT ITS LIMITS AND FIND THE AMOUNT
C   OF TIME THAT THE TEMPERATURE WAS WITHIN THE COMFORT RANGE
C
      TR = AMAX1(TMIN,AMIN1(TMAX,TR))
      IF(ABS(TR-TRLAST) .GT. 1.E-06) GO TO 445
      DT = 0.0
      TRBAR = TR
      GO TO 450
445  IF(AA .GT. 0.0) GO TO 446
      DT = 0.0
      IF(BB .GT. 0.0) DT = (TR-TRLAST)/BB
      GO TO 448
446  DT = ALOG((TR+BB/AA)/(TRLAST+BB/AA))/AA
C
C   DETERMINE THE AVERAGE TEMPERATURE OVER THE TIMESTEP
C
448  CALL DIFFEQ(TIME,AA*DT/DELT,BB*DT/DELT,TRLAST,TRF,TRBAR1)
      TRBAR = TRBAR1*DT/DELT + TR*(1.-DT/DELT)
C
C   CALCULATE ENERGY FLOWS FOR EITHER MODE
C
450  QBASE = UAB*(TSBAR-TRBAR)
      QVENT = FLVENT*CPAIR*(TVENT-TRBAR)
      QINFL = FLINF*CPAIR*(TAMR-TRBAR)
      QLD = OUT(10)
      QLOAD = (QL+QLD+QVENT+QINFL+QBASE)
      DELU = CAPAC*(TR-TSTART)
      IF(MODE .EQ. 2) GO TO 500
      QLOAD = QLOAD - CAPAC*(TR-TRLAST)/DELT
C
C   CHECK IF TR WAS WITHIN THE COMFORT RANGE FOR THE ENTIRE TIMESTEP
C
      IF(DELT - DT .LE. 0.0) QLOAD = 0.0
C
C   SET OUTPUTS FOR MODE 1
C
      QLAINT = (FLINF*(WAMB-WR)+FLVENT*(WVENT-WR)+WDOT+WDOTI)*HTVAP
      OUT(1) = QLOAD
      OUT(2) = TRBAR
      OUT(3) = QBASE

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      OUT(4) = QLATNT
      OUT(5) = DELU
      OUT(14) = TR
      RETURN
C
C  MODE 2 - ROOM TEMPERATURE AND HUMIDITY FREE TO FLOAT
C
500  IF (FLWH .GT. 0.) GO TO 550
      TOUT = THI
      QTRANS = 0.
      GO TO 600
550  QTRANS = ECMIN*(THI-TRBAR)
      TOUT = THI-QTRANS/FLWH/CP
600  QEXS = QLOAD + QTRANS
C
C  ANALYTICAL SOLUTION TO DIFFERENTIAL EQUATION FOR
C  ROOM HUMIDITY RATIO
C
      CC = -(FLINF+FLVENT)/CAPHUM
      DD = (FLINF*WAMB+FLVENT*WVENT+WDOT+WDOTI)/CAPHUM
      IF (INFO(7) .EQ. 0) OUT(17) = OUT(16)
      WRLAST = OUT(17)
      CALL DIFFEQ(TIME,CC,DD,WRLAST,WROOM,WRBAR)
C
C  SET OUTPUTS FOR MODE 2
C
700  OUT(1) = TOUT
      OUT(2) = FLWH
      OUT(3) = QEXS
      OUT(4) = TRBAR
      OUT(5) = WRBAR
      OUT(6) = QTRANS
      OUT(7) = QBASE
      OUT(8) = DELU
      OUT(14) = TR
      OUT(16) = WROOM
      RETURN
      END

```

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