TYPE 529: RADIANT SLAB

General Description

This component is intended to model a radiant floor-heating slab, embedded in soil, and containing a number of fluid filled pipes. The heat transfer within the slab and surrounding soil is assumed to be conductive only and moisture effects are not accounted for in the model. The model relies on a three-dimensional finite difference method, solving the resulting inter-dependent differential equations using an iterative approach. The user may define any number of pipes within the slab and surrounding soil through a separate data file containing information about the path that the pipes follow through the slab/soil for each pipe. The slab is assumed to be embedded in the soil and the user may define bottom and/or perimeter insulation that extends below the slab if desired. The slab may also be exposed to incident radiation whether from interior lights or from the sun. Default numbers of maximum slab nodes, maximum pipe nodes and maximum number of pipes are set, but may be increased by modification of the Fortran code.

Nomenclature

m	-	Mass.
C_p	-	Specific heat.
$\dot{Q}_{in}$	-	The rate at which energy enters a fluid, soil or slab node.
$\dot{Q}_{out}$	-	The rate at which energy exits a fluid, soil or slab node.
$\dot{m}$	-	Mass flow rate of fluid entering a fluid node.
$T_{fluid,i}$	-	The temperature of fluid in a given fluid node.
$T_{x,y,z}$	-	The temperature of a given soil or slab node.
x	-	The distance along the length direction.
y	-	The distance along the width direction.
z	-	The depth dimension (node 1 is at the surface, increasing downwards).
R_{inside}	-	The thermal resistance between the fluid and the pipe.
R_{pipe}	-	The thermal resistance of the pipe material.
$R_{contact}$	-	The thermal contact resistance between the pipe and surrounding soil or slab.
k	-	Thermal conductivity.
dx	-	The x direction dimension of a soil or slab node.
dy	-	The y direction dimension of a soil or slab node.
dz	-	The z direction dimension of a soil or slab node.
$T_{ambient}$	-	The temperature of a node's surroundings. For an indoor node, $T_{ambient}$ is the temperature of the zone. For an outdoor node, $T_{ambient}$ is the outdoor dry bulb temperature.
$T_{surface}$	-	The surface temperature of a node exposed to ambient conditions.
$T_{surroundings}$	-	The effective temperature with which a node radiatively transfers energy. For an indoor node, $T_{surroundings}$ is the surface temperature of zone walls. For an outdoor node, $T_{surroundings}$ is the effective sky temperature.
$h_{convective}$	-	The convective heat transfer coefficient for the surface of a node.

σ		The Stefan-Boltzmann constant (5.67×10^{-8} W/m^2.K^4).
ε	-	The emissivity of a surface.
$Q_{ambient}$	-	Additional radiation incident on a node.

Detailed Description

1. Slab and Surroundings

Type 529 divides the slab and its surroundings into three parts. First is the slab itself, for which the user specifies length, width, depth, thermal properties, and the number of nodes in the x and y directions as shown in Figure 1.

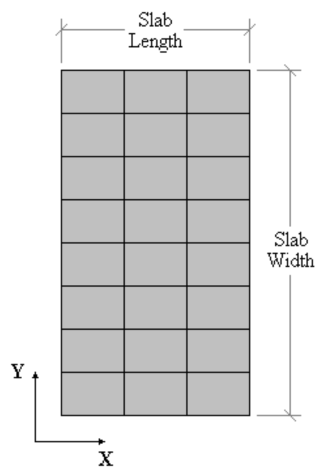

Figure 1: Slab Dimensions and Coordinate System

The slab is embedded in (surrounded on its sides and back by) soil for which the user must also specify the thermal properties. This zone surrounding the slab in the x, y and z directions as shown in Figure 2 is referred to as the near field. Nodes contained in the near-field can vary in size in all three dimensions (user-specified), usually becoming larger as they get farther from the slab, and are included in the slab energy balance. Because they are included in the energy balance, users may define pipes that pass through the near field soil nodes and may obtain information about temperature distributions within those nodes. The near field is in turn surrounded by the far field, which is assumed to be an infinite energy sink/source (energy transfer with the far-field does not result in a temperature change of the far-field). Users have a number of options as to how they wish to treat the soil in both the near and far fields as discussed in Section 2.

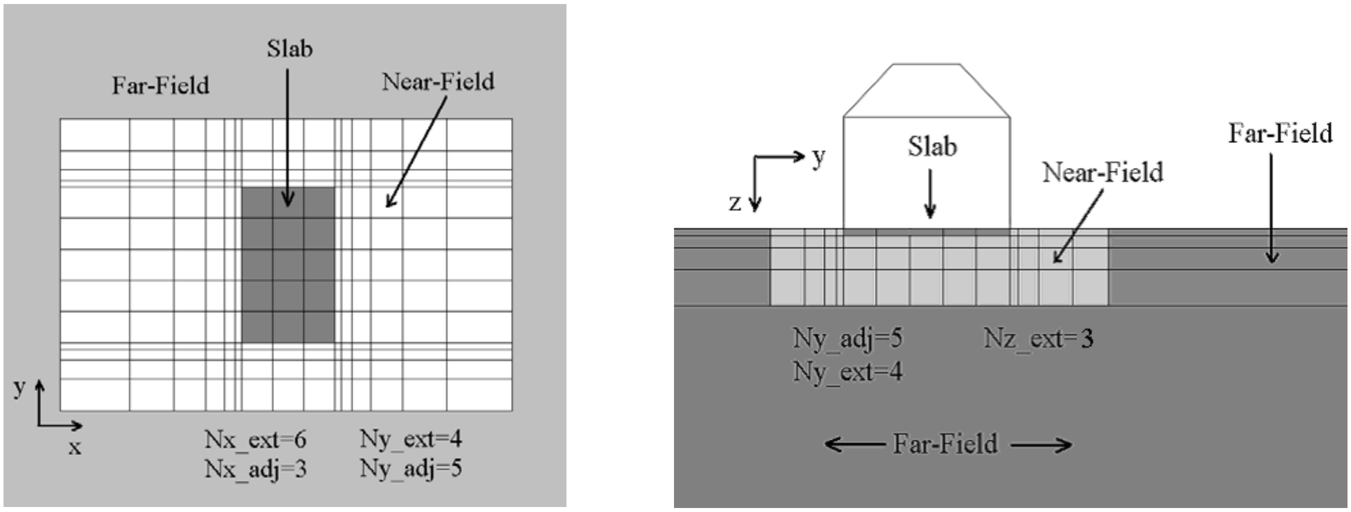

Figure 2: Slab, Near and Far Field Dimensions

The sizes of the slab nodes are automatically determined by the length and width of the slab and by the user specified number of nodes along the length and along the width of the slab. The slab is assumed to have a single node in the z direction. For near-field soil nodes, the user not only specifies how many nodes are to be considered in the x, y, and z-directions, but the length, width and depth of the nodes starting at the slab-to-soil boundary and moving outwards and downwards.

2. Ground Temperature Options

Users are asked to specify one of three near and far-field ground temperature options using Parameter 24.

The first option is to employ an energy balance on the soil surface to calculate the surface temperature. In this option, the user provides the ambient temperature, sky temperature, and the amount of incident solar radiation on the soil surface to the model as Inputs. An energy balance is applied to the surface to determine the surface temperature and conduction to the soil nodes provides the heat transfer to the ground from the surface. The Kasuda ground temperature profile [1] (which calculates a depth-dependent soil temperature based on the average annual surface temperature, the amplitude of the annual surface temperature, and the day of the year at which the minimum annual surface temperature occurs) sets the initial ground temperature profile for the near-field and for the far-field.

The second option is to use the Kasuda correlation to set the surface temperature as a function of the time of the year. In the near field, the Kasuda correlation is used to set the initial temperature profile in the soil and to obtain a time dependent surface temperature. The temperature of near field soil nodes depends upon conduction effects from neighboring nodes and from the Kasuda calculated surface temperature. In the far field, the Kasuda correlation is used to set the temperature of all nodes. The temperature of these nodes will change, but only as a function of depth and time of year.

The third option is for the user to supply the surface temperature as an Input to the model. In the near field, and in the far-field, the Kasuda correlation is used to set the initial temperature profile in the soil. The near and far-field nodes then change temperature due to conduction effect from other nodes (vertical direction only in the far-field) influenced by the surface temperature provided by the user.

3. Pipe Layout

Type529 allows users a great deal of flexibility in describing how heating and/or cooling fluid is passed through the slab. Users are first asked to specify (as TRNSYS input file parameters) the number of pipes that pass through the slab and the number of fluid nodes into which the pipe should be broken. The isothermal nodes must be specified from the start of the pipe to the end of the pipe – along the direction of flow. The greater the number of nodes, the greater the accuracy – but at the cost of simulation speed. Users may specify multiple pipe nodes per slab node but by definition a pipe node cannot extend over multiple slab nodes. For each pipe, the user must provide an external data file that describes that pipe's path through the slab and, if they choose, also through the near field soil nodes. The data file must contains the fraction of pipe length assigned to that pipe node and the coordinates of the soil/slab node in which the pipe is embedded. Figure 3 shows an example in which a twelve-node pipe (uniform pipe nodal lengths in this example) passes through a twelve-node slab. In this example, the boundaries between pipe nodes correspond to the slab node boundaries.

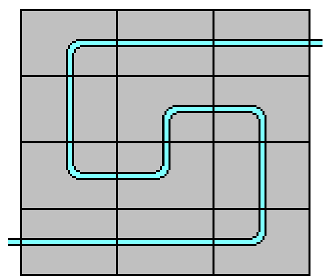

Figure 3: Example Pipe Layout with One Pipe Node Per Slab Node

The data file describing this pipe path is shown in Figure 4. The data file contains as many rows as there are slab nodes in the pipe. In each row, the first value is the fraction of the total pipe length assigned to the pipe node followed by the x, y, and z-coordinates of the slab/soil node containing the pipe node (slab nodes have by definition a z-coordinate of 1).

0.0833	1,1,1
0.0833	2,1,1
0.0833	3,1,1
0.0833	3,2,1
0.0833	3,3,1
0.0833	2,3,1
0.0833	2,2,1
0.0833	1,2,1
0.0833	1,3,1
0.0833	1,4,1
0.0833	2,4,1
0.0833	3,4,1

Figure 4: Data File Describing Figure 3 Pipe Path

Figure 5 shows a slightly more complex example in which two pipes pass through a single slab. This second example is meant to illustrate the method for specifying multiple pipe nodes per slab node (the perimeter pipe in Figure 5) and the method for defining different lengths of pipe contained in a slab node (the interior pipe in Figure 5). As with the previous example, both pipes in this example are contained entirely within the slab and do not dip below the slab into the soil nodes below.

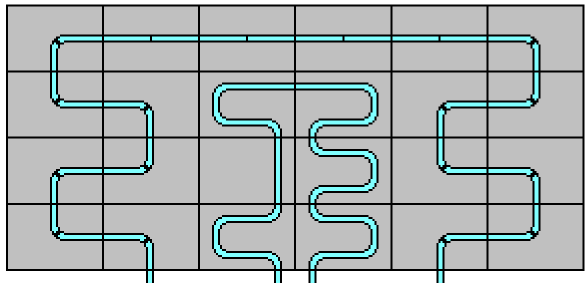

Figure 5: Slab Containing Two Pipe Paths

Data File Describing Flow Path for Perimeter Pipe	
0.0277	5,1,1
0.0277	5,1,1
0.0277	6,1,1
0.0277	6,1,1
0.0277	6,2,1
0.0277	6,2,1
0.0277	5,2,1
0.0277	5,2,1
0.0277	5,3,1
0.0277	5,3,1
0.0277	6,3,1
0.0277	6,3,1
0.0277	6,4,1
0.0277	6,4,1
0.0277	5,4,1
0.0277	5,4,1
0.0277	4,4,1
0.0277	4,4,1
0.0277	3,4,1
0.0277	3,4,1
0.0277	2,4,1
0.0277	2,4,1
0.0277	1,4,1
0.0277	1,4,1
0.0277	1,3,1
0.0277	1,3,1
0.0277	2,3,1
0.0277	2,3,1
0.0277	2,2,1
0.0277	2,2,1
0.0277	1,2,1
0.0277	1,2,1
0.0277	1,1,1
0.0277	1,1,1
0.0277	2,1,1
0.0277	2,1,1

Data File Describing Flow Path for Interior Pipe	
0.182	3,1,1
0.091	3,2,1
0.182	3,3,1
0.182	4,3,1
0.182	4,2,1
0.182	4,1,1

Figure 6: Data Files for Figure 5 Slab

When creating pipe layout files it should be born in mind that the origin (node 1,1,1) is in the very lower left hand corner of the diagram. Unless there are no near field soil nodes surrounding the slab, the lower left hand corner node of the slab will not be numbered 1,1,1. The user may specify there to be more than one pipe in a given soil/slab node. However, these pipes will not “talk” to each other directly due to a temperature difference between them. They will each affect the same slab node, however, and so affect each other indirectly.

4. Energy Balances

The basic energy balance on a fluid node or node contained in the slab or the soil is shown in Equation 1:

$$mC_p \frac{dT}{dt} = \dot{Q}_{in} - \dot{Q}_{out} \qquad \text{(Eq. 1)}$$

The equations describing Q_{in} and Q_{out} change depending upon the type of node and its location. Four types of nodes exist: fluid nodes, internal nodes, boundary nodes (between the slab and the near field or between the near field and the far field for example), and surface nodes.

4.1 Fluid Nodes

It is assumed that energy enters and leaves a fluid node through fluid flow into that section of pipe and through heat transfer with the soil/slab. Conduction due to a temperature difference between fluid nodes is not considered. Thus when flow ceases, the fluid tends toward a steady state temperature only though conduction with the surrounding soil/slab nodes and not through fluid conduction along its length. The appropriate Q_{in} term for flow is then:

$$\dot{Q}_{in} = \dot{m}C_p \left(T_{fluid,i-1} - T_{fluid,i}\right) \qquad \text{(Eq. 2)}$$

Energy is transferred from the pipe node to the surrounding soil/slab node via conduction according to Equation 3 where $\overline{T}$ denotes the average soil/slab temperature over the timestep.

$$\dot{Q}_{out} = UA\left(T_{fluid,i} - \overline{T}_{x,y,z}\right) \qquad \text{(Eq. 3)}$$

It should be pointed out that the heat transfer out of the pipe node is then by definition the heat transfer into the soil node.

The UA term in equation 3 is made up of three parts; an internal convection resistance from the fluid to the pipe, a pipe wall resistance, and a conductive resistance in the slab/soil.

The internal heat transfer resistance is simply:

$$R_{in} = \frac{1}{h_{inside} A_{surface,inner}} \qquad \text{(Eq. 4)}$$

where:

$$h_{inside} = \frac{Nusselt\ k}{d_{pipe,i}} \qquad \text{(Eq. 5)}$$

and for laminar flow (Reynolds number < 2300):

$$Nusselt = 4.36 \quad \text{(Eq. 6)}$$

and for turbulent flow (Reynolds number ≥ 2300):

$$Nusselt = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{(1/3)} \quad \text{(Eq. 7)}$$

The Reynolds number and Prandtl numbers are based on the inside diameter of the pipe and the fluid properties. The pipe wall resistance is defined by the radial conduction equation:

$$R_{wall} = \frac{\ln\left(\frac{d_{pipe,o}}{d_{pipe,i}}\right)}{2\,\Pi\,k\,l} \quad \text{(Eq. 8)}$$

In order to determine the conduction resistance from the pipe outer surface to the soil node, the conduction path length must be specified. In the case of a radiant floor, it is difficult to know this length since the exact path that the pipe follows through the soil/slab node is not known. Instead, a contact resistance ($R_{contact}$) that describes the conductance between the pipe and the surrounding soil/slab node is requested of the user as a parameter. This parameter can be thought of as a tuning parameter; used to adjust the heat transfer to measured data (if available).

The expression for the overall U value is:

$$U = \frac{1}{R_{inside} + R_{pipe} + R_{contact}} \quad \text{(Eq. 9)}$$

4.2 Internal Slab/Soil Nodes

If a node is contained within the near field and does not have any pipes passing through it, its energy balance is made up of purely conductive terms. The equations for Q_{in} and Q_{out} are:

$$\dot{Q}_{in} = \frac{k_{soil}\,dy\,dz}{x_i - x_{i-1}}\left(\overline{T}_{x-1,y,z} - T_{x,y,z}\right) + \frac{k_{soil}\,dx\,dz}{y_i - y_{i-1}}\left(\overline{T}_{x,y-1,z} - T_{x,y,z}\right) + \frac{k_{soil}\,dx\,dy}{z_i - z_{i-1}}\left(\overline{T}_{x,y,z-1} - T_{x,y,z}\right) \quad \text{(Eq. 10)}$$

$$\dot{Q}_{out} = \frac{k_{soil}\,dy\,dz}{x_{i+1} - x_i}\left(T_{x,y,z} - \overline{T}_{x+1,y,z}\right) + \frac{k_{soil}\,dx\,dz}{y_{i+1} - y_i}\left(T_{x,y,z} - \overline{T}_{x,y+1,z}\right) + \frac{k_{soil}\,dx\,dy}{z_{i+1} - z_i}\left(\overline{T}_{x,y,z} - T_{x,y,z+1}\right) \quad \text{(Eq. 11)}$$

It should be noted that some internal soil nodes may have an additional resistance term added because Type 529 allows insulation on the perimeter of the slab to extend down below the slab as far as desired.

4.3 Boundary Nodes

Two types of boundary nodes exist: nodes on the boundary between the near and far fields, and nodes on the boundary between the slab and the near field.

Nodes in the near field on the boundary with the far field are governed purely by conduction as long as they do not also contain fluid pipes. The far field temperature to which the node conducts is not affected by energy passing from the node to the far field, only by time of year and depth as given by the Kasuda correlation (for mode 2) or by conduction in the vertical direction (modes 1 and 3). Nodes at the bottom corners of the near field can conduct to two different far field temperatures while nodes only along the depth of the near field conduct to a single far field temperature (along the horizontal plane). Conduction between the near and far fields may be neglected, if desired, by setting parameters 20 and 21 to the appropriate values.

Nodes on the boundary between the slab and near field are also governed by conduction but may have an additional pure resistance built into their energy balance. The resistive term is used to account for insulation around the perimeter of or beneath the slab.

In the examples above, the slab thermal conductivity and soil thermal conductivity are assumed equal. If this is not the case (which it usually isn't), the $\dot{Q}_{in}$ and $\dot{Q}_{out}$ terms are slightly modified to account for the change in thermal conductivity due to the two materials. Refer to the source code for the governing equations if further clarification is needed.

The conduction length between the node and the far-field is one-half of the corresponding nodal distance. For example, the conduction length between a node along the bottom of the near-field and the far-field is dz/2.

4.4 Surface Nodes

Surface nodes are handled in a slightly different manner from other nodes because they interact with the surroundings, or with a known surface condition. For surface mode 1, the surface temperature is calculated from an energy balance on the surface (assuming the surface has negligible mass). The energy balance on the surface reveals that the energy conducted to the surface by the node must equal the energy convected (to the ambient) and radiated (to the surroundings) from the surface minus any incident solar radiation (or incident radiation from lighting etc.). The energy balance on the surface is shown schematically in Figure 7.

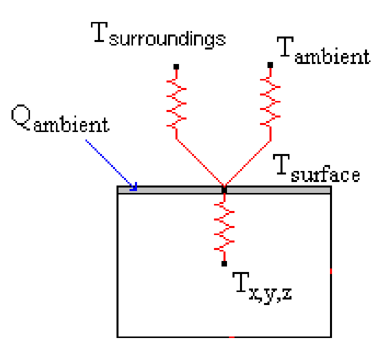

Figure 7: Energy Balance on a Surface Node

The energy transferred between from the surface to the environment contains terms for both radiative heat transfer between the surface and surrounding temperature and a standard convection term between the surface and the ambient. The radiation term may be linearized with ambient temperature using equation 12.

$$h_{radiation} = \left(\varepsilon\sigma\left(T_{ambient} + T_{surroundings}\right)\left(T^2_{ambient} + T^2_{surroundings}\right)\right) \quad \text{(Eq. 12)}$$

In the outdoor case, $T_{surroundings}$ is the effective sky temperature and $T_{ambient}$ is the outdoor dry bulb temperature. In the indoor case, $T_{surroundings}$ is the effective temperature of the zone walls and $T_{ambient}$ is the zone dry bulb temperature. The user can choose to neglect the radiation exchange calculations by setting either the slab or the soil emmisivity and absorptance to zero.

The energy balance on the surface is then:

$$0 = \left(h_{radiation} + h_{convection}\right)\left(T_{surface} - T_{ambient}\right) - Q_{ambient} - \frac{k\,dx\,dy\left(T_{x,y,z} - T_{surface}\right)}{dz/2} \quad \text{(Eq. 13)}$$

This energy balance is solved for the surface temperature which is then used for the conduction equations for the soil/slab node. Surface mode 2 uses the Kasuda correlation to set the surface temperature while surface mode 3 relies on an input surface temperature to the model.

The resistance network is shown in Figure 8 for a surface node. For purposes of clarity, conduction in the y direction has been neglected from the figure. The conduction length for heat transfer to/from the surface is half the nodal height (dz/2).

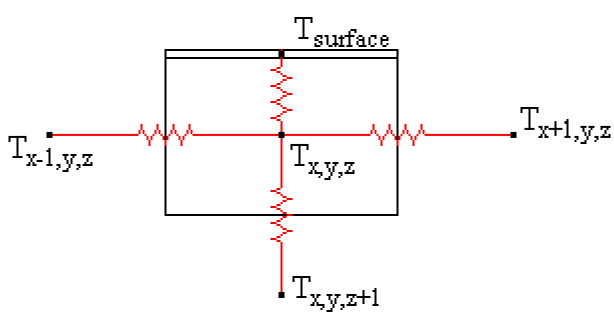

Figure 8: Energy Balance on a Surface Node

5. Solving the Resulting Equations

In this type of system, heat is transferred from a fluid in a pipe into an adjacent soil/slab node and then this heat is conducted to other soil nodes or convected/radiated to the surroundings. This is certainly not an easy problem to solve and compounding the problem is the fact that the effects of the pipe fluid mass must be considered for these types of systems. The problem breaks down into the required solution of two coupled differential equations of the form of Equation 1; one for each soil/slab node and one for each pipe node. The equations are coupled in that the pipe nodal equations depend on the pipe fluid temperatures and the pipe nodal equations depend on the soil/slab nodal temperatures. While there are other available methods to solve coupled differential equations, we decided to solve the problem with an approximate analytical solution. The analytical solution has several inherent advantages over numerical solutions. First, the subroutine solves its own mathematical problem and does not have to rely on non-standard numerical recipes that must be attached to the subroutine. In this way, the subroutine can be imported into any FORTRAN compiler without problems. Secondly, some of the other solution methods (mainly the numerical solutions) are extremely dependent on the simulation timestep and may not converge under certain circumstances commonly encountered in these types of systems (namely high flow rates for example). The analytical solution is timestep independent but does require an iterative solution inside the subroutine to solve the coupled differential equations. While solving two coupled differential equations iteratively can sometimes lead to convergence problems, this does not seem to be the case with this model under all operating scenarios tested.

To solve the differential equations analytically, each of the nodal the equations are placed into the form:

$$\frac{dT}{dt} = aT + b \tag{Eq. 14}$$

where T is the dependent variable (temperature in this case), t is time, a is a constant and b may be a function of time or the dependent variable. If b is a constant, than the solution of this differential equation can be readily solved. If b is not constant, then a reasonable approximation to the analytical solution can be found by assuming that b is constant over the timestep and equal to its average value over the timestep.

At any time (for **a** not equal to zero):

$$T_{final} = (T_{initial} + \frac{\bar{b}}{a})\ e^{a\Delta t} - \frac{\bar{b}}{a} \tag{Eq. 15}$$

where

$$\bar{b} = b(\bar{T}) \tag{Eq. 16}$$

and

$$\bar{T} = \frac{1}{a\,\Delta t}\left(T_{initial} + \frac{\bar{b}}{a}\right)\left(e^{a\,\Delta t} - 1\right) - \frac{\bar{b}}{a} \tag{Eq. 17}$$

With this assumption, the problem becomes straightforward to solve. Simply write the differential equation for each node in the correct form, determine a and $\bar{b}$ for each node and solve for T_{final} and T_{ave}. Then recalculate $\bar{b}$ for each node and iterate until all temperatures converge.

While the assumption that b is constant over the timestep (and equal to its average value) is not technically correct in our case (b for the pipe node is a function of the soil/slab temperature for example), it is a reasonable approximation for the small timesteps we are using in the TRNSYS simulation (maximum timestep=1 hour). The b terms hold the temperatures of other soil/slab nodes and pipe nodes. These temperatures in the b term are assumed to be constant for the solution of the nodal differential equations at their average value over the timestep.

References

[1] Kasuda, T., and Archenbach, P.R. "Earth Temperature and Thermal Diffusivity at Selected Stations in the United States", ASHRAE Transactions, Vol. 71, Part 1, 1965